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ANALYSIS OF RUFENER'S METHOD FOR THE ATMOSPHERIC EXTINCTION REDUCTION

BY

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ABSTRACT

Theoretically obtained stellar intensities as measured from the Earth are used in the observational reduction programme proposed by Rufener (1964) to examine the accuracy of the atmospheric extinction reduction procedure. Code's (1960) spectrophotometric calibrations for a representative sample of spectral types were used to obtain the theoretical stellar intensities. The results for the differences between the theoretical and observational extinction coefficients show that there is a high degree of correlation. The standard deviations of the differences between the theoretical and observational extinction coefficients obtained through a study of a number of stars are small (less than 0.007 mag). This indicates the accuracy of the observational reduction procedure. A quantitative analysis of the dependence of the colours and the passbands of the system on the residuals of the extinction coefficients has been attempted. The study also shows that it is not necessary to modify the observational procedure.

1. INTRODUCTION

RUFENER'S METHOD OF EVALUATING THE ATMOSPHERIC EXTINCTION

The method used by Rufener (1964) for obtaining the instantaneous extinction cœfficients is an improvement over the method of Bouguer which uses a relationship that is valid for monochromatic radiation. For details concerning the reduction technique the original reference [Rufener (1964)] should be consulted. A few essential relationships for the study of the atmospheric extinction will first be considered.

The heterochromatic magnitude of a star $m_{\lambda 0}(z)$ as observed from the Earth at a certain zenith distance z, and through each passband is given by:

$$m_{\lambda 0}(z) \cong m(\lambda_0) - 0.543\left(\frac{\mu}{\lambda_0}\right)^2\left[\frac{E''(\lambda_0)}{E(\lambda_0)}\lambda_0{}^2\right] - 2.5\log\int\varphi(\lambda)\,d\lambda$$
$$+ 1.086\,k(\lambda_0)F_z\left[1+\left(\frac{\mu}{\lambda_0}\right)^2 n\left(\frac{n+1}{2}\right) - n\left(\frac{\mu}{\lambda_0}\right)^2\frac{E'(\lambda_0)}{E(\lambda_0)}\lambda_0\right]$$
$$- 0.543\,n^2k^2(\lambda_0)\left(\frac{\mu}{\lambda_0}\right)^2 F_z^2 \qquad (1)$$

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where $m(\lambda_0)$ is the monochromatic magnitude outside the atmosphere, μ is the bandwidth, λ_0 is the mean wavelength of the passband, n is the exponent of the atmospheric extinction law, $E(\lambda_0)$ is the stellar energy distribution outside the atmosphere, and $\varphi(\lambda)$ is the response curve of the filter.

The cœfficient of the airmass F_z, for the monochromatic case is given by 1.086 $k(\lambda_0)$ and for the heterochromatic case by the term

$$1.086\, k(\lambda_0)\left[1 + \left(\frac{\mu}{\lambda_0}\right)^2 \left\{\frac{n(n+1)}{2} - n\,\frac{E'(\lambda_0)}{E(\lambda_0)}\,\lambda_0\right\}\right]$$

Expression (1) shows that the relationship between the heterochromatic magnitude and the air mass is not linear but that the divergence from such a relationship is proportional to the square of the bandwidth μ.

Bouguer's relation

$$m(\lambda_0, z) = m(\lambda_0) + 1.086\, k(\lambda_0)\, F_z$$

is thus a first order approximation of equation (1).

Changes in the stellar flux are brought about by the atmospheric extinction and scintillation, and thus the precision of a stellar photoelectric measurement is limited. According to Rufener, certain other factors given below influence the method of obtaining the magnitude of the star outside the atmosphere:

1. Large passbands of a photometric system, wherein the extinction cœfficient varies with the spectral energy distribution of the star (displacement of the effective wavelength).
2. When stellar measurements are made through different passbands it causes the observation time for each star to be sufficiently large (about 20 minutes per star in the case of the seven colour photometric system of the Geneva Observatory).
3. The observational site having only a few good photometric nights and these being displaced far apart in time.
4. If a system is newly created then the choice of a photometric standard will not exist.

The following hypotheses have to be considered to obtain the extinction cœfficient:

1. The radiation of the star is constant.
2. The instrument sensitivity remains the same over large intervals of time.
3. An isotropic atmosphere in the solid angle of observation.
4. The diffusing and absorbing properties of the atmosphere during the night remain the same.

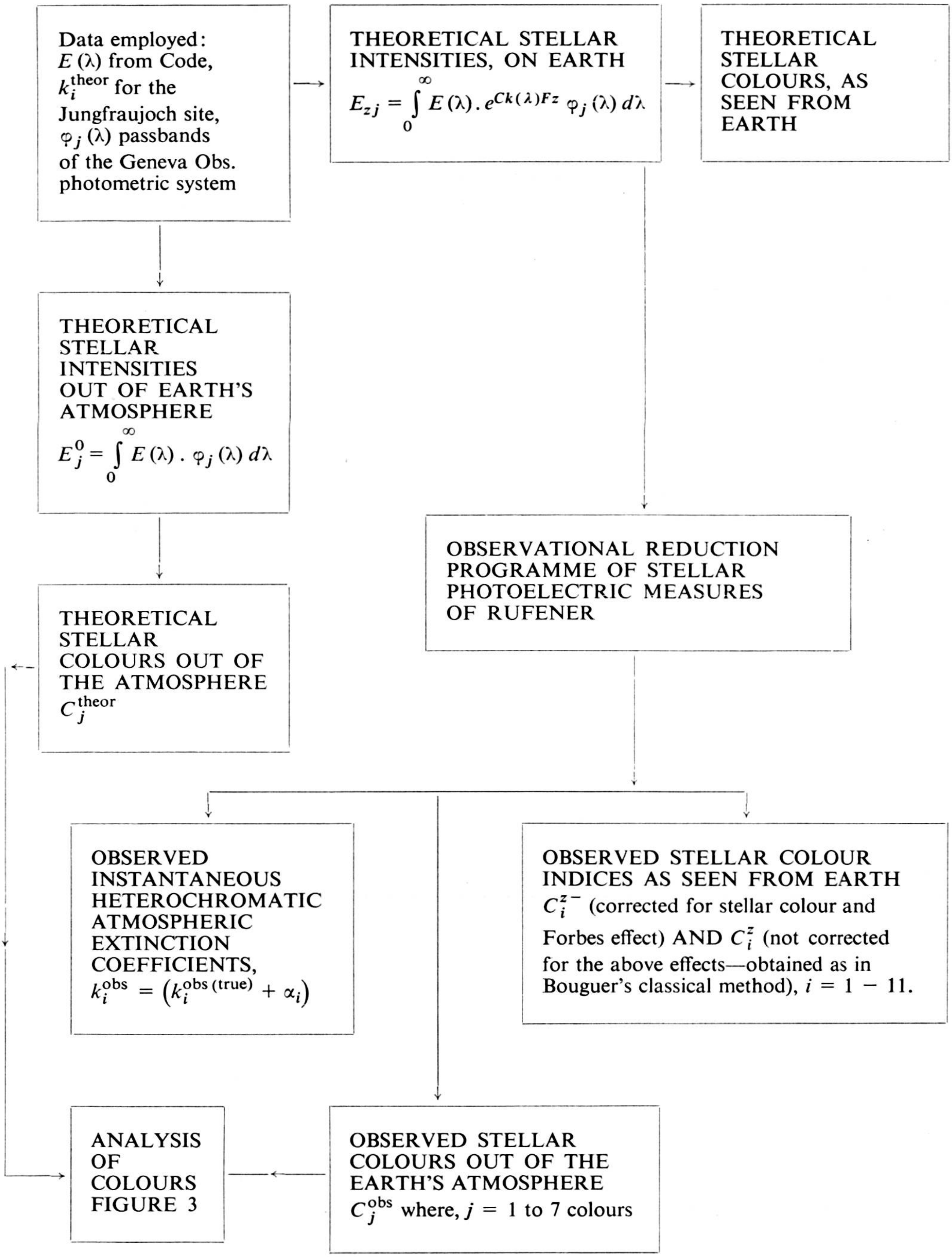

FIG. 1. — Schematic diagram showing the principle involved in the simulation study.

While a judicious choice of stars can be made to obtain the extinction cœfficient, it is often difficult to guarantee the absolute constancy of the instrument sensitivity. One can partially eliminate the latter difficulty by applying the Bouguer's method for the colour indices. It is seen that the ratio of the spectral sensitivities of the two passbands remains constant over a larger interval of time and the absolute spectral sensitivity stays nearly constant during the time of one complete measurement. Meteorological conditions present major difficulties in adopting the latter two hypotheses, and the method of Rufener is particularly useful in this respect as well as for the factors indicated earlier that influence the method to obtain the stellar magnitude outside the atmosphere.

The detailed expression for the colour index of a star as measured from the Earth through filters 1 and 2 follows from equation (1):

$$C^z_{1-2} = C^{\circ}_{1-2} + F_z \left[\underbrace{k_{1-2} + \alpha_{1-2}}_{k_{1-2\,(t)}} + \beta_{1-2} C^{\circ}_{1-2} + \gamma_{1-2} F_z \right] \tag{2}$$

where

$k_{1-2} = k(\lambda_1) - k(\lambda_2)$, gives the difference between the monochromatic extinction cœfficients

$$\alpha_{1-2} = \frac{n_1(n_1+1)}{2} \left\{\frac{\mu_1}{\lambda_1}\right\}^2 k(\lambda_1) - \frac{n_2(n_2+1)}{2} \left\{\frac{\mu_2}{\lambda_2}\right\}^2 k(\lambda_2) - \beta_{1-2}(\Phi_1 - \Phi_2) \tag{3}$$

$$\beta_{1-2} = -\frac{1}{1.086\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)} \left[n_1 \left\{\frac{\mu_1}{\lambda_1}\right\}^2 \frac{k(\lambda_1)}{\lambda_1} - n_2 \left\{\frac{\mu_2}{\lambda_2}\right\}^2 \frac{k(\lambda_2)}{\lambda_2} \right] \tag{4}$$

$$\gamma_{1-2} = -0.543 \left[n_1^2 \left\{\frac{\mu_1}{\lambda_1}\right\}^2 k_2(\lambda_1) - n_2^2 \left\{\frac{\mu_2}{\lambda_2}\right\}^2 k_2(\lambda_2) \right] \tag{5}$$

n is a quantity characterising the atmospheric extinction law, $k(\lambda) = a\lambda^{-n}$

$$n = -\frac{d \log k(\lambda)}{d \log \lambda} \tag{6}$$

μ_j represents the bandwidth of the passband j

$\Phi_j = -2.5 \log \int_0^\infty \varphi_j(\lambda)\, d\lambda$ represents the response curve for the passbands in magnitudes

C^z_{1-2} is the colour index of the star as observed from the Earth

C^0_{1-2} is the colour index of the star as seen outside the atmosphere

The above equations indicate only the colour index (1-2). Similar equations can be given for various other colour indices i, obtained from colours j.

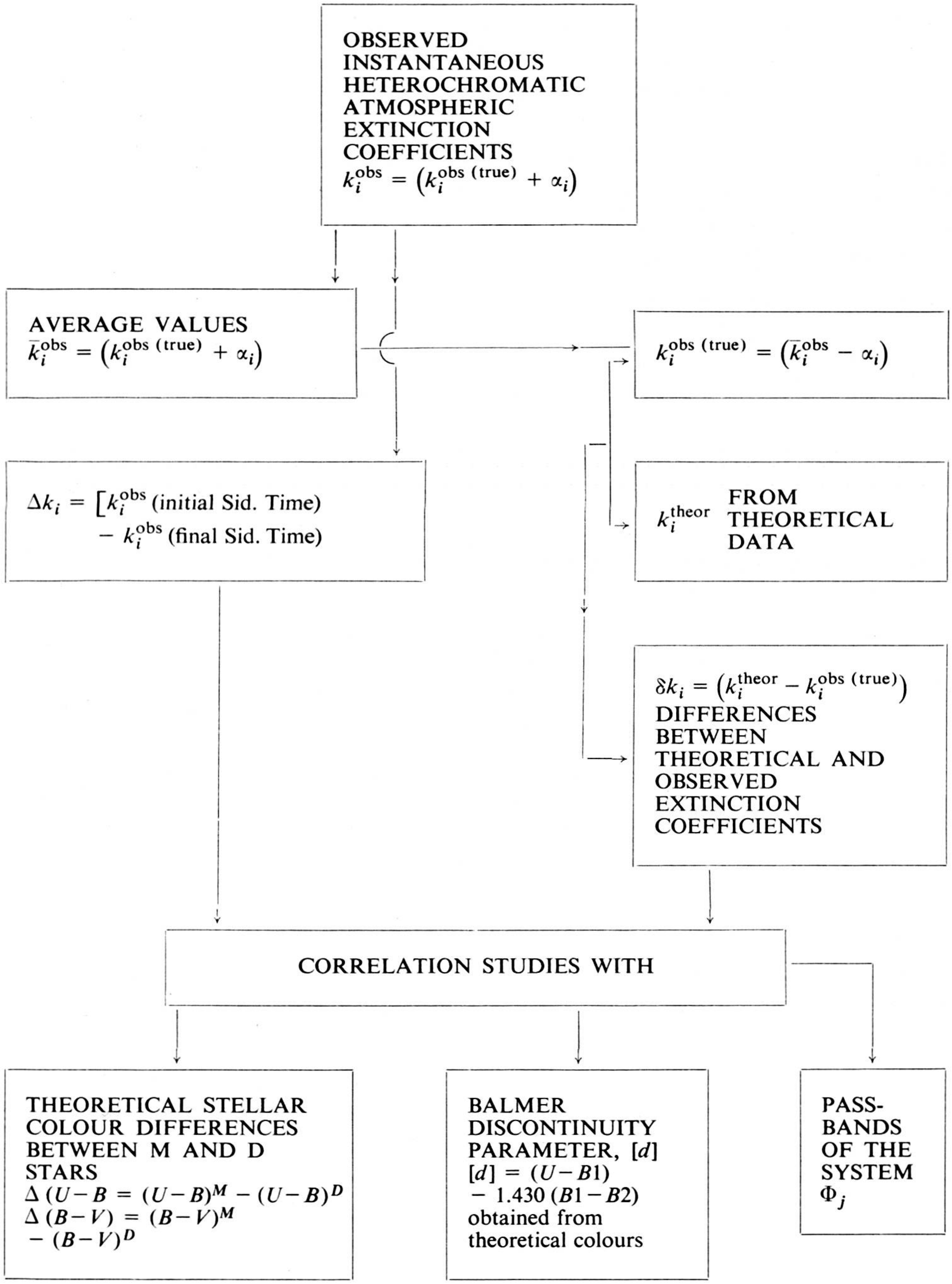

FIG. 2. — Schematic diagram showing the parameters used in the analysis of the theoretical and observed values of atmospheric extinction coefficients.

Equation (2) shows that C^z_{1-2} depends on the colour of the star and it is applicable at each instant of time. If the condition of continuity of the energy distribution of the star received at the receiver having a response curve $\varphi_j(\lambda)$, is not fulfilled then the slope of the line can also depend upon the extent of the discontinuity. It is also seen that the exponent n of the atmospheric extinction function varies with the wavelength. Therefore, if the variations of the terms inside the brackets of equation (2) are not considered there will be an error of a few percent in the determination of the magnitude. The extinction law also plays an important role for the terms of $F^2{}_z$ which is sometimes known as the Forbes effect.

Rufener indicates that if one admits a slow variation of the atmospheric extinction cœfficient then k_{1-2}, α_{1-2}, β_{1-2}, and γ_{1-2} will be functions of time. However, $\alpha_{1-2}, \beta_{1-2}$ and γ_{1-2} are small when compared to k_{1-2} and hence it can be assumed that their variations are of the second order with respect to k_{1-2}. Furthermore, he makes the hypothesis that α_{1-2}, β_{1-2} and γ_{1-2} have constant values, and that only k_{1-2} is a function of time. The factors k_{1-2} and α_{1-2} are grouped under one symbol $k_{1-2}(t)$ in the above equation.

The above discussion indicates the need for a thorough knowledge of the instrumental and atmospheric parameters for the evaluation of stellar colours.

Rufener's method consists of observing an ascending star (M) at regular intervals of time from a zenith distance of 70° at the beginning of the night up to its meridian passage towards the end of the night. Similarly a second star (D) with the same colour is observed as it descends from the meridian to approximately 70° zenith distance. A few such pairs of M and D stars are observed at regular time intervals during the course of the night. The above stars are chosen sufficiently distant from the pole so that n observations of each of the M and D stars enable a colour index C^z_i, to be derived by Bouguer's method, viz.,

$$(C^z_i) = C^\circ_i{}^* + (F_z)\, k^*_i \tag{7}$$

Thus approximate values of $C^\circ_i{}^*$ and k^*_i can be obtained by applying the least squares method. $C^\circ_i{}^*$ and k^*_i represent respectively, the colour index outside the atmosphere and the extinction cœfficient for each M and D star separately and i indicates various colour indices. The above colour indices C^z_i permit the calculation of the colour indices C^{z-}_i, which are corrected for colour effect of the star and the Forbes effect for each of the M and D pairs of stars observed during the course of the night. The relationship obtained with the aid of equation (2) is as follows:

$$(C^{z-}_i) = (C^z_i) - (F_z)\,[\beta_i\, C^\circ_i{}^* + \gamma_i (F_z)] \tag{8}$$

For each of the stars M and D and for various such pairs, separate air masses will be associated. The seven colour system of the Geneva Observatory allows us to obtain indices $i = 1$ to 11, and these are indicated in various tables of this article.

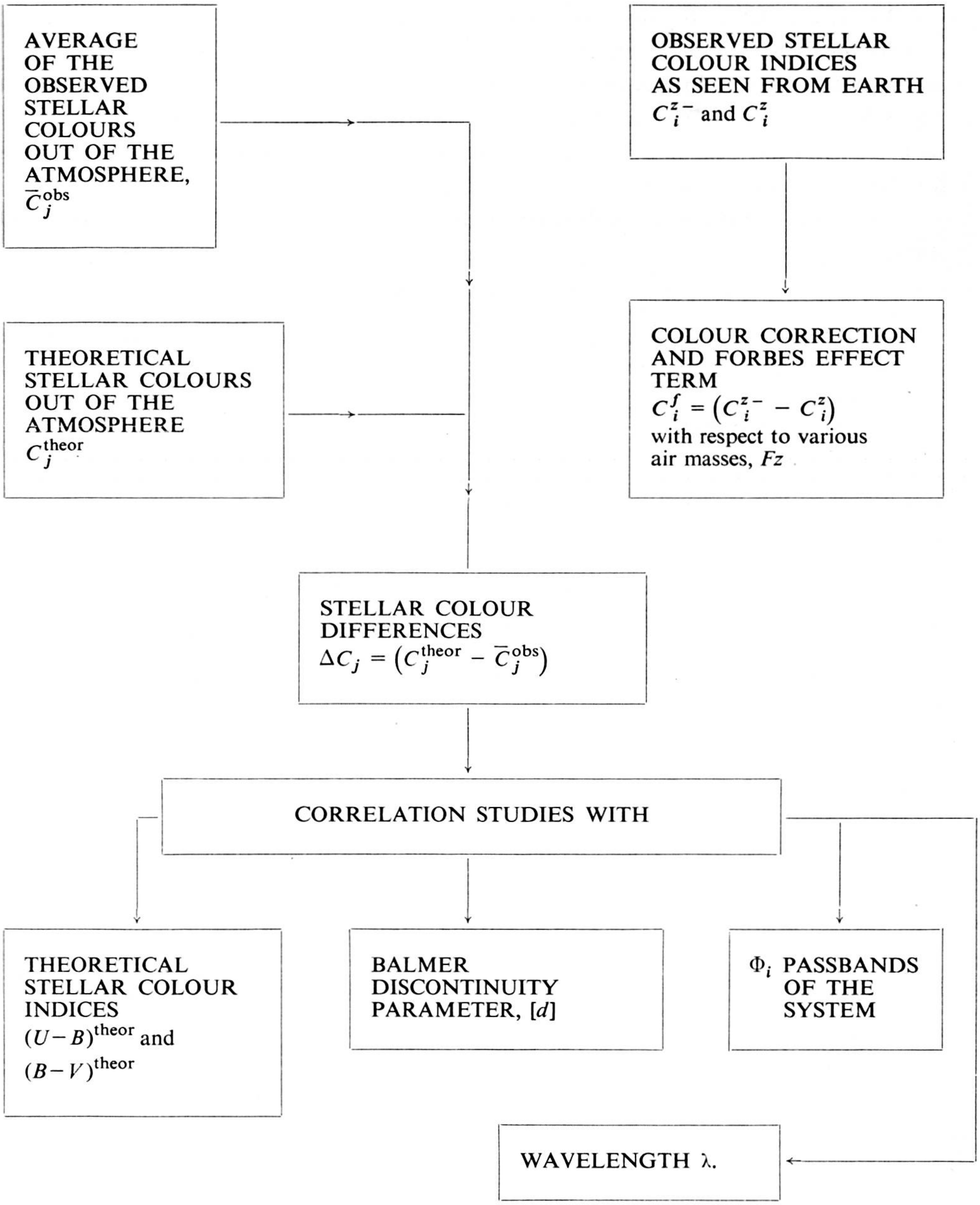

FIG. 3. — Schematic diagram showing the parameters used in the analysis of the theoretical and observed values of stellar colours.

In the evaluation of equation (8) we can further take into account the correction due to the fact that the measurements are not made simultaneously through two different filters. For low lying stars, the difference between the air masses is not negligible, and such a source of error is removed by introducing a correction term, such as, $k_2\ (F_{z1}\text{-}F_{z2})$. This term is subtracted directly from C_i^z using average values for the extinction cœfficients.

Therefore, equation (8) will take the following form for each of the M and D stars:

$$(C_i^{z-}) = C_i^{\circ} + F_z\, k_i(t) \tag{9}$$

where C_i° represents the colour of the star outside the atmosphere corrected for the colour effect and the Forbes effect.

The observational measurements with respect to M and D stars are not made simultaneously, but consecutively. Hence, the times of observation between two consecutive measurements for these stars are not the same, and their differences amount to less than 30 minutes. Moreover, the assumption of an isotropic atmosphere within the solid angle used ($z \leqslant 70°$) allows one to put for the M and D stars:

$$[k_i(t)]^M = [k_i(t)]^D = k_i(t) \tag{10}$$

The instantaneous extinction coefficients are obtained by Rufener from the stellar colour indices evaluated outside the atmosphere for both M and D stars, and these are given by:

$$k_i(t) = [C_i^{z-} - C_i^{\circ}] / (F_z) \tag{11}$$

The method of determining the average of various values of normalised magnitudes, and also the treatment for the observations of M and D stars has been clearly indicated by Rufener (1964). He has also discussed the method of establishing a new photometric standard and the means of estimating the α, β, and γ values.

In the following section we shall describe the procedure to obtain theoretically the intensities of stars. These simulated intensities are introduced into the above mentioned observational programme to look at the consistency of values of the instantaneous extinction coefficients.

2. PROCEDURE

Consider a site (Jungfraujoch) whose altitude and monochromatic atmospheric extinction coefficients are known. Next, choose a fictitious pair of M and D stars giving their respective right ascensions and declinations. A duration period of 11 hours is assumed for the simulated night. The time intervals and coordinates of stars are so chosen that they simulate consecutive observations of M and D stars as described in Section 1. These will then give air masses in the range of 4 to 1 for the

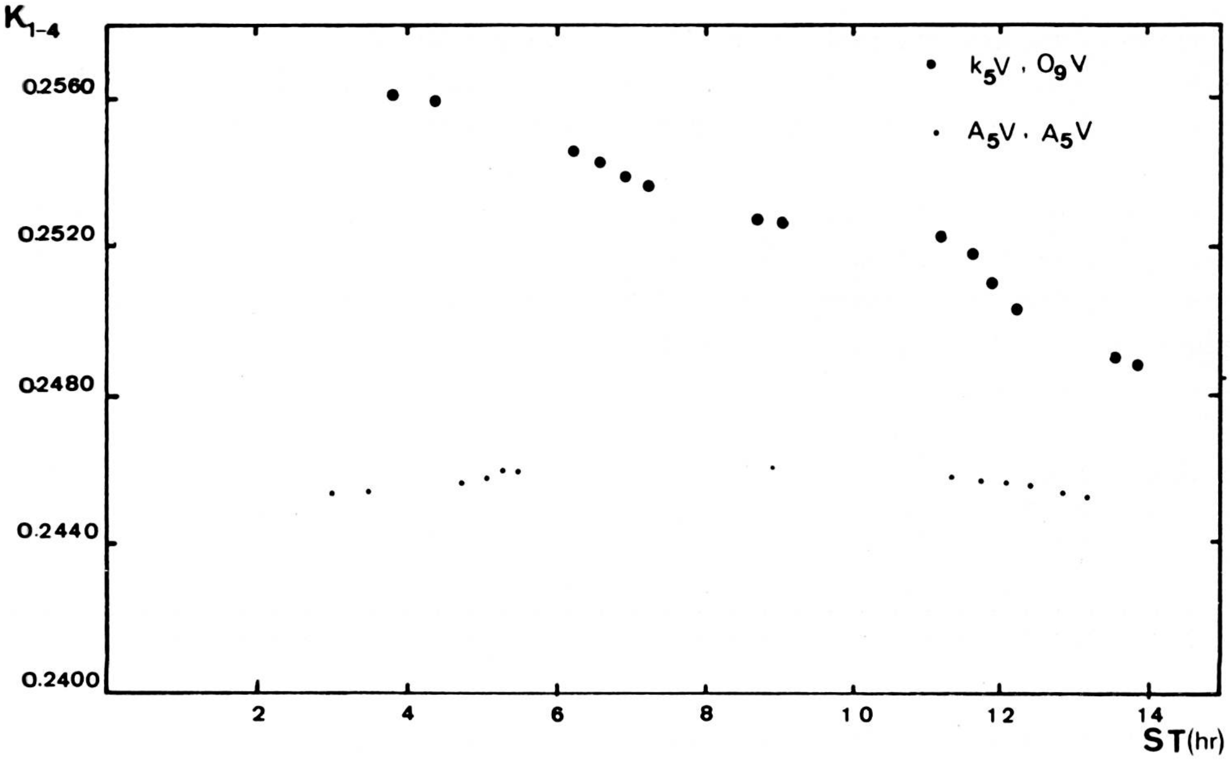

FIG. 4. — Observed instantaneous extinction cœfficients (in magnitude) for the M and D stars plotted against sidereal time.

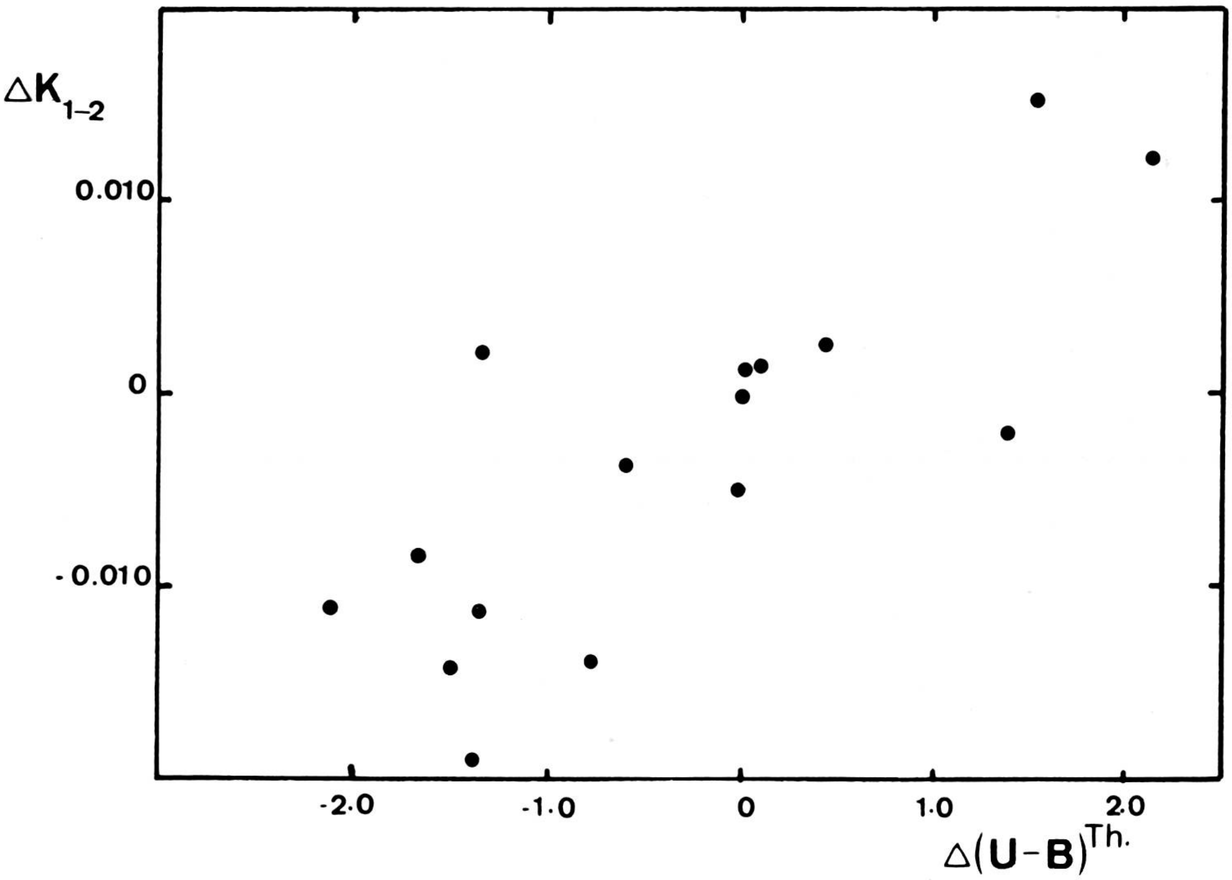

FIG. 5. — Variation of the instantaneous extinction cœfficients (in magnitude) with stellar colour differences $\Delta(U\text{-}B)^{\text{theor}}$.

corresponding time intervals of the M star. Air masses in the range of 1 to 4 are similarly obtained for the D star. Each of the M and D stars has a spectral type assigned, and many pairs with different spectral types are chosen. Stellar energy distributions given by Code (1960) are used for these spectral types. The atmospheric function $A(\lambda, z)$ is calculated from the above values of the air masses and from the average monochromatic extinction coefficients for the site given by Rufener (1973). The spectral response curves $\varphi_i(\lambda)$, of the passbands of the Geneva Observatory photometric system were taken from a paper by Rufener and Maeder (1971).

The theoretical stellar intensities are obtained through an integration of the energy distribution of the star, the atmospheric function and the passbands of the system. These intensities obtained through each passband for the M and D stars together with their right ascensions and declinations and sideral times are fed into the programme of observational reduction of stellar measures (Section 1) and the instantaneous extinction coefficients are obtained. These extinction coefficients should not involve any variation during the course of the simulated period because of the fact that they do not correspond to the actual observing conditions. Further, they should correspond to the original values of the average extinction coefficients used in the theoretical evaluation of stellar intensities. A validity test of the technique of reduction of atmospheric extinction is thus established.

The first part of this study describes the numerical evaluation of the theoretical intensities. It is followed by an analysis of the differences between the theoretical and observed atmospheric extinction coefficients as well as the colours of stars outside the atmosphere. Theoretical quantities are the ones used in the evaluation of stellar intensities from formula (12) given below. Observed quantities are the ones obtained from Rufener's atmospheric reduction procedure after the theoretical stellar intensities are fed in it. The conclusion recapitulates the main findings from the various diagrams and establishes the accuracy of the observational method for the reduction of stellar measurements.

A schematic diagram given in Figures 1, 2 and 3 illustrates the procedure involved in the analysis of the atmospheric extinction coefficient and the colours.

3. NUMERICAL METHOD

The intensities E_{zj}, of the stars measured from the Earth at various zenith distances z, and through each passband j, are given by:

$$E_{zj} = \int_0^\infty E(\lambda)\, A(\lambda, z)\, \varphi_j(\lambda)\, d\lambda \tag{12}$$

where,

$E(\lambda)$ represents the energy distribution of the star outside the atmosphere

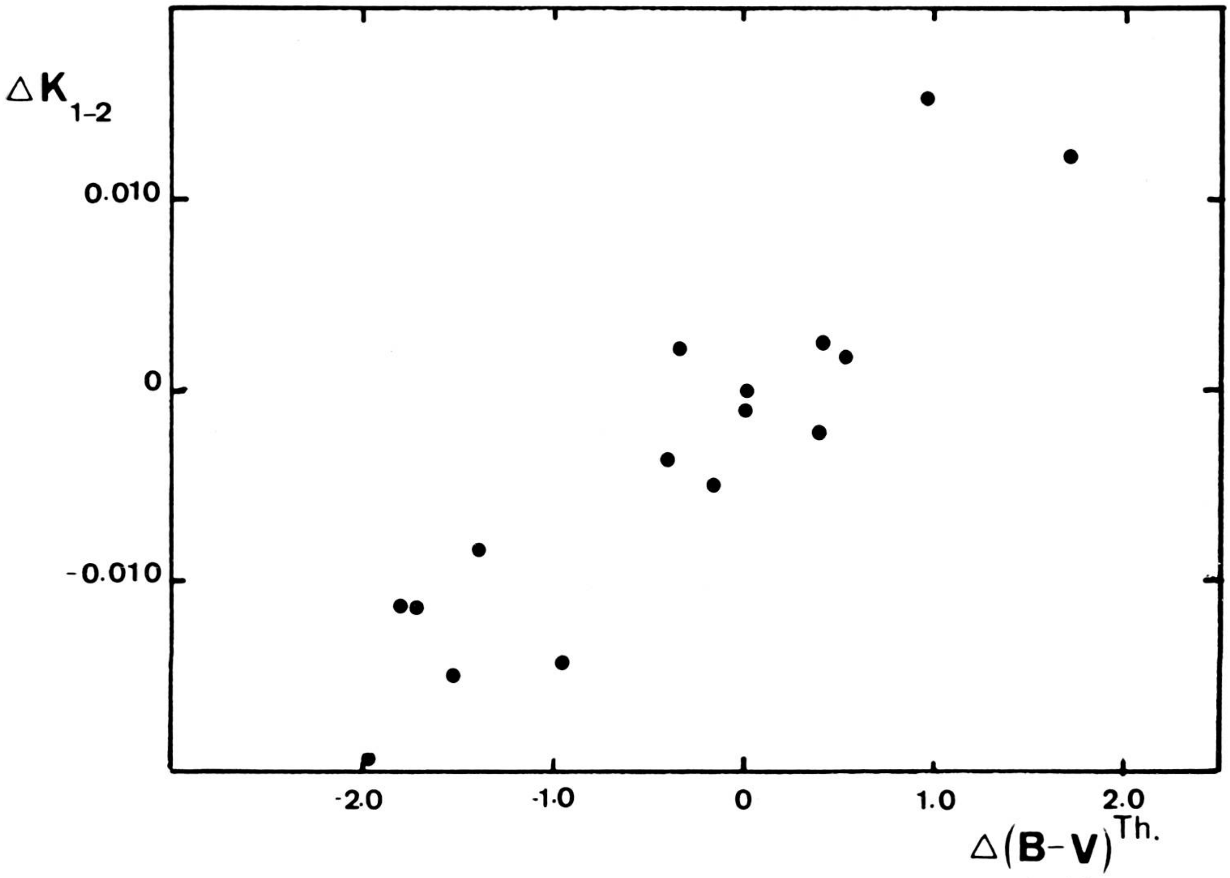

FIG. 6. — Variation of the instantaneous extinction cœfficients (in magnitude) with stellar colour differences $\Delta(B\text{-}V)^{\text{theor}}$.

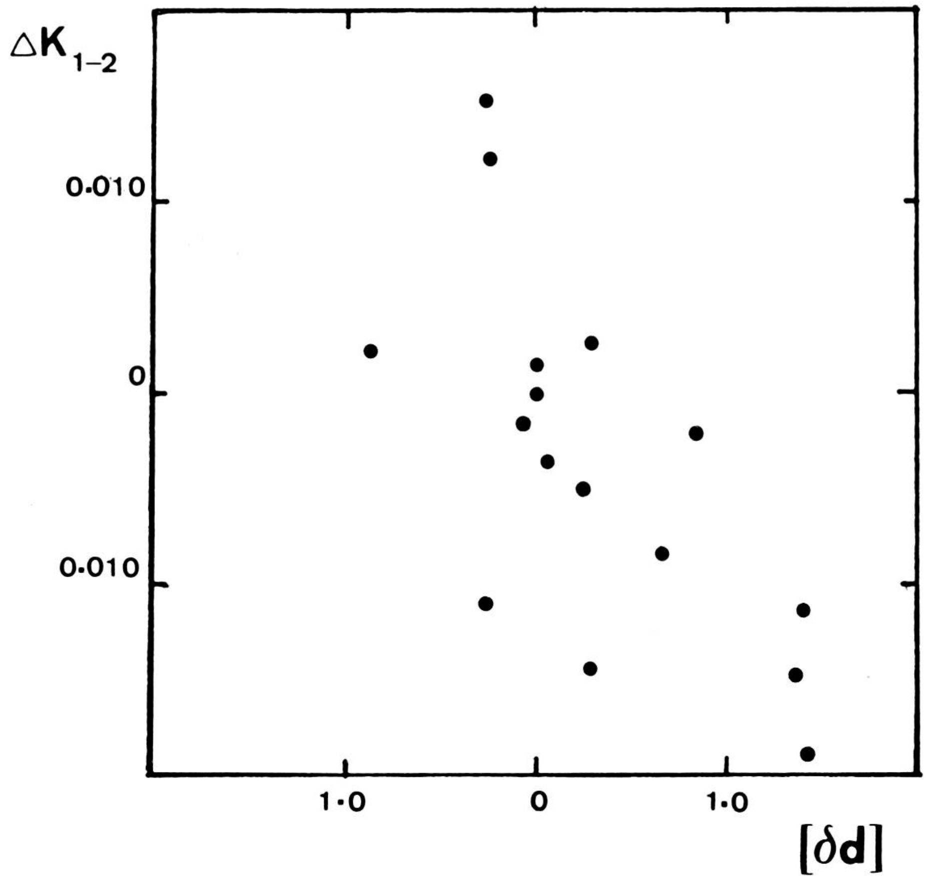

FIG. 7. — Variation of the instantaneous extinction cœfficients for the colour index (1-2) plotted against the differences for the Balmer discontinuity parameter [δd] for the M and D stars.

$A(\lambda, z)$ is Lambert's law of atmospheric extinction.

$\varphi_j(\lambda)$ describes the spectral response of the passbands j, in arbitrary units, of the photometric system used. These are established experimentally.

Theoretical evaluation of the stellar intensities proceeds as follows:

(a) The spectral energy distributions given by Code (1960) are tabulated as monochromatic magnitudes, $m(1/\lambda)$, relative to $(1/\lambda) = 1.80$. Converting these into magnitudes per unit wavelength interval one obtains:

$$m(\lambda) = m(1/\lambda) - 2.5 \log \left[\frac{1/\lambda}{1.8}\right]^2 \quad (13)$$

The monochromatic fluxes $E(\lambda)$, per unit wavelength interval relative to $\lambda = 5560$ A° are then obtained from equation:

$$E(\lambda) = 10^{-0.4m(\lambda)} \quad (14)$$

TABLE 1

List of stars used in the simulation study

Name	Sp. Type	Name	Sp. Type
10 Lac	09 V	σ Boo	F2 V
55 Cyg	B 3 Ia	λ Ser	GO V
η U Maj	B 3 V	16 Cyg A	G2.5 V
α Lyr	AO V	α Tau	K5 III
β Ari	A5 V	61 Cyg A	K5 V

Table 1 gives the various stars and their spectral types used in the simulation study. The energy distributions corresponding to these stars were taken from Code's paper.

(b) The atmospheric function $A(\lambda, z)$ is calculated from the relationship:

$$A(\lambda, z) = \exp\left[-C\, k(\lambda)\, F_z\right] \quad (15)$$

where,

C is a constant given by $2/(5 \log e)$

F_z is the air mass

$k(\lambda)$ is the monochromatic extinction coefficient expressed in magnitudes per unit mass for the Jungfraujoch site.

Table 2 gives the values of $k(\lambda_0, j)$ which represents the average monochromatic extinction coefficients for the mean wavelength λ_0, of the passband, j. These values were taken from Rufener (1973). Table 7 shows the heterochromatic extinction coefficients for the various colour indices i, and these are called k_i^{theor} values (thus $i = 1$ represents the colour index (1-4), etc...). In order to perform the integration of equation (12) to obtain the intensities, we interpolate between the $k(\lambda_0, j)$ values. A value of $1^m.100$ is taken for the extinction coefficient at $\lambda = 3000\,A$, through a smooth fit to the extinction curve.

TABLE 2

Adopted mean values for the extinction coefficients

Passband, j	1 U	2 B	3 V	4 B1	5 B2	6 V1	7 G	
Mean wave-length, λ_0, j (Å)	3456	4245	5500	4024	4480	5405	5805	3000
k (λ_0, j) (mag)	0.522	0.235	0.120	0.281	0.198	0.124	0.116	1.100

With the help of the quantities obtained from paragraphs (a) and (b) and using the spectral responses for the passbands, we obtain the stellar intensities, E_{zj}, from equation (12). These theoretical intensities together with the time intervals and the coordinates of the stars are introduced into the observational reduction programme to give the instantaneous extinction coefficients and the colours of the stars as seen from the Earth and outside the Earth's atmosphere. The latter results will be termed the observed quantities. In Table 3 are given the observed instantaneous atmospheric extinction coefficients, k_i^{obs}, for each M and D star. The example indicated is only for one pair, where the M star has a spectral type *A O V* and the D star, *K*5 III. Likewise, for various spectral type pairs the k_i^{obs} values are obtained. The above pair of spectral types as well as 16 other samples show small variations of the instantaneous extinction coefficients, k_i^{obs} with time. Therefore, an average for each index i can be established separately for each pair of spectral types and these are called $\bar{k}_i^{\text{obs}}$. The latter values will enable a comparison to be made with k_i^{theor} values.

In section 1 the observed instantaneous extinction coefficients were grouped together with a quantity α_i under the symbol, $k_i(t) = k_i + \alpha_i$, and therefore, this quantity must be eliminated to effect a true comparison between the observed and theoretical values.

Table 4 gives the values of $n(\lambda_0, j)$ [Rufener (1964)] for the Jungfraujoch site and $(\mu/\lambda_0)^2$ [Rufener (1971)] for various passbands j, and also the values of α_i [Rufener (1964)] for various indices i.

The values of α_i in Table 4 are indicated in parenthesis for the colour indices (1-4) and (1-2) because fomula (3) is itself in error due to the effect of the Balmer

TABLE 3

k_i^{obs} values obtained from Rufener's method for reducing the atmospheric extinction coefficient using M and D stars

No.	i	F_z	1 1-4	2 1-2	3 4-2	4 4-5	5 2-5	6 2-3	7 5-6	8 5-7	9 6-3	10 6-7	11 3-7
1	M*A0V	4.4062	0.2428	0.2884	0.0147	0.0789	−0.0282	0.1109	0.0748	0.0839	−0.0018	0.0091	0.0023
2	D*K5III	1.0398	0.2432	0.2889	0.0147	0.0788	−0.0282	0.1108	0.0748	0.0839	−0.0017	0.0090	0.0022
3	M*	3.3534	0.2448	0.2913	0.0144	0.0785	−0.0284	0.1105	0.0748	0.0838	−0.0017	0.0089	0.0020
4	D*	1.1044	0.2453	0.2920	0.0142	0.0783	−0.0284	0.1103	0.0748	0.0838	−0.0016	0.0088	0.0019
5	M*	1.8871	0.2470	0.2954	0.0119	0.0764	−0.0284	0.1088	0.0747	0.0839	−0.0017	0.0090	0.0022
6	D*	1.4255	0.2473	0.2961	0.0115	0.0762	−0.0284	0.1086	0.0747	0.0839	−0.0017	0.0090	0.0022
7	M*	1.3860	0.2492	0.2997	0.0101	0.0755	−0.0280	0.1077	0.0746	0.0837	−0.0017	0.0090	0.0022
8	D*	1.9451	0.2492	0.3003	0.0098	0.0754	−0.0279	0.1075	0.0744	0.0836	−0.0016	0.0090	0.0022
9	M*	1.2259	0.2486	0.3007	0.0096	0.0757	−0.0276	0.1071	0.0741	0.0833	−0.0016	0.0091	0.0023
10	D*	2.3707	0.2490	0.3016	0.0093	0.0755	−0.0275	0.1068	0.0740	0.0832	−0.0015	0.0091	0.0022
11	M*	1.1045	0.2515	0.3043	0.0083	0.0747	−0.0275	0.1065	0.0740	0.0831	−0.0015	0.0090	0.0021
12	D*	3.0264	0.2524	0.3053	0.0079	0.0743	−0.0275	0.1063	0.0740	0.0830	−0.0015	0.0090	0.0021
13	M*	1.0152	0.2538	0.3071	0.0068	0.0736	−0.0273	0.1059	0.0738	0.0828	−0.0014	0.0089	0.0020
14	D*	4.1508	0.2540	0.3073	0.0067	0.0735	−0.0272	0.1059	0.0738	0.0828	−0.0014	0.0089	0.0020
	$\bar{k}_i^{obs}$		0.2484	0.2985	0.0107	0.0761	−0.0279	0.1081	0.0744	0.0835	−0.0016	0.0090	0.0021

discontinuity. For a better appreciation of these quantities, the paper by Rufener (1964) should be consulted. The calculation of α_i. β_i and γ_i depends also on a correct estimation of n.

TABLE 4

Values of n (λ_0, j), $(\mu/\lambda_0)^2$, α_i

j	U 1	B 2	V 3	B1 4	B2 5	V1 6	G 7
λ_0 (A°)	3456	4245	5500	4024	4480	5405	5805
n (λ_0, j)	4.3	3.6	1.05	3.75	3.4	1.8	0.2
$(\mu/\lambda_0)^2 10^3$	2.41	4.44	2.90	2.36	1.32	1.39	1.23

i	1 1-4	2 1-2	3 4-2	4 4-5	5 2-5	6 2-3	7 5-6	8 5-7	9 6-3	10 6-7	11 3-7
$\alpha_i . 10^4$	(90)	(140)	−280	−20	−640	40	20	20	−40	0	−50

Therefore, subtracting the α_i values from $\bar{k}_i^{obs}$ (defined above) gives the true extinction coefficients which should be compared with the k_i^{theor} values. Thus, we have defined:

$$k_i^{obs(true)} = \bar{k}_i^{obs} - \alpha_i \tag{16}$$

The observational procedure is judged accurate if the instantaneous extinction coefficients, k_i^{obs} values, remain essentially the same during the simulated time period of 11 hours, and if the $k_i^{obs(true)}$ values correspond well with the k_i^{theor} values.

4. COMPARISON BETWEEN THE OBSERVED AND THEORETICAL VALUES OF THE ATMOSPHERIC EXTINCTION COEFFICIENTS

(a) Results for the observed instantaneous extinction coefficients, k_i^{obs}, show small variations during the simulated period for each of the 16 pairs of spectral types. Table 3 is an extract of the variation of k_i^{obs} with respect to a pair of spectral

TABLE 5

Variation of extinction coefficient with the time period for each case and for various indices, i

$$\Delta k_i^{obs} = [k_i^{obs}\,(\text{initial S.T.}) - k_i^{obs}\,(\text{final S.T.})]$$

	(M*, D*) / *i*	1 1-4	2 1-2	3 4-2	4 4-5	5 2-5	6 2-3	7 5-6	8 5-7	9 6-3	10 6-7	11 3-7
1	A0V, K5III	−0.0112	−0.0189	0.0080	0.0054	0.0010	0.0050	0.0010	0.0011	−0.0004	0.0002	0.0003
2	A5V, K5III	−0.0038	−0.0112	0.0069	0.0038	−0.0010	0.0042	0.0011	0.0013	−0.0001	0.0002	0.0001
3	K5V, K5III	−0.0025	−0.0026	0.0037	0.0022	0.0003	0.0007	0.0004	0.0004	−0.0004	0	0.0004
4	A0V, A5V	−0.0038	−0.0049	0.0005	0.0008	0	0.0010	0	0.0001	0	0.0001	0.0001
5	A5V, O9V	−0.0011	−0.0022	−0.0013	−0.0022	−0.0001	−0.0018	−0.0001	−0.0004	0.0001	−0.0003	−0.0004
6	O9V, A5V	0.0012	0.0023	0.0013	0.0025	0.0004	0.0019	−0.0002	0.0001	0.0001	0.0003	0.0001
7	A5V, A5V	0.0001	0.0001	0	0.0001	0.0001	0.0001	−0.0003	0	0.0004	0.0004	0
8	B3Ia, K5V	−0.0109	−0.0144	0.0053	0.0044	−0.0001	0.0064	0.0013	0.0016	−0.0001	0.0002	0.0002
9	K5V, B3Ia	0.0123	0.0152	−0.0058	−0.0041	0.0009	−0.0058	−0.0017	−0.0017	0.0004	0.0001	−0.0001
10	K5V, O9V	0.0072	0.0124	−0.0061	−0.0052	0.0014	−0.0053	−0.0013	−0.0009	0.0005	0.0003	0.0001
11	O9V, K5V	−0.0060	−0.0111	0.0062	0.0055	−0.0013	0.0053	0.0007	0.0010	0	0.0003	0
12	K5V, K5V	0.0013	0.0013	0	0.0003	0.0003	0.0002	−0.0006	0	0.0004	0.0005	0.0001
13	F2V, K5III	−0.0037	−0.0084	0.0082	0.0048	−0.0006	0.0054	0.0016	0.0014	−0.0004	−0.0002	0.0001
14	B3V, O9V	0.0034	0.0026	−0.0008	−0.0013	−0.0003	−0.0011	−0.0001	0.0002	0.0002	0.0003	0.0002
15	G2V, G0V	0.0018	0.0016	−0.0009	−0.0010	−0.0002	−0.0008	−0.0003	−0.0003	0.0001	0	−0.0002
16	A0V, K5V	−0.0073	−0.0148	0.0046	0.0029	−0.0018	0.0040	0.0010	0.0013	−0.0002	0.0003	0.0003

types (M star = AO V, D star = K5 III) calculated by the observational reduction programme described in Section 1.

Other cases of different pairs of spectral type combinations processed through the above programme also exhibit small variations in these coefficients.

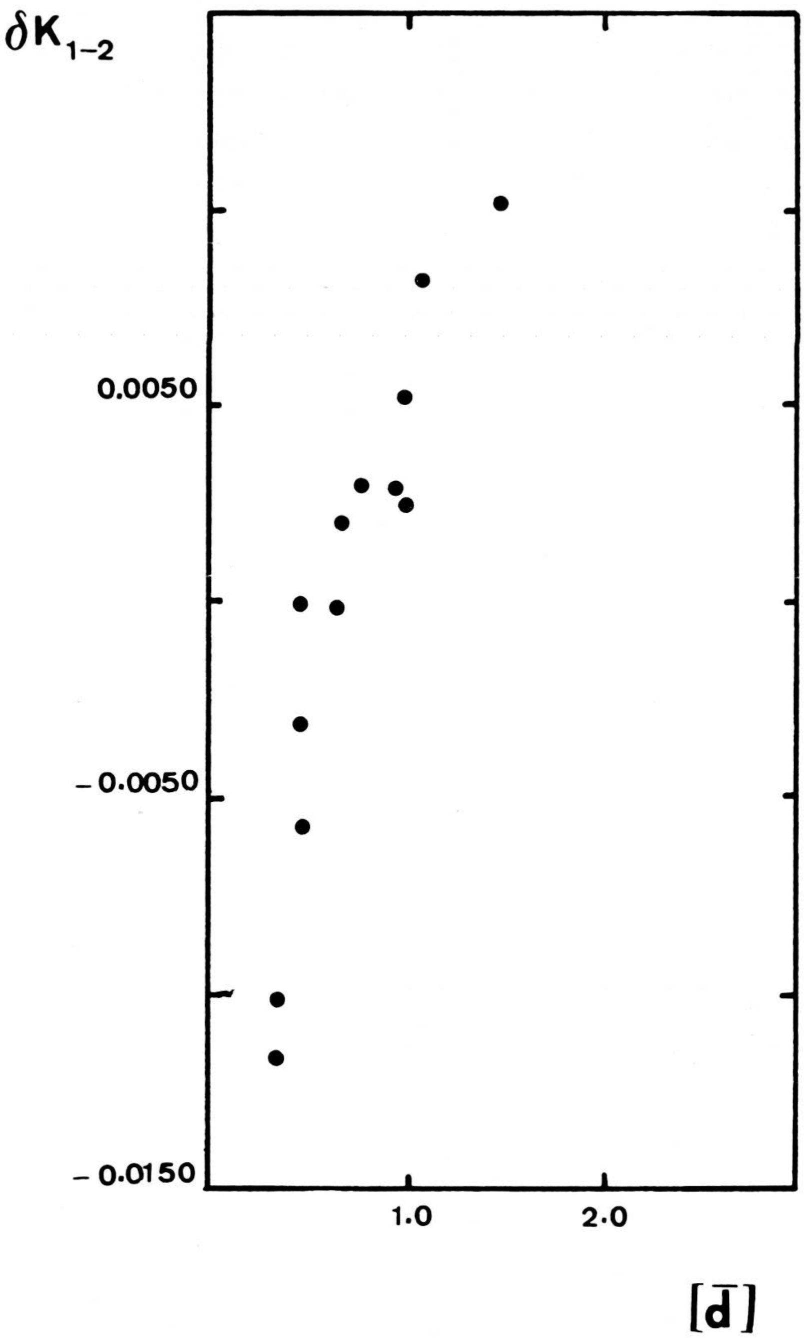

FIG. 8. — Differences between the theoretical and observed extinction cœfficients (in magnitude) plotted against the mean Balmer discontinuity parameter [$\bar{d}$] for the *M* and *D* stars.

We define Δk_i to be the variation of k_i^{obs} as:

$$\Delta k_i = [k_i^{obs}\,(\text{initial time}) - k_i^{obs}\,(\text{final time})] \qquad (17)$$

The initial and final times are the sidereal times for the total period of 11 hours of the simulated night. We have seen that the coefficients k_i^{obs} depend on α_i, but upon taking the differences as indicated in equation (17) these disappear, giving us the variation in the instantaneous extinction coefficients separate for each index i and for each pair of spectral types. The Δk_i values calculated for various pairs of spectral types for the 11 colour indices are given in Table 5. It can be seen that they are different for each pair, but remain essentially small. Figure 4 gives examples of the variation of the instantaneous extinction coefficients k_{1-4} with time for different pairs (7, 10 of Table 5) of spectral types.

(b) In order to compare the observed extinction coefficients with the theoretical extinction coefficients, we define a parameter δk_i as follows:

$$\delta k_i = [k_i^{theor} - k_i^{obs(true)}] \qquad (18)$$

where $k_i^{obs(true)}$ is defined in (16)

δk_i values are shown in Table 6 along with the standard deviations for various pairs of spectral types. This table shows that the values of δk_i are small and vary with respect to the pairs of M and D stars. The maximum standard deviation of these values is 0.007 magnitude which indicates that there is a good correlation between the observed and theoretical extinction coefficients. This can also be seen from Table 7 where we have indicated the average of the $k_i^{obs(true)}$ values established from all the 16 pairs studied (which we have called $\bar{k}_i^{obs(true)}$) and the values of the theoretical extinction coefficients.

5. ANALYSIS OF THE OBSERVED EXTINCTION COEFFICIENTS

In the preceding section we saw that small differences exist between the observed and theoretical extinction coefficients. We shall investigate if these differences could furthermore be minimised. An elimination of these is not possible because of the difficulties in determining the proper values for the constants, α_i, β_i and γ_i, especially with respect to the colour indices (1-4) and (1-2) as explained in Section 3. Therefore, after ascertaining that the best possible values are obtained for the above constants, we shall make a correlation study between the extinction differences and the colours of the stars, passbands of the system, etc...

$\delta k_i = (k_i^{theor} - k_i^{obs\,(true)})$ (magnitude)

	i Sp. types	1 1-4	2 1-2	3 4-2	4 4-5	5 2-5	6 2-3	7 5-6	8 5-7	9 6-3	10 6-7	11 3-7
1	A0V, K5III	0.0014	0.0025	0.0073	0.0049	0.0009	0.0109	0.0016	0.0005	0.0016	−0.0010	−0.0031
2	A5V, K5III	0.0028	0.0029	0.0073	0.0049	0.0005	0.0118	0.0023	0.0011	0.0015	−0.0010	0.0031
3	K5V, K5III	−0.0053	−0.0101	0.0112	0.0072	0.0009	0.0141	0.0020	0.0019	0.0012	−0.0010	−0.0029
4	A0V, A5V	0.0067	0.0137	0.0012	0.0013	0.0025	0.0074	0.0010	0	0.0017	−0.0009	−0.0031
5	A5V, O9V	0.0030	0.0082	0.0003	−0.0003	0.0013	0.0067	0.0012	0	0.0017	−0.0011	−0.0031
6	O9V, A5V	0.0031	0.0082	0.0003	−0.0002	0.0013	0.0067	0.0010	0.0001	0.0018	−0.0009	−0.0031
7	A5V, A5V	0.0043	0.0102	0.0015	0.0019	0.0016	0.0093	0.0010	0.0002	0.0018	−0.0009	−0.0029
8	B3Ia, K5V	0.0032	0.0003	0.0023	0.0006	−0.0009	0.0078	0.0015	0.0004	0.0014	−0.0010	−0.0031
9	K5V, B3Ia	0.0026	0	0.0032	0.0013	−0.0010	0.0083	0.0017	0.0006	0.0014	−0.0010	−0.0031
10	K5V, O9V	−0.0027	−0.0030	0.0036	0.0015	−0.0001	0.0093	0.0018	0.0008	0.0016	−0.0010	−0.0030
11	O9V, K5V	−0.0018	−0.0022	0.0034	0.0016	0.0003	0.0093	0.0017	0.0008	0.0016	−0.0009	−0.0028
12	K5V, K5V	−0.0065	−0.0115	0.0080	0.0056	−0.0008	0.0136	0.0025	0.0017	0.0012	−0.0008	−0.0029
13	F2V, K5III	0.0020	0.0002	0.0060	0.0034	−0.0014	0.0105	0.0025	0.0016	0.0014	−0.0009	−0.0029
14	B3V, O9V	−0.0019	0.0030	−0.0001	−0.0011	0.0013	0.0064	0.0014	0.0002	0.0016	−0.0012	−0.0031
15	G2V, G0V	0.0025	0.0019	0.0010	0.0010	0.0001	0.0081	0.0020	0.0009	0.0016	−0.0010	−0.0031
16	A0V, K5V	0.0040	0.0052	0.0044	0.0029	0.0002	0.0102	0.0017	0.0008	0.0017	−0.0008	−0.0031
	Standard deviation of residuals $\delta k_i = \sigma_i(\delta k_i)$ (mag)											
	$\sigma_i(\delta k_i)$	0.0037	0.0067	0.0033	0.0023	0.0011	0.0023	0.0005	0.0006	0.0002	0.0001	0.0001

TABLE 7

Average of $k_i^{obs\,(true)}$ values obtained from 16 pairs of spectral types

	1 1-4	2 1-2	3 4-2	4 4-5	5 2-5	6 2-3	7 5-6	8 5-7	9 6-3	10 6-7	11 3-7
k_i^{theor} (mag.)	0.2410	0.2870	0.0460	0.0830	0.0370	0.1150	0.0740	0.0820	0.0040	0.0080	0.0040
$\bar{k}_i^{obs\,(true)}$ (mag.)	0.2400	0.2852	0.0422	0.0807	0.0368	0.1056	0.0723	0.0813	0.0023	0.0090	0.0070

5.1 INVESTIGATION INTO THE POSSIBILITY OF MINIMISING THE DIFFERENCES, δk_i

We saw in Section 3 that the factor α_i is coupled with the instantaneous extinction coefficient, k_i^{obs}. It will now be our aim to see if a re-calculation of α_i values and their subsequent subtraction from the k_i^{obs} values, giving $k_i^{\text{obs(true)}}$ values, could result in further narrowing of the differences between the theoretical and observed extinction coefficients. Such a calculation is made by estimating the values of $n(\lambda_0, j)$ from equation (6). This was done by assessing the slopes through a plot of logλ versus log $k(\lambda)$ at the mean wavelength λ_0 of the passband, j. Through a choice of two different values of $n\ (\lambda_0, j)$ two values of α_i (indicated as calculation I and II in Table 8) are obtained. Table 8 shows that there is a somewhat larger difference between the calculated and adopted values of n for the mean wavelength, $\lambda_0 = 5805.Å$ This is due to the interpolated curve beyond $\lambda\lambda$ 5805Å up to 6500Å. The re-calculated values of α_i and β_i, given in Table 8 together with the adopted observed values, show that the differences are small. We continue to give the values corresponding to the indices (1-4) and (1-2) in parenthesis because equations (3), (4) and (5) are in error due to the effect of the Balmer discontinuity. Thus we see that a re-estimation of $k_i^{\text{obs(true)}}$ values is not essential since the δk_i differences will not be effectively minimised even if the new α_i, β_i and γ_i values are used. Therefore, it can be stated that the adopted values of the observational reduction programme are sufficiently well chosen and can be retained for all practical purposes.

5.2 CORRELATION STUDIES OF Δk_i

(a) *Dependence of stellar colours on the variation of the extinction coefficients*

The α_i, β_i and γ_i factors are sufficiently well determined, and hence it would be interesting to examine whether there is any dependence of the stellar colours on the extinction coefficients. Stellar colours are obtained from the observational programme as well as the simulation programme as described in Section 1 and 2, respectively. We choose the (U-B) and (B-V) indices of the M and D stars obtained outside the atmosphere and assume that the colour index difference between the pairs of spectral type influence the observed instantaneous extinction coefficients. We thus define,

$$\Delta(U-B)^{\text{theor}} = [(U-B)^M - (U-B)^D] \tag{19}$$

$$\Delta(B-V)^{\text{theor}} = [(B-V)^M - (B-V)^D]$$

TABLE 8

Constants α_i and β_i re-calculated

i	1 1-4	2 1-2	3 4-2	4 4-5	5 2-5	6 2-3	7 5-6	8 5-7	9 6-3	10 6-7	11 3-7
Adopted values of $\alpha_i 10^4$	(70)	(184)	−280	− 20	−640	40	20	20	−40	0	−50
Calculation: I	(155)	(169)	−216	− 54	−636	41	22	16	−23	− 3	−51
$\alpha_i 10^4$ II	(136)	(170)	−297	− 42	−601	30	26	16	−43	− 5	−54
Adopted values of $\beta_i 10^4$	(−271)	(−138)	255	− 81	−488	−120	−24	−25	44	−27	−57
Calculation: I	(−229)	(−133)	192	−161	−487	−138	−32	−31	36	−30	−50
$\beta_i 10^4$ II	(−225)	(−134)	178	−151	−454	−129	−32	−31	56	−29	−54

n : Value characterising the atmospheric extinction law, $k(\lambda) = a \cdot \lambda^{-n}$

j	U 1	B 2	V 3	B1 4	B2 5	V1 6	G 7
λ_0 (A^0)	3456	4245	5500	4024	4480	5405	5805
Adopted values of $n(\lambda_0, j)$	4.3	3.6	1.05	3.75	3.4	1.8	0.2
Calculation: I	4.55	3.6	1.10	3.75	3.3	1.8	0.65
II	4.3	3.4	1.2	3.6	3.3	1.8	0.7

λ_0: Indicates mean wavelength; Calculation I, II: These indicate attempts at assessing the slopes n at the corresponding mean wavelengths of the pass-bands, through a plot of log λ versus log $k(\lambda)$. The constants α_i, β_i were thus calculated twice as indicated by I and II values through formula (21) of Rufener (1964).

There are slight fluctuations in the observed colours obtained outside the atmosphere for the M and D stars for each time interval of the simulated night, which is due to the effect of extinction coefficients. The standard deviations of observed colours outside the atmosphere for each simulated night of M and D stars are small. For example, a pair of M and D stars having spectral types AO V and K5 III respectively, has the following standard deviations for the colours:

			U	V	B1	B2	V1	G
M	star	(AO V)	$0^m\!.0024$	0.0001	0.0005	0.0005	0.0003	0.0003
D	star	(K5 III)	$0^m\!.0040$	0.0003	0.0011	0.0003	0.0002	0.0003

The difference between the observed and theoretical colours are small (see Section 7) so that theoretical colours can be used for the above parameter.

Figures 5 and 6 show evidence for the dependence of the colours on the instantaneous extinction coefficients Δk_{1-2}. Similar evidence is found for Δk_{1-4}. The Δk_i values of other indices are more or less constant, consequently there is no dependence of stellar colours on such variations.

(b) *Dependence of the Balmer discontinuity on the variation of Δk_i*

We are aware that the indices (U-B) and (U-B1), indicated as (1-2) and (1-4) are correlated with the Balmer discontinuity, and this motivated us to examine the dependence of the measurement of this discontinuity on the variation of the instantaneous extinction coefficients obtained from different pairs of spectral types. The Balmer discontinuity parameter defined by Golay (1974) for the seven colour photometric system of the Geneva Observatory is as follows:

$$[d] = (U - B1) - 1.430(B1 - B2) \tag{20}$$

The above parameter is calculated for various pairs of spectral types from the theoretical colours, and an average value, $[\bar{d}]$, is obtained for each pair. These average values of the Balmer discontinuity showed that there is no clear dependence of the measured discontinuity on the variation of the extinction coefficients. However, if a parameter $[\delta d]$ is defined giving the difference of the Balmer discontinuity between the pairs of stars as,

$$[\delta d] = [d]^M - [d]^D \tag{21}$$

and plotted against Δk_{1-4} for all the pairs, we find from Figure 7 that, indeed, there could be a dependence of $[\delta d]$ on the instantaneous extinction coefficients.

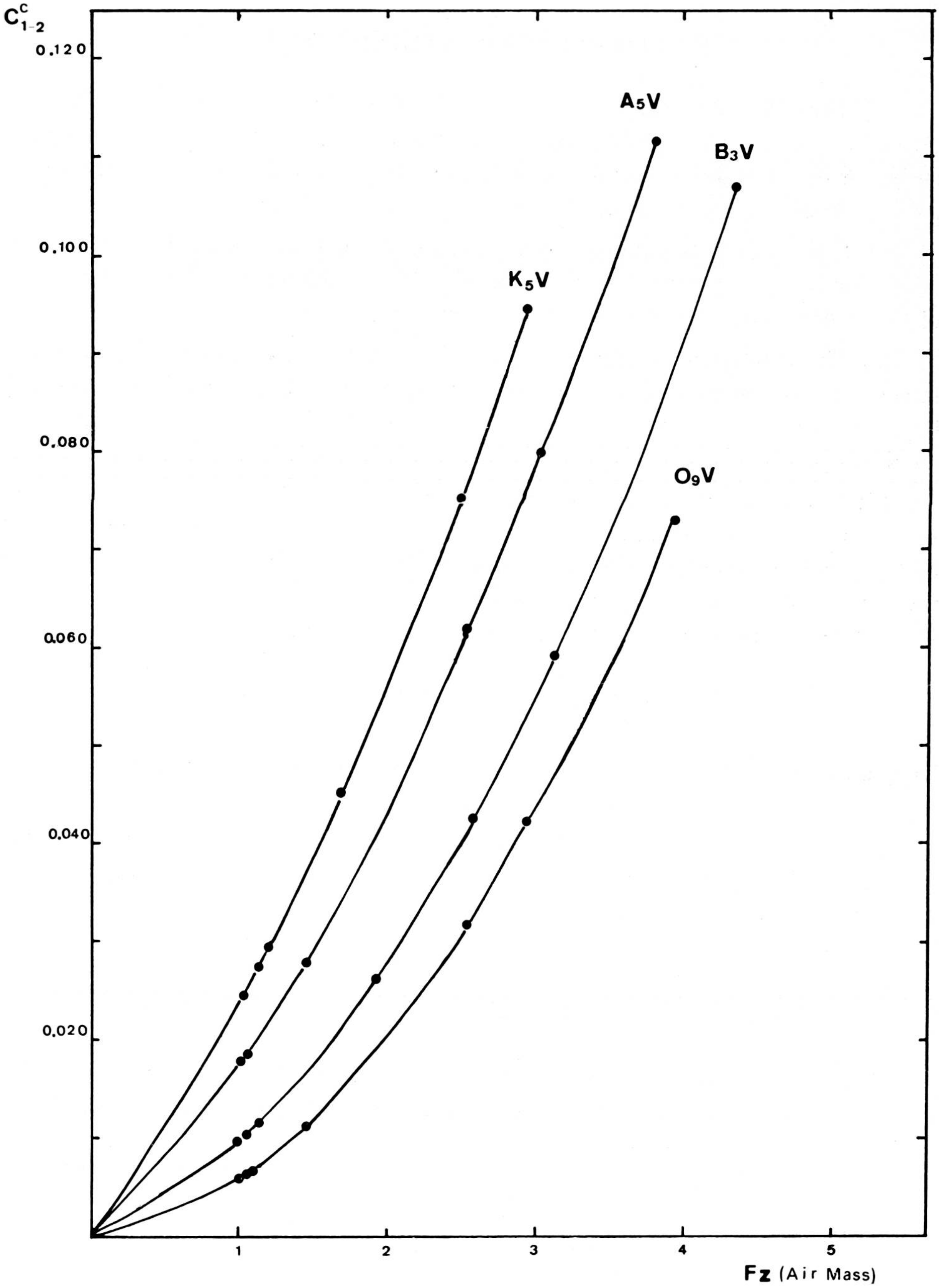

FIG. 9. — The colour correction and Forbes effect term, C^c_{1-2} for a few spectral types plotted against the air mass, F_z.

5.3 CORRELATION STUDIES OF δk_i

(a) Table 6 shows that the difference between the theoretical and observed extinction coefficients δk_i obtained for various pairs of spectral types are slightly larger for the (2-3) index than for other indices. Hence, the dependence of the stellar colours on δk_{2-3} was examined but no correlation was found.

b) The standard deviations $\sigma_i\,(\delta k_i)$ given in Table 6 show that there is a larger dispersion of δk_{1-2} values. A further analysis of δk_{1-2} did not reveal any dependence of the stellar colours on the extinction coefficients.

(c) The extinction differences δk_{1-4}, δk_{1-2} and δk_{2-3} in the same table are larger than the others and a dependence of the Balmer discontinuity on these differences is found. An example of such a correlation for δk_{1-2} versus $[\bar{d}]$ is indicated in Figure 8.

(d) It was shown in Section 1 that the evaluation of the factors α_i, β_i and γ_i proceeds from a knowledge of various quantities contained in equations (3), (4) and (5), and one such quantity involved is the bandwidth of the system, μ. We have therefore tried to see if there exists a correlation between δk_i and Φ_j, but find none. Here Φ_j represents the response curve in magnitudes and is given by:

$$\Phi_j = -2.5 \log \int_0^\infty \varphi_j(\lambda)\, d\lambda - 2.5 \int_0^\infty \varphi_B(\lambda)\, d\lambda \tag{22}$$

The above function is normalised with respect to the B colour of the system, and its values were taken from the paper by Rufener and Maeder (1971).

(e) It also appears from Table 6 that systematic errors in δk_i exist for the indices (6-3), (6-7) and (3-7), which could be attributed to the second order Taylor series expansion of $E(\lambda)$ used in equation (6).

6. ANALYSIS OF THE STELLAR COLOUR DIFFERENCES

The observational reduction programme as well as the theoretical simulation programme allows one to obtain the colours of stars as measured from the Earth and outside the atmosphere. The equation given below is used in computing the theoretical intensities, E_j^0, of stars outside the atmosphere for the passband j:

$$E_j^0 = \int_0^\infty E(\lambda)\, \varphi_j(\lambda)\, d\lambda \tag{23}$$

Changing the above equation into magnitudes we obtain,

$$m_j^0 = -2.5 \log E_j^0$$

TABLE 9

Residuals $\Delta C_j^o = (C_j^{otheor} - C_j^{obs})$ (mag.)

for each pair of M and D stars

j Sp. Ty.		1 U	2 B	3 V	4 B1	5 B2	6 V1	7 G
1. A0V	M*	0.0075	0	0.0046	−0.0044	0.0010	0.0034	0.0067
K5III	D*	−0.0109		−0.0069	0.0010	−0.0028	−0.0061	−0.0065
2. A5V		0.0020		0.0037	−0.0043	0	0.0034	0.0034
K5III		−0.0112		−0.0062	0.0025	−0.0038	−0.0052	−0.0061
3. K5V		−0.0005		−0.0008	−0.0043	−0.0012	−0.0010	−0.0006
K5III		0.0077		−0.0038	−0 0015	−0.0027	−0.0026	−0.0031
4. A0V		−0.0036		0.0007	−0.0010	−0.0006	0.0001	0.0007
A5V		−0.0099		−0.0020	−0.0004	−0.0015	−0.0016	−0.0012
5. A5V		−0.0017		−0.0028	0.0013	−0.0021	−0.0021	−0.0018
O9V		−0.0086		0.0015	−0.0018	−0.0011	0.0019	0.0009
6. O9V		−0.0089		0.0015	−0.0017	0.0012	0.0018	0.0015
A5V		−0.0014		−0.0027	0.0014	−0.0017	−0.0023	−0.0016
7. A5V		−0.0053		−0.0009	−0.0004	−0.0006	−0.0005	0.0001
A5V		−0.0051		−0.0008	−0.0004	−0.0005	−0.0006	0.0001
8. B3Ia		0.0139		0.0064	−0.0034	0.0026	0.0052	0.0047
K5V		−0.0116		−0.0085	0.0029	−0.0047	−0.0076	−0.0074
9. K5V		−0.0138		−0.0077	0.0020	−0.0043	−0.0070	−0.0068
B3Ia		0.0138		0.0063	−0.0041	0.0020	0.0049	0.0045
10. K5V		−0.0072		−0.0063	0.0014	−0.0033	−0.0056	−0.0055
O9V		0.0041		0.0043	−0.0030	0.0014	0.0044	0.0034
11. O9V		0.0032		0.0046	−0.0029	0.0019	0.0047	0.0039
K5V		−0.0069		−0.0061	0.0017	−0.0022	−0.0054	−0.0047
12. K5V		0.0015		−0.0012	−0.0016	−0.0011	−0.0012	−0.0010
K5V		0.0028		−0.0013	−0.0014	−0.0014	−0.0013	−0.0008
13. F2V		0.0030		0.0046	−0.0051	0.0010	0.0035	0.0041
K5III		−0.0017		−0.0076	0.0031	−0.0046	−0.0065	−0.0066
14. B3V		−0.0107		−0.0018	0.0007	−0.0015	−0.0009	−0.0011
O9V		−0.0007		0.0014	−0.0010	0.0011	0.0021	0.0012
15. G2V		−0.0055		−0.0009	−0.0002	−0.0017	−0.0015	−0.0012
G0V		−0.0011		0.0010	−0.0016	−0.0004	0.0003	0.0003
16. A0V		0.0045		0.0037	−0.0019	0.0001	0.0025	0.0032
K5V		−0.0162		−0.0053	−0.0009	−0.0028	−0.0053	−0.0044

and these are normalised with respect to the B colour, i.e., $j = 2$. The observed colours outside the atmosphere are evaluated as indicated in Section 1.

We define a parameter ΔC_j^0 giving the difference between the theoretical and observed colours outside the atmosphere,

$$\Delta C_j^0 = (C_j^{0\text{theor}} - C_j^{0\text{obs}}) \qquad (24)$$

The residuals ΔC_j^0 obtained for each pair of spectral types are given in Table 9 which shows that they are considerably small for pairs having nearly the same spectral type and for similar spectral types.

Another parameter $\overline{\Delta} C_j^0$ is defined giving the average value of the residuals ΔC_j^0 belonging to the same spectral type (Table 10).

The standard deviations of the colour differences, $\sigma_j(\Delta C_j^0)$, of spectral types ocurring repeatedly in various pairs of stars studied, are given in Table 11. These standard deviations are small (less than $0^m_.003$) for all colours except for the *U* colour (less than $0^m_.009$).

In the following paragraph we analyse the above colour residuals to indicate any possible dependence of quantitative factors on them.

6.1 COLOUR CORRELATION STUDY

(a) Tables 9, 10 and 11 show that the colour residuals are larger for the *U* colour than for the other colours. However, no dependence of the $(U\text{-}B)^{theor}$ and $(B\text{-}V)^{theor}$ indices on $\overline{\Delta} C_j^0$ is found, and these diagrams do not reveal any correlation.

(b) Furthermore, no dependence of the Balmer discontinuity on the colour differences ΔC_j^0 exists. The discontinuity parameter [*d*] was calculated from equation (18) using the theoretical colours.

6.2 EFFECT OF PASSBANDS

Since the analysis is of a purely simulated nature not involving stellar measurements, the systematic erros which come into play are not caused by measurements

TABLE 10

Residuals $\overline{\Delta} C_j^0$ (average of ΔC_j^0 values of Table 9 for similar spectral types)

j Sp. Ty.	1 U	2 B	3 V	4 B1	5 B2	6 V1	7 G
O9V	$-0^m_.0022$	0	$0^m_.0027$	$0^m_.0021$	$0^m_.0009$	$0^m_.0030$	$0^m_.0022$
B3V	−0.0107		−0.0018	0.0007	−0.0015	−0.0009	−0.0011
B3Ia	0.0139		0.0064	−0.0038	0.0023	0.0051	0.0046
A0V	0.0028		0.0030	−0.0024	0.0002	0.0020	0.0035
A5V	−0.0036		−0.0009	−0.0005	−0.0011	−0.0006	−0.0002
F2V	0.0030		0.0046	−0.0051	0.0010	0.0035	0.0041
G0V	−0.0011		0.0010	−0.0016	−0.0004	0.0003	0.0003
G2V	−0.0055		−0.0009	−0.0002	−0.0017	−0.0015	−0.0012
K5V	−0.0069		−0.0047	0	−0.0026	−0.0045	−0.0039
K5III	−0.0040		−0.0061	0.0013	−0.0035	−0.0051	−0.0056

of stars. Hence, these errors appearing in our analysis may have been introduced during the reduction of atmospheric extinction. Now, in the determination of the passbands of a system a residue always exists since the Fourier series of $\varphi(\lambda)$ is truncated. Therefore, an attempt is made to find this error by studying the dependence of wavelength and the function characterising the passbands Φ_i (in magnitudes) on ΔC^0_j. Both these analyses do not reveal any correlation, suggesting that the passbands of the system do not depend on the colour residuals.

TABLE 11

Standard deviations of colours of various sp. types

$\sigma_j (\Delta C^o_j). 10^2$ (Magnitude)

j Sp. Ty.	1 U	2 B	3 V	4 B1	5 B2	6 V1	7 G
O9V	0.63	0	0.16	0.09	0.12	0.14	0.14
A0V	0.57		0.20	0.18	0.08	0.17	0.30
A5V	0.41		0.24	0.21	0.08	0.21	0.19
K5III	0.90		0.17	0.21	0.09	0.18	0.17
K5V	0.68		0.31	0.24	0.14	0.25	0.28

7. OVERALL CONCLUSIONS

The small differences between the theoretical and observed extinction cœfficients demonstrate the high accuracy of the atmospheric reduction process through a study of ascending and descending stars. This means that the average values of the monochromatic atmospheric extinction coefficients that are adopted correspond best with the actual observational conditions. The results may be summarised as follows:

(a) An analysis of the differences between the theoretical and observational extinction coefficients shows that there is a dependence of the Balmer discontinuity on these coefficients.

(b) There is no dependence of the passbands of the system on the extinction coefficients.

(c) An analysis of the cœfficients α_i, β_i, and γ_i shows that the adopted values of these cœfficients are indeed well chosen.

(d) Studies of colour residuals obtained from the theoretical and observed stellar colours shows that neither the passbands of the system nor the (U-B), (B-V) colours depend upon these residuals. It is also unlikely that there is a dependence of the Balmer discontinuity on the colour residuals.

(e) The atmospheric extinction coefficient consists of the monochromatic extinction at the mean wavelength, and a term which depens on the colour of the star defined in the spectral interval covered by the passband of the response curve, $\varphi_i(\lambda)$. Hence the slope of Bouguer's line actually depends on the colour of the star. The observational reduction programme takes due consideration of the above effect.

These conclusions justify the use of the atmospheric reduction programme of Rufener (1964) and also shows that it is not necessary to modify the programme.

The following paragraph demonstrates the importance of the colour correction term and the Forbes effect.

7.1 COLOUR CORRECTION AND FORBES EFFECT

Equation (2) of Section 1 permits the evaluation of the colour correction term and the Forbes effect, and we can write:

$$C^{c}_{1-2} = (C^{z-}_{1-2}) - (C^{z}_{1-2}) \qquad (25)$$

A graphical representation of the air mass versus C^{c}_{1-2} (for M or D star) is shown in Figure 9 for the colour index (1-2). This figure demonstrates the importance of the stellar colour correction and the Forbes effect for various spectral types as we go towards higher air masses.

Figure 10 shows the colour index C_{1-2} obtained from Earth (C^{z-}_{1-2}&C^{z}_{1-2}) versus the air mass F_z for four examples of M and D pairs of stars. It is possible to estimate the extinction cœfficients without ambiguity through the slope of the line after the colour correction and Forbes effect are applied (points through which a straight line has been drawn in the above Figure). The linearity is not well shown if the C^{z}_{1} colours obtained through Bouguer's method are used (shown by points lying below the line).

Therefore, it is clear that one is in error if one uses the classical Bouguer's analysis to estimate the extinction cœfficients. Furthermore, it can also be judged that the extinction cœfficients are less dispersed using the proposed reduction of measures if the two stars, M and D, are of the same spectral type. Through practical applications of the observational reduction method, Rufener (1973) was able to arrive at the same conclusion.

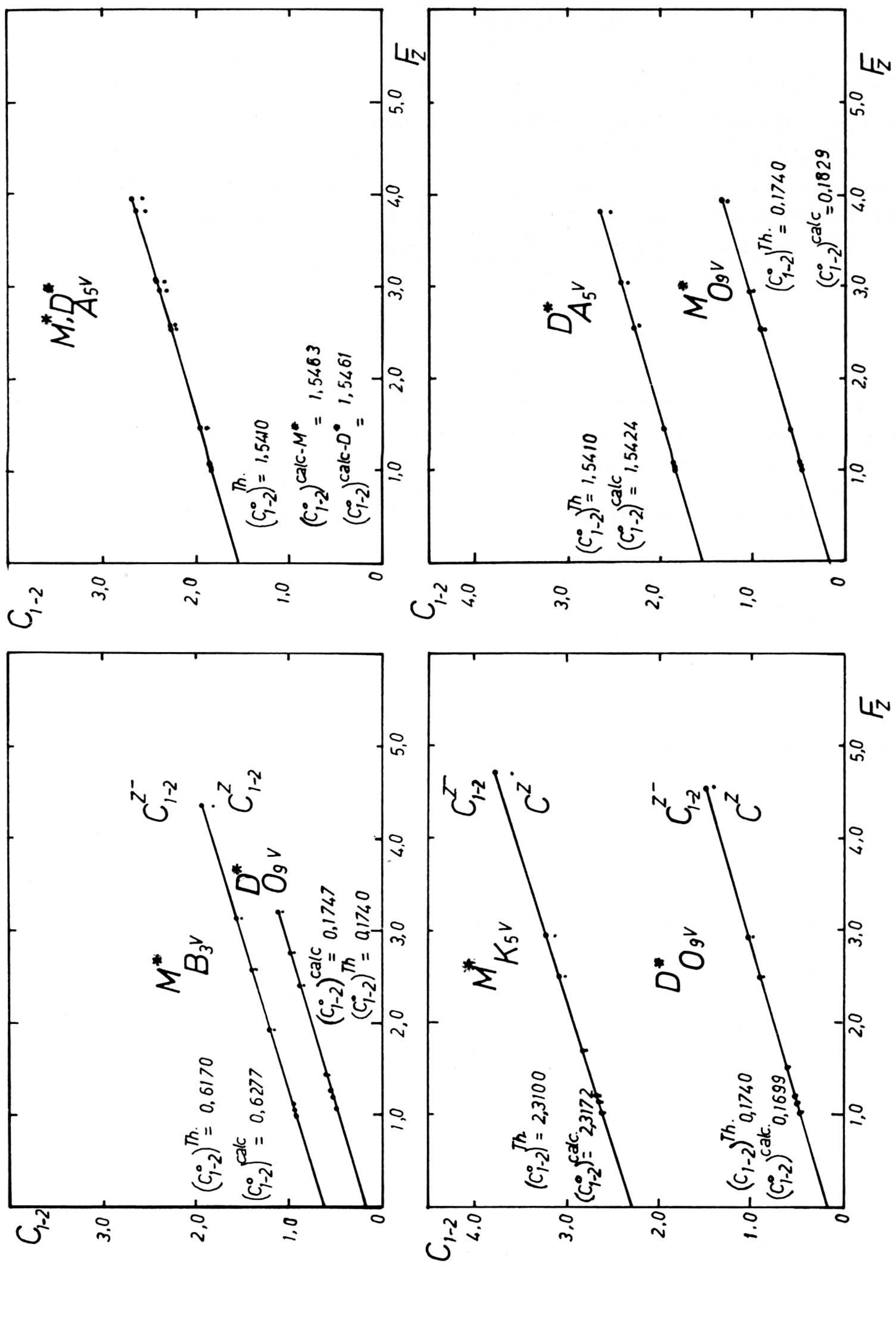

FIG. 10. — Some examples of the colour index (1-2) for M and D stars plotted against the air mass, F_z. Here C^z- indicates the observed colour index corrected for colour and the Forbes effect, and C^z indicates values from Bouguer's classical method. $(C^0_{1-2})^{\text{theor}}$ and $(C^0_{1-2})^{\text{obs}}$ are values obtained by the theoretical and observational procedures.

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