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QUANTUM KINETICS: A STRUCTURAL REINTERPRETATION OF THE QUANTIZATION IN DYNAMICAL SYSTEMS [1]

CINÉTIQUE QUANTIQUE: UNE RÉINTERPRÉTATION STRUCTURALE DE LA QUANTIFICATION DANS LES SYSTÈMES DYNAMIQUES

BY

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ABSTRACT

The *structural reconstruction*, first of Quantum Mechanics, and then, of (Gibbsian) Statistical Mechanics allows for a *structural reinterpretation* of the phenomenon of quantization in the more extended class of the theories of *Kinetics*. Thus, Quantum Kinetics is based on three structural characters: 1) the duality between tangent and cotangent bundles on a Differentiable Manifold, 2) the presence of a physical dimension or megethos, and 3) an active projection. Some methodological considerations are offered concerning the rather unusual approach to this type of problems.

I. QUANTIZATION: THE STRUCTURAL APPROACH

1. Introduction

Within the larger framework of a *structural reconstruction* (1) of the theories of *Kinetics* (i.e. of change as motion: *kinesis*), it is permissible to perform a structural reinterpretation of the two major scientific revolutions in physics at the dawn of this century: the Theory of Special Relativity and the Quantum Theory. The result of such process is that they are dislodged from their majestic epistemic status of scientific revolutions and reduced to the more modest, yet adequate rank of convenient responses to two major deficiencies in Classical Mechanics. The Theory of

[1] In homage to E.E.G. Stueckelberg de Breidenbach, from whom I have learned to conceive of physics not only as a science, but primarily as a *theoria*, an invaluable conceptual instrument to probe reality.

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Special Relativity lifts the ubiquitous ambiguity of Classical Mechanics with regard to physical time, by the explicit suppression of the classical confusion of the two different mathematical roles of time, resp. as parameter of evolution and as dimensional coordinate. (2) On the other hand, as will argued here, the Theory of Quantum Mechanics aims at remedying the inherent incapacity of Classical Mechanics to treat *vectors* and *covectors* as such, i.e. as they are intrinsically, and not merely in terms of their components in a given frame, which totally blurs the fundamental duality existing on a differentiable manifold (DM) M between its tangent bundle $T(M)$ and its cotangent bundle $T^*(M)$. This still holds on the mathematical level. Moreover, the use of the mathematical language of the DM's to express a physical discourse also blurs the fact, that in a physical discourse one deals not only with classical numbers (called *c-numbers* by Dirac) but also with *μ-numbers*, i.e. numbers endowed with a *megethos* (in homage to Eudoxus), that is to say, with a physical dimension (for short: phy-dimension). These two facts: 1) the duality $T(M)/T^*(M)$ and 2) the existence of a megethos enable the Quantum structure to manifest itself. By now it should be clear that quantization is a phenomenon more general than is usually thought, since it can be found in a priori arbitrary DM's and/or with quanta of megethos not restricted only to action. (27)

The following is a brief but illuminating exemplification of the present point of view. Let

$$[1] \qquad \frac{d}{dt}\vec{v}\,(\vec{x}\,(t), t) = -\frac{1}{\rho}\overrightarrow{\text{grad}}\; p$$

be Euler's fundamental equation of hydrodynamics (1755 !), with the speed $\vec{v}$, the density ρ and the pressure p as functions on a streamline of the fluid.

For our purpose, only the total derivative written in its linear operator form matters:

$$[2] \qquad \frac{d}{dt} = \frac{dx^i}{dt}\frac{\partial}{\partial x^i} + \frac{\partial}{\partial t}$$

Mathematically, this is a trivial expression of the tangent mapping (Leibniz' chain rule and difference between total and partial derivative).

But its usage within a physical discourse conceals formidable ambiguities, since the expression covers both the tangent to a relativistic worldline and a Legendre transform between Quantum Lagrangian and Hamiltonian, leading directly to the Schrödinger equation! A detailed exposition of the argument is given in (2); it will therefore suffice to indicate the operations to be performed on [2] as well as the results.

a) Multiply [2] with the factor $\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$. One obtains:

[3] $$\frac{d}{d\tau} = u^{\mu} \frac{\partial}{\partial x^{\mu}} \quad , \quad \mu \in \{1, 2, 3, 4\}$$

with $d\tau = \sqrt{1 - \frac{v^2}{c^2}}\, dt$ the proper time, $u^i = \dfrac{v^i}{\sqrt{1 - \frac{v^2}{c^2}}}$ and $u^4 = \dfrac{c}{\sqrt{1 - \frac{v^2}{c^2}}}$ the 4-celerity. This suppresses the confusion in [2] between t as a parameter (on the streamlines) and t as a coordinate.

b) Multiply [2] with the factor $-ih$ (or $-i\hbar$: no distinction is made here between h and $\hbar$). One obtains:

[4] $$\hat{L} = v_i \hat{P}^i - \hat{H}$$

the (Quantum) expression of the momentum-energy vector. As $v^i \partial_i = (\vec{v} \cdot \overrightarrow{\text{grad}})$, the rule for treating the vectors intrinsically makes v^i to become $\hat{v}^i = \hat{P}^i/m$ and gives: (28)

[5] $$\hat{L} = g_{ik} \frac{\hat{P}^i \hat{P}^k}{m} - \hat{H} = 2\hat{T} - \hat{H}$$

whence the Schrödinger equation:

[6] $$\hat{H} = \hat{T} + \hat{V} = g_{ik} \frac{\hat{P}^i \hat{P}^k}{2m} + \hat{V}$$

c) The application of a) and b) combined yields the Klein-Gordon equation:

[7] $$-\hat{H}_0 = -ih \frac{d}{d\tau} = u_{\mu} \hat{P}^{\mu} = g_{\mu\nu} \frac{\hat{P}^{\mu} \hat{P}^{\nu}}{m_0} \quad \text{with } \hat{H}_0 \psi = E_0 \psi$$

d) The substitution in c) of the use of the spinor tangent mapping for that of the vector tangent mapping produces the Dirac equation:

[8] $$-\hat{H}_0 = -ih \frac{d}{d\tau} = \gamma_{\mu} \hat{P}^{\mu} \quad \text{with } \hat{H}_0 \chi = E_0 \chi$$

The aim of the remainder of this paper, is to present a justification of this very effective structural reinterpretation of quantization in dynamical systems.

2. KINETICS

Kinetics is change as motion. Something, call it a physical system, changes from an actual state (or position) into another one. In some way, the states accessible to the system virtually form a definite domain, a multiplicity of states, something like a set, or a field, or a space. In order to be more specific, it is necessary to add to this domain some mathematical structure: topological space, vector field, lattice, phase space, *DM*, etc., depending on the required specification. Furthermore, the succession of the actual states in this domain is recorded by means of a parameter of evolution, a c-number, or, as is often the case in physics, a μ-number. There too, more specification may be required: one-dimensionality, continuity or discreteness, the kind of megethos, etc.

In our case, the specification consists in the fact that motion is described by *differential equations*, which in turn implies a deterministic (in the Einsteinian sense of the word) interpretation of evolution, and which means moreover that motion is described in terms of the (very rich) structure of a *DM*. (3) This reading of the concept of Kinetics completely agrees with Stueckelberg's preference to speak of thermokinetics, rather than of thermodynamics. (4)

While physicists are usually concerned with limited but rather well delimited problems (the negative result of the Michelson-Morley experiment, the black-body radiation, the spectrum of the hydrogen atom, etc.), the problem here is less sharply defined, but more fundamental. This explains why the theory related to the problem is relatively unusual. This resort to an unusual theory, however, can be supported by referring to Einstein's distinction (relatively early, in 1919) between two kinds of theories. (5) There are *theories of construction*, built up from "a relatively simple formal scheme", such as the kinetic theory of gases starting from the hypothesis of molecular motion; and there are *theories of principle*, based on "empirically discovered elements, general characteristics of natural processes, principles that give rise to mathematically formulated criteria". Examples of the latter are the science of thermodynamics (starting from the universally experienced fact that perpetual motion is impossible) and the theory of relativity, with its two postulates. *Theories of structure*, however, such as the theory of Kinetics, presented here, demand a third, intermediary class of theory: indeed, they are theories both of construction, since the structure constitutes really a "simple formal scheme" and of principle, since the choice of the relevant structure is dictated by an empirical principle. For Kinetics, indeed, the structure is that of a *DM*, and the empirical principle is the very observation of change as motion. This empirical principle is certainly one of those most deeply rooted in our understanding. It was already very well expressed by Aristoteles' saying that "Time is the number (or the measure) of motion". The difference between this ancient expression and its modern counterpart lies in the fact that the latter contains a powerful language of structures, of which *DM*'s structure is among the strongest. (6)

3. Epistemological approach

One of the goals of (6) was precisely to give a detailed exposition of new epistemological concepts, born from the present research in (quantum) Kinetics, and conversely stimulating the development of further research in that area: concepts such as the language of structures versus the language of forms, the slow emergence of structuring reason, (7) the exploration of the field of possibles (see Dirac's favorite game: play with formulas and see what comes out of it (8)), the historical distortion with regard to the structural reconstruction; the dialectic of the seen and the unseen in the seen, etc. Although it is impossible to do more here than list these topics, it is necessary to insist on the opposition made in (6) of language and discourse. In the analysis of physical theories, the coupling of language/discourse is more useful than that of syntax/semantics cherished by linguists. Thus physical theory is conceived as a discourse fixed in a (mathematical) language, whose constraints a priori determine what can and what cannot be said by that language. This opposition between language and discourse can be reinforced and made even more meaningful by the consideration of what could be called the *double coding*. Indeed, from the empirical data given by observation and experiment to the propositions of a theory, there is, not as Einstein claimed, a single, but a double jump. (9) First, we code the data into concepts of a discourse (e.g. into physical entities), and then we recode these concepts into the (mathematical) entities of some mathematical structure. For example, the standard opposition between absolute time and relative time concerns the conceptual level of discourse, while the opposition between parameter time and coordinate time is relevant to its mathematical coding. Similarly, the duality of the wave-particle pertains to the first coding, while the formal asymmetry between total derivative and partial derivative as first observed by Einstein (10) expresses this duality through the second coding. Within the present point of view of the structural reinterpretation of Kinetics, quantization and relativization operate only after this double coding, and thus they represent structural characteristics of a DM, while the physical megethos calibration of the geometrical unit becomes meaningful only on the level of conceptual discourse. All in all, this situation is very well known to mathematicians and physicists (11): indeed, the classical question of formalism and its interpretation is the problem of decoding in reverse. Furthermore, the strategy of the geometrical process of quantization here advocated closely resembles the fundamental strategy of B. Kostant and J. M. Souriau (12), except for the fact that their discourse does not sufficiently respect the demands of elementarity and intuitiveness.

4. Intuition *(Anschaulichkeit)* of the duality tangent *vs* cotangent in a DM

After first coding and accepting Newton's First Law, it is clear that the vector momentum $\vec{p}$ is tangent to the trajectory and the position $\vec{x}$ is fixed by its coordinates. By second coding, this time on the full structure of a DM, $\vec{p}$ is coded into the tangent vector operating on the parametrized curve representing the trajectory, and so $\vec{p} \in T(M)$. Similarly, $\vec{x}$ is coded into the numerical functions of the coordinates, and becomes an element of $T^*(M)$.

Now in a frame of natural coordinates, the basis vectors of $T(M)$ are given by $\frac{\partial}{\partial x^i}$, and the basis covectors of $T^*(M)$ are given by the differentials dx^i. The fundamental duality between tangent and cotangent, consequently, amounts to the following condition of "orthonormality":

$$\left\langle \frac{\partial}{\partial x^i}, dx^j \right\rangle = \delta_i^j, \text{ the Kronecker index.} \tag{9}$$

This, as has been recognized now for at least 10 years (13), is also the expression of the (algebraico-geometrical) quantization on any DM (cf. § 6). It is the most elementary and intuitive way to present the duality responsible for a quantum structure. At the other, most abstract pole, there is what is (improperly) called Quantum Logic. The structure of lattices encompasses various sorts of duality present in a large variety of mathematical structures: in particular in projective geometry, combinatorial topology, probability theory, mathematical logic, theory of functional spaces, etc. (14). Specifically, C. Piron has shown that the structure of a *CROC*-lattice provides an adequate foundation of Quantum Physics (15).

5. Intuition lost and regained (16)

At this point, the question why such simple interpretation of quantization was not seen before imposes itself. Part of the answer is purely historical. Up to the late forties, the structure of the DM's was not completed, ans was restricted mainly to mathematical research. The other important part of the answer, however, is relevant to the dialectic of the seen and the unseen in the seen, and its corollary: the historically grown distortion between the real course of discovery or invention (or even creation) and the development along a structure. (It is impossible to give a detailed account of those marvelous adventures: they will however be told in a forthcoming essay *A critical conceptual History of the Invention of Quantum Mechanics*, which aims at revealing some of the unseen in Jammer's famous book which bears almost the same title, except for the word "critical"! (8).) In a nutshell: When rejecting as inobservable the concept of trajectory in space and time of a quantum particle, Heisenberg by the same token gave up the (natural) concept of a tangent vector altogether. However, using differential equations of motion, he could not avoid having integral curves as solutions of these differential equations.

The impulse in that direction was provided by Bohr's momentous paper of 1918 on the correspondence principle, in which he made use of the Fourier transform (see 8 for this, as well as for the rest of this part). In the years 1922-1924, Kramers took over this Fourier transform and worked out his dispersion relation. In the famous year 1925, Heisenberg realized that a coordinate x given by a Fourier expansion could no longer pretend to represent an observable position, and thus, it was reasonable to give up classical kinematics while retaining the observable dynamical entities: whence his rejection of the space-time frame as inappropriate for the description of quantum motion, and the concomitant claimed abandonment of all appeal to intuition. In a sense, Heisenberg was perfectly right. To him, as well as to his German fellow-scientists, time and space were still the philosophical constructs of Kant: a priori forms of pure intuition. But now, the rejection of the Kantian space and time and of the forms of intuition (correct as it may be from the point of view of the present analysis, i.e. on the level of concepts), obviously is of no consequence on the level of mathematical entities: indeed, even Heisenberg continued to work in terms of parameter, coordinate and the like.

Between 1925 and 1927, Heisenberg's momentous breakthrough gave way to a variety of new formalisms (soon recognized as being more or less equivalent): the first matrix mechanics of Heisenberg, Born and Jordan shared the stage with Dirac's q-numbers theory, the Born-Wiener theory of operators (they found the commutator $\left[\frac{d}{dt}, t\right] = \mathbf{1}$, [now advocated by Prigogine as giving a new complementarity, although he fails to recognize that the Liouville operator L is nothing but id/dt operating on the phase-space], but incomprehensibly missed the corresponding commutator $\left[\frac{\partial}{\partial x}, x\right] = \mathbf{1}$), and Schrödinger's wave mechanics. Then, while Born gave his probabilistic interpretation of $\psi^*\psi$, Jordan and Dirac formulated their respective transformation theories which offered the first acceptable physical interpretation of all these formalisms. In pursuit of ever more abstraction (Dirac had already invented his famous δ distribution!), J. von Neumann, in 1927, objecting to the "unlösbare mathematische Schwierigkeiten" of this δ function, provided the by now standard theory of observables as operators acting on the vectors of an abstract Hilbert space, while Weyl, with his isomorphism, taking the opposite course, considered the observables as simple functions on phase-space (17). Finally, in 1932, the same von Neumann, in association, with G. Birkhoff, launched the most abstract theory of lattices of (physical) propositions.

All this was a far cry from the so simple (and, why not, so naive) intuition of the duality of $T(M)$ and $T^*(M)$ on a DM! For example, in order to recover the Schrödinger picture of the linear momentum $\hat{P}^i$ as $-ih\frac{\partial}{\partial x^i}$, Dirac had to proceed through the derivative of his δ distribution. What a detour! But even Heisenberg, the initiator of this race toward abstraction, considering the fact that electrons leave traces of a trajectory in a Wilson chamber, was forced to finally (in 1927) return to some sort of intuition in his famous uncertainty relations (abbreviated as HUR). To this, one should add that in the present gauge theories of elementary particles, space and time play a part more valid than ever.

II. QUANTIZATION: MATHEMATICAL LANGUAGE

6. The fundamental duality on a *DM*

Structurally, a *DM M* (of dimension n) is an atlas of charts, a collection of differential mappings of overlapping local pieces of M into Euclidean spaces of the same dimension n, with differential maps to connect the relevant overlapping charts. Anyway the reader is supposed more or less familiar with such a structure. Of special interest here is *the fundamental duality* existing on M between its *parametrized curves* and its *numerical functions*.

Consider the following table:

C: parametrized curve	numerical mapping f (e.g. coordinate x^i)
$C:\ \mathbf{R} \to M$	$f:\ M \to \mathbf{R}$
$t \mapsto x$	$x \mapsto f(x)$

By convenient equivalence relations one obtains equivalence classes:

v: tangent vector	df: differential
$C \in v$	$f \in df$

In a frame of natural coordinates (on a chart), with parameter t

$v = \dfrac{d}{dt} = v^i \dfrac{\partial}{\partial x^i}$	$df = \dfrac{\partial f}{\partial x^i}\, dx^i \qquad i \in \{1, 2, \ldots, n\}$
$\dfrac{\partial}{\partial x^i}$: basis vectors of $T(M)$	dx^i: basis covectors of $T^*(M)$

The combination of both yields the usual derivative

$$f \circ C:\ \mathbf{R} \to \mathbf{R}$$
$$t \mapsto f(x(t))$$

Here, one sees a flaw of differential geometry: its multiplicity of formulas for the same relation:

$$\frac{d}{dt} f(x(t))\,\Big|_{t=0} = C(f) = v(f) = \langle C, f \rangle = \langle v, f \rangle = \langle v, df \rangle$$

The brackets indicate the contraction of the vector and the covector, which in the last expression becomes a real *product* (bilinearity). For the basis vectors of $T(M)$ and covectors of $T^*(M)$ one obtains the relation [9] (cf. § 4):

[9] $$\left\langle \frac{\partial}{\partial x^i}, dx^j \right\rangle = \delta_i^j \quad \text{the Kronecker index}$$

7. The Leibnizian Quantum Structure (18)

The bracket [9] is already a *product* expressing the duality between the tangent and cotangent bundles of M. The more standard formulation of this relation can be found in terms of a commutator between operators. This may be stated simply in what can adequately be called the Leibnizian Quantum Structure (or LQS).

Let us firstly remark that the Minkowski World Postulate allows us to always find a frame in which the system in question is at rest. This means that all change can always be made uniquely timelike. Thus it is convenient to begin with the one-dimensional $DM\,\mathbf{R}$ of the parameter t (a c-number). Let $f\,(t)$ and $g\,(t)$ be two derivable functions of t, and $\frac{d}{dt}$ the derivation operator. It is very easy to transform the standard Leibniz' rule (derivation of the product of two functions) into a commutator of operators. Let

$$\frac{d}{dt}\,(f\cdot g) = \frac{df}{dt}\cdot g + f\cdot\frac{dg}{dt} \tag{10}$$

Here we have one differential operator, and two functions, symmetric in their role. But now, introduce a break of symmetry between f and g. Singularize g as a function expressing the state of the system, and f as an observable of the system, a characteristic of the system. $\frac{d}{dt}$ also represents such an observable. We now have two characteristics and one state function. It is then possible to conceive of f as an operator too, a multiplicative operator, or better, a differential operator of degree 0. Thus we obtain two operators acting on the same function. Taking care to place the latter always to the right of the operators, one easily obtains:

$$\left[\frac{d}{dt}, f\right] g = \frac{d}{dt}\,(f)\,g \tag{11}$$

or, dropping the arbitrary function g,

$$\left[\frac{d}{dt}, f\right] = \frac{d}{dt}\,(f) \text{ as operators.} \tag{12}$$

Substituting t for f, one gets

$$\left[\frac{d}{dt}, t\right] = \mathbf{1} \text{ (the Born-Wiener relation!)} \tag{13}$$

Thus we dispose of three equivalent formulas:

$$[14] \qquad \frac{d}{dt}(t) = 1 ; \quad \left\langle \frac{d}{dt}, dt \right\rangle = \mathbf{1} = \left[\frac{d}{dt}, t\right]$$

with $\frac{d}{dt} \in T(\mathbf{R})$ and $dt \in T^*(\mathbf{R})$ (19).

Now, it is easy to unfold the manifold M from the parameter manifold. If the parametrized curves are chosen so as to be the different axes, then the coordinates x^i function themselves as a parameter on their respective axis.
Thus one obtains

$$[15] \qquad \left[\frac{\partial}{\partial x^i}, x^j\right] = \delta_i^j$$

where x^i is still a c-number.

The proper noun LQS finds its origin in an application of the methodology of research programs of the late I. Lakatos. "Leibnizian program" does not mean the program of the historical Leibniz; it refers to the structural potentialities of this program, some of which were not even conceived by Leibniz. But by 1677, Leibniz had almost everything necessary for the development of the LQS. He already had identified and coined the term function. He knew of operators, using of d for *differentia* and S (transformed later into the integral sign $\int$ by Bernouilli) for *summa.* He also had correctly obtained the Leibniz' rule. Still missing, however, was the recognition of the function as a multiplicative operator and, last but not least, the real incentive of a puzzling physical problem (as would be the puzzle of the line spectrum of the hydrogen atom).

Everyone should by now be convinced that the Quantum commutator proceeds directly from the *DM* structure. This, together with the fact that the Quantum commutator is responsible for the existence of the *HUR*, throws a new light on the status of indeterminism of Quantum Mechanics. Differential equations of motion induce (Einsteinian) determinism; but they by themselves also produce the duality in which the *HUR* are rooted, and consequently, Quantum indeterminism as well. This is a real problem for epistemologists which, however, I shall not pursue within this paper.

III. QUANTIZATION: PHYSICAL DISCOURSE

8. Intervention of the megethos

It was already pointed out that Aristotle defined time in his *Physics IV* as follows: "Time is the number (or the measure) of motion." A present day interpretation of this sentence should insist that Aristotle was indeed right to assert that time is not a pure number, a c-number, but the number of something, i.e. a μ-number (20). Up to here, the symbols x, t were only c-numbers; whenever this was not the case, it was clearly indicated. But now, a μ-number is given as both a c-number (its measure) and a μ-unit. For instance, for time t, one should read

16]
$$t = \underline{t}\,[t] \qquad \begin{array}{l} t:\ \mu\text{-number} \\ \underline{t}:\ c\text{-number} \\ [t] = \text{unit of } \mu\,(t) \end{array} \tag{21}$$

Using once more Minkowski's World Postulate, we can restrict the study of change to the study of something purely timelike. In other words, it will suffice to study first what happens to the parameter, i.e. on the one dimensional $DM\,\mathbf{R}$, and then to unfold the results onto the manifold M by the tangent and the cotangent mappings:

[17]
$$C : t \mapsto x \quad \begin{cases} dc: & \dfrac{d}{dt} \mapsto v^i \dfrac{\partial}{\partial x^i} \\ c^*: & p_i dx^i \mapsto E\,dt \end{cases}$$

In so doing, one obtains the relation: (for the sake of simplicity, here $M = \mathbf{R}$)

[17 *bis*]
$$\left\langle \frac{d}{dt}, dt \right\rangle = \left\langle v \frac{\partial}{\partial x}, \frac{1}{v}\,dx \right\rangle = \mathbf{1}$$

where t, x, and v are now megethe. This relation immediately discloses the effect of duality on the megethos, viz. an inversion relative to the megethos, and for this reason, it will be noted here a μ-inversion. Thus, comparatively, we have:

[18] c-numbers: inversion (trivial) $\quad l \cdot \dfrac{1}{l} = 1$

μ-numbers: μ-inversion $\quad L \cdot \dfrac{\mathbf{1}_\mu}{L} = \mathbf{1}_\mu$

For example, the unit of the megethos action $\mathbf{1}_\mu$ is the Planck Quantum of action, written h:

[20]
$$\mathbf{1}_{\text{action}} = h \cong 6 \cdot 10^{-27} \quad \text{C.G.S.}$$

If $\mathbf{1}_{\text{action}}$ does not have the value 1 as it should, this is only because, historically, there were more cases of determining "anthropomorphic" megethe and their units.

Now, the megethos interferes with the expressions of the basis vectors and covectors. These must remain c-numbers. Thus, the megethos of the coordinate x^i must be neutralized by a factor λ of the same megethos and the fundamental mathematical expression [9] must now be read:

[21]
$$\left\langle \frac{\partial}{\partial\left(\frac{x^i}{\lambda}\right)}, d\left(\frac{x^j}{\lambda}\right) \right\rangle = \delta_i^j$$

in order to be adaptable to a physical discourse, while, of course, remaining mathematical.

The following table illustrates how to treat μ-vectors intrinsically. For instance, for the linear momentum $\vec{p}$, $\mu(\vec{p})$ is "momentum" and must remain so, through basis and components:

1) Euclidean space $\vec{p} = p^i \vec{e}_i$ all μ is in p^i

2) DM, x^i a c-number $\vec{p} = p^i \dfrac{\partial}{\partial x^i}$ idem

3) DM, $\mu(x^i) =$ "length" $\vec{p} = p^i \lambda \dfrac{\partial}{\partial x^i}$ $\mu(p^i \lambda) =$ "action"

4) idem, prepared for Quantum Mechanics $\vec{p} = \dfrac{v^i}{v}(p\lambda)\dfrac{\partial}{\partial x^i}$ idem.

This last expression is of the highest interest. There still is no quantum: it only appears with de Broglie's relation $p\lambda = h$. But that $p\lambda$ is a global constant in any chart of the DM, remains to be shown. Note the *superposition of the operators* $\left(\text{clearly } \sum_i \left(\dfrac{v^i}{v}\right)^2 = 1\right)$.

5) Quantum Mechanics $\vec{p} = \dfrac{v^i}{v}\left(-ih\dfrac{\partial}{\partial x^i}\right) = \dfrac{v^i}{v}\hat{P}^i$

(imaginary i put for hermiticity)

It is enough now that all the components of $\vec{p}$ be 0, except for the kth which is equal to one. Then

$$\vec{p} = \hat{P}^k \qquad [22]$$

This process of reduction to only one non-zero component is here called *active projection*. The measurement of $\vec{p}$ requires, in order to produce a physical interaction, that the apparatus be lined up with the direction of $\vec{p}$. In contrast with this, the geometrical projection of the momentum onto the axis of a predetermined frame constitutes a mere *passive projection* which, physically speaking, is a mere shadow. The situation is very reminiscent of this old pseudo-paradox: Let a light ray proceed at an angle of 45° relatively to two orthogonal axes x and y. We know that c is the speed of light (in vacuo) along the ray. But what is its component along axis x? The answer is $c/\sqrt{2}$. This is the value obtained by passive geometrical projection. But this value is physically invalid. In order to know the real value of the light ray along x, one must rotate the frame until it coincides with the direction of the ray, or one must produce another ray along x. In both cases, the answer is the same: c.

Now, in both cases, one did something physical before measuring, i.e. one did perform an active projection.

In conclusion, there are *three facts* which together are required to obtain the 6) *Quantum linear momentum* (Schrödinger picture)

$$\hat{P}^i = -ih\frac{\partial}{\partial x^i}:$$

the *duality* $T(M)/T^*(M)$, the *megethos* and the *active projection.*

9. DE BROGLIE RELATION $p\lambda = h$ AS A GLOBAL CONSTANT (relatively to the manifold space-time)

1) Again, one begins on the one-dimensional parameter. Here we assume the existence of an unique megethos, time, but rather, for the general case of Kinetics, mass.

Assuming moreover there exists a fundamental mass m_0, one obtains by μ-duality ($\mathbf{1}_{\text{action}} = h$)

[23] $$m_0 \cdot \frac{h}{m_0} = h \quad \text{then} \quad \frac{h}{m_0 c^2} = \tau_0 \quad \text{or} \quad E_0\tau_0 = h \quad \text{(Einstein)}$$

Let it be noticed that, in Mechanics, the μ-inversion alters the types of the dual megethe: energy versus time. Such is not always the case. In Kinematics (which is a Kinetics as well, albeit of a particular type), as $\mathbf{1}_\mu = c^2$, thus both megethe are velocities. For $v \cdot \frac{c^2}{v} = c^2$ makes $V_\varphi \equiv \frac{c^2}{v}$ to be another velocity: the phase velocity. There is another anomalous characteristic: the commutator $[v, V_\varphi] = -ic^2$ admits $\frac{c^2}{2}$ for its *HUR* least bound, which is not at all microscopic. Are these facts evidence for a kind of degeneration of Kinematics compared to the rest of Kinetics (due to its no rest mass)?

Now take the commutator with t a c-number. Then $\left[E_0\frac{d}{dt}, \tau_0 t\right] = h\mathbf{1}$. In order to pass to μ-numbers for the operators as well, it is convenient to put $\tau = t\tau_0$ (confer [16]). One obtains

[24] $$\left[h\frac{d}{d\tau}, \tau\right] = h\mathbf{1}$$

Up to the hermiticity (factor imaginary i), this is the standard Quantum Commutator.

2) One proceeds now from the parameter to an axis x. Here, one can use a light ray along x. As $x = \tau t$, one can say that t parametrizes x, and vice versa. Consequently, one can adapt the result on the parameter proper time $E_0\tau_0 = h$ both to the axes t and x, and hence

[25] $$ET = p\lambda = h$$

3) For a massive particle, the parameter is unfolded into space-time. By derivation of the relativistic energy (22)

$$d(p_\mu p^\mu) = 0, \text{ then } c^2\vec{p} \cdot d\vec{p} = EdE \text{ and hence}$$

[26] $$pV_\varphi = E$$

This is nothing else than [25] divided by a period T.

4) The extension to a configuration space follows immediately.

10. Duality on the phase-space

Up to here, the discussion was centered on the linear momentum as a vector. In phase space mechanics, it is well known that $\overleftarrow{p}$ is covector. What happens to the duality in such a space?

A detailed answer is kept for another paper. It can be said, however, that the symplectic structure of M^{2n} has probably been overestimated. The 1-forms (both the canonical Darboux form $p_i dq^i$ and the other Lagrangian ones (23)) seem to be of more interest within the present approach: they are elements of $T^* (M^{2n})$, as the Liouville operator id/dt is an element of $T(M^{2n})$. But the application of the duality on this pattern has not yet been worked out.

11. Phenomenology of the various Quantum Kinetics

We are free to choose the DM and/or the parameter and/or the calibration of the unit of the megethos. Here is a list of the most usual realizations:

Type	Quantum of megethos $\mathbf{1}_{\mu}$	*DM*	Commutator	*HUR* bound	π-relation
Quantum Mechanics	of action of Planck h	space-time or configuration space or abstract space	$\left[-ih\frac{\partial}{\partial x}, x\right] = -ih\mathbf{1}$	$\frac{h}{2}$	$p\lambda = h$ de Broglie
(Gibbs) Statistical Mechanics	of entropy of Boltzmann k	thermodynamic space	$\left[-k\frac{\partial}{\partial \vartheta}, \vartheta\right] = -k\mathbf{1}$	$\frac{k}{2}$ *	$E\vartheta = -k$ Stueckelberg
Quantum Electro-magnetism	of e.m. action of Maxwell $\frac{e^2}{c} = \alpha h$	space-time	$\left[\hat{J}, Q\right] = \frac{-ieh}{mc}\mathbf{1}$	μ_B Bohr magneton	$J\lambda = 2\mu_B$ Bohr
Quantum Kinematics	kinematical of Einstein c^2	space-time	$[\hat{v}, V_{\varphi}] = -ic^2\mathbf{1}$	$\frac{c^2}{2}$ **	$vV_{\varphi} = c^2$ Brillouin

* About the inequality sign, see (24).
** This is not microscopic!

From this table, three theses appear.

IV. CONCLUSION

I. Due to the variety of the megethe, the Quantum Structure is not restricted to the quantum h of the megethos action.

II. Due to the value of the bound in Kinematics, the Quantum Structure is not restricted to microscopicity.

III. Due to the interdependence of the natural constants (e.g. E is equal to $h\nu$ or kT or $ce^2r^{-1} = \alpha ht^{-1}$ or c^2m, or ...), in Quantum Kinetics there exists an unique megethos.

For the sake of completeness, one should also recall the role of the *DM* structure in the Quantum indeterminism.

IV. The Quantum indeterminism is rooted in the determinism of Kinetics itself.

V. APPENDIX

This could be our conclusion. Some topics presented in (18): Anti-Leibnizian Quantum Structure, cotangent vectors such as $\overleftarrow{p}$ will not be treated here. (29) One final comment, however, with regard to an extraordinary case of the unseen in the seen should be added. In order to restore continuity against quantum jumps, Schrödinger proposed his famous relation $S = K \log \psi$; but he did not notice that by the same token, he introduced a probabilistic theory analogous to Boltzmann's $S = -k \log$ W. This explains the isomorphism between theories of these two authors; that isomorphism has very good reasons. Indeed, action and entropy are extensive entities, and the independence of events and the separation of the variables calls for a multiplication of factors. But the only isomorphism between resp. an additive and a multiplicative law of composition is the exponential mapping! Without giving any more justification (here!), we can only list the main stages of the development of the argument.

Boltzmann	Schrödinger
canonical ensemble	wave mechanics
entropy $S = -K \log W$	action $S = -ih \log \psi$
(25) $\dfrac{\partial S}{\partial \vartheta} = -E$	$\dfrac{\partial S}{\partial t} = -E$
$= -k \dfrac{\partial}{\partial \vartheta} \log W$	$= -ih \dfrac{\partial}{\partial t} \log \psi$

whence eigenvalues problem

$k \dfrac{\partial}{\partial \vartheta} W = EW$	$ih \dfrac{\partial}{\partial t} \psi = E\psi$

Sommation over the states

partition function Z (Zustandssumme)	different possible realizations Ψ
$Z = \sum_i e^{\frac{\vartheta E_i}{k}}$	$\Psi = \int_{-\infty}^{+\infty} e^{-\frac{iHt}{h}} dt = \delta(t)$
	or $\Psi = \sum_{-\infty}^{+\infty} e^{-\frac{iHnt}{h}}$ Fourier series
	or $\Psi = \sum_{-\infty}^{+\infty} c_n e^{-\frac{iE_n t}{h}}$
	completeness on a functional space

mean value of E

$$- k \frac{\partial}{\partial \vartheta} \log Z = < E >$$

$$< E > \equiv ih \frac{\partial}{\partial t} \log \Psi \cdot \frac{\overline{\Psi}}{\overline{\Psi}}$$

$$= \sum_i \frac{\overline{\Psi} E_i \Psi}{\overline{\Psi} \Psi}$$

(usually, axiom of the mean value of an observable)

fluctuations of E

$$k^2 \frac{\partial^2}{\partial \vartheta^2} \log Z = \Delta E^2$$

$$- h^2 \frac{d^2}{dt^2} \log \Psi \equiv \Delta E^2$$

(possibility to interpret mass as given by fluctuations!)

then quadratic operator of fluctuations

hyperbolic

$$\left(k^2 \frac{\partial^2}{\partial \vartheta^2} - \varepsilon^2\right) Z = 0$$

$$\left(k \frac{\partial}{\partial \vartheta} - \varepsilon\right)\left(k \frac{\partial}{\partial \vartheta} + \varepsilon\right) Z = 0$$

with

$$\varepsilon \vartheta < 0$$

by Stueckelberg's 2nd b) principle of equilibrium.

So: reversal of ϑ entails reversal of ε

Therefore *irreversible evolution*

Irreversible evolution is given by a factor $e^{\frac{E\vartheta}{k}}$, with $\vartheta < 0$ if $E > 0$. This is in accordance with *Stueckelberg's arrow of time*, given by a factor $e^{-\alpha t}$ with $\alpha > 0$ whose sign is given by the two parts of the 2nd principle: 2a) evolution and 2b) equilibrium.

elliptic

$$\left(h^2 \frac{d^2}{dt^2} + \varepsilon^2\right) \Psi \overline{\Psi} = 0$$

$$\left(ih \frac{d}{dt} - \varepsilon\right) \Psi \overline{\Psi} \left(-ih \frac{d}{dt} - \varepsilon\right) = 0$$

So: $\left(-ih \frac{d}{dt} - \varepsilon\right) \overline{\Psi}$ is the time reversal of $\left(ih \frac{d}{dt} - \varepsilon\right) \overline{\Psi}$ (26)

Therefore *mechanical reversibility*

Reversible time must appear in a factor $e^{\frac{-iHt}{h}}$. (Cf. Aristoteles: time is measured by periodical phenomena.)

Horizontal passage:

W probability	ψ amplitude of probability (Pauli)

$$W = \bar{\psi}\,\psi \text{ (Born)} \qquad (30)$$

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NOTES AND REFERENCES

(1) This is a sophisticated version of Lakatos' rational reconstruction avoiding its bad treatment of the real course of history.

(2) In the same journal, see the companion paper of this one about the structural reinterpretation of relativization. The overlapping between the two is kept to as little as possible.

(3) This structure of DM is by no means necessary for the description of motion. One can even drop the continuity both of the parameter and of the domain. For example, in Quantum Mechanics, it is not necessary that the time evolution operator be the derivative d/dt. Clearly, one has the commutator $[d/dt, t] = \mathbf{1}$. But what is relevant in d/dt for the quantum structure is that it is a derivation operator. One can conceive of a *step inverse derivative operator* $\widehat{\frac{1}{t}}$ defined by

$$\widehat{\frac{1}{t}} \cdot t^n = \left(\widehat{\frac{1}{t}} \cdot t\right) \cdot t^{n-1} + t\left(\widehat{\frac{1}{t}} \cdot t\right) t^{n-2} + \ldots + t^{n-1}\left(\widehat{\frac{1}{t}} \cdot t\right) = nt^{n-1}.$$

For eigenfunctions proportional to t^n, there is

$$\left[\widehat{\frac{1}{t}}, t\right] t^n = \mathbf{1}\, t^n .$$

(4) E. C. G. Stueckelberg and P. B. Scheurer, *Thermocinétique phénoménologique galiléenne*, Birkhäuser, Basel, 1974.

(5) A. Einstein, *Out of my later years*, Greenwood Press, Westport, CT, 1970 [2].

(6) P. B. Scheurer, *Révolutions de la science et permanence du réel*, P.U.F., Paris, 1979. Spanish translation, 1982.

(7) An essay *L'émergence de la Raison structurante* is under way.

(8) M. Jammer, *The Conceptual Development of Quantum Mechanics*, McGraw-Hill, New York, 1966, p. 300. For the sake of brevity, all quoted pioneering works in this field are referred to through Jammer's book.

(9) G. Holton, "Constructing a Theory: Einstein's Model", *The American Scholar*, *48* nr. 3, 1979, pp. 309-340.

(10) A. Einstein, „Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt", *Annal. d. Phys.* (4) *17* (1905) p. 132 ff. See also P. H. Byrne, "The Origins of Einstein's Use of Formal Asymmetries", *Annals of Science*, *38* (1981), pp. 191-206, esp. p. 201.

(11) For example, see Th. VOGEL, *Pour une théorie mécaniste renouvelée*, Gauthier-Villars, 1973, and D. Hilbert (8, p. 310).

(12) J.-M. SOURIAU, *Structures des systèmes dynamiques*, Dunod, Paris, 1970.

(13) P. B. SCHEURER, Comptes rendus séances SPHN Genève, NS *7* (1972), 9-23 and 89-96; and NS *8* (1973), 32-37. See also (6).
P. B. SCHEURER, "Cohomology of the action differential forms", in *Lecture Notes in Physics No. 50*, Springer, Berlin-New York, 1976, p. 609-613.

(14) A. LAUTMAN, « Symétrie et dissymétrie en mathématique et en physique » (1946!) in *Essai sur l'unité des mathématiques*, U.G.E., 10/18, Paris, 1977.

(15) See the relevant works of J. VON NEUMANN, G. W. MACKEY, J. M. JAUCH, all quoted in C. PIRON, *Foundations of Quantum Physics*, Benjamin, Reading, Mass., 1976.

(16) Arthur I. MILLER, "Visualization lost and regained: the genesis of the Quantum theory in the period 1913-27", in *On Aesthetics in Science*, ed. J. Wechsler, MIT Press, Cambridge, Mass., 1978.

(17) For the Weyl isomorphism and the Wigner function, see: S. R. DE GROOT and L. G. SUTTORP, *Foundations of Electrodynamics*, North-Holland Pub. Co., Amsterdam, 1972, part D. See also: J.-P. AMIET and P. HUGUENIN, *Mécaniques classique et quantique dans l'espace de phase*, Univ. Neuchâtel, 1981.

(18) P. B. SCHEURER, "Leibnizian Quantum Structure, Irreversible Dynamics, Quantum Kinematics, and all that", in *Arch. Sc. Genève 34*, No. 3 (1981), p. 383-388. The present text gives only a survey of this construction and then supplies some semantics to this paper too restricted to syntax.

(19) In one dimension, all distinctions are blurred:

$$\mathbf{R} \approx T(\mathbf{R}) \approx T^*(\mathbf{R})$$

This is one of the reasons for the delay in the perception of the duality between $T(M)$ and $T^*(M)$.

(20) For this reason, Aristotle places physics higher than mathematics, and nearer to the study of Being qua Being. In the same manner, in the present view, physics is more complete in its description of reality than mathematics.

(21) Cf. (4) Appendix A.1. We speak there of t as the abstract entity. It seems better instead to speak of intrinsicness.

(22) This brings to mind Pauli's method to obtain the non-relativistic wave equation. See *Pauli Lectures on Physics. Vol. 5. Wave Mechanics*, ed. Ch. P. Enz, MIT Press, Cambridge, Mass., 1973, p. 1-4.

(23) See V. ARNOLD, *Méthodes Mathématiques de la Mécanique classique*, MIR, Moscow 1976, p. 456 and ff.

(24) This sign is not reversed, as I have erroneously written in (18). I thank Prof. Ch. P. Enz for pointing out this error to me.

(25) Stueckelberg's natural temperature $\vartheta \equiv -\frac{1}{T}$ with $m\vartheta < 0$ plays here a fundamental heuristic role. Otherwise, how can $-kT^2 \frac{\partial}{\partial T} \log W$ be thought to compare with $ih \frac{\partial}{\partial t} \log \psi$?

(26) PAULI, in (22), betrays in a short sentence an extraordinary insight (p. 4): "The imaginary coefficient assures that there is no special direction in time; [the non-relativistic wave equation of ψ'] is invariant, under the transformation $t \to -t$, [then] $\psi' \to \bar{\psi}'$, whereby $\bar{\psi}' \psi'$ remains unchanged."

Notes added on proofs:

(27) Thus Quantum Kinetics appears as a General Quantum Theory.

(28) The constant h contains p by de Broglie's relation, hence the emergence of a quadratic form!

(29) Nevertheless, here are the covariant expressions corresponding to the contravariant ones of § 8: 1) $\overleftarrow{p} = p_i \overleftarrow{e}^i$; 2) $\overleftarrow{p} = p_i\, dx^i$; 3) $\overleftarrow{p} = \frac{p^i}{\lambda}\, dx^i$; 4) $\overleftarrow{p} = \left(\frac{v_i}{v}\right) \frac{p}{\lambda}\, dx^i$; 5) $\overleftarrow{p} = \left(\frac{v_i}{v}\right) \frac{p^2}{h}\, dx^i$

$$= \left(\frac{v^i}{v}\right) \frac{h}{\lambda^2}\, dx^i \text{ ; and 6) } \hat{P}_i = i \frac{h}{\lambda^2}\, dx^i \text{ (quark potential!).}$$

(30) This correspondence plays more fully if the real action S is allowed to become a complex one: $S = S_1 + iS_2$, with S_1 mechanical action and S_2 entropy. This would lead to considering a complex time too: $t = t_1 + it_2$, with t_1 reversible mechanical time and t_2 irreversible evolutive time. This study will be presented elsewhere.