

Possible consequences of a fluid dynamic origin for columnar joints in basalts

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POSSIBLE CONSEQUENCES OF A FLUID DYNAMIC ORIGIN FOR COLUMNAR JOINTS IN BASALTS

BY

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ABSTRACT

The merits and deficiencies of the thermal contraction and the fluid finger hypotheses for the formation of columnar joints in basalts are examined. Although thermal stresses are recognized as the necessary initiating mechanism, they are probably not sufficient, by themselves, in controlling the growth of these fractures into elongated patterns as observed. Double diffusive fingering provides a plausible mechanism to produce such a pattern of pre-existing weakness that fractures can propagate preferentially to form columnar joints. If, indeed, fluid fingers play a role in the formation of columnar joints, many morphological, structural and chemical consequences can be derived. These include the plan-form geometries of the columns, their width to height ratios, the rate of crystallization (hence, the rate of cooling) of the magma body, and the compositional variations across and along a column. Many of these predictions can serve as tests to verify the validity of the fluid fingering hypothesis. However, data available at the present are inadequate for such an analysis. More appropriately chosen samples together with improved chemical analyses are necessary to gain a more definitive conclusion.

RÉSUMÉ

Les avantages et les inconvénients des hypothèses de contraction thermique et de digitation fluide dans la formation des joints prismatiques dans les laves sont examinés. Bien que les tensions thermiques soient reconnues comme le mécanisme initial nécessaire, elles sont probablement insuffisantes par elles-mêmes pour contrôler la croissance longitudinale de ces fractures telle qu'elle est observée. La double digitation diffusive offre un mécanisme plausible pour la production d'un tel système de faiblesse préexistant que les fractures peuvent propager préférentiellement en formant les joints prismatiques. Si les digitations fluides jouent en fait un rôle dans la formation des joints prismatiques de nombreuses conséquences morphologiques, structurales et chimiques en résultent. Ces conséquences sont les formes des prismes en plan, leurs rapports hauteur-largeur, la vitesse de cristallisation (donc de refroidis-

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sement) du magma, et les variations de composition transversale et longitudinale dans un prisme. Plusieurs de ces prédictions peuvent servir de test pour vérifier la validité de l'hypothèse de la digitation fluide. Cependant, les données existantes ne sont pas encore suffisantes pour une telle analyse. Un échantillonnage plus sélectif combiné avec des analyses chimiques sont nécessaires pour atteindre une conclusion plus définitive.

INTRODUCTION

Columnar jointing is one of the most spectacular features in many lava flows and sills. It has received much attention from geologists for over a century. Present understanding of the general features of columnar jointing perhaps was summarized best by Spry (1962), which is mostly a qualitative account. It is generally accepted that columnar joints are formed by thermal contraction during cooling. Recent studies in this area concentrate on the analyses of microscopic evolution of jointing surfaces (e.g. Ryan and Sammis, 1978; DeGraff and Aydin, 1987; Aydin and DeGraff, 1988) assuming thermal stresses to be the driving mechanism. Although thermal contraction is undoubtedly a very important contributing factor for the generation of columnar joints, it is not clear whether this mechanism alone is responsible for the jointing formation. Kantha (1981) has pointed out some of the features that cannot be explained satisfactorily by the thermal contraction hypothesis. The well documented two- or three-tier structures within fractured lava flows that consist of colonnades and entablatures (Spry, 1962) are good examples. While the colonnades possess well developed columns, the entablatures are often found to have highly irregular and sometimes chaotic fracture patterns. The transition zone between these two structures is defined by a strikingly narrow discontinuity. Formation of such a narrow discontinuity within a uniform flow cannot be explained easily by thermal cracking.

Furthermore, thermal stresses within a cooling elastic layer are by no means tensional throughout the whole layer. Timoshenko and Goodier (1970) have demonstrated that only approximately the outer fifth of the layer is under tension. The interior of the cooling lava layer is in fact under compressive thermal stresses. Propagation of a tensional crack into a compressive regime in the context of columnar jointing formation has not been examined. It is not intuitively apparent how jointing surfaces can continue their propagation into the interior without changing directions. Experimental studies indicate that fractures produced by compressive loadings tend to follow a direction that is about 30° - 45° offset from the compressive stress direction. Therefore, columnar joints are expected to be deflected by a similar amount when they propagate into the compressional interior if, indeed, they are thermal cracks. An alternative is to assume that these jointing surfaces do not propagate

instantaneously into the interior of a lava flow. Instead, they pause during their inward migration. This assumption will also pose problem however. Once the initial cracks are formed, thermal regime surrounding the cracks will be altered greatly due to air and groundwater circulations. As a result, thermal regime within the lava flow can no longer be ensured to have isotherms parallel to the surface of the magma body which is necessary to produce tensional stresses in the horizontal direction. Therefore, subsequent jointing surfaces cannot be expected to continue uniformly in the same direction. Thus, formation of columnar joints within the interior of a lava flow becomes a difficult problem to explain for the thermal contraction hypothesis.

The polygonal plan-form geometry of columnar joints has also been well documented. Because of its similarity to the plan-form of many convective systems, columnar jointing has been speculated to have a fluid dynamic origin. The concept has been examined by Spry (1962). It is discarded largely because thermal convective cells usually have a horizontal dimension that is comparable to their vertical dimensions. Columnar joints obviously have a vertical length that is many times larger than their horizontal scales. Therefore, the possibility of a fluid dynamic origin for columnar joints was discarded. It is not until recently that Kantha (1981) reiterated the possible fluid origin of columnar joints based on a double diffusive fingering hypothesis. Double diffusion is a flow process involves a multi-component fluid. Within such a dynamic fluid system, many unusual fluid behaviors can occur because of the different diffusive characteristics of heat and composition (Huppert and Turner, 1981). These include fluid layering and fingering. Fingering represents the narrow elongated plumes rising within the fluid. It is hypothesized that within a cooling magma body, double diffusive fingering can also occur. When this dynamic structure is frozen into the igneous formation as the magma cooled through its solidus, it provides the structural framework for the subsequent development of columnar joints initiated by thermal cracking. Although the general concept has been discussed by Kantha (1981), many details associated with this hypothesis have not yet been examined. The purpose of this paper, therefore, is to examine in detail the relationship between double diffusive fingering and formation of columnar joints, to test the feasibility of this phenomenon as a geological process, and to evaluate the consequences of this hypothesis so that further studies can be planned to test its validity.

DOUBLE DIFFUSION IN A COOLING MAGMA BODY

Double diffusion is a fluid dynamic process consisting of two diffusive components. In terms of a cooling lava, they are the thermal energy and the composition of the lava. Temperature variations within a lava sheet are caused by cooling through

the surfaces both on top and at the bottom. Compositional variations, on the other hand, are mainly produced by crystal fractionation. Because residual liquids expelled during crystallization are found generally to be less dense than the unfractionated liquids (Stolper and Walker, 1980), crystallization will cause a stabilizing effect at the top of the cooling lava while a destabilizing configuration is produced at the bottom. However, within this cooling body, the top surface is thermally unstable because cold materials are on top, while at the bottom, it is thermally stable. Hence, the dynamic behavior within a cooling magma body is governed by the competing effects from thermal and compositional instabilities. A stability analysis of a double diffusive system incorporating the effect of crystallization has been carried out (Riahi and Hsui, 1986). The dynamic system is characterized by two parameters: R , the thermal Rayleigh number and R_s , the compositional Rayleigh number. They are defined as

$$R \equiv \frac{g\alpha\Delta Td^3}{\nu\kappa} \quad \text{and} \quad R_s \equiv \frac{g\beta\Delta Cd^3}{\nu\kappa} \quad (1)$$

where g is the gravitational acceleration; d is the thermal boundary layer thickness; ν is the kinematic viscosity of the lava; κ is the thermal diffusivity; $\alpha\Delta T$ is the multiplier of density change across d due to thermal expansion and $\beta\Delta C$ is the corresponding multiplier due to crystallization. Results of the stability analysis is shown in Figure 1. They are plotted in a traditional R - R_s space (Baines and Gill, 1969). When the system is thermally unstable ($R > 0$) and compositionally stable ($R_s > 0$), instability can occur above line CD. Laboratory studies (Chen and Turner, 1980) showed that transient convective layers can take place within this regime. This phenomenon was used to explain the layered intrusive formations found in many igneous provinces (McBirney and Noyes, 1979; Irvine, 1980). However, for this analysis, the region characterized by a stable thermal gradient (i.e. $R < 0$) but unstable compositional gradient ($R_s < 0$) is of interest. This region is represented by the lower left quadrant of the stability diagram. This diagram demonstrates that almost the entire quadrant is unstable dynamically. Stability occurs only to the right of line CE. Line OF is a 45° line which indicates the boundary of static stability where compositional instability is exactly balanced by thermal stability. Thus, the region bounded by lines GF and GE represents a region where the system is dynamically unstable but statically stable. Such a condition is most favorable for finger formation (Turner, 1974). Indeed, laboratory observations showed that fingering does take place under this condition (Chen and Turner, 1980).

If double diffusive fingers are indeed responsible for the formation of columnar joints, where will they occur within a cooling magma body? As a magma sheet is being emplaced at the surface or within the country rocks, it will be cooled from both above and below. It follows that the top of the magma sheet will possess an unstable thermal gradient whereas the bottom part possesses a stable gradient.

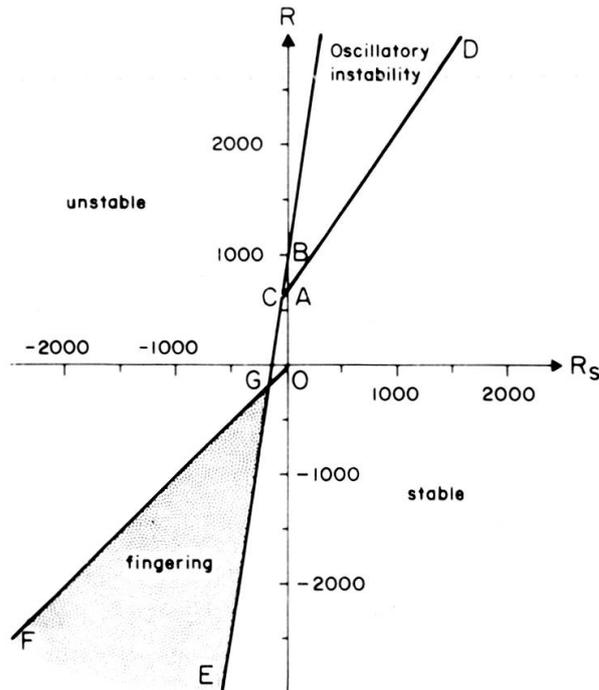


FIGURE 1.

Stability diagram for double diffusive convection with crystallization. Compositional Rayleigh number is plotted against thermal Rayleigh number.

Because both surfaces of the magma sheet are at temperatures below the liquidus of the magma, crystal fractionation will occur immediately adjacent to the surfaces. The result of crystallization is to produce less dense residual liquids. Thus, a stable and an unstable compositional gradient will be established near the top and the bottom surfaces respectively. An examination of the combined compositional and thermal gradients suggests that the bottom of the cooling magma sheet possesses the essential attributes for the production of double diffusive fingers. On the basis of the stability diagram and the definitions of thermal and compositional Rayleigh numbers, double diffusive fingering will take place when $\beta\Delta C < \alpha\Delta T$. This condition can be satisfied generally at the beginning of a crystallization process because the amount of residual liquids will be small resulting in a smaller change of bulk density. In order to cause a breakdown of fingering, a 30-60% of crystal fractionation of the bulk liquid is required if residual liquids are 10-5% less dense respectively than the original liquid. Such a high percentage of fractionation can be accomplished only when the magma body is buried at great depth where cooling is slow. Near the surface, a cooling magma body would become subsolidus before such a large amount of crystal fractionation takes place. Therefore, it is reasonable to expect the existence of double diffusive fingering within cooling magma bodies when they are sufficiently thick that crystal fractionation can occur before they are frozen into solids. It should be

emphasized that unlike classical Rayleigh-Bernard convection, fingering is a distinct fluid dynamic phenomenon. It has been observed in many double diffusive laboratory systems. Because lava flows and sills can be represented by such a system prior to solidification, it is quite plausible that fingering is a dominant process taking place within these systems. As a magma body cools below its solidus temperature, the fingering pattern will be frozen into the rock formation. As the subsolidus magma body continues to cool towards the ambient temperature, elastic thermal stresses start to accumulate at the surfaces. When the thermal stresses exceed the yield strength of the rock formation, failure will occur and tensional fractures will form near the surfaces. It is hypothesized that these fractures will propagate inward preferentially along the boundaries of the fingers to form columnar joints. The reason for these fractures to grow along the finger boundaries is that the composition there is less homogeneous when compared with the depleted residual liquids that form the center of the fingering structure. As a result, it represents an area of structural weakness where fractures would prefer to propagate.

HORIZONTAL AND VERTICAL SCALES

If double diffusive fingering, indeed, plays a role in the formation of columnar joints, it is of interest to examine the vertical and the horizontal scales of the fingers and to compare them with the observed dimensions of columnar joints. According to the double diffusion stability study, columnar jointing can be inferred to initiate from the bottom of a flow or a sill where stable thermal gradient exists. Once fingers are formed, their extent can occupy a substantial portion of a magma body. Theoretically, they can exist throughout the complete region below the lower boundary of the upper layer where an unstable thermal gradient exists. However, the exact vertical dimension of the fingers can not be determined without using nonlinear theories. Nevertheless, it is clear that they may occur within the complete unit of the cooling body. It has been documented (Williams and McBirney, 1979) that cross sections of columnar joints range from a few centimeters to 3 meters in "diameter" and up to 30 meters in length. Thus, horizontal scales of columnar joints cannot be larger than 1/10 of its length. This characteristic aspect ratio was used to argue against any convective origin of the fracture pattern (Spry, 1962). Indeed, classical Rayleigh-Bernard type of convective structures are insufficient to explain the observation. However, double diffusive fingering is able to generate the necessary elongated structures. Typical horizontal scales of fingering can be estimated according to the following relationship (Stern, 1960; Baines and Gill, 1969):

$$L \sim \pi \left| \frac{g\alpha\beta_T}{4\nu\kappa} \right|^{\frac{1}{4}} \quad (2)$$

where α is the coefficient of thermal expansion; β_T is the thermal gradient; g , v and κ are defined previously. For a given cooling body of 30 m thick, a temperature difference of 1000°C between the magma body and its surrounding country rocks, L is estimated to be about 0.5 m using typical values for other physical parameters (e.g. $g = 10 \text{ m/sec}^2$; $\alpha = 3 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$; $\kappa = 10^{-6} \text{ m}^2/\text{sec}$ and $v = 1 \text{ m}^2/\text{sec}$). This value is substantially smaller than the 1/10 upper limit established from observation. This length scale is somewhat sensitive to the viscosity of the cooling magma. Generally, the smaller the viscosity, the thinner will be the fingers. The value of $10^4 \text{ cm}^2/\text{sec}$ used for the above estimation is only a representative value for most molten magma at their liquidus temperature (Murase and McBirney, 1973). Even if the viscosity of the cooling magma is increased up to $10^3 \text{ m}^2/\text{sec}$, the width of the fingers will still remain within the 1/10 upper bound. Therefore, horizontal scales of double diffusive fingers are in good agreement with observations.

A comparison can also be made on the plan-form of the fingers and the observed columnar joints. The cross section of columnar joints is generally composed of polygons that vary from four sides to eight sides (Spry, 1962). Although hexagons have been suggested to be the preferred mode, MacDonald (1967) indicated that squares are most abundant. Field observations, therefore, do not show a single dominant plan-form geometry for the jointing structure. A theoretical study on the plan-form of double diffusive fingers has been carried out by Stern (1976) who found that no preferred geometry exists, and thus concluded that the plan-form of double diffusive fingers is determined by a stochastic process. This conclusion appears to be consistent with field observations discussed earlier. It should be pointed out, however, that laboratory observations indicate that squares are the preferred mode (Williams, 1975; Linden, 1978). It should be noted that generation of fingers in laboratories is a transient phenomenon. Transient fluid behaviors generally are dependent strongly on initial perturbation and boundary conditions. Because a rectangular tank with flat walls is used usually in laboratory experiments, the square plan-form of fingers as observed under laboratory conditions is perhaps simply a manifestation of the boundary conditions. Whether it is indeed a preferred mode remains to be studied.

COMPOSITIONAL GRADIENTS WITHIN A COLUMN

As pointed out earlier, fingers are formed because of the different diffusion rates of composition and thermal energy. Consequently, presence of both compositional and thermal variations within a cooling magma body are expected. Because thermal energy is able to escape to the ambient in time, whatever temperature variations existed during finger formation would have dissipated from all the exposed formations today. As a result, temperature signature during the formation of fingers is

not expected to be found. However, compositional variations are expected to be frozen into the formation during cooling. Consequently, compositional variations within a column can be studied to test if double diffusion does play any role in the formation of the jointing patterns. To determine the compositional variation within a double diffusive finger, it is necessary to understand the fluid dynamic process of fingering. A finger represents the uprising of a fluid parcel of residual liquids produced by crystal fractionation at the bottom boundary due to cooling. Because this fluid parcel is generally less dense than the bulk fluid, it will be subjected to an upward buoyancy force. As a result, it will rise in the form of a plume. Although the prevailing temperature gradient is a stable one, density changes due to thermal variations are generally small when compared with that due to compositional fractionations. Thus, the stable thermal gradient will only slow down the upward motion somewhat, it will not be sufficient to inhibit the motion completely. As the fluid parcel rises through the bulk fluid, both thermal and compositional diffusion will take place between the residual liquid and the bulk liquid. In principle, the upward motion of a fluid parcel will cease when the composition of the parcel and its surrounding fluid is completely homogenized such that no buoyancy force can be realized. Because rate of compositional diffusion is generally slow, this state is not expected to occur within a cooling finger. Usually, other fluid instability will set in before this stage can be reached. For example, a finger can rise into a thermally unstable regime such that fingering is no longer possible and the system will turn into a turbulent convective regime instead. Thus, the transition from fingering to turbulent convection will mark the top boundary of the fingers for this case.

On the basis of this description, it is apparent that horizontal compositional variations are expected to exist within a column. The central part of the column will be depleted of heavy minerals because it represents the depleted residual liquids while the rims of the columns are the undepleted bulk fluids. Additionally, the contrast of compositional variations is expected to be most prominent at the base of a column. As the fingers rise, diffusion will gradually smooth out the compositional variations. As a result, compositional contrasts are expected to decrease as one moves upward along a finger. This direct implication on the compositional variation within columnar joints is illustrated in Figure 2. The schematic curves for compositional variations are given in terms of the relative abundance of the residual liquids. If the concentration of depleted elements is plotted, these schematic curves should be inverted such that lower concentrations are located at the center of a column.

Chemical analyses across a column have been studied by many investigators. Their results have been summarized by Spry (1962) who reported that, in general, no detectable chemical variations exist across a column. However, when significant variations occur, results show consistently that the rim is composed mainly of high temperature mineral assemblages while the center is composed of relatively low temperature mineral assemblages and enriched in volatiles. This latter result is sur-

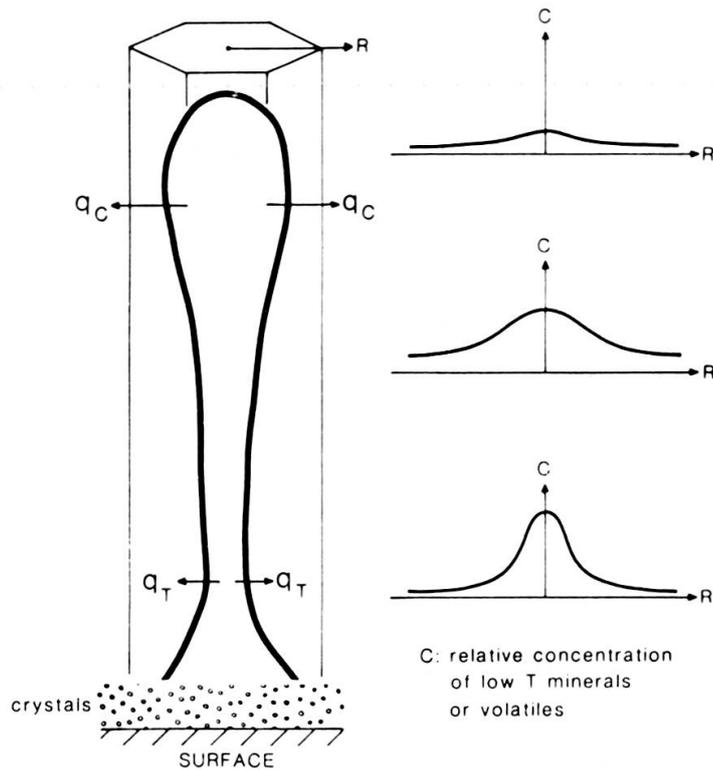


FIGURE 2.

Schematic diagram illustrating the relationship between a fluid finger and a fractured column. q_T and q_C represent the diffusive fluxes of temperature and composition respectively. On the right, relative concentrations of the residual liquids are plotted as a function of 'radius' of a column at three different heights. They illustrate schematically the effects of compositional diffusion as the uprising finger is moving away from its origin near the bottom surface.

prisingly consistent with the consequences derived from the double diffusive fingering hypothesis. According to the hypothesis, the rims of a column represent the solidification of unfractionated bulk magma whereas the center is the frozen residual liquid. Thus, the concentration of high and low temperature mineral assemblages at their respective observed locations is immediately explainable. As to the lack of chemical variations across most of the columns that have been analyzed, a speculative explanation can be offered. Perhaps, this lack of evidence is simply a result of a systematic bias of sample collection. Due to the ease of accessibility, it is understandable that laboratory samples are mostly collected from the top of a column, or from fragments that have already fallen down from their original positions. Consequently, they represent samples where large compositional variations are not expected. Therefore, to verify the fluid dynamic hypothesis, new chemical analyses of samples obtained from more appropriate positions are necessary.

Finally, it is also of interest to point out that the thickness of fingering layers is controlled by the flux of the residual liquids (Linden, 1978). At a slow rate, a single layer of fingers will form. However, as the rate of crystallization increases (i.e. as

the flux of residual liquids increases), the single layer fingering structure will break down and turn into a three-layer structure where a chaotic convective layer is sandwiched between two orderly fingering layers. This multi-layer fingering structure has strong resemblance to the three-tier colonnade-entablature-colonnade structure observed in many lava flows and sills (Spry, 1962; McDonald, 1967). Consequently, the rate of crystallization within a cooling magma body can be estimated from the thicknesses of the multi-tier structures if fluid fingers can be demonstrated to influence the formation of columnar joints.

SUMMARY

In this article, the merits and the shortcomings of the classical thermal contraction hypothesis and the recent, but more controversial, fluid finger hypothesis for the origin of columnar joints have been examined. Thermal stresses are undoubtedly needed to initiate the fractures. However, thermal contraction alone does not appear to be sufficient to explain the propagation of these fractures into the compressive interior of a thick cooling magma body. On the other hand, the double diffusive fingering hypothesis provides a plausible mechanism to establish a pattern of elongated structures within these magma bodies. The solidified fingers yield a pattern of pre-existing weakness where fractures induced by thermal stresses will subsequently propagate. Double diffusive fingering not only provides the morphological framework for jointing formation, it will also produce compositional variations both across and along the columns. Additionally, multi-tier structures can be related directly to the rate of crystallization at the bottom of the magma body. Consequently, observed columnar structures can be used to estimate the physical parameters during the cooling of magma bodies. Such information are generally not obtainable directly. However, these advantages can be realizable only if the hypothesis can be demonstrated to be valid. Unfortunately, rigorous tests cannot be carried out with the presently available data. More appropriate sampling and better chemical analyses are necessary in order to provide more conclusive evidences on the origin of columnar joints.

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