General theory of plasticity, fields of equal yield lines

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General Theory of Plasticity, Fields of Equal Yield Lines.

Allgemeine Plastizitätstheorie, Gleitlinienfelder.

Théorie générale de la plasticité. Champs des lignes de cession.

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Introduction.

Though the theory of plasticity has advanced with such rapid strides during recent years that a separate working meeting has been devoted at this Congress to the consideration of the results and effects of its development, yet there is still a considerable amount of unclarity attached to its basic principles. It is true that the changes that have taken place in the views of modern physics have made us revise many of our traditional ideas on the theoretical strengths of materials (and above all from a structural point of view), still the unclarity that predominated in various basic conceptions of the theory of plasticity far outweighs the effects of these modifications. This unclarity is mainly caused by unclear reasoning in connection with phenomenological facts.

The mechanical aspect of solid bodies is for the great part governed by *Hooke*'s Law. But as this law, which enabled a comparatively exhaustive theory of elastic continuum to be elaborated, is only valid up to a certain point, it has long been the endeavour of mechanical science to discover new and similar laws of general import applying to the conditions prevalent beyond this point. Unfortunately, this aim has been thwarted by numerous great difficulties, since, although the elastic behaviour of various materials — at least from a phenomenological point of view — is more or less the same, the behaviour of the material after the limit of elasticity has been passed is fundamentally influenced by its internal structure. The actual beginning of the mathematical investigation of the theory of plasticity was the employment, based on the affinity between *Mohr*'s enveloping curves for non-cohesive matter and those of various solid bodies, of methods for the calculation of solid matter which proved applicable to the theory of conditions at the limit of equilibrium of non-cohesive matter. Owing to the entirely different composition of these bodies, however, the method was bound to fail, i. e. lead to results deviating substantially from reality. For it should never be forgotten that a crystaline body must first undergo more or less elastic changes in shape before attaining the plastic condition in which elastic and plastic fields practically always exist side by side and overlap along certain areas, whereas the non-cohesive mass is usually subjected to equilibrial disturbances through attaining the "plastic condition". In treating theoretical problems of elasticity *Hencky*'s[1] distinction between "statically determinate" and "statically indeterminate" conditions of equilibrium is essential. By a statically determinate case *Hencky* means one in which the conditions necessary for equilibrium and that for plasticity are together sufficient to determine the tensile stresses at every point, while the solution of a "statically indeterminate" case necessitates the examination of deformations. When investigating the plastic conditions of materials practically all the cases encountered are bound to be "statically indeterminate" ones, since generally, in the ultimate conditions analysed, plastic fields are not to be found where large elastic areas exist, so that in the transition regions there must be compatibility between the two conditions and these can thus only be considered independently of each other. Mathematical treatment of such conditions is rendered extremely difficult by this connection. Yet no assumptions may be made which are contradictory to the actual behaviour of the materials, simply for the sake of simplifying calculation and arriving at a mathematical solution.

The most important simplification of this kind, which governs the whole mathematical theory of plasticity, is the assumption that the elastic deformations can be neglected on account of their relative smallness in comparison with the plastic ones. This assumption, which is nothing but an analogical conclusion drawn between the behaviour of amorphous and crystalline substances, is inadmissible for conditions of equilibrium in which both elastic and plastic fields exist. In the well-known work by *Haar* and *Kármán*[2] proof is already to be found that in the semi-plastic zone, i. e. in the zone in which $(\sigma_1 - \sigma_2)^2 = 4k^2$ ($2k =$ yield limit), while $(\sigma_2 - \sigma_3)^2 < 4k^2$, $(\sigma_3 - \sigma_1)^2 < 4k^2$, the plastic deformations are of the same magnitude as the elastic and it is therefore not feasible to neglect the latter for the former where both kinds of deformation occur.

All attempts hitherto made to solve problems of plastic equilibrium for crystalline bodies have, however, been more or less based on this assumption. The exceptions are few and far between. The fundamentally most important work discarding this assumption is one by *Hencky*,[3] which, on the other hand, leads to such involved mathematical investigations for the simplest case that the treatment of less straightforward cases rendered impossible with the mathematical resources at our disposal today.

In consideration of the unclarity surrounding the basic principles of the theory of plasticity, we shall now proceed to analyse briefly these basic principles and to compare the significance of the phenomena of plastic deformation of crystalline bodies, paying special attention to those phenomena termed in literature "yield patterns". — As is customary in the theory of plasticity, we shall consider processes of such a gradual nature that they may be regarded as a consequence of equilibrium conditions, so that in general there will be no need to discuss the rapidity of deformation in the various cases.

1) *Conditions necessary for plasticity.*

The first question to be answered in the theory of plasticity is: under what circumstances the yield limit of a material is passed. Before giving a brief review of the existant yield hypotheses, let us first quote a theorem by *Roš* which is very important when considering both the rupture and the yield hypotheses: — "A general theory of rupture which makes no allowance for the texture of the material is not possible owing to the fact that the behaviour of materials of different internal structure is often fundamentally different. Each material requires its own theory of rupture, a consequence of its internal structure and behaviour under deformation". The fact that this theorem had never before been formulated so precisely and that there was thus a tendency to generalise the results of experiments carried out when testing a certain material, explains the existence of so many hypotheses.

The materials used in engineering are generally crystalline substances which, though composed of individual crystals, yet behaves in a quasi-isotropic manner in consequence of the amorphous arrangement of the latter. — As regards the structure of the single crystals, in the metals used in engineering these are almost exclusively in stereometrical lattice arrangement, of which there are three kinds:

1) The simple type of lattice, which is singularly determined by stating the distance between molecular accumulations (characteristic distance);
2) the plano-centric lattice, with additional molecular accumulation in the planes of cubes;
3) the stereo-centric lattice, with central molecular concentration.

α, β and δ irons crystallise in stereo-centric, γ iron, nickel and manganese steel, as well as copper, aluminium, etc., crystallise in plano-centric lattice formation.

The type of lattice is extremely important from the technical standpoint also, as the manner of transition to the plastic state and the characteristic phenomena of the latter are decisively influenced by the crystal lattice.

The most important of the yield hypotheses are the following:

1) *Guest-Mohr*'s shear stress hypothesis[4] in the form of

$$\tau_{max} = f(\sigma_x + \sigma_y)$$

developed from *Coulomb*'s old theory of internal friction[5]. The function $f(\sigma_x + \sigma_y)$ can be adapted to the results of experiments.

2) *Beltrami*'s hypothesis of constant deformation energy, which regards a definite amount of accumulated deformation energy as a criterion for the attaining of the yield limit, but which did not tally with the results of experiments, and was newly formulated and improved by *Huber*[6] and independently by *Mises* and *Hencky*[7] to become

3) Hypothesis of constant deformation energy

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 8k^2.$$

4) *Schleicher*'s improvement on this hypothesis[8], which perhaps represents the most general form, runs

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = \sigma_e(p),$$

$$\text{whereby } p = {}^1/_3(\sigma_1 + \sigma_2 + \sigma_3).$$

The many experiments that have been carried out to test the correctness of all these hypotheses, and various others that have been entirely abandoned today,[9] have shown that for plastically deformable metals, whether they possess a pronounced yield limit or not, the *Huber-Hencky-Mises* hypothesis characterises their transition to the yield zone and subsequent behaviour in the latter, whereas for brittle materials and conditions of stressing at yield limit *Mohr*'s hypothesis gives the best mean values.

When assessing the value of these statements it must always be borne in mind that they are provisional results which may yet be altered when further research work is carried out.

2) *Yield Limit.*

The yield hypothesis is the condition that must be fulfilled by the main stresses so that yielding is attained in one point. This condition applies (an essential fact) for the state occuring immediately after the yield limit has been passed. It gives no information, however, as to the manner in which this transition takes place. Not only with regard to different metals (steel and copper) are there differences in the process of deformation, but also in the various kinds of one metal and even in absolutely similar kinds of different antecedents.

The principal difference is between metals with and without a pronounced yield limit. In the case of the latter the transition from elastic to plastic state takes place in quite constant manner, for even slight stressing is sufficient to cause plastic deformations. In the former, on the other hand, the deformations are completely reversible up to a certain point; suddenly, however, the material, which has till now offered such resistance, suddenly collapses, plastic deformations at once begin to appear and develop rapidly. Often, however, the load does not remain constant, but decreases considerably, so that there would seem to be an "upper" and a "lower" yield limit.

Bach[10] was the first to point out that this upper yield limit depends to a very great extent upon the shape of the test bar, and he recognised the nature of this limit as one of a typical symptom of instability (overturn of the loading). Modern research has gone a step further and also declared the "lower" yield limit to be a symptom of retardation such as may be seen in other branches of physics (delayed boiling, undercooling) and which are distinguished in that the change of state to be expected in accordance with the physical laws is considerably retarded and then, suddenly, sets in and develops with rapid strides. When the stress-deformation curve gives a straight line, it is considered — for instance by *Moser*[11] as a phenomenon of retardation, namely, as the expression of a retardation of the permanent deformations, brought about by internal resistance to yield. The shock-like development of plastic deformation at yield limit is then to be regarded as an outward effect of retardation. This opinion is also supported by the fact, confirmed by experience, that for steel of the same granulation and processing, the limit of proportionality approaches the yield limit, the more homogeneous the material is and the more undisturbed the state of stressing that can be produced in it.

The proper manner of regarding the nature of the yield limit is extremely important for the theory of plasticity in that it forms a basis without which it is impossible properly to judge the fundamental significance of the individual-plastic phenomena in the transition stage.

3) *Distortion Patterns.*

In smoothly polished test pieces of soft iron relatively fine relief patterns become apparent in the initial stage of deformation and become more and more corwded as stressing increases. These dull lines, which represent intersections of the more deformed layers with the polished surface, are either crested (in compression) or troughed (in tension) or else shed-roofed. These lines, called *Lüder* or *Hartmann* Lines after their first observers and to-day generally described as distortion wedges in the theory of plasticity, are the most striking distortion patterns. Their most important property is their intersection with the shear-tension trajectories. It is because of this property that these lines are regarded as extremely valuable aids in the investigation of tension conditions of solid bodies in the plastic zone.

The fields of equal yeld lines are actually distinguished by a number of important properties from a mathematical point of view, properties which make it possible completely to resolve conditions of tension in the plastic zone from the knowledge of distrotion wedges[12]. The most important of these properties is the identity of these distortion wedges with the characteristics of the condition necessary for plasticity. Proof of this identity was first brought by *Massau*, though it was put into a general form by *Reissner*[13]. In view of this property it is possible to compute unanalytically various integrals along distortion wedges, a fact which greatly facilitates the application of solutions to the actual conditions prevailing. The few existent solutions to the mathematical theory of plasticity are practically all based on this property of distortion wedges.

When judging how far the above method may be applied for the real solution of technical problems in the realm of the plasticity theory, it must, however, be considered that mathematically perfect transitions to states above their admissible limit allow *a priori* of no judgment being formed. For if we start with a mathematically defined hypothesis and within it allot definite ultimate values to definite magnitudes, this procedure is indubitably admissible from a mathematical standpoint. Physically, however, it is possible that the physical behaviour of the material is considerably altered by these ultimate values and that the factors which formed the basis of the hypothesis have lost much if not all of their validity. This is the case as regards the condition necessary for plasticity.

The condition necessary for the plasticity of a generally plastic body for two-dimensional stressing is

$$\sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau^2} + \sin \varphi \frac{\sigma_x + \sigma_y}{2} = C.$$

ρ being the angle of friction, C a value dependent on cohesion. For the non-cohesive mass on which the investigation of distortion wedges was based, $C = 0$. The appearance of a distortion wedge results primarily in disturbance of equilibrium; the reversible deformations preceding entry into the disturbance zone are quite negligible compared with the "plastic" deformations. For metals, however, $C =$ constant and $\rho = 0$. Owing to the great cohesion the appearance of distortion wedges is only a local and transitory disturbance of equilibrium, the elastic state of stressing and deformation preceding the beginning of yield phenomena is of the utmost importance as regards the character of the yield, while the magnitudinal arrangement of the plastic deformations is equal to that of the elastic deformations.

It will be clear from the above that no importance can be attached to the results of the so-called mathematical theory of plasticity when it is a question of the plasticity theory of crystalline materials, for the necessary conditions are not fulfilled. This also means that less importance must be placed on distortion wedges in the investigation of plastic conditions in metals of technical interest. They only become of any value when deformation is so far advanced that there are no longer any elastic zones whatever in the whole field. These cases are not of frequent occurrence, being principally confined to problems of processing.

Discarding the generally accepted view that distortion wedges are of great importance in the theory of plasticity, and utilising without prejudice the numerous results of experiments already carried out, it will be found that the phenomenon of distortion wedges is not connected with plastic deformation as such, but only with the character of the transition from the elastic to the plastic state. Just like the yield limit, they are typical phenomena of instability. This fact is proved by quite a number of observations, such as those of *Ludwik*[14], showing that yield lines are particularly liable to appear when the body begins to yield under decreasing stress, i. e. when the formation of distortion wedges is restricted to the downward slope of the peaks of the stress-deformation diagram. This observation has also been confirmed by *Nadai*[15] and often referred to by *von Kármán*. In this connection mention should also be made of *Nadai*'s observation that the pattern of yield lines was much more crowded when compression tests were rapidly carried out than when more leisurely tests were made. This is a further proof that instability in general, whether caused by stressing or by the texture of the actual material, favours the formation of distortion wedges. It is thus quite obvious that such formation must also be favoured by boring and notching.

In this connection reference should be made to the extremely interesting measurements of hardness taken by *Moser*[16] in distortion patterns. His results show that metals increase in hardness in the yield zone, and reveal interesting details concerning the character and process of plastic deformations. *Moser* observed that permanent deformations at first only occurred in zones (deformation wedges), only a definite degree of hardness being attained in each zone. A general increase in hardness only takes place when the whole bar (tension test) is covered with a network of deformation wedges. The reason for this phenomenon lies in a kind of "blocking" of the yield surfaces. When this phase is reached in a particular zone, a further increase of loading will result in slipping (yielding) in another zone, not yet deformed. Before the yield resistance of this next zone is overcome, the loading always increases somewhat and drops again as soon as the distortion wedges have formed. Each peak of the $\sigma - \varepsilon$ diagram therefore corresponds to a local upper yield limit at which a yield line forms under decrease of loading. — Contrasted with the phenomena which occur in steel, a copper rod revealed a steady increase in hardness from the moment when loading is applied, whereby no zonal yield lines but only a general dulling of the surface was observed.

The above tests are an obvious corroboration of the view that the yield limit of steel is a "retardation" of the yielding process; they furthermore explain the deformation wedges as a phenomenon, herewith connected, characteristic only for steels with varying yield limits.

This conception is further confirmed by the results of tests carried out by *Ititaro Takaba* and *Katuni Okuda*[17] showing that

1) the appearance of deformation wedges and the sudden break in the stress-strain line are results of one and the same occurrence, viz. the displacement in groups of large quantities of crystal granules;
2) all metals in which deformation wedges can occur belong to the stereocentric lattice crystalline structure. It is shown that formation of deformation wedges was not observed in steels of Austenite texture, which belong to the plano-centric lattice type of structure.

It may therefore be stated that in the investigation of elastic-plastic conditions of crystalline materials — and above all of metals — the observation of fields of equal yield lines is not a suitable method, but that on the contrary the fundamental occurrences that really matter are often rendered even more unclear by the deformation wedges. This chiefly applies for the development of the true limit between the elastic and the plastic state.

4) *Ultimate Limits of the Range of Plasticity.*

Employing one of the known methods for determining the plastically deformed areas in metals — the best of which is that of recrystallisation[18] — the limits between elastic and plastic areas can be definitely ascertained (Fig. 1). The form of

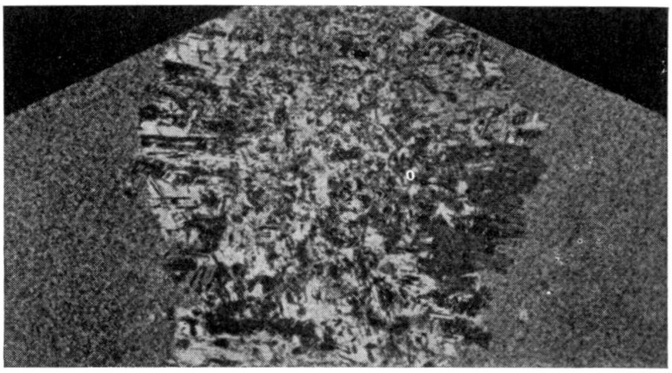


Fig. 1.

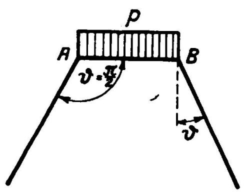
these limit lines, as all observations have indisputably proved[19], has nothing to do with corresponding deformation wedges, but is composed of those lines corresponding both to the plastic and to the elastic state of tension. The only group of lines fulfilling this stipulation are the τ_{max} = constant lines of the elastic state. This type of limit line, which is independent of that of transition from the elastic to the plastic zone, are to be observed in all elastic-plastic states and form the most essential symptom of the latter. No solutions of the plastic field of tension will correspond to reality other than those which can always be applied to the corresponding elastic field of tension along every line τ_{max} = constant. Every solution of the plastic problem must therefore be preceded by that of the elastic problem, and here it must be borne in mind that the limit between elastic and plastic range is not a fixed one, but varying as its loading varies. The latter must, however, always keep to the lines τ_{max} = constant of the elastic field.

The mathematical treatment of elastic-plastic problems under the above conditions is not easy; up to the present it has only been successfully applied in very few simple cases. A certain alleviation may, however, lie in the fact that by carrying out so-called optical investigation experiments for stresses in models in such a manner that the lines of constant difference of greatest tension appear primarily as isochromes, it becomes possible to get at the limit of plastic range *a priori*.

5) *Resistance to Penetration.*

As an example of the solution of a technical problem in the above manner we shall now treat that of resistance to penetration as a duodimensional problem. The case is of especial interest because it represents the best known example of a plastic solution with the assistance of the field of equal yield stresses, and because it was its publication that directly gave rise to the development of the modern mathematical theory of plasticity[20].

Fig. 2.



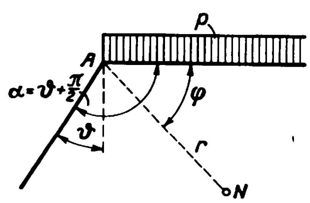


Fig. 3.

The problem is to find the uniformly distributed load stress p which (Fig. 2), acting along A B, causes yielding inside the zone under consideration. This load, which we shall define as resistance to penetration, may be represented as a function of the yield limit and of the angle of inclination of the lateral delimitation of the zone. Considering the plane state of distortion ($\varepsilon_z = 0$), the conditions necessary for yield laid down in the *Huber-Hencky-Mises* theorem are:

$$(\sigma_x - \sigma_y)^2 + 4\,\tau^2 = \frac{16}{3}\,k^2,$$

the yield limit being $\sigma = 2k$.

As the solution of the "blunt wedge" is not possible either as a plastic problem or in a closed form, assistance may be obtained by only considering the corner A and taking into account the fact that in this corner the lines τ_{max} = constant of the problem shown in Fig. 3 are tangents to the lines τ_{max} = constant of the blunt wedge. In determining the critical load it is unimportant whether we deduce the lines τ_{max} = constant themselves or their tangents.

From the elastic solution of the corner, using *Airy*'s stress function

$$F = ar^2 + br^2\varphi + cr^2 \sin 2\varphi + dr^2 \cos 2\varphi \tag{1}$$

the four constants a, b, c, d can be obtained from the four support conditions

$$\text{for } \varphi = 0 : \sigma_t = -p, \quad \tau = 0 \text{ (without friction)}$$
$$\text{for } \varphi = \alpha : \sigma_t = 0, \quad \tau = 0$$

we get the stresses

$$\begin{aligned} \sigma_r &= p\,(Q-1) - 2P \cdot p \cdot \varphi - p \cdot P \cdot \sin 2\varphi + p \cdot Q \cos 2\varphi \\ \sigma_t &= p\,(Q-1) - 2P \cdot p \cdot \varphi + p \cdot P \cdot \sin 2\varphi - p \cdot Q \cos 2\varphi \\ \tau &= p \cdot P - p \cdot P \cos 2\varphi - p \cdot Q \cdot \sin 2\varphi \end{aligned} \tag{2}$$

whereby

$$P = -\frac{1}{2(\alpha - \operatorname{tg}\alpha)}, \qquad Q = -\frac{1}{2(\alpha \operatorname{ctg}\alpha - 1)}.$$

With the abbreviation

$$x = \frac{\tau_{max}{}^2 - p^2 \cdot Q^2}{4p^2 \cdot P^2}$$

a brief calculation yields the equation of the lines τ_{max} = constant in the form of

$$y = -\frac{x}{2(x-1)}\left[\operatorname{tg}\alpha \pm \sqrt{\operatorname{tg}^2\alpha - 4x^2}\right] \tag{3}$$

This is the equation of a pair of straight lines passing through A, which remain real as long as $\operatorname{tg}^2\alpha \geqq 4x^2$.

The principal shear stress becomes a maximum on that line for which

$$\frac{\delta \tau_{max}}{\delta \varphi} = 0.$$

This is satisfied for $\varphi = \frac{\alpha}{2}$. The calculation leading to the second deduction further shows that only for $\frac{\pi}{2} < \alpha \frac{3\pi}{2}$ along the line $\varphi = \frac{\alpha}{2}$ is a maximum value created, while for $\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ a minimum is given there. The latter values of α are, however, not of technical interest. For $\varphi = \frac{\alpha}{2}$ we get the magnitude of the principal shear stress

$$\tau_{max}{}^2 = p^2\,[Q^2 - 2P \cdot Q \cdot \sin\alpha + 2P^2\,(1 - \cos\alpha)]. \tag{4}$$

Introducing the condition for yield, we obtain for the critical load stress

$$p = \sigma_F \cdot \frac{\left(\vartheta + \frac{\pi}{2}\right)\sin\vartheta + \cos\vartheta}{1 + \sin\vartheta} \tag{5}$$

This is the relation between resistance to penetration, angle of wedge and yield limit[21].

Sachs[22] has studied the problem of resistance to penetration in metals with great thoroughness, and he, also by recrystallisation, ascertained that the plastically deformed zone is delimited by the lines τ_{max} = constant of the elastic stress area. In Fig. 4 the resistances to penetration in steel, as obtained by *Sachs*, are compared with the values deduced from Eq. 5 for various wedge angles. There is a satisfactory amount of coincidence.

The solution of the same problem, as worked out by *Prandtl* with fields of equal stress lines, gave the equation

$$p = \sigma_F \, (1 + \vartheta) \tag{6}$$

as a function of the resistance to penetration from the wedge angle and the yield limit. By way of comparison this equation has also been entered Fig. 4 with the result that coincidence is revealed solely for extremely acute wedge angles, but that the course is quite different.

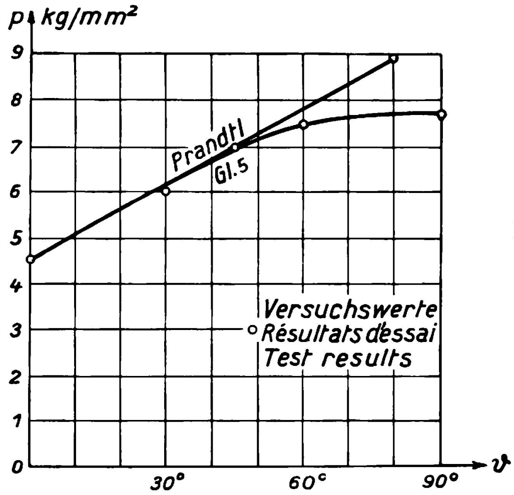


Fig. 4.

The example cited shows that the treatment of plastic problems for crystalline materials must always be based on the delimitation curves of the plastic zone. The assumption of stress lines a delimitation of this kind, and the obtaining of solutions with the assistance of the properties possessed by these stress lines will always lead to results which do not correspond to reality.

Reference works:

1 *Hencky:* Zeitschrift f. ang. Mathem. u. Mech. 1923, p. 241.
2 *Haar* und *Kármán:* Göttinger Nachrichten, math. nat. Klasse, 1909, p. 204.
3 *Hencky:* Zeitschrift f. ang. Mathem. u. Mech. 1924, p. 323.
4 *Guest:* Philosophical Mag. 1900, vol. 5, p. 69. ***Mohr:*** periodical VDI. 1900, vol. 44. p. 1524.
5 *Voigt:* Verh. d. Ver. z. Bef. d. Gewerbefl. 1907, p. 4.
6 *Huber:* Czasipismo Techniczny 1904, Lemberg.
7 *Mises:* Zeitschrift f. angew. Mathem. u. Mech. 1925, p. 147.
8 *Schleicher:* Zeitschrift f. ang. Mathem. u. Mech. **1926**, number 3.
9 *Kármán:* Verh. 2. Int. Kongr. f. techn. Mech., Zürich 1936. *Böker:* Forschungsheft VDI. 175/176. *Roš* und *Eichinger:* Diskussionsberichte der E.T.H. Zürich, 1926—1929.

[10] *Bach:* Forschungsarbeiten d. VDI., Nr. 29.
[11] *Moser:* Forschungsheft d. VDI., Nr. 295.
[12] *Cartheodory* und *Schmidt:* Zeitschrift f. ang. Mathem. u. Mech. 1923, p. 468.
[13] *Reißner:* Proceedings I. Int. Congr. Mechanics, Delft. 1924.
[14] *Ludwik:* Technologische Mechanik, Berlin 1909.
[15] *Nadai:* Schweizer Bauzeitung 1924, p. 157.
[16] *Moser:* Stahl und Eisen 1928, p. 1601.
[17] *Okuda* und *Takaba:* Werkstoffaussch. d. Ver. D. Eisenhüttenleute, Bericht 119.
[18] *Oberhoffer:* Das technische Eisen, Berlin 1925.
[19] *Nadei:* Zeitschrift f. ang. Mathem. u. Mech. 1921, p. 20. *Sachs:* Naturwissenschaften 1926, p. 1219. *Sachs:* Zeitschrift f. Techn. Physik, 1927, p. 132. *Körber* und *Siebel:* Mitt. d. Inst. f. Eisenforschung in Düsseldorf 1928. *Siebel:* Ber. d. Werkstoffaussch. deutscher Eisenhüttenleute, Nr. 122.
[20] *Prandtl:* Göttinger Nachrichten, 1920, p. 74.
[21] *Freudenthal:* Dissertation, Prag 1930.
[22] *Sachs:* Zeitschrift f. Techn. Physik 1927, p. 132.

Summary.

The fundamental principles of the general theory of plasticity are still unclear in certain essential points owing to the fact that the phenomena of yielding are not judged and evaluated in a clear and uniform manner.

The most essential conceptions of the theory of plasticity — condition for yield, yield stress limit and deformation wedges (stress lines) are therefore submitted to a brief analysis, the most important result of which is the conclusion that both the yield stress limit and the yield lines are essentially phenomena of instability dependent upon the internal structure of the material and that they specifically influence the manner of transition from elastic to plastic but are nevertheless of much less importance for general plastic deformation than is commonly believed. The more so, as both phenomena can only be observed in the case of very definite crystalline structures, namely, those of stereo-centric lattice formation, while for materials of a different type of crystalline structure the transition from elastic to plastic takes place in a continuous, uninterrupted manner.

The limit of the plastic zone is formed independently of the manner of transition by lines $\tau =$ constant in the elastic field of stress.

The example of resistance to penetration shows the distinctions to be made in the treatment of plastic problems — on the one hand from the point of view defined above, and on the other from the standpoint of the mathematical theory of plasticity, which in fact is a theory of deformation wedges — and proves that for metals the results obtained with the mathematical theory of plasticity do not correspond with reality.

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