Calculation of welds under consideration of constant deformation energy

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Calculation of Welds under Consideration of Constant Deformation Energy.

Berechnung der Schweißnähte unter Berücksichtigung konstanter Gestaltsänderungsenergie.

Calcul des soudures basé sur de la conservation de l'énergie de déformation.

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The experiments of Professor *Jensen* have demonstrated that the theory of constant deformation energy furnishes exactly the relation between the loads in different direction on a fillet weld. The calculation of statically endeterminate connections is based on the theory of plasticity. The author's conclusions from theory and tests are given at the end of this paper.

According to the German standards (DIN 4100) concerning welded steel structures, and in the American Code of Fusion Welding and Gascutting in Building Construction, and as well as in other Regulations the permissible amount of stress in fillet welds was taken the the same for all directions in which the force is capable to act. The strength offered by the weld is much greater if the force is acting at right angles to the face of the weld (line CD in Fig. 1)

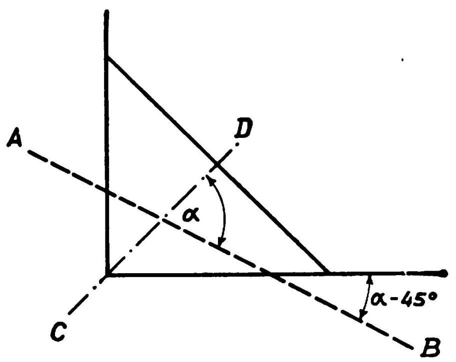


Fig. 1.

than when acting in the face of the weld. Professor *Cyrill D. Jensen* (U.S.A.) has published a series of interesting tests on the strength of front fillet welds for different values of the angle α formed between the direction of the force AB and the fillet CD (Fig. 1). The rupture point stresses $\sigma_{B\alpha}$ found by Professor *Jensen* are represented in Fig. 2 in such a way that the length of the vector represents the magnitude of the rupture point stress and the angle between vector and the axis of abscissae is equal to angle α. Professor *Jensen,* however, has not examined if the results of his tests are compatible with the theory of constant deformation energy. If this examination is carried out it is found that excellent agreement exists. The dotted curve gives the points on which the vectors should end if they would fully agree with the theory.

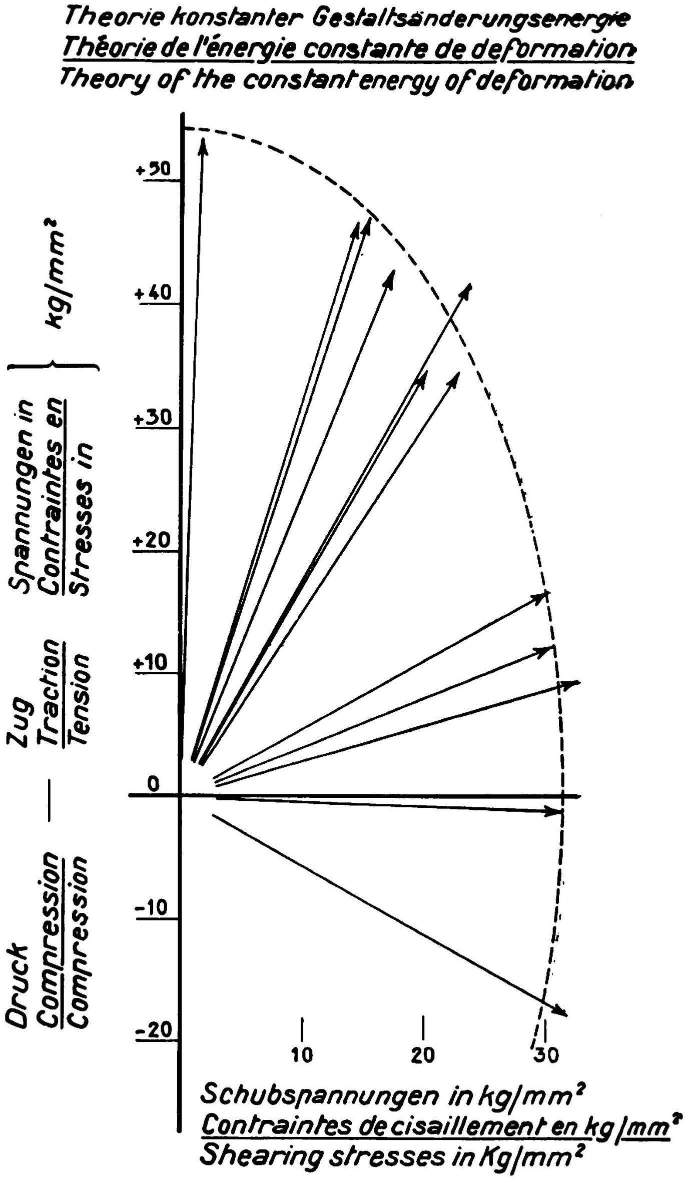


Fig. 2.

The following notations are used:

$\sigma_{B\alpha}$ represents the rupture point stress for forces acting under an angle α to the bisecting line of the fillet.

σ the component of $\sigma_{B\alpha}$ perpendicular to the plane of rupture.

τ the component of $\sigma_{B\alpha}$ in the plane of rupture.

$\sigma_{B\,zug}$ the rupture point stress for normal loading.

According to the theory of constant deformation energy fracture occurs when

$$\sqrt{\sigma^2 + 3\tau^2} = \sigma_{B\,zug} \text{ and hence}$$

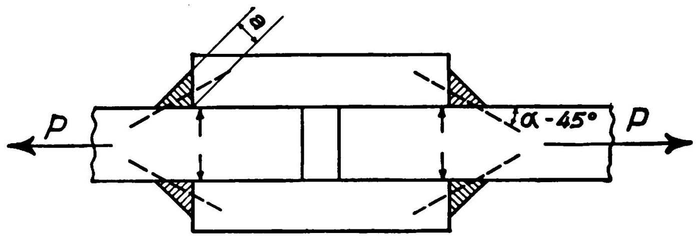
$$\sigma_{B\alpha} = \sigma_{B\,zug} \cdot \frac{1}{\sqrt{\sin^2\alpha + 3\cos^2\alpha}}.$$

The mean value of $\sqrt{\sigma^2 + 3\tau^2}$ for which rupture occurs has been deduced from the tests of Professor *Jensen.* With these mean values the values of $\sigma_B\,\alpha$ were formed, which are represented by the dotted line curve of Fig. 2.

The agreement of Professor *Jensen's* test results with the theory of constant deformation energy is so much the more interesting since the object of Professor *Jensen*'s tests was to examine other theories of rupture. He found that either the results of his tests do not agree with any of the theories of rupture known to him or that his tests were not accurate enough. We find, however, that the results of his tests agree exceedingly well with the theory of constant deformation energy and that they were of great accuracy.

The researches of Professor *Jensen* refer to front fillet welds only (line CD, Fig. 1) stressed either by tension and shear, or shear and very little compression; they do not concern welds stressed chiefly by compression.

The specimens were constructed in such a way that the direction of the force could be exactly determined. This was necessary should the angle α be determined accurately. As a rule the direction of the force is statically not determined. The calculations of $\sigma_B\,\alpha$ shall therefore be extended by determining first the angle α for the action of forces whose direction is unknown statically.



Fig, 3.

We propose, based on the theory of plasticity to choose the direction of the force in the welds (angle α) such, that the greatest force transmitted to the structure, but compatible with the equilibrium of forces, can be considered in calculation.

These conditions shall be elucidated by an example. In Fig. 3 is shown a specimen with front fillet welds, and stressed by a tensile force P. The lines of action (shown dotted) for the forces to be transmitted by the welds can form, from the point of view of equilibrium, any angle with the horizontal line. This angle with the horizontal line is $\alpha - 45^0$ (see also Fig. 1). If F represents the area along line CD in Fig. 1 for two welds we receive for the oblique rupture force of one weld the expression:

$$\frac{1}{2} F \sigma_{B\alpha} = \frac{1}{2} F \cdot \frac{\sigma_{B\,zug}}{\sqrt{\sin^2\alpha + 3\cos^2\alpha}}.$$

In consequence of the obliquity of the forces at the joint the lateral pieces are pressed against the central piece. Fracture of the specimen will only result when the friction between the central piece and the lateral pieces are overcome. The breaking force P is therefore the sum of the horizontal components of the oblique forces and the resistance due to friction. The horizontal components of the oblique breaking forces are

$$\frac{1}{2} F \frac{\sigma_{B\,zug}}{\sqrt{\sin^2 \alpha + 3 \cos^2 \alpha}} \cos(\alpha - 45^0)$$

and the vertical components,

$$\frac{1}{2} F \frac{\sigma_{B\,zug}}{\sqrt{\sin^2 \alpha + 3 \cos^2 \alpha}} \sin(\alpha - 45^0)$$

If the coefficient of friction be denoted by μ we can write.

$$P = 2 \cdot \frac{1}{2} F \frac{\sigma_{B\,zug}}{\sqrt{\sin^2 \alpha + 3 \cos^2 \alpha}} \left[\cos(\alpha - 45^0) + \mu \sin(\alpha - 45^0)\right]$$

The central piece is severely compressed by the lateral pieces, from wich it results that the coefficient of friction can be chosen much higher than in the case of moderate compression. We allow a coefficient equal to that of a riveted joint, that is to say, 0.2. Our formula then becomes,

$$P = F \frac{\sigma_{B\,zug}}{\sqrt{\sin^2 \alpha + 3 \cos^2 \alpha}} \left[\cos(\alpha - 45^0) + 0.2 \sin(\alpha - 45^0)\right].$$

Based upon the law of plasticity the value of α must be such as to give P a maximum value. This is the case if $\alpha = 79^0$ and

$$P = 0.91\, F \sigma_{Bzug}$$

(Type of construction of Fig. 3).

The effect of the force is essentially different in the case of Fig. 4. The compression between the pieces does not exist in this case and in order that the

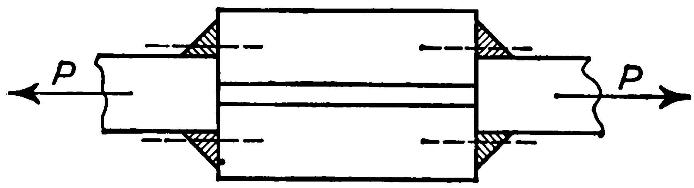


Fig. 4.

system may be in equilibrium, it is necessary that the forces acting on the upper and lower welds should be directly opposed. For reasons of symmetry it is further necessary that the forces should be horizontal as is shown in Fig. 4. The angle is therefore in this case 45^0 and

$$P = F \frac{\sigma_{B\,zug}}{\sqrt{\sin^2 \alpha + 3 \cos^2 \alpha}} = 0.71\, F \sigma_{B\,zug}$$

(type of construction shown in Fig. 4). If in the specimen of Fig. 3 tension is replaced by compression, the pressure between the lateral pieces and the central piece no longer exists. In this case we have again $\alpha = 45^0$ and

$$P = 0.71\, F\, \sigma_{B\,zug}.$$

We will now try a method of calculation based on the following hypothesis:

1) Fracture occurs at the smallest section of the weld; at least, let us make our calculations as if such were the case.

2) According to the theory of constant deformation energy, the ultimate stress $\sigma_B\,\alpha$ occasioned by a force acting at an angle α is equal to the normal tensile stress which leads to fracture multiplied by

$$\frac{1}{\sqrt{\sin^2\alpha + 3\cos^2\alpha}}.$$

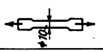
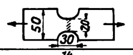
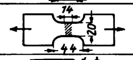
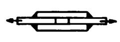
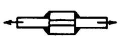
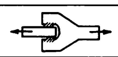
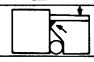
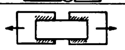
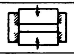
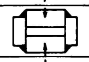
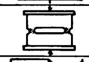
3) According to the theory of plasticity, the hyperstatic value will be chosen as favorable as equilibrium permits. This concerns quite as much the angle at which the force acts at a weld as the distribution of the forces over several welds.

If the calculation according to the third hypothesis is too complicated, a practical supposition concerning the hyperstatic value should be made. This supposition may lead us to calculate a state of load which is smaller than the true ultimate load. It must yet be remarked that the hypothesis 3 does not apply to alternating loads (repeated, for example, one million times) because the material is no longer plastic in such a case. This method of calculation can only be applied to structures in which the loads vary but slightly or not frequently, for example, frame work. In all cases it must be verified by tests.

Under agreement with the Dutch Standards Committee for Welded Metallic Structures (36 C) and with the willing collaboration of "Willem Smit & Co's Transformatorenfabriek", Nimegue, "Arcoselectrofasch" Amsterdam, and the "Nederlandsche Kjellberg Electrodenfabriek" Amsterdam, the Polytechnic School at Delft (Holland) we have undertaken a series of experiments on electrically welded specimens to investigate the hypotheses we have indicated above. In the table, one column gives sketches of the specimens, another indicates the type of weld and the following, the angle formed by the direction of the force on the one part, and by the smaller transverse section of the weld on the other part.

Three specimens of each type were prepared. One specimen was welded with Smit's "Resistanz" rods, another with "Stabilend" rods from the Maison Arcos, are the mean of the quotient of the greatest load supported by the specimen and the smallest transverse section of the weld (if there are collaborating welds — the sum of the smallest sections of these welds). The breaking loads are compared with the mean breaking load of a standard bar (10 mm diameter for a length of 50 mm) composed entirely of the deposited metal. The average stress of nine of these bars (three prepared from the rods each mark) was determined and is given behind XVI in the table. It amounts to 48.3 kg/mm^2.

We have used steel 37 for the preparation of the specimens. The ratio between the mean breaking stress of the welded joint and the mean breaking load of bars of the deposited metal are given in the 6th column of the table. In column 7 are shown the results that should be obtained if the hypotheses under inve-

№	Test piece	Type of weld	Nature of stressing	Rupture point Stress in kg/mm²	Ratio to XVI observed	Ratio to XVI calculated	Stress due to impression of 0,2‰ kg/mm²	Stress due to impression of 0,2‰ Ratio to XVI	Stress due to impression of 1,0‰ kg/mm²	Stress due to impression of 1,0‰ Ratio to XVI	Bessel kg/mm²	Vogel kg/mm²	Dresden Welding Tests kg/mm²	Boston kg/mm²	Hohn kg/mm²	Mac Gibbon kg/mm²	E.T.H. Zürich kg/mm²	R.S.W. kg/mm²	
XVI		Entirely of weld metal	Normal tension	48,3															
I		Butt weld	Normal tension	49,3	1,02	1,00					38,0[1]							34,0[1]	not machined
II		Butt weld	Normal tension	50,1	1,04	1,00											40,0		
VII		End fillet weld	Normal tension	57,3	1,19	1,00													
VIII		End fillet welds	Tension under 79° and friction	51,9	1,07	0,91						46,5		45,0	41,0		49,5	35,0	
IX		End fillet welds	Tension under 45°	33,4	0,69	0,71													
VI		Butt welds	Shear	31,1	0,64	0,58													
X		End fillet weld	Shear	29,9	0,62	0,58													
XIV		Side fillet welds	Shear and secondary tension	30,9	0,64	0,58					32,0	40,0		34,0	28,0	39,0	35,4	29,0	
XV		Side fillet welds	Shear and secondary compression	35,6	0,74	0,58						50,0				48,0			
XI		End fillet welds	Compression under 45°	40,6	0,84	0,71						42,5							
XII		End fillet welds	Compression under 72°	> 64,1	> 1,33	0,82	37,8	0,78	55,0	1,14			> 53,0						
V		Butt welds	Compression	> 78,8	> 1,63	1,00	37,8	0,78	48,1	1,00			> 46,0						
XIII		End fillet welds	Compression	> 70,7	> 1,46	1,00	61,6	1,27											

stigation were exact. The comparison of the figures in columns 6 and 7 gives the degree of exactitude of the hypothesis. The photograph, Fig. 5, shows the several specimens before the tests; specimens I, II, V, and VI are butt welded and accurately machined (thickness 10 to 14 mm). The other specimens are fillet welded and were measured with Dr. Ing. *H. Schmuckler*'s instrument (an instrument which proved itself very useful for these measurements). The

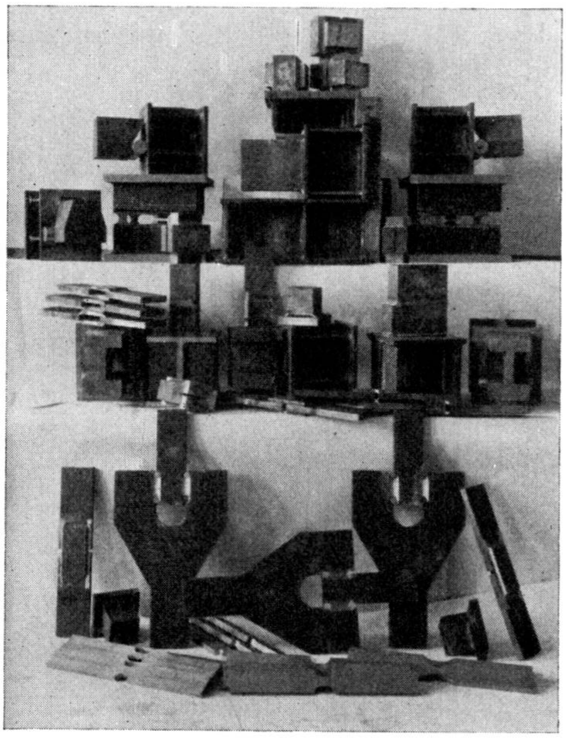


Fig. 5.

breaking loads are referred to a measured section (thickness "a" multiplied by the length of the weld). In general the thickness of the weld was intended to be 4 mm but measurements have shown that this thickness was exceeded.

Fig. 6 represents a specimen of type VII reinforced by a stirrup to enable the placing in position on the press. Fig. 7 shows the method by which the weld was

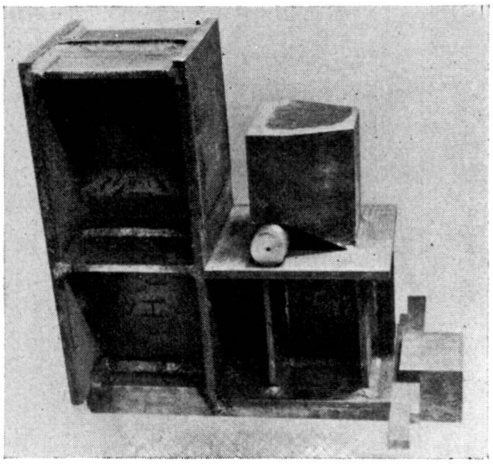


Fig. 6.

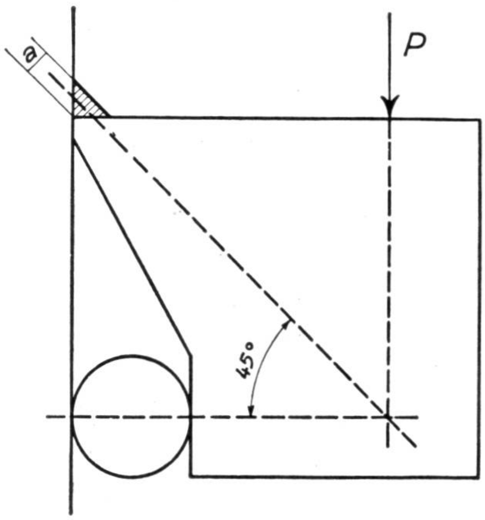


Fig. 7.

submitted to tension. If P is the breaking load, "b" the width of the weld and "a" its thickness, the ultimate stress is equal to P $\frac{\sqrt{2}}{a \cdot b}$. The shape of this specimen as well as that of the specimen X conformed to the experiments of *Jensen*. The play of forces in specimens of type VIII and IX have already been explained above. The photograph of Fig. 8 represents a specimen of type VI after the tests. The specimens of X are similar to the specimens of type VII with the sole difference that the welding fillet is placed below the top flange of the Tee girder; it follows that the weld will be subjected to shear.

In the case of specimens XII, V and XIII, the welds were subjected either to compression at an angle of 72^0 or to normal compression. In this case

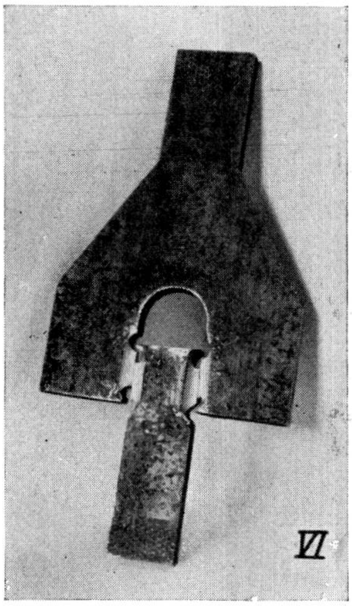


Fig. 8.

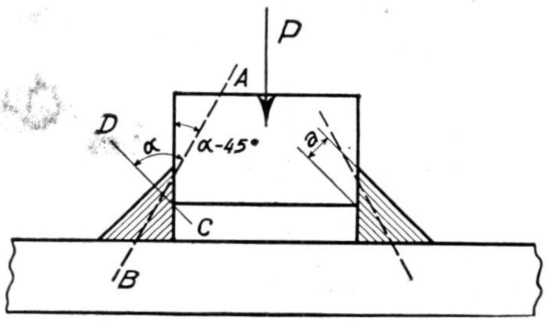


Fig. 9.

there had not been fracture because the weld was stretched without being broken. The distortion of the open welds were measured and the stresses for distortions of 0.2 and 1 mm are recorded in the table. In the column "Ultimate stress" the stresses are recorded at the point where the tests could be carried no further.

During the welding of the specimens XII and XIII, copper plates were introduced into the spaces left free so as to prevent the welding metal flowing there. In the calculations we have introduced the tickness "a" (see Figs. 9 and 11). In the case of specimen XII the angle α contained between the direction of the load to be supported by the weld (line AB of Fig. 9), and the smallest section of the weld, is not statically determinate. If the sum of the smallest sections (CD) of two welds, is represented by F, the ultimate oblique force for a weld is

$$\frac{1}{2} F \sigma_{B\alpha} = \frac{1}{2} F \frac{\sigma_{B\,zug}}{\sqrt{\sin^2 \alpha + 3 \cos^2 \alpha}}.$$

The direction of this force makes an angle of $\alpha - 45^0$ with the vertical. From the fact that the horizontal components are in equilibrium in the weld, the load

P supported by the specimen is equal to the sum of the vertical components, therefore

$$P = 2 \cdot \frac{1}{2} F \frac{\sigma_{B\,zug}}{\sqrt{\sin^2 \alpha + 3 \cos^2 \alpha}} \cos (\alpha - 45^0)$$

this load is a maximum for $\alpha = 72^0$ and is equal to

$$P = 0.82\, F\, \sigma_{B\,zug}.$$

Photograph 10 represents a specimen of type V after the tests. The welds are found in the weak places at in the corners. The webs of the girder which were 9 mm apart, before the tests, are touching. The play of the loads in the case

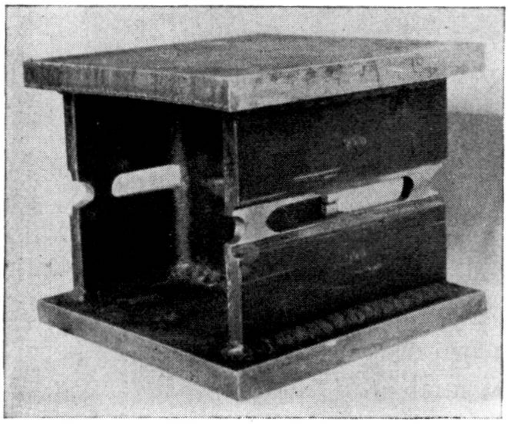


Fig. 10.

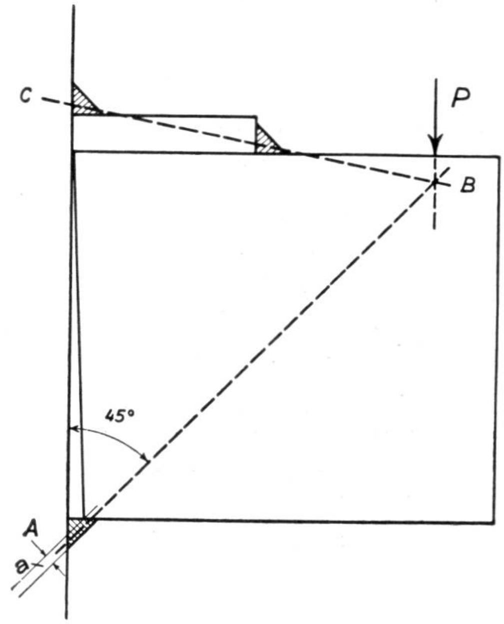


Fig. 11.

of specimen XIII is explained by Fig. 11. The lower weld is loaded in the direction BA, consequently under normal compression. The force is equal to

$$P \frac{AB}{AC}, \text{ and the stress to } P \frac{AB}{AC} : ab,$$

where b represents the length of the weld. In spite of the thickness, 8 mm of the upper welds and 3 mm of the lower, the top ones were the first to fail. The tests which were made of the lower weld could therefore not be extended to a point where the two parts must meet, or even to a shortening of 1 mm. For a shortening of the gap by 0.2 mm however, the stresses were measured as recorded in the table.

We have also examined whether the values obtained by other observers agree with the theory enunciated above. These tests vary but little in the case of load. It is therefore difficult to gauge the theory by these observations; the experiments of the several observers cannot be compared. We have only taken into consideration the experiments which extend over several cases of load. They are the following experiments:

C. Bessel. Tests published in Stahlbau, 1931, N° 23. The results given in the table under "Bessel".

Andrew Vogel. "Journal of the American Welding Society" April, 1929, in the table under "Vogel".

Dresden Tests of Welds. "Der Stahlbau", 1931, N° 12.

Report of tests „Boston Section, Journal of the American Welding Society" in the table under "Boston".

E. Hohn. International Congress on metallic structures at Liege, September, 1930, in the table under "Hohn".

F. P. Mac Gibbon. First Communications of the new International Association for testing materials, page 155. In the table under "*Mac Gibbon*".

Federal laboratories for testing material associated with the E. T. H. Zurich, report N° 86, page 5. In the table under "E. T. H. Zurich".

Report of Structural Steel Welding Committee of the American Bureau of Welding. In the table under R.S.W.

The values given in the table under the designations "Bessel", "Vogel", etc., are the mean of the extreme values in the series.

Let us now compare the above theory with the values given in the table. It will be noticed first of all that in the series of experiments, those specimens have been put first in which the welds are subjected to tension, normal to their smallest section (I, II and VII), subsequently dealing with specimens in which the welds are subjected to an oblique tension acting at a gradually diminishing angle (VIII and IX). Specimens are then given demonstrating resistance to shear, and finally, specimens subjected to normal compression.

If the ratio of the measured ultimate stress of the specimens to that of the standard bars of deposited metal (6^{th} column) be compared, to the calculated ratio (7^{th} column), it will be noticed that the former are a little greater than the latter. Up to specimen XI these ratios are essentially similar. In the case of welds compressed at an angle of 72^0 (XII) as well as in that of welds compressed normally (V and XIII) this ratio is clearly greater than that calculated. The theory is not valid for a compression acting at an angle greater than 45^0. This agrees with the use made in practice of allowing higher stresses for compression than for tension, since the resistance of the weld to compression is greater; this is valid not only for butt welds but also for fillet welds. The compressed welds XIII are very strong; in fact, for a penetration of 0.2 mm the stress measured is already 1.27 times greater than the breaking stress in tension of the deposited metal. In practice the resistance of a joint such as that of specimens XII and XIII will still be much greater because the penetration of the deposited metal in the gap will not be hindered by the introduction of copper plates as was the case for the specimens. It will be important to make tests with welded specimens without preventing the penetration of the deposited metal into the gap which occurs in practice with the joints of columns in which the lower piece has not been properly machined, or for the attachment of girders to the column. (At Dresden some tests of this sort were made with a specimen of type XII, but we only know that the breaking stress[1] was more than 52.5 kg/mm^2.)

Setting aside with specimens XII, V and XIII in which the welds are submitted to compression at an angle of 72^0 or normally, specimens XV present the greatest deviation from the theory. The ratio of the measured breaking stress to the breaking stress of the deposited metal is equal to 0.74, although it should be equal to 0.58 according to the theory. The specimens are therefore $\frac{0.74}{0.58} = 1.24$ times too strong. They have lateral welds subjected to shear and they differ from specimens XIV in that the welded parts are submitted to compression and not to tension, in consequence the welds elongate instead of contracting. The tests of *Vogel* and of *Mac Gibbon* demonstrate that lateral welds between compressed pieces offer greater resistance to compression than to tension. This is explained by the fact that the welds are shortened during cooling, if they are not prevented by the pieces to be joined. The weld is thus subjected before the test to a tension stress, which diminishes when the specimen is submitted to tests of compression. This is not conform to the theory of plasticity, according to which temperature stresses should not have any influence on the resistance to breaking.

The greatest deviations from the theory are found with specimens VIII and VII. The strength in these cases is $\frac{1.07}{0.91} = 1.19$ and $\frac{1.19}{1.00} = 1.19$ respectively, which is more than calculated. The deviation of the remaining specimens I, II, IX, VI, X and XIV amounts up to 10 % between measured rupture stress and the calculated stress from the tensile strength of the weld metal. With one exception (IX) the measured stress was higher.

In general it can be said that the measured rupture stresses are somewhat higher than those derived by calculation from the rupture stress of standard test bars made of weld metal. This applies to both fillet welds and butt welds. The rupture stress varies in the same way as the angle varies under which the force is acting on the smallest section of the weld. A fillet weld of the type VII, if stressed normally, is of great strength.

Conclusions.

From the preceding, we can draw the following conclusions concerning the calculations for welded joints of framework submitted to a statical live load.

1) It is not correct to calculate all fillet welds with the same low permissible stress as prescribed in the German standards "DIN 4100" and the Code for fusion welding and gas cutting in Building Construction.

2) The standards should prescribe the same permissible stresses for fillet welds and for butt welds; the value of these permissible stresses should depend upon the angle formed by the force and the smallest section of the weld.

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[1] By breaking stress is understood the total force acting on the joint divided by the sum of the least sections of the weld.

3) The theory of constant deformation energy gives in an exact manner the ratio existing between normal ultimate tension and oblique ultimate tension; welds submitted to compression, normally or inclined at an angle greater than 45° are much more resistant than the calculations of ultimate tension show.

4) We have made the two following hypotheses:

a) The failure of a fillet weld occurs at the smallest section (CD Fig. 1).

b) In the hyperstatic structures the direction of the force is according to the theory of plasticity, as favocurable as equilibrium permits.

These two hypotheses furnish practically well applicable results.

Summary.

The author starts his report with the theory of application of welded connections. Afterwards a number of tests are described which are compared with the result of other tests, and by this the validity of the theory of constant deformation energy is proved for this field of engineering.