# Wide-span reinforced concrete arch bridges

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# IVb 3

## Wide-span Reinforced Concrete Arch Bridges.

## Weitgespannte Eisenbeton-Bogenbrücken.

## Arcs à grande portée, en béton armé.

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### I. General remarks. Properties of materials. Working stresses. Shape of cross section.

The construction of long span arched bridges in reinforced concrete, and the possibility of further increasing their span, is governed by a variety of considerations. In the first place great importance attaches to the properties of the cement, the strengths obtained in the concrete and the behaviour of the latter (as regards shrinkage and plastic deformation, etc.) after the arch is completed. The form of cross sections adopted for the arch, the relation between the arch and the decking construction from a structural point of view, the ratio of rise to span, the deformation undergone by the arch, the manner of construction and removal of shuttering, and the false arch work, are all matters that demand attention.

Each of these factors is subject to some limit which is used as a theoretical basis for the design of the arch to guarantee the latter against failure. The various factors all become considerably more important than is the case in bridges of medium span and this makes it necessary that all the operative influences should, like the bearing value of the ground, be rigorously checked. Up to a certain point the permissible stresses set a limit to the increase in span, and the answers to the various questions that arise in the construction of long-spanned bridges in reinforced concrete must, therefore, be gathered from investigations of theory, of practice, and of testing technique.

In this paper new suggestions are put forward for the stricter calculation and execution of long span reinforced concrete bridges, and a description is given of the author's design for a bridge of 400 m span for which a new method of construction is proposed. In addition only a few of the effects mentioned above as being relevant to the design and construction of long span bridges are discussed in greater detail. The question of economic comparison with steel arch bridges of large spans is not dealt with.

*a) Properties of materials.*

One thing is certain: if reinforced concrete arch bridges are to be built with still greater spans this can be done only through the use of concrete of considerably higher strength than hitherto, and this in turn implies high-quality cement. It may have been possible, up to now, to equalize the extreme fibre stresses by the adoption of special procedures for relieving the arches, and by this to ensure a better distribution of stress over the arch as a whole; future again looses its importance, because the dead weight of the bridge itself considerably increased, but if so its tensile strength will have to be increased also.

With very large spans, however, this increased tensile strength of the concrete again looses its importance, because the dead weight of the bridge itself considerably exceeds the live load and if the rise is great enough it becomes possible, by adopting a suitable design, to obtain a purely "compression arch".

It is important also to endeavour that the consistency of the concrete should be kept as uniform as possible, although it is impossible to avoid climatic effects on concrete of varying age in the arch. In this paper uniformly worked concrete will be assumed for the purpose of the mathematical investigation of the arch.

By the use of special cements it is already possible to obtain compression strengths in the concrete as high as 600 kg/cm²; in the Traneberg bridge, for instance, the concrete was made with 400 kg/m³ Portland cement and this gave a compressive strength of 620 kg/cm². "Ciment Fondu", which is a rapid hardening cement, appears particularly well adapted to the purpose: in France, by adding 300 kg of cement to a mixture of 1200 liters of sand and gravel, this gave an elastic modulus of 350,000 kg/cm² after 7 days and 450,000 kg/cm² after 28 days (Lossier, Génie Civil, 1923/II, p. 205). These cements have a shrinkage figure of 0.4 mm/m after 30 days and 0.5 mm/m after 6 months, which, however, is greater than the corresponding figure for ordinary Portland cements.

According to No. 227 of Research Work in the Field of Engineering (Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, Beton und Eisen, 1923, p. 4), Graf obtained the following results for elastic moduli of concrete:

| | | | | |
|---|---|---|---|---|
| Strength at 28 days: | 300 | 400 | 500 | 600 kg/cm² |
| $E_b$ | 360,000 | 418,000 | 440,000 | 463,000 ,, |

It is clear from these figures that high cube strengths may be associated with high values for elasticity, and the application of the latter to the construction of arches will be mentioned later.

*b) Permissible stresses.*

On the basis of the foregoing, the permissible stress in the concrete may be increased to 200 kg/cm² assuming that a suitable proportion of high-strength cement or special cement is used. Naturally, however, if stresses of the order of 150 to 200 kg/cm² are to be allowed in the concrete, the repercussions of these high values on the remaining properties of the concrete and particularly on the elastic modulus must first be ascertained. The detailed calculations for the design of an arch of 400 m span which are given here in Section VIII indicate the possibility of working to a permissible stress of 160 kg/cm², and for spans of less than 400 m — unless the arch is particularly flat — an even lower value will suffice. Dischinger has succeeded in designing a three-hinged arch of 260 m span, and the exceptionally low rise ratio of 1/15.4, with a permissible stress in the concrete of 150 $kg/cm^2$ (Bautechnik, 1934, p. 658). Freyssinet, in designing an arch of 1000 m span, went up to a permissible stress of 280 $kg/cm^2$ — a value which, for the near future at least, appears rather high, but 200 $kg/cm^2$ might be justifiable even at present.

*c) Form of cross-section.*

It is clear that the hollow form of cross section for the arch — or in very long spans possibly an open frame arch — are the only types calling for consideration, since the heavy extreme fibre stresses are confined to the top and bottom slabs of the cross section. Whether Freyssinet's "béton traité" involves a qualification of this statement is a question which can be decided only when more detailed information in regard to it is available.

In the construction of arch bridges of long span for which the design follows the pressure line, it is possible to reduce or partially to equalize the maximum stresses under consideration of the three following points:

1) By establishing an axis deviating from the pressure line but keeping the rise unchanged.

2) By consideration of the deformation theory as means for calculation, whereby the rise may undergo a slight change.

3) By special modi operandi, involving the use of hydraulic jacks, when striking the false arch work.

Point 1) is to be chiefly considered for small bridges, and either point 2) or points 1) and 2) together in the case of long span arches.

## II. Reduction of maximum stress in arch bridges by adjusting the axis of the arch (elastic theory).

An adjustment of the axis of the arch to take account of the elastic compression suffered by the latter may easily lead to nil values being obtained for the incremental bending moments at the springing and crown, but, if so, it will also lead to greater moments than exist in an unadjusted arch at a distance of about $^1/_6$ l in the case of a two-hinged arch or at distances of $^1/_{12}$ and $^1/_3$ l in the case of an encastré arch.

In an fixed arch the compression of the arch axis causes an additional horizontal thrust $\Delta$ H, and additional moments which are given by

$$M_x = - H\eta - \Delta H(y - \eta) + \Delta M$$

the latter being particularly important in the case of flat and stiff arches. If the arch is assumed to be cut at one end it is possible to establish factors K for the correction $\eta$ of the arch axis, by calculating the horizontal displacement of the free end of the arch due to the loading $\left(g + \frac{p}{2}\right)$, change of temperature, shrinkage and plastic deformation.

The correction $\eta$ of the ordinates y ot the arch axis can be calculated from a function $\eta = KF(x)$ wherein F(x) represents the equation for the thrust line. For parabolic axes this is a function of the second degree; for arches conforming in shape to the line of thrust it is a function of the fourth or higher degrees or may even be an angular hyperbolic function. The additional moments to be added at the springing and crown remain zero at all these cases, if the correction is chosen: $\eta = 0$.

The maximum values of the additional moments can be reduced if $\eta = \alpha K F(x)$, wherein $\alpha < 1$. Small moments will then be set up in the crown and springing and this leads to a better distribution of the additional moments.

The problem is to determine the amount of adjustment which will cause the additional moments — including those due to the least favourable position of the live load — to be a minimum. This problem is of an indeterminate character since there is a free choice of both $\alpha$ and $F(x)$.

In the case of an encastré arch the adjustment expressed by this function is subject to the condition that the adjusted axis intersects the original axis at the level of the elastic centre. It is impossible, however, to compensate the moments completely at all points of such an arch, and this fact has its analogy where the striking of the false arch work is carried out with the aid of hydraulic jacks.

The solution $\eta = KF(x)$ is due to Campus (International Congress on Reinforced Concrete, Liége 1930, p. 163)[1] and reference may also be made to Chwalla in Mitteilungen des Hauptvereins Deutscher Ingenieure in der Č.S.R., 1935. A different solution has been put forward by M. Ritter (International Congress on Bridge Construction, Zürich 1926), whereby the axis of the arch is determined according to the line of thrust due to dead load with the addition of virtual loads acting upwards, and the moments and normal thrust are calculated by the complementary force method of Mörsch. The values for these virtual loads are found from the pre-determined position of the axis of the arch at the springings and crown. The additional loads occur between the zero points of the summated influence lines appertaining to two symmetrically placed point loads, or in the case of flat arches Ritter makes use of distributed loads. In consequence of the virtual loads the elastic centre of gravity comes a little higher than it otherwise would.

Other methods are described in the literature here cited[1]. Generally speaking, it may be said that the value of $\eta$ is arbitrary, and that the degree of improvement that can be attained, in pursuit of economy in material, depends on the greater or lesser approximation to the ideal.

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## III. Closer calculation of arches and deformation theories.

If the permissible stresses in arch bridges of long span are to be fixed as high as is envisaged here, it follows that especially rigorous methods of calculation must be applied — taking account, for instance, of the real values for the elastic properties of the concrete. This requirement is in no way inconsistent with the fact that the calculated stresses are mere approximations and not mathematical quantities, for the method of dimensioning to be adopted is one of the relevant factors in the problem, and by more rigorous scrutiny of experiments carried out on large bridges it will be possible to build up a progressively clearer understanding of the conditions actually obtaining in arches of this kind.

Known methods of calculation will, therefore, be discussed, together with new investigations on the part of the author.

1) *Calculation of arched bridges by the exponential law.*

The quest for a closer understanding of reinforced concrete arch bridges through the Bach-Schüle exponential law, by applying this in conjunction with elastic theory, leads to calculations of some complexity. For small spans, as Dr. M. Ritter has shown (Schweizerische Bauzeitung 1907/I. p. 25), there is no occasion to apply the exponential law as variations in the extreme fibre stress amount to 2.5% at the most, and are on the safe side $\left(\frac{f}{l} = \frac{1}{10}\right)$.

As yet no numerical evaluations have become known for large spans, but it may be anticipated that their results would show large deviations from those obtained by the ordinary methods of calculation for fixed arches.

The equation for elastic deformation $\varepsilon = \alpha \cdot \sigma^m$ (taking $m = 1.1—1.14$) results in greatly increased values for elastic deformation at the crown of an fixed arch, and this fact is important from the point of view of more rigorous investigation. Straub (Proc. Am. Soc. Civ. Eng., Jan. 1930), considering small spans with an excessively high value of $m = 1.3$, obtained somewhat large deviations of the deformation angles and deflections by comparison with $m = 1$. With $m = 1.3$, however, the sum of the angular changes worked out at practically zero, as is true when $m = 1$. The horizontal displacements of the ends of the arch resulting from the compression of the latter were considerably larger with $m = 1.3$ than with $m = 1$. (Straub: Trans. Am. Soc. Civ. Eng., 1931, p. 665.)

With the arch fully loaded the line of thrust, taking $m = 1.3$, comes closer to the axis of the arch (assumed by Straub as a parabola in all cases). Non-uniformly distributed loads have a greater effect on the shortening of the axis with low values of m than with high values.

Straub's treatment is given for a generalised form of arch of rectangular cross section. In view of the parabolic shape of axis assumed in his illustrative examples, the conclusions reached are valid only for flat arches.

*2) Assumption of an elastic modulus which is uniform over the cross section but varies along the axis of the arch.*

A method which is simpler to apply than the exponential law and is more practical even for large spans — though not quite so accurate — is to assume the validity of Hooke's law with a variable elastic modulus in successive elements of the arch. Such a variation can be justified by the length of time that elapses in the concreting of the arch and, therefore, in the different age of the concrete as between the springing and the crown. Again, the measurements made by Prof. Dr. Roš on the Baden-Wettingen Bridge (Schweiz. Bauzeitung, 1929/I, 2nd March) disclosed variations in the elastic modulus across the bridge: in this pure arch structure the value for $E_C$ was 343,000 kg/cm² at the springing and 284,000 kg/cm² at the crown, but it was not found possible to establish a law governing the variation. The elastic modulus determined from the mean of the extreme fibre stresses was smaller at quarter span of the arch than at either the springing or the crown. The eccentricities of the line of thrust as measured were throughout smaller than as calculated.

In the Hundwiler arch E was 541,000 kg/cm²; at the quarter span points the values found from the stresses at the intrados were 725,000 and 624,000 kg/cm² respectively, comparing with 362,000 kg/cm² at 9 weeks as determined in the laboratory. (Schweiz. Bauztg. 1929/II. 10th Aug.).

It is not clear how far these differing results of measurements may be attributable merely to incidental deviations. At present the measured results at the quarter span points vary too much to admit of systematisation, though the measurements of elongations at these points are fairly consistent. It would be valuable to obtain a check on the determination of E by applying the exponential law to the measured deformations.

The simplest and best grounded assumption for the variation of the elastic modulus — a uniform quality of concrete being assumed — is to relate this quantity to the age of the concrete and to the length of time needed for building the arch. In this way the elastic modulus at the springing $E_K$ works out higher than that at the crown $E_S$ and the change can be taken as linear over half the length of the arch. $E_K$ and $E_S$ can be determined from preliminary experiments.

For an encastré arch the consequence of this assumption is to raise the elastic centre of gravity. The author has obtained the following values of moments for the centre of gravity of sections in an arch of 400 m span with 1:4 rise (shaped to the line of thrust):

With permanent load of 1 ton per m over whole span:

At springing, + 381.56 tm, as against
+ 374.7 tm, when E is constant
$\Delta = +1.8\%$.

At crown, + 119.76 tm, as against
+ 134.7 tm, when E is constant
$\Delta = -11.2\%$.

With load of 1 t/m over half the span:

At left-hand springing, — 2210.00 tm, as against
— 2092.6 tm, when E is constant
$\Delta = +5.8\%$.

At right hand springing, + 2549.56 tm, as against
+ 2467.3 tm, when E ist constant
$\Delta = +5.2\%$.

At crown, + 59.88 tm, as against
+ 67.35 tm, when E is constant
$\Delta = -11.2\%$.

The ordinates of the influence lines for the crown when the arch is cut at the crown, work out as follows:

$X = 0.942$ t (as against 0.923 t)
$Y = 0.500$ t (as against 0.500 t)
$Z = 50.800$ tm (as against 53.400 tm)

corresponding to

$E_K = 470{,}000$ kg/cm$^2$
$E_S = 350{,}000$ „
$E_m = 410{,}000$ „

This shows that, under consideration of a variable E, the moments at springing become greater, and those at the crown smaller, than for a constant value of E.

3) *Variation of the elastic modulus in an arch girder of hollow cross section.*

The construction of large arch bridges in reinforced concrete proceeds by first concreting the lower face, possibly with parts of the walls, over the whole of the span, and the remaining portions of the cross section which then follow are built in the same sequence from the springing towards the centre. Owing to the intervening lapse of time the elastic modulus will, therefore, vary over the depth of the cross section, being greater towards the bottom and smaller towards the top.

If methods of de-centering which have the effect of reducing the maximum stresses are applied to the arch, due regard must be taken of these different elastic moduli, since, when the arch is closed, the weight of the decking and its supports as well as the stresses due to temperature, shrinkage, plasticity and live load will all be imposed on the arch itself.

The relevant calculation will now be given in reference to an encastré arch. $E_1$ and $E_2$ represent the mean values of the elastic moduli at the intrados and extrados respectively and the transition from the one to the other is assumed to follow a straight-line law. The hollow cross section has a total depths 2v and is symmetrical about the horizontal axis.

Writing $K_1 = \frac{E_1}{E_2} + 1, \qquad K_2 = \frac{E_1}{E_2} - 1, \qquad K = \frac{E_1 - E_2}{E_1 + E_2};$

we have for the angular change $\gamma$

$$\tan\gamma = \frac{ds}{2vE_1}\left[\frac{N}{F}K_2 + \frac{Mv}{I}\cdot K_1\right]$$

$$\Delta\, dx = \frac{ds\cdot\cos\varphi\, K_1}{2E_1}\left[\frac{N}{F} + \frac{Mv}{I}\cdot K\right]$$

$$\Delta\, dy = \frac{ds\cdot\sin\varphi\, K_1}{2E_1}\left[\frac{N}{F} + \frac{Mv}{I}\cdot K\right]$$

and the three unknowns are girder by setting $Q_o = \sum_x^{\frac{1}{2}} G$

$$H = \frac{\int\frac{M_o y}{I}\cdot ds - K\int\frac{M_o v\, ds\cos\varphi}{I} + \int\frac{Q_o\, ds\sin\varphi\cos\varphi}{F} - K\int\frac{Q_o y\, ds\sin\varphi}{Fv} + \frac{2E_1\omega t}{K_1}}{\int\frac{y^2 ds}{I} + \int\frac{ds\cos^2\varphi}{F} - K\left[\int\frac{v\, y\, ds\cos\varphi}{I} + \int\frac{y\, ds\cos\varphi}{Fv}\right]}$$

$$V = \frac{\int\frac{M_o x}{I}\cdot ds - K\int\frac{M_o v\, ds\sin\varphi}{I} + \int\frac{Q_o\, ds\sin^2\varphi}{F} - K\int\frac{Q_o x\cdot ds\sin\varphi}{Fv}}{\int\frac{x^2 ds}{I} + \int\frac{ds\sin^2\varphi}{F} - K\left[\int\frac{v\, x\cdot ds\sin\varphi}{I} + \int\frac{x\cdot ds\sin\varphi}{Fv}\right]}$$

$$M = -\frac{\int\frac{M_o\, ds}{I} - K\int\frac{Q_o\, ds\sin\varphi}{Fv}}{\int\frac{ds}{I}} + HK\frac{\left[\int\frac{v\, ds}{I}\cos\varphi - \int\frac{ds\cos\varphi}{Fv}\right]}{\int\frac{ds}{I}}.$$

In the same way account can be taken of a variation in E in every panel from the springing to the crown, if K remains with the $\int$, and if the term $+ 2\omega t\int\frac{E_1\, ds}{K_1}$ expresses the effect of temperature.

These values allow the moments, normal thrusts and stresses in the arch to be calculated.

4) *Deformation theory of the arch under varying E and I.*

A further refinement in the calculation of the arch consists in allowing for variations both of E and of I. This is now given for the first time, constant values for E and I having been assumed in all previous publications on the subject. Here only the final results will be stated and not their derivation, which will be published elsewhere.

It is necessary first to make some assumption as to the mode of variation either of E alone or of E and I together.

If $E_{\varphi o}$ and $I_{\varphi o}$ denote respectively the elastic modulus and the moment of inertia at the springing, E and I the corresponding quantities at the crown, then the transition for an intermediate point x, y, $\varphi$ can be calculated according to a parabolic law, and at any given point in the arch:

$$E_\varphi I_\varphi = E I \left[\frac{E_{\varphi o} I_{\varphi o}}{E I} - \frac{4}{l}\left(\frac{E_{\varphi o} I_{\varphi o}}{E I} - 1\right) x + \frac{4}{l^2}\left(\frac{E_{\varphi o} I_{\varphi o}}{E I} - 1\right) x^2\right]$$
$$= E I [A - B x + D x^2].$$

The axis of the arch is assumed to be a parabola and the origin of the coordinates is taken to be at the left hand springing.

The following is the differential equation for the displacement $\eta$ of the arch:

$$\eta'' = -\frac{H \eta}{E_\varphi I_\varphi} - \frac{H}{E_\varphi I_\varphi} F(x) \qquad \text{with } \frac{H}{E I} = c^2 \text{ this becomes}$$

$$\eta'' + \frac{c^2 \eta}{(A - Bx + Dx^2)} + \frac{c^2 F(x)}{(A - Bx + Dx^2)} = 0 \tag{1}$$

whence F(x) for a fixed loading on any type of arch (three-hinged, two-hinged or fixed may be expressed as follows:

$$F(x) = m + n x + k x^2.$$

The homogeneous equation belongs to the type known as a hyper-geometric differential equation, and as it entails calculations with complex quantities an exponential series has been introduced.

The solution to differential equation (1) is:

$$\eta = -\left(m - \frac{2 A k}{c^2 + 2 D}\right) - \left(n + \frac{2 B k}{c^2 + 2 D}\right) x - \frac{c^2 k}{c^2 + 2 D} x^2 + c_1 \eta_1 + c_2 \eta_2. \tag{2}$$

the values $\eta_1$ and $\eta_2$ being expressible by rapidly convergent exponential series thus:

$$\eta_1 = 1 - a_2 \xi^2 + a_4 \xi^4 - a_6 \xi^6 \ldots$$
$$\eta_2 = \xi - a_3 \xi^3 + a_5 \xi^5 - a_7 \xi^7 \ldots$$

wherein $\xi$ is of the form

$$\xi = rx - r_1 = x \sqrt{D} - \frac{B}{2 \sqrt{D}}.$$

Another possible statement is by means of Fourier series, the unknown quantity H being calculated from an equation of least work:

$$g \int \eta \, dx = \frac{1}{EI} \int \frac{M_x^2 \, ds}{A - Bx + Dx^2} + \frac{1}{EF_m} \int \frac{N_x^2 \, ds}{A - Bx + Dx^2}. \tag{3}$$

H, however, can also be determined from the horizontal displacements of the springings, and this method of calculation can be applied to all forms of arch.

In equation (3), $M_x^2$ is of the form

$$M_x^2 = H^2 (S + S_1 x + S_2 x^2 + S_3 x^3 + S_4 x^4).$$

In the case of a load completely covering the span, the very important second term on the right hand side of equation (3) becomes

$$\frac{H\Phi}{EF_m} \cdot \frac{2\,l\nu}{\varepsilon^2}\left[\left(a-\frac{1}{2}\right)\ln\frac{\nu_2}{\nu_1}+4\nu\sqrt{1+16\nu^2}+\frac{a^2}{8\,(u_1-u_2)}\ln\frac{(\nu_1^{\,2}-u_1)(\nu_2^{\,2}-u_2)}{(\nu_1^{\,2}-u_2)(\nu_2^{\,2}-u_1)}\right]$$

where $\quad \nu_{1,2}=\pm 4\nu+\sqrt{1+16\nu^2}; \quad \varepsilon=\frac{E_{\varphi 0}}{E}-1; \quad a=\frac{16\nu^2}{\varepsilon^2}-1; \quad \nu=\frac{f}{l}$ .

and $u_1$, $u_2$ are the roots of the equation

$$u^2+2u\,(1+2a)+1=0$$

$\Phi$ is the plane of the H-line.

Matters are considerably simplified if the first term on the right hand side of equation (3) is absent, $M_x$ being then related to the un-deformed axis and so becoming zero.

5) *Deformation theory of an fixed arch with axis following the line of thrust.*

So far a parabolic arch axis has been assumed for the purpose of stating the theory of deformation. Since, however, the arch becomes deformed according to the line of thrust even over short spans, rigorous calculation is necessary in respect of all loads and influences arising subsequent to its closure. This calculation, also, is now set forth for the first time.

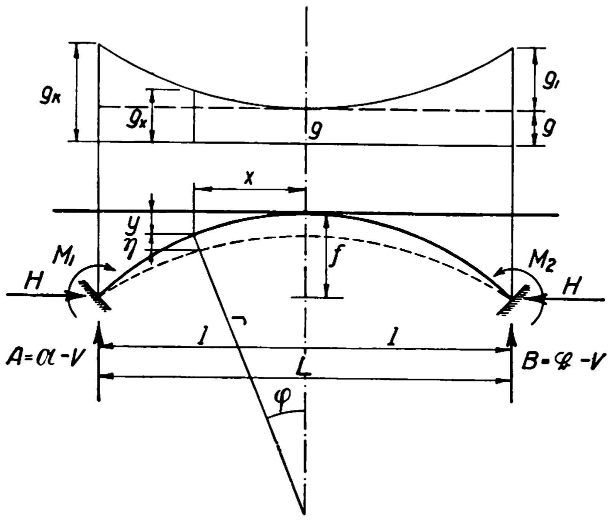


Fig. 1.
left aboutement:
$\mathfrak{A}+V$ instead of
$\mathfrak{A}-V$.

Let the following be assumed as the equation to the arch axis:

$$y=\frac{f}{m-1}(\cosh\alpha x-1)=f\nu\,(\cosh\alpha x-1) \tag{1}$$

wherein $\quad m=\frac{g_x}{g}=\cosh\varkappa; \quad \varkappa=\operatorname{arc\,cosh} m; \quad \varkappa=\alpha\, l; \quad \alpha=\frac{\varkappa}{l}.$

The loading curve is determined by the loads $g$; $g_k$ at the crown and springings respectively (Fig. 1)[1], and a load $g_x$ is assumed to fix the law of transition for any given point x, y, $\varphi$ referred to an origin of co-ordinates at the springing: $g_1 = g_k - g$

$$g_x = (g - g_1 v) + g_1 v \cosh \alpha x = g \cosh \alpha x \tag{2}$$

In a bridge with a large rise the difference between the loads $g_x$ and $g$ can be very large, and for this reason the calculation given below is strongly to be recommended.

The moment is generally

$$M_x = \mathfrak{M}_x + V(l - x) - H[f - (y + \eta)] + M_1 \tag{3}$$

where
$$H\,F(x) = \mathfrak{M}_x + V(l - x) + M_1 - H(f - y) + H \cdot \frac{2\,I}{r\,F_m} \tag{4}$$

$r = \frac{l^2}{8\,f}$ and $c^2 = \frac{H}{E\,I}$ the differential equation is

$$\eta'' + c^2 \eta + c^2 F(x) = 0 \tag{5}$$

and the solution becomes

$$\eta = A \sin c\,x + B \cos c\,x - F(x) + \frac{1}{c^2} F''(x) - R \cosh \alpha x \tag{6}$$

whereby
$$R = \frac{\alpha^4 \left(f\,v - \frac{g}{\alpha^2 H}\right)}{c^2(\alpha^2 + c^2)}$$

Both F(x) and F''(x) are hyperbolic functions. This calculation, while somewhat tedious, involves no great difficulty, and it is one which may well be adopted in the case of a long-span bridge. To facilitate integration it has been deemed preferable to fix the origin of coordinates at the crown.

The moment $\mathfrak{M}_x$ becomes

$$\mathfrak{M}_x = \frac{g}{\alpha^2}(\cosh \alpha\,l - \cosh \alpha\,x) \tag{7}$$

The restraint moment is:

$$M_1 = H\left[B \cos c\,l + f(l + v) - \frac{2I}{r\,F_m} - \frac{\cosh \alpha\,l}{\alpha^2 + c^2}\left(\frac{g}{H} + f\,v\,c^2\right)\right] \tag{8}$$

and the moment $M_x$ is given by

$$M_x = H\left[A \sin c\,x + B \cos c\,x - \frac{2\,I}{r\,F_m} + \left(f\,v - \frac{g}{\alpha^2 H}\right)\frac{\alpha^2}{\alpha^2 + c^2} \cdot \cosh \alpha\,x\right] \tag{9}$$

Under symmetrical loading $V = 0$.

The horizontal thrust H may again be found by trial from the work equation

$$\int_0^l g_x\,\eta\,d\,x = \frac{1}{E\,I}\int_0^l M_x^2\,d\,s + \frac{1}{E\,F}\int N_x^2\,d\,s \tag{10}$$

By means of calculations according to equations (3) to (5) it is possible to ascertain the deformations and statically indeterminate quantities more closely than hitherto.

## IV. Stability of arches against buckling.

In flat three-hinged arches of varying cross section the resistance to buckling can be calculated according to the method of Dischinger (Bautechnik, 1924, p. 739) which is particularly applicable where the moment of inertia in the neighbourhood of the crown or springing is not uniform. In arches of this type the limiting span is smaller than in arches with a larger rise, because of the heavy horizontal thrust developed and the difficulty of adequately anchoring this in the ground; the most suitable final state will, therefore, be an encastré type of arch. As is well known, however, there is some advantage, from the point of view of more uniform distribution of stress, in first building the arch as a three-hinged structure and later converting it permanently to the encastré type. For this purpose, and also for application to arches with a large rise, it is recommended to use the three-hinged arch of constant cross section with an adjusted line of thrust, the hinges being subsequently locked. This method applies especially to very large spans wherein the live load is small compared with the dead load; it is impracticable only for very flat arches of constant cross section.

Since a relatively small thickness of the arch may at first appear to be sufficient, it is important before finally determining this to examine the resistance against buckling, having due regard to the operations it is proposed to carry out with the arch as such, before its resisting moment is increased by the completion of the decking.

The author's formula, as follows, may then be used as a first assumption for the thickness of a rectangular hollow reinforced section:

$$\rho^3 \{B[1-(1-2\gamma)^3] + r\rho l(1-2\gamma)^3 + 3\beta^2 \alpha Bn\} = \frac{Ns(1+4v^2)}{8 E_b A l}$$

from which $\rho = \frac{h}{l}$ is to be determined. Here h denotes the overall height of the hollow cross section of width B. If $2fe = \alpha B h$, the reinforcement percentage is $\alpha = \frac{2fe}{Bh}$; $\beta = \frac{h'}{h}$, h' being the spacing of the steel bars. $\gamma h$ is the thickness of upper and lower slabs and of the side wall; r is the number of walls occurring in a width B; s is the required factor of safety against buckling; N is the thrust at the springing, $v = \frac{f}{l}$; $A = \frac{2+k^2}{8-k^2}$ and in the case of a parabolic arch k is found approximately from the buckling formula $k = \frac{1}{2(1+4v^2)}$.

In the case of arches on which a decking slab is later to be superimposed, a factor of safety of $S < 3$ may be chosen — say $S = 2$ to 2.5 — provided that acceptable experimental values for E have been obtained and that the three-hinged arch is later to be converted to an fixed arch.

If the elastic deformation of the three-hinged arch is taken into account for the purpose of exact calculation, the degree of resistance to buckling under uniformly distributed loading can be found by the method of Fritsche (Bautechnik, 1925, p. 465), which is valid for a flat parabolic arch axis.

The buckling load $H_K$ due to the horizontal force at the crown is

$$H_K = \frac{4\varkappa^2 EI}{l^2}$$

wherein $\varkappa$ can be calculated from the equation $\vartheta = \gamma v^2$

$$\tan\varkappa + 3\vartheta \frac{[\varkappa^2(2\varkappa^2+1) + 16(\sec\varkappa - 1)]}{\varkappa[\varkappa^2(6-7\vartheta) - 120\vartheta]} = 0.$$

In the case of an fixed arch, $\varkappa$ in the equation for $H_K$ may be found from

$$\tan\varkappa - \frac{\varkappa(12+7\vartheta)}{12+\vartheta(6\varkappa^2-12)} = 0.$$

Freyssinet recommends that the crown thickness of an encastré arch should be taken as 1/80 l, having regard to the possibility of buckling in the plane of the bearing walls, and Mesnager has proposed 1/100 l. Maillart made the thickness of the full arch in the Landquart bridge in Klosters equal to 1/115 l at the crown and 1/88 l at the springing (Bauingenieur, 1931, No. 10).

As regards safety against buckling, a hollow form of arch is naturally superior to a solid form. With a large rise it is safe to make the thickness of the arch even smaller than the values given above, and this applies of course to long-spanned bridges.

Safety against buckling must also be checked for the arch as finally completed, assuming unfavourable combinations of live loads.

A more rigorous treatment of buckling conditions the publication of which is pending) can be derived from the author's solutions in the theory of defor-mation given here under III—4, 5.

The problem of buckling is also dealt with by F. Bleich in his Theorie und Berechnung der eisernen Brücken (Theory and Calculation of Steel Bridges), p. 213; Fritsche in Bautechnik, 1925, p. 484; E. Gaber in Bautechnik, 1934, p. 646; and F. Dischinger in Bautechnik, 1934, p. 739. There is need for further investigations of the problem of buckling with E changing in time.

## V. Shrinkage and plastic deformation (creep) of the arch.

In arch bridges of large span an important part is also played by the plastic deformation of the concrete under load — known as flow or creep — because this effect is associated with a sinking of the axis of the arch which gives rise to parasitic stresses.

The significance of shrinkage and plastic flow may be understood from the publication by C. C. Fishburn and J. L. Nagle describing experiments carried out on the Arlington Memorial Bridge (Research Paper R. P. 609, U. S. Standards Journal of Research, Vol. 11, Nov. 1933). In this bridge, an fixed arch of 57.24 m span, the movement at the crown due to this cause was at the end of a year 68 % greater than the temperature effect.

This is a reason in favour of adopting high-strength cements, particularly ciment fondu. The physics of shrinkage and creep has hitherto not been completely explained, though ample numerical data are available on shrinkage and its consequences. Since the time relationships of the contractions due to shrinkage and creep are extremely similar, it would appear that both these properties of concrete derive from a single physical principle, whereof creep constitutes the general case and shrinkage a special case corresponding to the load $P = 0$.

Straub, in his paper (Trans. Am. Soc. Civ. Eng., 1931), has put forward a theory of arches under plastic deformation in which account is taken of the time t and according to which the plastic deformation $\varepsilon_p$ is governed by the law $\varepsilon_p = k\,\sigma^p t^q$. Here $p = 2$, $q = 0.15$ is taken for concrete two weeks old, and $p = 1.25$, $q = 0.4$ for 1 : 2 : 4 concrete after 4 months' hardening. It would be more correct to put p equal to m in the exponential law. For an arch, however, the mathematical developments of this are much more complicated than those of the deformation theory; they are of scientific interest and could be made use of in loading tests. A further point in this connection is the assumption of a superposition, which does not in fact arise: that is to say the angular changes suffered by different points in the arch in consequence of elastic and plastic deformations are added together.

For the present, then, it is better to abstain from introducing the time element into the theory of arches and to frame the calculations on experimentally determined laws for the increase of deformation with time. As regards the amount of the plastic strain, $\varepsilon_p$, it will be possible, according to the anticipated lapse of time in constructing and closing the arch and completing the bridge, to form some impression of the stage likely to be reached in the development of plastic strain at each of these stages of the work, and, therefore, of the balance of plastic effects still to be expected after the construction has been finished.

After a certain lapse of time these deformations come to an end, but at present no accurate information is available regarding the precise time that the creep condition is terminated. The researches of Gehler and Amos, very fully recorded in N. 78 of the German Commission for Reinforced Concrete (Deutscher Ausschuß für Eisenbeton) indicate that the termination occurs after one year, whereas according to Whitney (Journ. Am. Concrete Inst., March 1932), Davis and Glanville, creep does not cease till after four or five years — only very small contractions occurring in the last two years.

According to Gehler and Amos as quoted above, at the end of three months some test specimens reinforced on one side only of the cross section opened out, through plastic deformation alone, by 142% of the amount due to shrinkage when the concrete was compressed to 40 kg/cm², and by as much as 408% of that amount when compressed to 120 kg/cm². After a year the corresponding values were 158% and 365%. It was hoped to arrive at an accurate numerical determination of the magnitude of the shrinkage and creep, but difficulties compression at the end of a year — the 28-day strength being 296 kg/cm². So far as could be determined from experiments made by assuming values of E and n varying in time, measurements being taken in the cracked tension zone of the concrete, the creep value after 150 days under a concrete strength of

40 kg/cm$^2$ amounts to 118% of the shrinkage value, and under a stress of 120 kg/cm$^2$ to 270%. After 270 days the corresponding values are 138% and 300%.

If the shrinkage measurement in reinforced concrete at the end of a year be taken as 0.2 mm/m, the creep measurement under a stress of 40 kg/cm$^2$ will be about 0.28 mm/m, and under 120 kg/cm$^2$ will be 0.6 mm/m.

In bridges of large span these relatively high values are, of course, important; the greater part if not the whole of them must, therefore, be eliminated by the adoption of suitable means of construction. Whatever remains outstanding will take effect after closing the arch. The magnitude of this remainder depends on the time of striking the centres, and therefore, on the span and construction time. From 2/3 to 4/5 of the main effect can always be eliminated.

No danger to the permanence of the arch is present, as the effect eventually comes to an end and the elastic modulus of the concrete increases.

Freyssinet has given the following limiting values for the amount of shrinkage:

With 350 kg cement per m$^3$: $\varepsilon_s = 4$ to $6.\ 10^{-4}$
With 400 kg cement per m$^3$: $\varepsilon_s = 5$ to $7.\ 10^{-4}$
With 450 kg cement per m$^3$: $\varepsilon_s = 6$ to $8.\ 10^{-4}$

(Génie Civil, 1921/II, p. 126) and he proposes to use the values $\varepsilon_s = 0.4$, 0.5 and 0.6 mm/m for reduction of stress in application to his method of construction.

In reference to long span bridges there is still a need for really searching experiments on the amount of shrinkage and plastic deformation occurring in concrete made with different admixtures of high-strength and other special cements.

The stresses caused by shrinkage and plastic deformation can be calculated according to the methods described by M. Ritter[1] or in the author's book.[2]

## VI. Methods of construction and de-centering.

The Spangenberg-Melan system of construction and the Freyssinet system of de-centering are well known. The difficulty with the first mentioned is the great mass of material for pre-loading, that has to be supported in case of long span arches, therefore the limits for the scope of this method lies at about 180 m. Fig. 2 shows a proposal by Melan for the suspension of the centering.

With the Freyssinet system it is impossible completely to balance the extreme fibre stresses on each face of a cross section as the ends of the arch have to be assumed as fixed from the start. Considerable reductions in the surface stresses can, however, be attained, as in the following examples.

Villeneuve-sur-Lot bridge: 31% top of crown, 30% underside of springing,
St. Pierre de Vauvray bridge: 25% top of crown, 29% underside of springing,
St. Bernand bridge: 25% top of crown, 43% underside of springing.

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[1] Dr. *M. Ritter:* Wärme- und Schwindspannungen in eingespannten Gewölben (Temperature- and Shrinkage Stresses in Encastré Arches). Schweiz, Bauztg., Vol. 95, March 1930.

[2] Dr. *A. Hawranek:* Nebenspannungen von Eisenberon-Bogenbrücken (Secondary Stresses in Reinforced Concrete Arch Bridges). W. Ernst and Sohn, Berlin, 1919.

In these bridges, therefore, the maximum stresses were only 57.5 and 76.9 kg/cm²; in the Plougastel bridge the maximum was 75 kg/cm² and at La Roche Guyon 80 kg/cm².

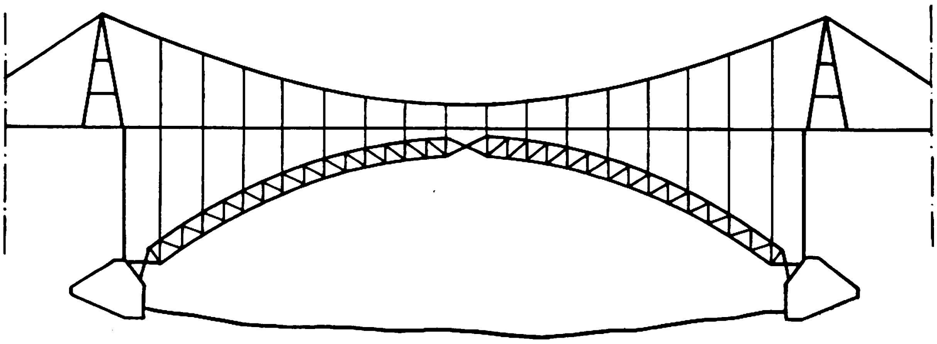

Fig. 2.

The system thus provides a means for increasing the attainable span, and it allows of building a 500 m span without the permissible stress having to exceed 159 kg/cm². Fig. 3 shows a design by Freyssinet (later abandoned) for suspending the shuttering for an arch of 350 m span from wire ropes.

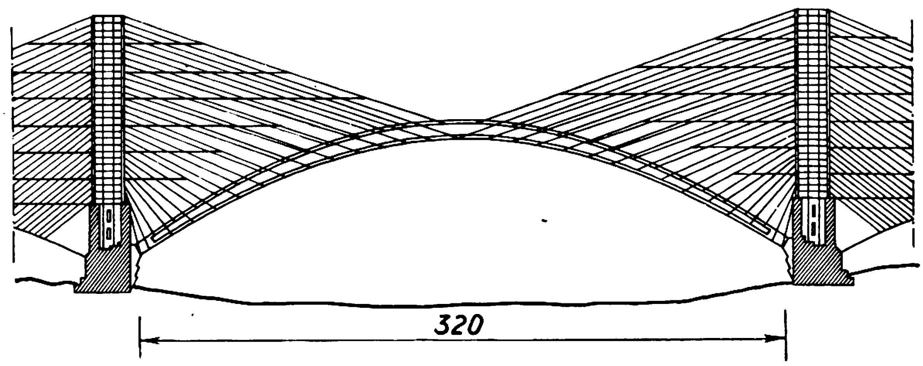


Fig. 3.

Another method of adjusting the position of the arch axis during construction is by the vertical action of hydraulic presses mounted on firm supports, as suggested by Lossier for an arch of 460 m over the Rance (Beton und Eisen, 1931, p. 370). This form of steel centering is shown in Fig. 4.

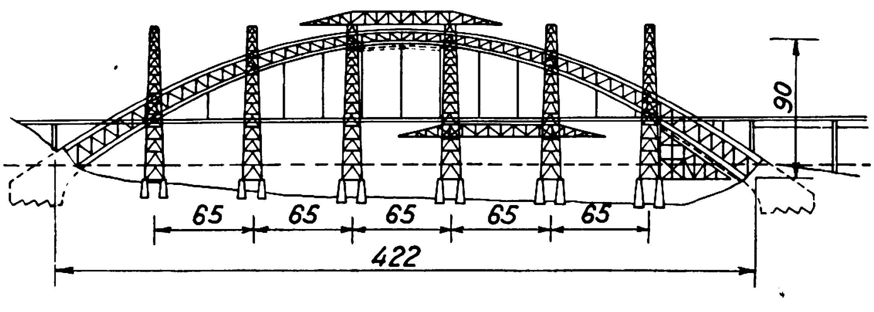


Fig. 4.

Another approach to the problem is that proposed by Dr. Fritz[3] whereby the undesirable stresses that arise in arched beams and arches are avoided by first building a three-hinged arch on fixed supports and later converting this

[3] Dr. *B. Fritz:* Vereinfachte Bestimmung des Einflusses der Systemverformung beim Dreigelenkbogen (Simplified Manner of Ascertaining the Influence of Axial Deformation of a Three-hinged Arch). (Bauingenieur, 1935, Nos. 15/16; Schweiz. Bauztg., 1935/II, p. 277).

to an fixed arch. In the final structure the line of thrust and the axis of the arch are brought practically to coincide by giving a slight excess of height to the two halves of the three-hinged arch before they are subjected to the pressure of the centering, the effect of arch shortening due to dead and live load, the shrinkage effects and the spreading of the supports.

Hinges are built into the skewbacks for later removal after the shrinkage gaps in the arch and crown have been filled with concrete. When the superstructure is complete the centering is struck; shrinkage is compensated by an additional imposed load of $\Delta p_s$ and likewise the displacement of the supports by a load $\Delta p_w$; finally the hinges are taken out of action by the insertion of appropriately shaped voussoir stones.

If the amount of shrinkage allowed for in the calculation does not actually occur, bending moments due to $g+\frac{p}{2}$ will arise in the arch. In the case of large spans the additional loads $\Delta p_s$ and $\Delta p_w$ would reach excessive values, and it is necessary, therefore, to await the occurrence of shrinkage and of displacement of the supports before closing the arch, whereupon the artificial loading required will be only $\frac{p}{2}$.

The waiting period is of relatively long duration, and the displacements of the supports are not completed until the full load has been imposed. To mitigate these objections the hinges may be fixed eccentrically at points corresponding to those through which the line of thrust in the abutments and the crown is to pass.

A similar method has been worked out by Dischinger (Bauing., 1935. Nos. 12—14).

These systems are available as a basis for further development of arch bridge construction, and one more will be described in Section VIII.

## VII. Centering.

The type and cost of the falsework plays a decisive part in the construction of long span bridges, representing as it does a significant part of the total cost of a reinforced concrete arch bridge. Not only the cost but also the nature of the material used for the falsework is of importance. Hitherto it has been the practice to use timber even for bridges up to 187 m span (as in the Elorn Bridge near Plougastel), and this material has been proposed even for still greater spans. Centering has been applied according to the methods usual in small spans, supported on poles arranged to suit the rise of the arch. Alternatively, special arrangements have been followed, such as the nailed segments and framing used by Freyssinet.

In building the Traneberg bridge at Stockholm, which has a span of 181 m, solid-webbed steel arches were used as centering; these were formed of high-tensile steel with a normal permissible stress of 1800 kg/cm² which in this instance was (quite justifiably) increased by 35% to 2430 kg/cm². This centering involved about 1000 tons of steelwork, used twice over by lateral displacement to form the twin arch ribs. Up to the present this represents the only instance of the use of steel centering in a reinforced concrete arch bridge of large span, as distinguished from certain American examples of relatively small span.

There can be no doubt that for fairly large spans (say up to 250 m) the use of timber must be confined to bridges of low rise built either over moderately shallow streams with a firm bottom, or over solid ground, for under any other conditions the weight of the concrete would be excessive. This remains true even if measures are adopted to lighten the structure, on the plan followed with advantage by Freyssinet in the Roche-Guyon bridge of 161 m span — for it will scarcely be possible to adopt that method in application to still greater spans and to particularly high rises of arch: Freyssinet's method was to subject the lower slab of the box-shaped cross section to pressure applied by jacks, thereby ensuring a temporary connection between this slab and the centering so that any tendency of the arch to buckle would be avoided and so that no additional load would be imposed on the centering as the result of subsequent stages in the construction of the arch.

Timber falsework is exposed to heavy wind stresses. Its weight is considerable and a great deal of labour is involved in its erection. In a strong current, or in deep water, it becomes expensive because if the span is a wide one several foundations for the falsework have to be formed and afterwards removed. This is so even if use is made of framed timber arches or of nailed solid-webbed girders on the Lembke system. If the height is considerable the spread of the supports will have to be developed laterally in order to give the necessary stability.

Hence the use of steel — for preference high tensile steel — will become essential for the centering if spans exceeding 200 to 250 m are required. This may take the form either of high tensile rolled steel or of steel cables, and in view of the merely temporary usage the permissible stress in St. 52 may be increased to 2500 kg/cm$^2$ or that in steel cables to 7000 kg/cm$^2$. The use of steel has the great advantage of being independent of all those various questions of bearing pressures at the joints, which have to be considered in the case of timber framing.

Of course steel centering, like any other, should be so designed as to pick up only the unavoidable minimum of the weight of the arch and so as to make the latter self-supporting as soon as possible.

A proposal put forward in reference to a design by the present author is reproduced in Section VIII.

If the use of steel arches as centering for spans even exceeding 200 m should prove feasible at all, it will be necessary to allow for say 4000 tons of steel in the case of 400 m span, even assuming that the centering is to be moved sideways for concreting twin arches in succession. The erection of such a structure, and still more this lateral displacement, will present difficulties. In the case of a flat arch the weight of steel will be greater still, and in any case additional stiffening will be required to resist buckling in either of the main dimensions, and wind loads.

It will be necessary, therefore, to replace the steel arch by a suspended construction or by some suitable combination of a supporting with a suspended structure. The relative advantage of any one such solution compared with another must be decided by cost. The sort of suspension, that may be envisaged, is an anchored cable, having a rise of $^1/_{10}$ to $^1/_{15}$ of the span, from which the suspenders and the shuttering would be hung: this, however, proved uneconomical when examined by complete calculations having reference to the design of a bridge of 400 m span with cable construction of the same span.

In this instance, however, it was found that economy could be attained by combining the use of a substructure projecting 88 m from each of the abutments with a suspended construction to carry the shuttering in the intervening 224 m. This is shown in Fig. 5, and it should be emphasized that the arch in question had a rise of $^1/_4$ of the span. The cable can thus be anchored directly into the abutment of the arch so that no special anchoring block is needed. Moreover, by taking the cable below the crown of the arch the height of the pylons is reduced, and the central portion between the intersections of the cable with the arch can be utilised for stiffening, besides having the effect of reducing the total length of cable.

The deformations that occur in the cable when the arch is concreted are, like the temperature effects, accurately calculable, and may be compensated as the work progresses by means of turnbuckles in the suspenders; alternatively they may be corrected by the action of hydraulic jacks under the cable saddles on the towers.

These conditions entail no difficulty in concreting, for similar deformations occur if steel arch centering is used, and if joints are left open in the concrete the shrinkage effects may be facilitated.

Finally, Fig. 6 shows a proposal for a stiffened suspension construction applied to smaller spans. This would take the form of a two-hinged or rigid frame, the top members of which could later be incorporated as main longitudinal girders in the stiffened decking of the bridge. Such frames offer the further advantage that they could be used to support the shuttering and to provide a horizontal passageway for the supply of concrete and other material. Moreover they would have a considerable effect in reducing the deformations.

The elastic-theory and the deformation-theory of such a suspension bridge with a rigid frame has been given by the author[1].

It will be clear from the above that the combined use of concrete and steel is not only promising, if greater spans than hitherto are to be obtained, but is both necessary and economical.

## VIII. A proposed new method of construction for long span arch bridges in reinforced concrete.

*Design for such a bridge of 400 m span.*

In order to reduce the costs of shuttering and concreting in an arch of this magnitude, the proposal is made that it should be built in the form of

[1] Dr. *A. Hawranek:* Hängebrücke mit einem Zweigelenkrahmen als Versteifungsträger (Suspension Bridge with two-hinged Frame Stiffening girden). (Deformation Theory of this System.) Stahlbau 1935.

Dr. *A. Hawranek:* Verformungstheorie dieses Systems. "Publications" of the Internat. Association for Bridge- and Structural Engineering. Vol. III, 1935.

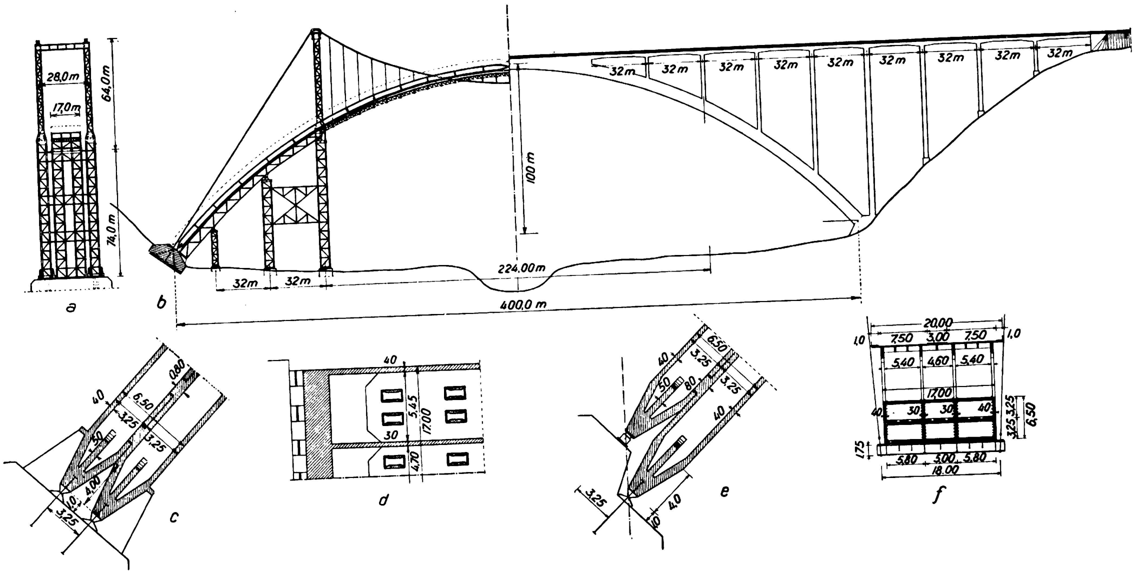
28,0m
17,0m
64,0m
74,0m
a
b
32m
32m
224,00m
400,0 m
100 m
32m 32m 32m 32m 32m 32m 32m 32m 32m
c
d
e
f
20,00
7,50 3,00 7,50
5,40 4,60 5,40
12,00
5,80 5,00 5,80
18,00

Fig. 5.

two parallel superimposed arches. The first of these would be concreted on centering; the centering would then be struck and the first arch used as a support for building the second; finally both arches would be connected together so as to work as a single encastré arch.

If each of the two arches were first formed as a three-hinged arch it would be free to adjust itself to the effect of axial contraction, and also — to an extent governed by the time of closing the arch — to the effects of shrinkage and plastic strain. As an example, an arch of 400 m span has been designed, having the same rise of 100 m as the design published by Prof. Dischinger in Bauingenieur, 1935, Nos. 11—14. The support of the decking has also

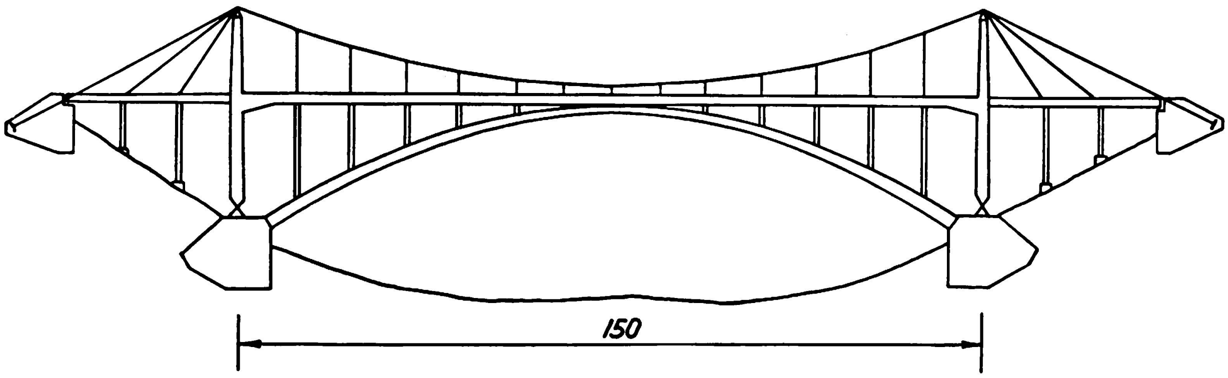


Fig. 6.

been arranged as in that example so as to make the results of the calculations fully comparable (Fig. 5). The proposed new form of falswork will be described presently.

1) *Lower arch.*

The first arch B must first be imagined as formed under its own weight or under a loading calculated to allow the maximum possible reduction of stress, and a constant height and shape of cross section will be assumed. The cross section F is symmetrical about each of the principal directions. The arch rests upon steel hinges in the springings, and provision is made for the temporary presence of hydraulic jacks in the crown. The arch centering is given an excess height in accordance with its own deformation, the elastic contraction of the arch axis, the shrinkage effect, and the plastic strain in the arch. When the arch has been completed it is lifted off the centering by the action of the jacks, thereby sustaining the elastic axial compression due to its own weight.

Alternatively, if the centering is of the suspended type, it may be struck by the direct use of the turnbuckles in the suspension bars, or if it is supported from below it may be operated by vertical jacks.

*Temperature effects.* It will not always be possible to ensure that closure of the arch takes place at a moment, when the temperature happens to be $10^{0}$ C, the average for Central Europe. If, then, the temperature effects in the fixed arch are to correspond to equal maximum positive and maximum negative variations in either direction, the difference between the mean temperature and the temperature obtaining at the time the centering is struck must be compensated by lifting or dropping the crown accordingly.

This will be successfully accomplished only if the three-hinged arch can be allowed to continue to act as such until some occasion when the mean temperature is actually attained, the closure of the arch being effected at that moment. Premature closure will alter the system, making it statically indeterminate like a two-hinged or an encastré arch, and causing a difference in the maximum values of temperature variation as between the positive and negative directions, so that compensation of stress will be more difficult to secure.

In practice only slight changes in pressure will be undergone by jacks inserted axially at the crown. The necessary alteration in the height of the axis of the three-hinged arch to correspond with these temperature effects may effectively be made by a vertical adjustment of the height of the centering, and if the latter is of the suspended type the turnbuckles will provide a simple means of doing so.

*Heat of setting* in concrete will probably be apparent only in those portions of the arch, which have most recently been concreted, but no certain means has yet been devised for taking account of this.

Given proper attention to all the measures here indicated in connection with the various effects, the lower arch will be the more favourably stressed of the two.

After plates have been inserted close to the jacks, symmetrically about the axis of the arch, and some of the load has been removed with these in position, the three-hinged arch thereby freed from the centering will itself become available to serve as shuttering for the second arch to be built above.

2) *Upper arch.*

The upper arch, of exactly similar dimensions, can now be concreted and bedded on top of the lower one. The necessary corrections must be applied in the same way, taking account, as before, of the age of the component portions, the relevant amount of shrinkage, and the temperatures in question.

Should it happen that the axis of the upper arch, on completion, fails to run parallel to that of the lower arch, contact between the two arches can be perfected by making suitable slight alterations in the pressures respectively applied in them.

The question whether, assuming opposed pressures in the haunches of the two arches, the pressure exerted by the upper jacks would be increased or that exerted by the lower jacks decreased if the distance between the arch axes were greater at the haunches than at the crown or springings, is one that depends on the degree of success attained in compensating the maximum stresses with the arch in its final condition.

When these processes of assimilation have been completed the two arches are connected by the casting of concrete in those portions which key into one another (Fig. 5d) so that they will work together. In this way a single arch of double thickness is constituted in which the stresses are fairly well distributed and the bending moments correspondingly small.

3) *The bonded double arch.*

By reason of the jacks still existing within the bonded double arch, the latter is susceptible to stress-adjustments, as required, in accordance with the loading later to be imposed. For this purpose the double hinges in the skewbacks continue to be useful as their arrangement confers rigidity on the ends of the arch.

Another possibility would be to build in jacks instead of hinges at the springings of the upper arch, so as to enable later adjustment of the line of thrust in the combined arch.

The decking and spandrels can now be concreted, a gap being left over the crown of the arch. With the aid of the jacks it becomes possible to level out the stresses very completely, taking due account of the loads arising after the completion of the bonded double arch. The relevant factors are the weight of the decking and spandrels, the live load, such further shrinkage effects as have not already been allowed for, further plastic deformation, wind and temperature effects.

Since two or more superimposed rows of jacks are available in the crown it is possible to reduce the maximum stresses by the application of different pre-calculated pressures in the several rows. If, in addition, jacks are provided in the springings, the line of thrust can be still more completely controlled.

The closure of the arch and the concreting of the crown, the skewbacks and the gap in the decking can then follow. At the same time the reinforcing bars necessary for the encastré action of the arch will be embedded in concrete at the places where the action of the hinges is to be discontinued.

The question of whether the full arch should be closed immediately on completion of the two component arches, or not until the decking has been built, must depend on the span and rise and the available forces of the jacks and mainly on the relation between the dead and live loads.

4) *The bond of the double arch.*

Since the two separately constructed arches are later to form a single whole it is equally necessary to prevent their mutual displacement along the line of contact and to ensure that this contact will hold good for ever.

To attain the first of these objects the extrados of the first arch is provided with several rows of transverse reinforced projections, dovetailed in section, which are concreted together with the arch itself (Fig. 5c, d). Corresponding gaps are left in the bottom slab of the upper arch, and the intervening space is filled with concrete only after both arches have been fully adjusted. This operation is made possible by the provision of manholes in the cross walls and in the top slab of the upper arch, so providing access and even making it possible to remove the inside shuttering.

There is no difficulty in forming these cross-pieces to act as dowels, as they are made on the extrados of the lower arch, and the same applies to the gaps left in the intrados of the upper arch which, since the operation is accomplished while the lower slab is being concreted, is easily accessible.

The vertical bonding of the arch is effected by radial anchor bolts placed close to the inside cross ribs and passing through both of the component arches.

The connection may also be ensured by means of the longitudinal walls themselves.

5) *Position of the skewback hinges.*

The skewback hinges may be arranged in a gap perpendicular to the axis of the arch. The spans of the two component arches will then be slightly different and their respective rises will also differ appreciably (Fig. 5 c).

Alternatively the hinges in each springing may be arranged vertically one above the other, in which case each of the component arches will have the same span (Fig. 5 e) and will receive almost the same stresses due to its own weight, so that the same procedure may be followed as regards its completion (apart from measures necessitated by shrinkage and creep.

Another possibility would be to arrange the skewback hinges, and perhaps also the crown hinge, eccentrically from the beginning.

6) *Corbelled hinges.*

For large spans there is great advantage in the use of projecting temporary hinges. Such projections from the abutments give the possibility of thickening the arch at its ends so as to make it better able to whitstand wind and temperature stresses after its closure; moreover, in a narrow bridge, stability can be increased by widening the corbel. The projecting portions can be built on fixed scaffolding which is relatively low. The arch itself may be made thinner, and the span of the three-hinged arch in the first phase of the construction is reduced. Finally, the saving in weight enables the dimensions of the falsework for the three-hinged arch to be made smaller.

The hinges are thus situated at points where the surface stresses in the final the fixed arch will be lower than either at the springing or at the crown, and where the full permissible stresses cannot be utilised: hence the additional reinforcing bars required when the hinges are taken out of action and the structure is converted to an encastré arch may be reduced in cross section. There is a saving in this respect also, and the operation of embedding the additional bars in the arch is rendered easier.

Finally, the steel hinges are reduced in weight in accordance with the lighter thrust at the springing resulting from the smaller span.

The advantage of corbelled hinges is particularly well marked in the case of bridges with a large rise, partly because in such cases the distance that the hinges can be brought forward is relatively great.

Corbelled hinges are equally effective as permanent hinges or as temporary hinges in such arches.

The many advantages offered by the system which is here proposed, and the variety of the adjustments it makes possible by suitable arrangement of the details, commend it as a means for the construction of long span arch bridges in reinforced concrete.

## Summary.

The paper deals chiefly with the properties of materials as required in the construction of long-span arch bridges. It is ascertained that at the present time an admissible stress limit of 200 $kg/cm^2$ is attainable in the concrete. The question of reducing maximum stresses in arch bridges by means of correcting the axis of the arch is discussed and new methods for the stricter calculation of arches on the deformation theory investigated. New general formulae are elaborated for the variability of the modulus of elasticity E in a hollow arch section, the deformation theory for arch bridges with variable E and I, and the deformation theory for an encastred arch with a funicular axis curve for effective loads. In every case the solutions to the differential equations and the formulae for the deformations and moments are published here for the first time. Then the influence of shrinkage and creep in concrete arch bridges is considered.

For the false archwork of large concrete arch bridges a steel construction, parts of which are suspended, is proposed and a new method of erecting an arch of 400 m span given. This method consists in constructing an arch of half the final thickness, which, on removal of the formwork, serves to carry a second, parallel arch which is superimposed upon it. When completed both arches are connected to give united action. By the combined use of provisory hinges and hydraulic jacks it is possible to eliminate the effect of shrinkage and creep in concrete. Finally, the advantage of provisional corbelled hinges is discussed.

No comparison is drawn between reinforced concrete and steel arch bridges.

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