Development of the analysis of the arch dams

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Development of the Analysis of Arch Dams.

Entwicklung der Berechnung von Bogen-Staumauern.

Le développement du calcul des barrages arqués.

Zd. Bažant,

Professor of Structural Mechanics, Czech Polytechnic School. Prague.

Introduction.

Masonry dams were in the beginning executed straight and calculated as vertical cantilevers between two vertical crossections, fixed in the foundations, loaded with the weight of masonry and water pressure; both loads produce combined compression and bending stresses which cause the strength of masonry to be but little utilised, especially as the tensile strength of masonry is neglected. To eliminate the dangerous effect of temperature changes, a slight curvature of dams was later used; but the stresses were computed as if the dam were straight. It was supposed that the curved dam accomodated itself to the changes produced in its length, which are the consequence of temperature changes, by a change in its curvature. The dam also being fixed at the abutments in a horizontal direction, it was supposed that there was additional safety both for the weight of masonry and for the water pressure. But a detailed statical investigation showed that a slight curvature of the dam has not the favorable consequences expected, because the usual computation gives great thickness. For, if the dam is considered as a horizontal arch under water pressure, the computation gives, with a slight curvature and a great thickness of arch, tensions at the abutments in the extrados and at the crown in the intrados, which can produce vertical cracks in the masonry of the dam.[1] Though the strenghtening of the dam will obviate cracks in horizontal joints, cracks in vertical joints may nevertheless occur; the strengthening of the dam with a surplus of masonry is only apparent as the masonry is not rightly located.

Analysis of Arch Dam as a System of Independent Horizontal Arches.

An arch is, in comparison to a cantilever, a much better structural element as it permits, given a right disposition, a much more uniform distribution of stresses on the masonry and a better use of its strength. The first conscious application of it was made about 1800 in the Meer Allum Dam at Hyderabad in India[2] with 21 horizontal arches between vertical buttresses and in 1845 in the dam built after the project of *M. Zola*[2] near Aix in France in a narrow valley and having the shape of one single horizontal arch.

The following considerations refer as a rule the up-stram face of the dam vertical.

The analysis of arch dams considered, at first approximately, the horizontal arches in different heights as independent arches, loaded with the whole radial water pressure, uniformly distributed along the length of the arch. This method makes no allowance for the mutual connection in the vertical direction; it therefore disregards the shearing stresses in horizontal planes between adjacent arches, which are the consequence of various horizontal displacements. When the reservoir is empty, the weight of the upper arches acts vertically upon the lower arches as in a straight dam; when the reservoir is full, this method considers horizontal elements as independent arches, each of which bears its full water pressure. If the up-stream face is inclined, the vertical component of water pressure adds to the weight of masonry[3]. *Deloore,* who made the first theoretical analysis of arch dams,[4] supposed approximately that the resultant of stresses caussed by water pressure in the crown and abutment joint passes through a point distant by one-third thickness from the up-stream face. *Pelletreas*[5] supposes for uniformly distributed radial water pressure the circular centre line of arch as a pressure line (an for thin cylindrical shells equally loaded), he therefore assumes a uniform pressure in all sections of the arch. This method was then customary, especially in America in the majority of cases, and the many arch dams in Australia were calculated in this manner, which is still advocated by *H. Hawgood*[6]. The dams calculated by this method proved very safe. The transmission of external forces by arch action causes a much better division of stresses and a very considerable diminution of thickness compared with dams opposing to water pressure only the weight of masonry as vertical cantilevers, which are therefore very uneconomical as regards the division of stresses and the utilisation of masonry strength.

R. Ruffieux[7] first calculated the horizontal arch of arch dams as an elastic arch with fixed ends (according to the theory of *J. Résal*), taking also into account the effect of normal stresses, which is very essential here, and using the theorie of the *thin arch.* The same method was used later by *E. Mörsch*[8], *H. Ritter*[9], *C. Guidi*[10], *W. Cain*[11], *R. Kelen*[12] and *G. Ippolito*[13].

In the analysis of an arch dam as a system of independent horizontal arches, the usual assumption was, as for thin cylindrical shells, that the stresses are uniformly distributed throughout the thickness t, that is, the circular centre line was supposed as pressure line to the uniformly distributed radial pressure p_2 on the extrados of arch with radius r_2 (fig. 1). That gives in each section a thrust $N_o = - p_2 r_2$ or a stress

$$v_0 = \frac{N_0}{A} = - \frac{p_2 r_2}{b t}, \tag{1}$$

for an arch of a length b, area of section $A = bt$; the thrust N and the arch stress v are positive, if they are tensions. Instead of pressure p_2 on the extrados, a radial pressure p uniformly distributed along the centre line with radius r can be considered; it is

$$p = p_2 \frac{r_2}{r} \tag{2}$$

This method also corresponds to the analysis of an elastic arch, neglecting the effect of thrust (the shortening of centre line), for then the stress centre coincide with the centre of sections. But a detailed investigation showed that in this case the effect of thrust cannot be neglected even for higher arches. The thrust shortens the centre line, which would change in a circle of shorter radius if the abutments were free; since the width of arch does not change with fixed abutments, the arch cannot remain circular and therefore the centres of stresses must depart from centres of sections. The centre line, deformed by the thrust N_o, if the abutments are assumed to be free, can be brought into shape where the abutments come into their original position, by adding a horizontal force ΔH, going through centre of gravity of the centre line, as an additional reaction acting in both abutments in the outward direction[8] (fig. 1); its value is generally

$$\Delta H = \frac{N_o \int \frac{\cos \varphi \, ds}{A}}{\int \frac{y^2 \, ds}{J} + \int \frac{\cos^2 \varphi \, ds}{A}} \tag{3}$$

for constant thickness

$$\Delta H = \frac{N_o \, l}{\frac{A}{J} \int y^2 \, ds + \int \cos^2 \varphi \, ds} = \frac{N_o \, l}{\left(\frac{12 r^2}{t^2} + 1\right)\left[\frac{l}{2r}(r - h) + \frac{s}{2}\right] - \frac{12 l^2 r^2}{s t^2}} \tag{3a}$$

if A = area of section, J = moment of inertia and $s = 2r\,\alpha$ = length of centre line.

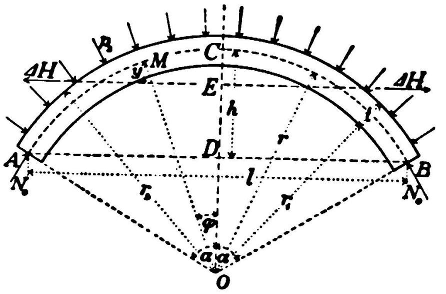

Fig. 1.

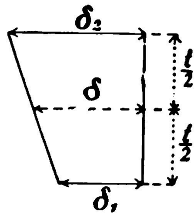

Fig. 2.

The temperature change, equal at all points of arch, gives a horizontal force

$$H_t = \frac{\delta \, \varepsilon \, E \, l}{\frac{1}{J} \int y^2 \, ds + \frac{1}{A} \int \cos^2 \varphi \, ds} \tag{4}$$

where δ = temperature change, ε = coefficient of temperature expansion, E = modulus of elasticity. If the change of temperature varies linearly from δ_1 at the intrados to δ_2 at the extrados (fig. 2) in all the sections, it produces in the abutments only a bending moment

$$M_t' = -(\delta_2 - \delta_1)\, \varepsilon \, E \, \frac{\int \frac{ds}{t}}{\int \frac{ds}{J}} \tag{5}$$

for constant thickness

$$M_t' = -(\delta_2 - \delta_1)\,\varepsilon\, E\,\frac{J}{t} \tag{5a}$$

A detailed analysis of the case, already made by *H. Ritter*[9] and later by *A. Stucky*[14], showed that also shear has an influence which can be of considerable importance for flat arches. The denominator in the formula (3) for ΔH has a general value

$$\int \frac{y\,y'\,ds}{J} + \int \frac{\cos^2\varphi\,ds}{A} + \beta\frac{E}{G}\int\frac{\sin^2\varphi\,ds}{A}$$

if β = reduction coefficient of shear $\left(\text{for a rectangular section} = \frac{6}{5}\right)$, G = shearing modulus, y' = ordinate of the antipole of the axis of gravity of the centre line with respect to the ellipse of elasticity of the element of arch. For isotropical substances $\frac{E}{G} = 2.5$, therefore $\beta\,\frac{E}{G} = 3$. For thin arches approximately $y' = y$ and for constant thickness the value of ΔH becomes

$$\Delta H = -\frac{p\,r\,t^2}{C_1 r^2 + C_2 t^2};\; C_1 = 6\left(\cos\alpha + \frac{\alpha}{\sin\alpha} - \frac{2\sin\alpha}{\alpha}\right);\; C_2 = \frac{2\,\alpha}{\sin\alpha} + \cos\alpha. \tag{3b}$$

H. Ritter[9] has computed tables for C_1, C_2 which facilitate the calculation. A constant temperature change produces the horizontal force

$$H_t = \frac{\delta\,\varepsilon\,E\,t^3}{C_1 r^2 + C_2 t^2} \tag{4a}$$

acting in the axis of gravity of the centre line. *Ritter* determines the effect of temperature change also in the case when temperature varies in the section continually, after a curve from zero at the extrados to maximum at the intrados. If (reservoir being empty), the temperature change in the section is symmetrical to its centre, the horizontal force H_t has the value (4a), for δ being the mean temperature change in the section.

Very detailed is the analysis of arches under radial loads in the article by *W. Cain*[11] and in the following discussion, further in the article by *F. A. Noetzli*[15] and in the discussion on it. *W. Cain* published in his article and in his conclusion of the discussion[16] the final formulas for calculation of fixed arches under uniformly distributed normal loads (fig. 3), as follows. The thrust H_c at the crown is given by

$$X = p\,r - H_c = \frac{p\,r}{\vartheta}\cdot 2\,\frac{i^2}{r^2}\,\alpha\sin\alpha, \tag{6}$$

where

$$\vartheta = \left(1 + \frac{i^2}{r^2}\right)\alpha\left(\alpha + \frac{1}{2}\sin 2\alpha\right) - 2\sin^2\alpha + 2.88\,\frac{i^2}{r^2}\,\alpha\left(\alpha - \frac{1}{2}\sin 2\alpha\right); \tag{6a}$$

i = radius of gyration $\left(i^2 = \frac{1}{12}t^2\right)$; the numerical factor $2.88 = \beta\,\frac{E}{G}$ with $\frac{E}{G} = 2.4$ for concrete (instead of $\frac{E}{G} = 2.5$ for isotropical substances) and $\beta = \frac{6}{5}$

for rectangular section. The member with the factor 2.88 comes from shear; the effect of shear can be neglected for central angles $90^0 < 2\alpha < 120^0$, but for smaller central angles and for large proportions t/r the effect of shear can be great enough. For the point M of the arc, given by the angle with the axis of symmetry, the thrust (positive for tension) is

$$N = X\cos\varphi - p\,r, \tag{7}$$

the shear

$$T = X\sin\varphi \tag{8}$$

and the bending moment (positive when clockwise for forces on the left side)

$$M = -X\,r\left(\frac{\sin\alpha}{\alpha} - \cos\varphi\right); \tag{9}$$

that is, the moment to the point M of a force X acting to the right in the centre of gravity E of the centre line, if it is the effect of the right portion, because the centre of gravity E is given by the distance $OE = \frac{r\cdot\sin\alpha}{\alpha}$. These results signify that in each section the force X, acting in the centre of gravity E, adds to the thrust $N_o = -p\,r = -p_2\,r_2$.

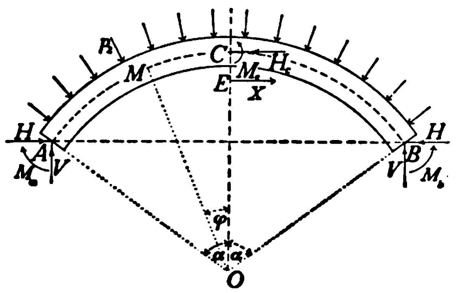

Fig. 3.

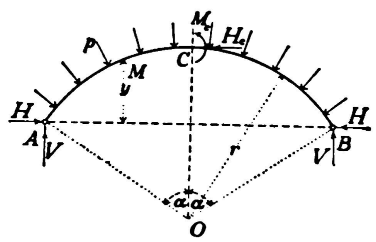

Fig. 4.

The uniform radial loads produce the deflection in the crown of arch (positive toward the centre of arch)

$$\eta = \omega\,\frac{p\,r^2}{E\,t}; \tag{10}$$

where

$$\omega = \frac{\alpha}{\vartheta}(1-\cos\alpha)\left[\left(1+\frac{i^2}{r^2}\right)(\alpha-\sin\alpha) + 2.88\,\frac{i^2}{r^2}(\alpha+\sin\alpha)\right] \tag{10a}$$

A temperature change δ, equal at all points of arch, gives a horizontal force

$$H_t = \delta\,\varepsilon\,\frac{E\,J}{r^2}\cdot\frac{2\,\alpha\sin\alpha}{\vartheta} \tag{11}$$

going through centre of gravity of the centre line, and a deflection of the crown section

$$\eta t = -\,\omega\cdot\delta\,\varepsilon\,r; \tag{12}$$

ω is the coefficient given by (10a). A good check of the foregoing equations is that for $\alpha = 0$ they become equations for a straight beam fixed at both ends; it follows by substituting infinite series for sin and cos and limiting for $\alpha = 0$.

Is the connection between arch and foundation not rigid (anchoring of reinforcing bars), the arch at the abutments can crack and it approches the arch with two hinges, especially in a thin arch. In this case (fig. 4), neglecting the influence of shear, as is possible with a thin arch, it works out that

$$X = p\,r - H_c = \frac{p\,r}{\vartheta'} \cdot 2\frac{i^2}{r^2}\sin\alpha, \tag{13}$$

$$\vartheta' = \alpha\,(2 + \cos 2\,\alpha) - \frac{3}{2}\sin 2\,\alpha + \frac{i^2}{r^2}\left(\alpha + \frac{1}{2}\sin 2\,\alpha\right), \tag{13a}$$

$$M = X\,y \tag{14}$$

For N and T we have equations (7), (8). The deflection of the crown of arch is

$$\eta = \omega' \cdot \frac{p\,r^2}{E\,t} \tag{15}$$

$$\omega' = 1 - \frac{\cos\alpha}{\vartheta'}\left[\sin\alpha + \alpha\,(1 - 2\cos\alpha) + \frac{i^2}{r^2}\,(\alpha - \sin\alpha)\right] \tag{15a}$$

A temperature change, equal at all points of the arch, produces in the abutments horizontal reactions

$$H_t = \delta\,\varepsilon \cdot \frac{E\,J}{r^2} \cdot \frac{2\sin\alpha}{\vartheta'} \tag{16}$$

and the deflection of the crown

$$\eta_t = -\,\omega' \cdot \delta\,\varepsilon\,r. \tag{17}$$

Cam. Guidi[10] transformed the equations for a hingeless arch, introducing lengths instead of goniometrical functions. To the thrust $N = -\,p_2\,r_2 = -\,p\,r$ in all sections there comes in both abutments an additional horizontal reaction going through the centre of gravity of centre line; its value is (see fig. 1)

$$\Delta H = -\frac{p\,r}{\vartheta''} \cdot 2\,\frac{i^2}{r^2}; \tag{18}$$

$$\vartheta'' = \frac{s}{l} + \frac{r - h}{r} - \frac{2\,l}{s} + 2\frac{i^2}{r^2}\left(2\frac{s}{l} - \frac{r - h}{r}\right) \tag{18a}$$

The result represents the effects of the bending moment, the thrust and the shear with $\beta\,\frac{E}{G} = 3$ (as for isotropical substances). An equal temperature change δ at all points of arch gives

$$H_t = \frac{\delta\,\varepsilon\,E\,t^3}{6\,\vartheta''\,r^2} \tag{11a}$$

acting in the axis of gravity of centre line. A uniform radial water pressure produces the deflection of the crown of arch

$$\eta = \frac{p\,r}{E\,t}\,h\left\{1 + \frac{1}{\vartheta''}\left[2\,\frac{l}{s} - \frac{l^2}{4\,h\,r}\left(1 - 2\,\frac{i^2}{r^2}\right)\right]\right\} \tag{10b}$$

The deflection produced by a constant temperature change is

$$\eta_t = \eta \cdot \frac{\delta \varepsilon E t}{p r}; \tag{12a}$$

this coincides with the equation (12) of *Cain. Guidi* facilitates the calculations by means of numerous tables giving for different values of the central angle 2α the values of $\frac{s}{r}, \frac{s}{l}, \frac{l}{s}, \frac{l}{2r}, \eta : \frac{pr}{E}$. He also analyses the non-uniform water pressure, which appears with inclined axes (surface lines) of arches in multiple-arch dams, and the effect of dead load for an arch with inclined axis, the arch of variable section and the butresses of multiple-arch dams. *H. Ritter*[9] analyses the arch of general form and with variable section.

A rapid preliminary calculation can be based on simple formulas given by *F. A. Noetzli*[17]. He neglects the effect of thrust and shear, replaces the centre line approximately with a parabola and neglects the difference between the length of arc and chord, assuming a low arch; thus he gets

$$\Delta H = -0.94\, p_2\, r_2 \frac{t^2}{h^2}. \tag{19}$$

More accurate would be, instead of 0.94, the coefficient

$$k_t = \frac{h^2 l}{t^3 \left(\int \frac{y^2\, ds}{J} + \int \frac{ds}{A} \right)}; \tag{19a}$$

its values are given by *Noetzli* for various central angles and for various proportions t/h in a diagram. The coefficient k_t is not yet exact, but it considers the thrust and the shear (with approximation, using 1 instead of $\beta \frac{E}{G} = 3$); it gives values very near to the exact ones, as *W. A. Miller*[18] proved. *Noetzli* gives for effect of temperature the approximate formula

$$H_t = 0.94\, \delta\, \varepsilon\, E \frac{t^3}{h^2} \tag{20}$$

on the same basis as equation (19); he supposes approximately H_t acting at a distance of $\frac{h}{3}$ from the crown of centre line as for a parabolic arch. The shrinking of concrete produces the same effect as a drop of temperature of -35° F; it gives, like a temperature change and in the same line of action, the horizontal reaction

$$H_s = -0.94 \frac{E \cdot \Delta s}{l} \cdot \frac{t^3}{h^2} \tag{21}$$

if Δs signifies the shortening of centre line with shrinking of concrete.

The normal stresses and their values at intrados and extrados are determined from M, N with

$$\nu_{1,2} = \frac{N}{A} \pm \frac{M e}{J} = \frac{N}{b t} \pm \frac{6 M}{b t^2} \tag{22}$$

$e = \frac{t}{2} =$ distance of intrados and extrados from the centre line. Or to the primary normal stress $v_o = -\frac{p_2 r_2}{b t}$, constant for all the arch, are added the additional stresses produced by the horizontal force ΔH acting in the line of gravity of centre line; this force gives in each section a moment M and a thrust N, and the extreme stresses $v_{1,2}$ are then determined by equation (22). *Guidi*[10] transforms for an arch of constant thickness the formulas for stresses in the crown and abutment joint into a very simple form and adds, to facilitate calculation, numerical tables of coefficients in the equations. The stress at the crown is:

in the intrados

$$v_1 = -p\left(\frac{r}{t} - \mu_1\right) - \varepsilon E\left(\delta \frac{t}{r}\mu_1 - \frac{\delta_2 - \delta_1}{2}\right), \quad \mu_1 = \frac{1}{\vartheta''}\left(\frac{s-1}{s} + \frac{t}{6r}\right), \quad (23\,a)$$

in the extrados

$$v_2 = -p\left(\frac{r}{t} + \mu_2\right) + \varepsilon E\left(\delta \frac{t}{r}\mu_2 - \frac{\delta_2 - \delta_1}{2}\right), \quad \mu_2 = \frac{1}{\vartheta''}\left(\frac{s-1}{s} - \frac{t}{6r}\right); \quad (23\,b)$$

the stress at the abutment:

in the intrados

$$v'_1 = -p\left(\frac{r}{t} + \mu'_1\right) + \varepsilon E\left(\delta \frac{t}{r}\mu'_1 + \frac{\delta_2 - \delta_1}{2}\right), \mu'_1 = \frac{1}{\vartheta''}\left[\frac{1}{s} - \frac{r-h}{r}\left(1 + \frac{t}{6r}\right)\right] \quad (24\,a)$$

in the extrados

$$v'_2 = -p\left(\frac{r}{t} - \mu'_2\right) - \varepsilon E\left(\delta \frac{t}{r}\mu'_2 + \frac{\delta_2 - \delta_1}{2}\right), \mu'_2 = \frac{1}{\vartheta''}\left[\frac{1}{s} - \frac{r-h}{r}\left(1 - \frac{t}{6r}\right)\right] \quad (24\,b)$$

These formulas assume the temperature change to vary in a section linearly (fig. 2) with a value δ_1 at the intrados, δ_2 at the extrados and δ at the centre line.

The thickness of arch dams atteins very high values in the lower portions, in proportion to the radius of curvature and the length of arch. Thus the main condition of the usual analysis of arch, that the dimensions of sections should be small in comparison with the radius of curvature and the length of arch, is not fulfilled. For *thick arches* (great curvature) one gets the known more exact, analysis leading to the variation of normal stresses according to the law of a hyperbola, as *H. Bellet*[19] remarks; he also tries a more exact calculation of the effect of thrust and shear, but comes for normal stresses to the formula (of *Lamé*) for a thick cylindrical shell because he supposes that the angle of two adjacent sections does not change with deformation, which is true only for a thick cylindrical shell loaded with uniform radial forces.

From the assumption that plane sections remain plane, which for thick arches leads to the hyperbolic law of normal stresses, *B. F. Jakobsen*[20] derived a solution for circular arch with constant sections, loaded with uniform radial pressures. *W. Cain*[21], in his contribution to the discussion on *Jakobsen*'s paper, transformed the final equations into a better form. He obtains (fig. 5)

$$X = p_2 r_2 - H_c = \frac{p_2 r_2}{\vartheta_0} \cdot 2 \frac{i^2}{r_0^2} \sin \alpha, \tag{25}$$

$$= \left(\alpha + \frac{1}{2} \sin 2\alpha\right)\left(1 + \frac{i^2}{r_0^2}\right) - \frac{1 - \cos 2\alpha}{\alpha} + 2.88 \frac{r}{r_0} \cdot \frac{i^2}{r_0^2} \left(\alpha - \frac{1}{2} \sin 2\alpha\right), \tag{25a}$$

if r_0 signifies the radius of the neutral line, which differs here from the centre line; the difference is

$$r - r_0 = c = r - \frac{t}{\log \text{nat.} \left(\frac{r_2}{r_1}\right)} \tag{26}$$

About any point M_0 of the neutral line, given with the angle φ of the radius OM_0 with the axis of symmetry OC, the external forces at one side of the section OM_0 give a moment

$$M = - X r_0 \left(\frac{\sin \alpha}{\alpha} - \cos \varphi\right); \tag{27}$$

it is the moment about the point M_0 of the force X, acting to the right, substituting the right half of arch, at a distance of $r_0 \frac{\sin \alpha}{\alpha}$ from the centre O, viz. in the centre of gravity of the neutral line. In the section given by the angle φ one

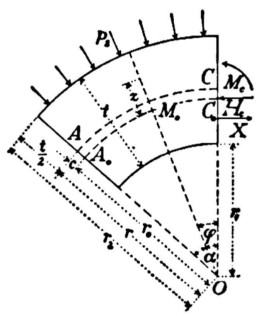

Fig. 5.

has also the thrust according to equation (7) $N = X \cos \varphi - p_2 r_2$ and the shear according to (8) $T = X \sin \varphi$; to the thrust $- p_2 r_2$ uniformly distributed in the section there comes the force X formerly mentioned.

Moment M and thrust N gives at a distance z from the neutral line the normal stress

$$v = \frac{N r_0}{(r_0 + z) t} - \frac{M}{J} \cdot \frac{r_0 z}{r_0 + z}; \tag{28}$$

N and v are positive as tensions, moment M is positive when acting clockwise for forces on the left of the section, and z is positive for the outer side of the neutral line. From (28) one gets the stresses at the extrados with $z = \frac{t}{2} + c$, $r_0 + z = r_2$ and at the intrados with $z = -\left(\frac{t}{2} - c\right)$, $r_0 + z = r_1$.

The water pressure produces a deflection of the crown (positive in the direction to the center O)

$$\eta = \omega_0 \cdot \frac{p_2 r_2 r_0}{E t}, \tag{29}$$

where *

$$\omega_0 = \frac{1}{\vartheta_0}(1 - \cos\alpha)\left[(\alpha - \sin\alpha)\left(1 + \frac{i^2}{r_0^2}\right) + 2.88\,\frac{r}{r_0}\cdot\frac{i^2}{r_0^2}(\alpha + \sin\alpha)\right]. \tag{29a}$$

With respect to formulas for thin arches the equations for thick arches give a lesser tension and a greater compression; the effect of great curvature of arch is therefore advantageous.

A constant temperature change gives a horizontal reaction

$$H_t = \delta\,\varepsilon\,E\,t\,\frac{i^2}{r_0^2}\cdot\frac{2\sin\alpha}{\vartheta_0}, \tag{30}$$

acting in the line of gravity of the neutral line. The deflection of the crown from temperature change is

$$\eta_t = -\,\omega_0 \cdot \delta\,\varepsilon\,r_0. \tag{31}$$

To facilitate the computation with *Cain*'s formulas, *F. H. Fowler*[22] elaborated for thin and thick arches *diagrams* for resulting normal stresses at intrados and extrados of the crown and abutment joint. The numerical results show that the shear can be neglected for $t/r = 0.02$ to 0.06.

The equations for thick arches give good results if the thickness of the arch is not too great. For too great dimensions, such as sometimes appear in the lower parts of arch dams, even this analysis is inexact. A correct calculation of stresses should be based on the mathematical theory of elasticity; *R. Chambaud*[23] showed that it gives in this case very good results. He proceeds from the mathematical theory of elasticity and introduces no other hypothesis than *Hooke*'s law. *Chambaud* gives the solution for an arch of rectangular section; it can be applied to all thick arches (arch dams, tunnels and underground conduits), further to thick cylindrical shells. This theory naturally gives complicated formulas, but numerous diagrams allow a quick and simple application. The results correspond very well to all surface conditions, except a small extent at the abutments; they can be adapted for any distribution of external forces on the intrados and extrados, and for any distribution of internal strains, therefore for various shrinkings in several places (caused for instance by the method of construction) or for irregular temperature changes. The solution is especially valuable, because it usually gives much more favourable results than the theory of thick arches previously mentioned. The usual theory of thick arches (and still more the usual theory of thin arches based on linear distribution of stresses in sections) leads as a rule to greater tensions on the intrados at the crown and especially on the extrados at the abutments, where this theory indicates the weakest point of dam. Great tensions would cause cracks in an arch without reinforcing and the consequence would be that the uninjured masonry would form a new arch able to resist safely the external forces; this was at first observed by *J. Résal*[1] (he supposed the "acting" arch parabolic), afterwards by *M. Malterre*[24] (with

* There is an error in Cain's paper (Transact A.S.C.E., vol. 90, p. 541, form. 109), as clearly shows comparison with the preceding equation.

the "acting" arch circular, of constant and variable thickness) and *L. J. Mensch*[25]. The exact calculation by the theory of *Chambaud* shows that the actual stresses are much more favourable; especially the tensions on the extrados disappear (which is particulary important for the impermeability of the dam), the tensions on the intrados are limited at most to a small portion at the crown. The exact solution gives on the whole few differences with respect to the usual theory of thick arches as regards the effect of bending moments; a considerable difference appears in the effect of thrust which outweighs the effect of bending moments in thick arches, if exactly calculated. The differences in the stresses mainly concern the neighbourhood of intrados. Moreover, the exact theory makes due allowance for the shearing force. The usual theory of thick arches does not give good results for too great thickness, because it is based on assumptions which are not correct: it neglects the normal stresses in radial direction and determines the normal stresses in sections, as though plane sections would remain plane after deformation. Especially the last hypothesis is not right for curved bars (arches), because there the determination of the effect of normal and shearing stresses cannot be divided as for straight bars. The exact theory gives for normal stresses (in the direction of radius v_1, of tangent to the arch v_2 and in the direction of the axis of intrados v_3) and for shearing stresses τ (perpendicular to the axis in the radial section and in the cylindrical section) altogether curves;

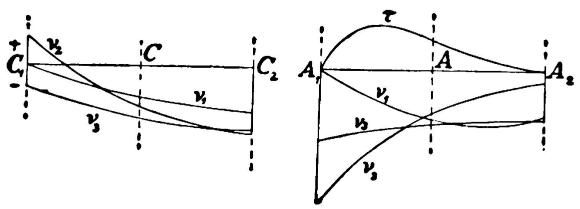

Fig. 6.

fig. 6 shows these curves for the crown section C_1 C_2 and for the abutment A_1 A_2 of an arch with radius $r = t = C_1 C_2 = A_1 A_2$. Chambaud made the analysis for an arch with external forces and stresses symmetrical to the plane of centre lines. The application for other cases naturally gives only approximate results.

The analysis of the arch dam as a system of horizontal arches independently withstanding the water pressure and the effects of temperature changes, shrinking and swelling of concrete, can be very good if for instance when constructing in layers the connection of layers in a vertical direction is destroyed; this can be seen at sudden breaks of deflection lines of vertical sections[15]. This analysis would be exact if the dam were actually divided into independent horizontal arches with horizontal contraction joints, filled with asphalt and bent copper sheets to obtain impermeability, as planned *by A. Peña Boeuf*[26]. Otherwise this analysis is only approximate.

Analysis of Arch Dam as a System of Horizontal Arches and Vertical Cantilevers.

In reality the horizontal arches hang together in a vertical direction and cannot deform quite independently; this causes a reciprocal action of horizontal arches in a vertical direction. A more exact analysis of arch dams considers the dam as divided by horizontal sections into horizontal arches and by vertical radial sections into vertical cantilevers. Between these two systems are distributed the external forces. The conditions of this distribution are given by the deformation of the dam, which must be equal at every point for the two systems. If we were to consider all the components of deformation at each point (three components of displacement in three perpendicular axes and three components of turning about these three axes), we should obtain an exact solution. Since this method of calculation is almost impossible practically, it is simplified by disregarding all turnings and the respective torsional stresses, by disregarding also the tangential component of horizontal displacement and the respective shearing stress. Moreover one can also disregard the vertical component of displacement, if one considers the dam after deformation by the weight of masonry is completed. There remains only the horizontal component of displacement perpendicular to the centre line of horizontal arch (radial displacement), and in consequence of this only one condition for each point where the centre line of supposed horizontal arch and the axis of vertical cantilever cross. Thus we substitute for the dam a system of vertical cantilevers and horizontal arches which simply (without restraining) support one another[27]. The torsional stresses, omitted by this method, in reality diminish a little the bending stresses and increase security.

An exact analysis by this method would be difficult, because the displacement of any point of the cantilever (or arch) depends on all loads acting on the cantilever (arch). The conditions of equal displacements of horizontal arches and vertical cantilevers in all points therefore give equations, each of which contains a great number of unknown quantities.

A. H. Woodard[28] simplifies the calculation regarding the deformation of the dam only in the vertical section through crowns of arches (where the dam is highest); he supposes the arch under simple compression, determines the deflection of the crown as for an arch with two hinges and takes the distribution of water pressure between the system of horizontal arches and vertical cantilevers, computed from the crown section, uniform along the arches. *R. Schirreffs*[29] endeavoured to improve the analysis by calculating the deflection of the arch crown as for a hingeless arch, otherwise using the same method of analysis; but he disregarded the effect of thrust and his formula is too complicated and incorrect, as *W. Cain*[11] showed. *H. Bellet*[19] determines the distribution of pressure between arches and cantilevers from a wrong supposition that the strain of centre line of arch at any point equals zero.

H. Ritter[9] in a numerical example (in 1913) proceeded approximately, supposing on each horizontal arch a uniform radial loading and determining its value by equating the deflection of arch crown and vertical cantilever in the middle vertical section. Analogically *L. R. Jorgensen*[30] examines only the middle vertical section, but computes the distribution of pressure only with a rough approximation; *L. J. Mensch*[31] uses for calculation of pressure distribution on cantilever and horizontal arches the unsuitable condition of equality of internal works. *J. Résal*[1] also considers only the middle vertical section.

H. Ritter[32] indicated the principle of a more exact calculation of load distribution on vertical cantilevers and horizontal arches thus: The deflection at any point M of the vertical cantilever A B (fig. 7) can be computed from its influence line (viz. deflection line of the cantilever A B loated with P = 1 in the point M): it has a value

$$\eta_m = \Sigma P'_n \eta_{nm} \tag{32}$$

if P'_n designates a load acting at the point N on the cantilever. This deflection equals the deflection of the horizontal arch at the same point with a load $P''_n = P_n - P'_n$; P_n is the total load at the point N. We thus obtain as many equations as we take horizontal elements, supposing that the loading of horizontal arches is uniformly distributed and that there is in consequence only one value P''_n for each horizontal arch to compute from these equations. In this manner we could proceed for any vertical section of the dam and we would find for different vertical sections various loads on the horizontal arches; the loading of these arches is therefore not uniform.

A. Stucky[14] is the first to consider actually (in the analysis of the dam on the river Jogue, made with the cooperation of prof. *A. Rohn*) all vertical cantilevers and horizontal arches (both of variable sections) and to take account not only of the different spans and rises of arches, but also of the different heights of vertical sections, which have an essential influence on their stiffness and therefore on the distribution of water pressure on vertical cantilevers and horizontal

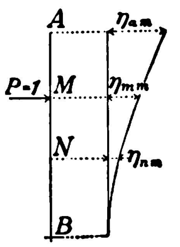

Fig 7.

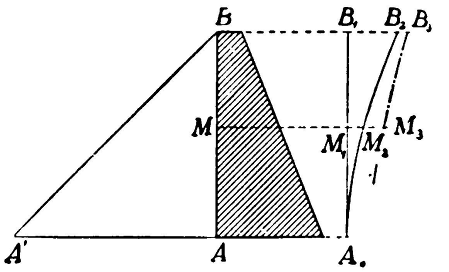

Fig. 8.

arches. The evolution of the resultant equations can be facilitated by solving separately the system of equations concerning each vertical cantilever (considering thereby only the loads on this cantilever). The approximate values calculated can thus be improved from original equations by iterative calculation. Since the exact fulfilment of suppositions of the analysis cannot be warranted for masonry dams with respect to the execution and the material used, each analysis of dam is to be considered as approximate; therefore the results of the first approximate solution are often sufficient. The results can be checked by calculating the deflections of vertical cantilevers and horizontal arches for the determined distribution of loads; it suffices if both deflections at the same point do not differ by more than 10 %.

A practical *trial method* was given by *F. A. Noetzli*[17] and completed by *W. Cain*[33]. It is first ascertained whether the horizontal arches act on the whole. To this end we determine the deflection line $A_1M_2B_2$ (fig. 8) of the vertical cantilever between two vertical radial sections in the middle of the dam, for the whole water pressure A A′B. In addition, we determine the deflections of horizontal arches, supposing them to bear the full water pressure. If the deflections of the vertical cantilevers are throughout smaller than the deflections of the arches (line B_3M_3), the cantilevers bear all the load; the arches could be stressed only if the temperature decreases and diminishes their deflection. This case occurs if the thickness of the dam is calculated by neglecting the influence of arches (as for a straight dam).

If the thickness of the dam is smaller, part of the water pressure is borne by vertical cantilevers, part acts on horizontal arches. The vertical cantilever bears at the base the full water pressure, because its deflection is very small there (smaller than the deflect on of arch with full water pressure). From the base to the top of dam the load of the arches increases approximately according to a straight line AB′ (fig. 9); in the upper part of dam the arches, being stiff enough, hinder the deflection of cantilever (deflect less than the cantilever and support it), therefore act on the cantilever with reactions opposed to water pressure. From the load diagram of water pressure AA′B the arches bear the part AB′B, the vertical cantilever the part AA′BB′ (AA′C′ is positive, C′B′B negative). We consider the highest vertical section and assume on the arches an approximately uniform loading. For the load diagram of the vertical cantilever it is easy to obtain (best by calculating) the bending moments and to determine the deflection curve of the cantilever as a funicular polygon to the loading diagram with ordinates $M\frac{J_0}{J}$; J_0 is a constant moment of inertia, J the moment of inertia of the section. At a chosen point C all the load is to be borne by the arch. We

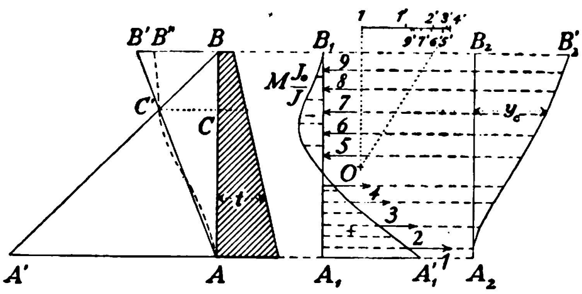

Fig. 9.

determine at C the deflection of the arch crown for full water pressure. If the cantilever has at C a greater deflection y_c than the arch, it is necessary to choose the point C lower and repeat the calculation. The exact position of C is determined with a linear interpolation between the two points C_1, C_2 formerly chosen (after fig. 10, where $C_1\,C'_1$, $C_2\,C'_2$ are arch deflections and $C_1\,C''_1$, $C_2\,C''_2$ cantilever deflections and the arch deflections throughout the height of dam. Usually there will not be complete accordance. To obtain equal deflections not only at C, but also on the top, we must change the load diagram for horizontal arches by substituting the straight line C′B″ for C′B′; the arches then support diagram AC′B″B (AC′A′ is positive, C′B″B negative); the vertical cantilever supports diagram AC′B″BA′. We change the point B″ until we have at C and B equal deflection of arch and cantilever. At other points the deflections need not be the same, because the broken line AC′B″ should be actually a curve. We determine it by assuming on the arches a smaller (greater) load, where the calculated arch deflection is greater (smaller) than the cantilever deflection.

The water pressure produces in cantilevers the greatest stresses in the lowest joint, where greater tensions can occur on the up-stream side. If there is no reinforcement, horizontal cracks on the up-stream face at the base of dam can appear. In this case vertical cantilever does not act as a beam perfectly fixed, but only as a beam partially fixed or hinged at the base. We can then find the right solution by trail, choosing the tangent to the deflection line at the base of the cantilever, otherwise calculating as formerly indicated and ascertaining whether the deflections of the cantilever coincide throughout with the arch deflections.

R. Chambaud[23] also indicates a method of finding the division of loading on horizontal arches and vertical cantilevers. He proceeds from any (approximate) law for the part of water pressure carried by the arches, supposes in each horizontal arch an approximately uniform loading and computes the deflections of arch crowns and the deflections of cantilevers under the load carried by them. For the second computation he introduces half the sum of these deflections, determines from it the division of loading between arches and cantilever and repeats the computation. Thus he can approach the exact values. He also considers approximately the normal stresses in the vertical direction (of the axis of arch) with their average value.

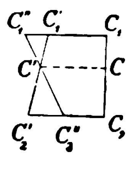

Fig. 10.

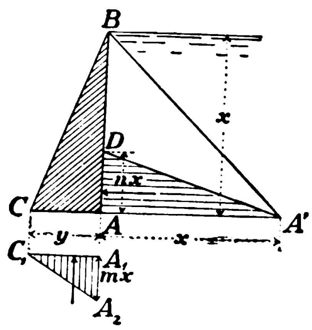

Fig. 11.

A. Rohn[34] recommends for the first calculation this *approximate method:* The vertical cantilever is supposed to carry from the load diagram AA′B (fig. 11) of the whole water pressure the triangular part AA′D with the base AA′ = x = height of dam, and $\overline{AD} = n \cdot x$, where $n = \frac{1}{5}$ to $\frac{1}{2}$ for $\frac{b}{h} = 1.1$ to 1.8; b is the length of dam at the top h its height. The rest of water pressure acts on horizontal arches. Besides he always recommends consideration of the *uplift* with a triangular load diagram $A_1C_1A_2$ (as for straight dams), where $\overline{A_1A_2} = m \cdot x$ for $m \leqq 1$; in the upper part of dam $m = 0.8$ suffices. For a triangular section of dam the necessary thickness at the base is

$$y = n \cdot x \sqrt{\frac{1}{\gamma - m}}; \tag{33}$$

γ = weight of masonry in proportion to the weight of the same volume of water. For $m = 1$, $n = \frac{1}{4}$, $\gamma = 2.3$ the result would be $y = 0.22\,x$.

The uniform distribution of radial pressures on horizontal arches assumed in the majority of approximate methods of analysis, is not sufficiently exact.

The division of water pressure on horizontal arches and vertical cantilevers depends very essentialy on the form of the cross-section of the valley. It is therefore necessary for an exact analysis to consider not only one (the highest) cantilever, but a greater number of vertical cantilevers and horizontal arches: it was thus that *A. Stucky*[14] proceeded. Another trial method was given by *C. H. Howell* and *A. C. Jaquith*[35], who choose a note uniform loading of arches, determine for this loading the deflections of arches and for the remaining loading the deflections of cantilevers and vary successive the loading of arches until they get at all points practically equal deflections of arches and cantilevers. It is necessary to make more trials in order to obtain a satisfactory coincidence. From the resultant loading the stresses in arches and cantilevers can be computed. In their analysis *Howell* and *Jacquith* omit the non-active extended parts of arches and cantilevers and limit the final calculation of the dam (without reinforcement) only to the parts working in compression: they always have arches of variable section which they calculate omitting the influence of shear.

Comparison of several cases showed that the analysis of arch dam as a system of independent horizontal arches is not exact and that it requires more masonry, especially for calculating the arches in a rough approximation as thin cylindrical shells, as was formerly the custom. The influence of vertical cantilevers should

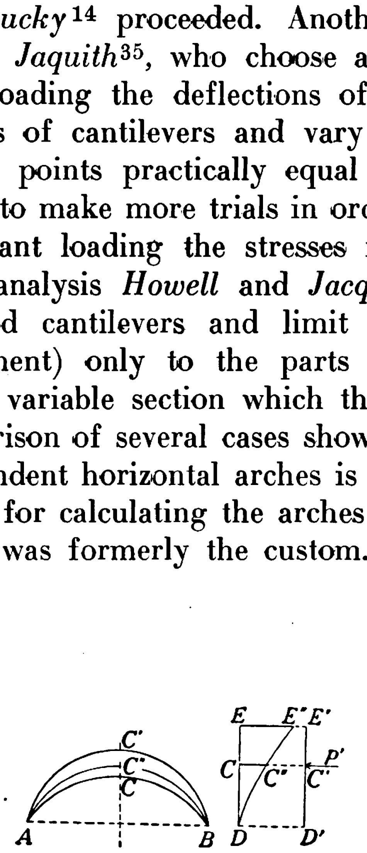

Fig. 12.

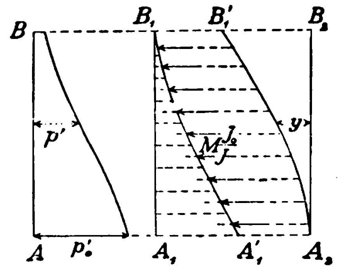

Fig. 13.

not be omitted, as it always appears and alters the loading and condition of stress of horizontal arches. The last method of analysis is available for any profile of the dam site, also for unsymmetrical profile.

The *influence of temperature changes,* which can produce greater stresses than water pressure can be computed in the same way as the latter. It can be even substituted (*Ritter*[9]) by a water pressure which gives the same deflections of horizontal arches as temperature change; this equivalent water pressure is to be divided over the system of vertical cantilevers and horizontal arches analogically as a real water pressure.

A trial solution of the influence of temperature change was given by *F. A. Noetzli*[17] and improved by *W. Cain*[21]. We again suppose the dam to be divided into vertical cantilevers and horizontal arches. The centre line of arch ACB (fig. 12), fixed at the ends and otherwise free, would deform by temperature change in AC′B; the displacement of the crown would be, according to the formula (31) for thick arches

$$\eta_t = \overline{CC'} = - \omega_0 \cdot \delta \varepsilon r_0 .$$

This displacement is hindered by the reactions p′ of vertical cantilevers DCE;

supposing them to be constant on the length of each arch, we have after (29) the displacement

$$\eta' = \overline{C''C'} = \omega_o \cdot \frac{p' r_2 r_o}{E t}$$

The resulting diplacement is

$$y = \overline{C C''} = \omega_o \cdot r_o \left(\frac{p' r_2}{E t} - \delta \varepsilon\right) \tag{34}$$

The resulting deflection curve of vertical cantilever is D C″ E″. The loading p′ will be determined by analysis of dam. In the base of dam there is

$$y = \omega_o r_o \left(\frac{p'_o r_2}{E t} - \delta \varepsilon\right) = 0$$

therefore

$$p'_o = \frac{\delta \varepsilon E t}{r_2}$$

We then choose at the crown a slight specific pressure (fig. 13) and in the vertical section a curve for distribution of pressures p′ (in the first attempt we can choose a straight line). For this loading we determine for the vertical cantilever the bending moments M and the values $M \frac{J_o}{J}$; J is the moment of inertia of the section of cantilever, J_o a constant moment of inertia. The line $M \frac{J_o}{J}$ gives the loading diagram for deflection curve as a funicular line. The analysis is correct if the deflections y of vertical cantilever coincide with the deflections of arches computed from equation (34); the loading of arches is given by p′ in the opposite direction as for vertical cantilevers. If there is no coincidence, it is necessary to correct the computation by altering the loading curve for p′.

The diminution of temperature can be combined with the *shrinking of concrete;* if ε′ is the shrinking for unit of length, the resulting deflection of arch crown is

$$y = \overline{C C'} = \omega_o r_o \left(\varepsilon' - \delta \varepsilon - \frac{p' r_2}{E t}\right); \tag{34a}$$

the temperature change δ is here negative, the reaction p′ of vertical cantilevers (in the last equation positive) acts from the centre of arch. The shrinking of concrete has the same influence as temperature change (diminution), which would cause a shortening equal to that caused by shrinking.

An increase of temperature causes a deflection of dam up-stream for empty reservoir; vertical sections also bend up-stream, which in vertical cantilevers produces tensions in the down-stream surface of the lower part of dam. In the arches, on the contrary, tensions are produced at the crown on the up-stream face; there cracks can develop if there is no reinforcement. With reservoir full and diminution of temperature the dam moves down-stream; there may be a tension in the cantilever in the lower part on the up-stream face in the arches tensions at crown on the down-stream face. For all tensions there should be adequate reinforcement; otherwise vertical cracks could occur gradually in the arch crowns on both sides, which would affect the stability of dam very unfavourably. It the distribution of loading on vertical cantilevers and horizontal arches is neglected (only the resistance of arches is considered), wrong construction can easily cause horizontal cracks, as the results of measurements on some dams appear to show[17].

As concerns the amount of temperature changes, *F. A. Noetzli*[17] therefore recommends for higher dams at the base thicker at the top greatest temperature change ($\pm 25^0$ F) be considered, at the base no change and between them linearly variable changes; for exact calculation we have not as yet enough results of actual measurements. At *Arrow-Rock* dam[36] the yearly change of temperature at the top was found to be 27^0 F, at the base only 6.5^0 F. There can also be several combinations of temperature changes on the up-stream and down-stream face; it is particularly necessary to consider for an empty reservoir the same largest drop of temperature on the up-stream and down-stream face, and for a full reservoir different diminutions of temperature on the up-stream face (to the lowest temperature of water) and on the down-stream face (to the lowest temperature of air).

In thicker dams the temperature changes do not penetrate the whole dam equally; a closer examination of it is given by *A. Stucky*[14]. *G. Ippolito*[13] examines in detail the masonry and derives simple formulas for distribution of temperature in the latter; they can be used for any masonry structure to determine daily and yearly changes of temperature. The same author also examines the influence of temperature rise on the hardening of concrete and gives results of temperature measurements on several dams; these are but few and do not permit safe conclusions to be arrived at. The calculations usually give too large stresses from temperature changes if one considers the temperature change constant or linearly variable through the thickness of dam, which does not correspond to reality. The deformations caused by temperature changes can also have a favourable influence on the stresses if there is unelastic yielding in the abutments or in the interior of dam.

A simple formula for the penetration of temperature changes in the interior of thick masonry, derived from American measurements, is given by *H. Ritter*[9]:

$$\delta = \frac{\delta_1}{3\sqrt[3]{x}} \tag{35a}$$

where δ is the temperature change in the masonry at a distance x from the surface. δ_1 the temperature change of air. *G. Paaswell*[37] develops for this case the formula

$$\delta = \delta_1 e^{-kx} \cos kx \tag{35 b}$$

k is a constant dependent on material and time: for concrete and the period of one day $k = 0.079$, for concrete and the period of one year $k = 0.00413$.

Too great influence of temperature changes and shrinking of concrete can be eliminated by *contraction joints.* For a dam calculated as gravity dam these joints are statically inoffensive. For an arch dam too many contraction joints are unfavourable with respect to stability.

Analysis of Arch Dam as an Elastic Shell.

An arch dam is in reality an elastic shell, free on top and supported or fixed on other parts of its circumference to the sides and bottom of the valley. But the analysis of an arch dam as elastic shell is very difficult. It is necessary to start from equilibrium and deformation of an infinitesimal element (as in the analysis of flat plates) and to satisfy the boundary conditions at the abutments and at the top of dam. The idea of this analysis was formulated generally by *G. Pigeaud*[3].

B. A. Smith[38] was the first to attempt to calculate an arch dam as an elastic shell. He simplified his analysis, considering only the highest part of dam and assuming in the horizontal direction throughout the dam the same conditions as for the highest section; he eliminated in this way the variability in horizontal direction (dependence on central angle φ). He considers the boundary conditions only for the top and base of the vertical section; this is in reality in accordance with the analysis of a vertical cylindrical shell of a reservoir. The connection of elements in a horizontal direction is considered in stresses, but not in deformation; it is only shown with a rough approximation that for central angles, smaller than 120^0, the deflection of the crown of horizontal arch can be computed as for a full circle, substituting the real modulus of elasticity E_0 for the arch with $\frac{2}{3}E_0$. *Smith* also considers the shearing forces in horizontal planes and from equilibrium conditions of forces acting on the element $t \cdot ds \cdot dy$ (between two horizontal planes, two vertical radial planes and the up-stream and down-stream face of dam), from deformation of vertical cantilever by bending moment and of horizontal arch by thrust (the bending moments in arches are neglected) develops the fundamental equation

$$\frac{d^2}{dy^2}\left(C_1 \frac{d^2 z}{dy^2}\right) + \frac{E_0}{r^2_2} t z = p ; \tag{36}$$

r_2 is the radius of up-stream face (fig. 14), p the external (water) pressure uniformly distributed along the horizontal arch, t = thickness of dam, $C_1 = E_1 J = \frac{1}{12} E_1 t^3$ is the flexural rigidity (for a vertical element of horizontal length = unit of length), E_1 modulus of elasticity for vertical cantilever (can be different from E_0 for horizontal arch, if there is another reinforcement), z = radial deformation (deflection and y = depth measured from water surface (at the top of dam) in the direction of vertical axis of dam surfaces. The analysis erroneously considers the vertical cantilever as an independent element, without connection with other elements; therefore *Poisson*'s ratio escapes from the resulting equations.

Smith gives the analysis for a dam of constant thickness and for a dam of trapezoidal vertical section. In the first case the solution is similar to the known solution for cylindrical shell of reservoir; only *Poisson's* ratio is not in the results. For a thickness linearly variable the solution contains series in the form of special *Michell's functions;* the author's paper gives numerical tables of these functions to facilitate the calculations and derives the connection with complex *Bessel's functions.*

W. Cain[33] showed in a numerical example that the methods of *Smith* and *Noetzli* give absolutely identical results, though *Noetzli* neglected the shearing forces; these forces can therefore be neglected. The accordance of both methods is natural enough, because their basis is in reality the same: both consider the vertical cantilever at the middle of dam and neglect the variation of values in a horizontal direction. The only difference is that *Smith* integrates a differential equation, also uses infinitesimal elements, whereas *Noetzli* considers finite elements. But this has no essential influence on the results if the number of elements of the vertical section is not too small.

G. Paaswell[37] derives from fundamental relations for deformation and from the energy expended on deformation the general flexural equation for an elastic shell

$$p = \frac{EJ}{1-\mu^2}\left(\frac{\partial^4 z}{\partial y^4} + \frac{2}{r^2}\cdot\frac{\partial^4 z}{\partial y^2\,\partial \varphi^2} + \frac{1}{r^4}\cdot\frac{\partial^4 z}{\partial \varphi^4} + \frac{2}{r^2}\cdot\frac{\partial^2 z}{\partial y^2} + \frac{2}{r^4}\cdot\frac{\partial^2 z}{\partial \varphi^2} + \frac{z}{r^4}\right); \quad (37)$$

z = deflection of shell, y = vertical distance from water surface (fig. 15), φ = angle measured in a horizontal plane from the plane of symmetry, r = radius of middle cylindrical surface, p = radial external pressure (water pressure) acting on the shell, and μ = *Poisson*'s ratio. For $r = \infty$ (and $r \cdot d\varphi = dx$) the equation (37) transforms itself into the fundamental equation for

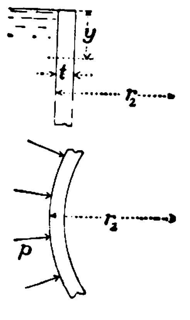

Fig. 14.

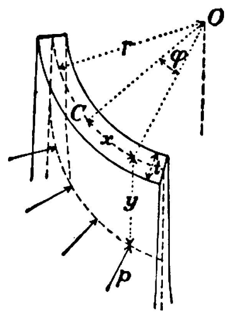

Fig. 15.

flat plates. The author does not determine the general integral of equation (37); he only gives a particular solution and derives from it the relation of bending moments in cantilever and arch. He shows that the bending moments in the cantilever are much greater at the base of dam than the moments in the arch, and that the arch moments change their sign in the lower part of dam.

In the excellent paper "Report on Arch Dam Investigation, Vol. I"[2] *H. M. Westergaard* deals theoretically with the analysis of arch dam as an elastic shell; he considers in radial and horizontal sections thrusts and two components of shear (in a radial and perpendicular direction), further bending moments for vertical sections and horizontal arches, and twisting moments; the distribution of stresses is supposed to be normal as for flat plates; and shearing stresses linearly variable through the thickness of dam, assuming a dam of small thickness. The author derives from the equilibrium and deformation of an element between two horizontal planes with a distance dy, two radial planes with a distance dx in the middle circle of radius r, and the down-stream and up-stream face of dam the equation of flexure

$$\frac{\partial^4 z}{\partial x^4}+2\frac{\partial^4 z}{\partial x^2\,\partial y^2}+\frac{\partial^4 z}{\partial y^4}+\frac{1}{r^2}\cdot\frac{\partial^2 z}{\partial x^2}+\frac{\mu}{r^2}\cdot\frac{\partial^2 z}{\partial y^2}+K\left(\frac{\partial^3 z}{\partial y^3}+\frac{\partial^3 z}{\partial x^2\,\partial y}+\frac{\mu}{r^2}\cdot\frac{\partial z}{\partial y}\right)$$

$$+k\left(\frac{\partial^2 z}{\partial y^2}+\frac{\mu\,\partial^2 z}{\partial x^2}+\frac{\mu}{r^2}\cdot z\right)-\frac{1}{N}\left(p-\frac{P_x}{r}+P_y\,r''+\gamma\,t\,r'\right)=0 \qquad (38)$$

and the differential equation of central forces

$$\frac{\partial^4 F}{\partial x^4}+2\frac{\partial^4 F}{\partial x^2\,\partial y^2}+\frac{\partial^4 F}{\partial y^4}-\frac{E\,t}{r}\cdot\frac{\partial^2 z}{\partial y^2}=0 \qquad (39)$$

In these equations z is the deflection of dam, r = the radius of cylinder of reference (fig. 15), x = distance measured on this cylinder in the horizontal direction from the vertical plane of symmetry (going through axis of symmetry O C at the top, y = the vertical distance from the top of dam, t = the thickness of dam: further there is $r'=\frac{d\,r_y}{dy}$, $r''=\frac{d^2 r_y}{dy^2}$, where r_y = the radius of the middle surface (dependent only on y), E = the modulus of elasticity of masonry, μ = *Poisson*'s ratio (for concrete $\mu = 0.15$), $N=\frac{E\,t^3}{12\,(1-\mu^2)}$ = the measure of stiffness of the dam in flexure,

$$K=\frac{2\,N'}{N},\quad k=\frac{N''}{N},\quad N'=\frac{d\,N}{dy},\quad N''=\frac{d^2 N}{d\,y^2},$$

p = the water pressure per unit of area of the cylinder with radius r, γ = the weight of masonry per unit of volume, P_x = the horizontal thrust per unit of length of vertical radial section, P_y = the vertical thrust per unit of length of horizontal section. Finally, F denotes the stress function determining the forces P_x, P_y, P_{xy} by means of equations

$$\frac{\partial^2 F}{\partial y^2}=P_x,\quad \frac{\partial^2 F}{\partial x^2}+\gamma\int_0^y t\,dy=P_y,\quad P_{xy}=-\frac{\partial^2 F}{\partial x\,\partial y}; \qquad (40)$$

P_{xy} is the vertical central shear per unit of length of radial section.

In the same paper *W. Slater* derives from the differential equation for a flat plate a simpler differential equation for flexure of arch dam

$$\frac{\partial^4 z}{\partial x^4}+2\frac{\partial^4 z}{\partial x^2\,\partial y^2}+\frac{\partial^4 z}{\partial y^4}+\frac{1}{r^2}\cdot\frac{\partial^2 z}{\partial x^2}-\frac{A}{J\,r}\left(\lambda_x+\mu\,\lambda_y\right)=p\cdot\frac{1-\mu^2}{E\,J} \qquad (41)$$

A = area of element in the vertical radial section, J = its moment of inertia; λ_x and λ_y are the horizontal and vertical strains in the direction x and y.

For an exact analysis of dam it would be necessary to solve the differential equations (38), (39), considering the boundary conditions on the top, where the dam is free, and at the abutments. In calculating the dam, we can according to *Fred. Vogt*[39] consider the deformation of bed-rock. *Fred Vogt*[40] determined the formulas and calculated numerically the influence of the *yielding of the rock foundation* in an arch dam. He came to the result that the yielding of the rock foundation can be very approximately computed by extending the dam to the imagined fixed foundation at a distance of 0.45 t from the abutment. The yielding of the foundations naturally alters the stresses and the deformations of an arch dam. For small thicknesses this yielding has no substantial influence. For great thickness (in the lower parts of high arch dams) the yielding of foundations diminishes the bending moment at the abutment of arch, thus diminishing the tension at the extrados of arch; on the other hand, the bending moment at the crown of arch and the tension at intrados increases. Stresses from temperature change, shrinkage and swelling of concrete become regularly smaller in consequence of yielding, the deflection of crown becomes considerably greater (up to a twofold value).

The solution of fundamental differential equations (38), (39) for an arch dam is very complicated and difficult. For ordinary practical cases this method of analysis is too laborious.

Shape of Arch Dams.

At first arch dams were generally constructed with vertical up-stream face and a constant radius of curvature in all horizontal sections or even with a radius increasing towards the bottom. Such a form is convenient for a constant width in all horizontal sections, therefore for an arch between vertical piers, although even there it is better to use a smaller radius with a greater thickness in the lower parts in order to obtain more flexible arches. If the dam is in a valley whose width diminishes downwards, the arches in the lower parts are very flat. In calculating the dam as a system of vertical cantilevers and horizontal arches (this is also assumed in the following cases), we get a relatively small portion of water pressure on the arches, the greater part of it being carried by the vertical cantilevers; because of bending in vertical cantilevers much masonry is required. It is therefore better to diminish the radius of curvature from top to bottom; this transfers the greater part of the load on the arches, where the stresses are more uniform and the strength of concrete is better utilised *(Stucky*[14]*)*. *L. R. Jorgensen*[30] therefore projects dams with constant central angle in all horizontal planes; these dams were often built in America in great dimensions. But the constancy of the central angle is not necessary and it cannot in practice be exactly attainded.

The idea of a constant central angle was definitely expressed as early as 1879 by *Pelletreau,*[5] who was also the first to determine the best value of the central angle if the effect of thrust be disregarded with the approximate value of 134^0, which gives the least volume of arch. If the effect of thurst is considered, the best central angle, according to *Ritter*[9] lies between 120^0 and 180^0; in this interval the volume of the necessary concrete varies but little. Tensile stresses in a thin arch with radial pressures uniformly distributed are obviated, if for a constant thickness the central angle is greater than 158^0; for a smaller central angle tensions on the extrados at the abutments can be excluded by reinforcing the arch at the abutments, if the central angle is greater than 115^0. As for the effect of temperature changes, *Ritter*[9] shows that a semicircular arch is the best.

The conditions of greatest possible economy were examined in detail by *Ippolito*[13] on the basis of the analysis of an elastic arch of relatively small thickness and calculating the arch dam as a system of independent horizontal arches. He shows that for an arch of constant thickness the best central angle is between 133 ^{0} and 180 ^{0} and that it depends on the depth of water; at a depth in meters equal numerically to the allowable stress k in kg/cm^2, the best angle is approximately 180 ^{0}. He further determines for an arch of constant width (i. e. for an arch between vertical piers) the best central angle for which the volume of the whole dam is smallest. For an arch dam in a valley whose breadth varies with the elevation, he construed graphical tables which permit the determination, for a given central angle at the top of dam, of the volume of arch rings in various elevations (for a constant radius of middle surface or of up-stream face), the volume of the whole dam and, by means of comparison of results for different central angles, the best central angle at the top leading to least volume of the whole dam.

For a uniform radial pressure the best form of central line is a circle, which is also convenient for construction. In reality the pressure on the arches is not uniformly distributed, because in consequence of the different resistance of vertical cantilevers differing in height (when calculating the dam as a system of horizontal arches and vertical cantilevers) the arches have to bear at various points different portions of the whole water pressure. It would naturally be possible to adjust the form of dam to this, choosing for the central line in each horizontal section the funicular line for the calculated loads on the arch (*Stucky*[14]). The possible saving of concrete would probably be outweighed by disadvantages regarding construction for which the best are circular arches also permitting the simplest calculations[41]. Moreover, the determination of the distribution of the pressure on the arch is complicated enough and cannot be performed exactly; if the form of arch is adjusted to the calculated division of pressure, it may be that the real division of load is different and does not suit the determined form of arch, so that the real stresses may exceed the calculated extreme values.

The triangular vertical section, suitable for straight or slightly curved dams wherein the arch action is disregarded, is not convenient for arch dams. With respect to stresses, it is convenient to make the dam very thin and to strengthen it towards the abutments and the sides and bottom of the valley (*Stucky*[14]), especially in case of dams of moderate height (up to 30 m), where one cannot attain the strength of material and where in the lower parts a considerable loading falls on the vertical cantilevers. A multiple statical indetermination in the distribution of loading on the system of vertical cantilevers and horizontal arches makes the arch dam very sensible to changes of dimensions; with convenient changes of thickness one can always improve the utilisation of strength of material or diminish the volume of masonry, for these changes can essentially alter the flexibility of the horizontal arches and vertical cantilevers and therefore the distribution of external forces on both systems, as *Howell* and *Jaquith*[35] have shown.

Even for very high arch dams strengthening at the abutments is advantageous, in larger valleys also a vertical up-stream face, giving a greater rigidity to vertical cantilevers[35]. The effect of uplift is much less dangerous to arch dams than to straight dams, since the abutments at the sides themselves prevent the overturning of dam. But it is necessary to consider the uplift especially where the greatest part of the loading acts on the vertical cantilevers.

Special care is necessary in determining the dimensions of the dam in places where the breadth of valley changes rapidly. Also the width of arches in adjacent horizontal elements is there very different and these arches would have very different deformations. To avoid too great shearing stresses, it is convenient to establish there massive abutments for arch dams, giving with the sides and bottom of the valley below them, a more regular form the circumference of the arch dam, and permitting full use of the limiting stress of concrete and excluding too high stresses (*Rêsal*[1]).

G. S. Williams[42] designed for the Six-Mile Creek dam at Ithaca a single form of dam in order to avoid the action of vertical cantilevers, so that all the load is carried by the arch action. The dam has at the bottom the form of an inverted dome which provides the dam, also at the bottom of the valley, with secure abutments; the water-pressure on the dome partly compensates the weight of dam.

Domes of great dimensions were used in the multiple-dome Coolidge dam on Gila River (Arizona)[43]. The dam was calculated as a system of independent arches, separated from the dome with plane sections, running perpendicularly to the inclined abutment lines.

The distribution of horizontal external forces on the system of vertical cantilevers and horizontal arches depends on the relation of the height to the total length of dam. With the increasing length of the arch dam, the length of the horizontal arches and their flexibility increases, but the vertical cantilevers remain equally rigid. Therefore the greater part of the horizontal loading acts on the vertical cantilevers and the dam gradually approaches in its statical action a straight dam of constant height, where all the load is borne by vertical cantilevers by means of compression and bending. On the other hand, in shorter dams the greater part of the horizontal load acts on the horizontal arches; with diminishing length the action of vertical cantilevers diminishes and that of horizontal arches increases. From constructed and calculated dams *Résal*[1] and *Stucky*[14] show that *the arch action is of value only in dams where the relation of the length at crownland of the height h is* $\frac{l}{h} \leqq 2.5$. Dams with $l > 2.5\ h$, where their thickness is great, are to be analysed as straight (gravity) dams. The arch action in this case can be disregarded, as it is of little importance; it is also useful for stability, because it relieves a little, especially in the upper parts, the vertical cantilevers. In relatively thin dams, even of greater length, the action of horizontal arches may be considerable[35].

Arch dams need of course secure abutments on the slopes of the valley; they can only be erected if the slopes are of solid rock. The arches at abutments should be approximately perpendicular to the contour lines of the ground.

If the analysis of the arch dam is more exact, the actual stresses calculated more in detail and the effect of temperature considered, then the limits of stresses can be (as in bridge construction) raised in respect to the ordinary brief calculation. *Stucky*[14] recommends in this case for concrete limiting stresses up to 35 kg/cm^2 in compression, 10 kg/cm^2 in tension. *Juillard*[41] objects that the working stresses used till now should not be tampered with before longer experience has proved the reliability of the new methods of analysis of arch dams.

The real stresses in arch dams can essentially depend on the mode of construction[44]. To realise the arch action it is necessary that the dam forms in vertical and horizontal direction a monolithic body; all preceding considerations as to stresses in arch dams therefore assume that during construction all layers are well connected among themselves or bound by greater stones. If there are vertical contraction joints in the dam, the arch action can be considerably reduced or even with opened joints fully eliminated. If the narrow contraction joints are afterwards filled, then horizontal transverse forces (in horizontal arches) can be transported; the friction in contraction joints produced by pressures acting on them, helps achieve this. Arch action can then be considered (at least partly).

Confirmation of the Analysis by Means of Measurements and Tests.

The stresses in a dam can be determined from measured deflection, as *A. F. Noetzli*[15] has shown. If Δs be the shortening of the centre line of arch (numerical value), then the horizontal thrust produced only by Δs is

$$H = -k_f \cdot \frac{E t^3}{h^2 l} \cdot \Delta s \tag{42}$$

for*

$$k_f = \frac{h^2 s}{t^3 \left(\int \frac{y^2 d s}{J} + \int \frac{\cos^2 \varphi \, d s}{t} + 3 \int \frac{\sin^2 \varphi \, d s}{t} \right)} \tag{42a}$$

we assume an arch with a section of the base $b = 1$, the signification of other quantities is as before (see fig. 1). The values of k_f for different central angles 2α and for different relations t/h are given by *Noetzli* in a graphical table. Approximately for a parabolic arch, if only the effect of bending moments is considered, $k_f = 0.94$; including the effect of thrusts and shears, $k_f = 0.75$. The shortening of centre line Δs gives approximately, supposing the deformed centre line to be circular (as for an arch with two hinges), the deflection at crown of arch (positive toward the centre) $\eta = \frac{3}{16} \cdot \frac{s}{h} \cdot \Delta s$. Substituting Δs from this equation in (42) we obtain

$$H = -\frac{16}{3} k_f \cdot \frac{E t^3}{h s^2} \cdot \eta \tag{43}$$

Approximately $k_f = 0.75$. $\frac{s^2}{h} = 8.3 \, r$, r is the radius of centre line; that gives

$$H = -0.48 \frac{E t^3}{h^2 r} \cdot \eta \tag{43a}$$

This force H acts in the gravity axis of the centre line, therefore approximately at a distance of $\frac{h}{3}$ from the crown; it is then easy to calculate the stresses at crown and abutments of arch.

* The numerator of k_f should be rightly $\frac{h^2 l^2}{s}$ instead of $h^2 s$.

The formulas of *A. F. Noetzli* can give, according to *W. Cain*[45], sufficiently good results for central angles 0—30° and thin arches; for greater central angles and thick arches the results will differ considerably from exact formulas.

If we consider the water pressure p (on the up-stream face) and the uniform radial loading p′ (positive toward the centre of arch) on the horizontal arch, produced by change of temperature and shrinkage of concrete, then the horizontal deflection of arch crown according to *W. Cain*[21] has a more exact value

$$y = \omega_0 r_0 \left[\varepsilon' - \delta \varepsilon + \frac{(p + p') r_2}{E t} \right] \tag{44}$$

with the signs used in equation (34a); for a fall of temperature, δ and p′ would be negative. From (44) we can calculate (p + p′), the whole radial loading of arch (uniformly distributed as we suppose) if we measure the actual deflection y and other quantities in the formula. Formulas (44), (34), (34a) are valid only if the whole considered arch is in action, therefore if there are no cracks from shrinkage of concrete or from too great tensile stresses.

If there are vertical cracks in the dam which alter the action of horizontal arches (they alter the arch sections), the measurements of deflections and temperature should be made at a time when the cracks are closed; from two observations at different times one can determine as a difference in the quantities observed the deflection y and the temperature change δ, which are to be included in the formula. It is also to be considered that the unfavourable influences of unequal modulus of elasticity (of heterogeneity of concrete) and of a non-uniform distribution of temperature are excluded. To attain this, deflections and temperature should be measured over a period of several days in which the temperature of the air does not change. If the calculation from observed values gives a pressure p almost equal to water pressure at the bottom of dam or greater, it is a sign that there were vertical cracks in the dam or that the temperature in the dam was distributed too unequally; the results of such measurements cannot be used.

From the measured radial deflection η at the crown of horizontal arch one can calculate the bending moment at crown after the formula

$$M_0 = - a \cdot \frac{E t^3}{h r} \cdot \eta, \tag{45}$$

applicable equally for the deflection from radial water pressure as for the deflection from temperature. The coefficient a is: for a hingeles arch[21] when the influence of shear is considered,

$$a = \frac{1}{6} \cdot \frac{\sin \alpha \left(1 - \frac{\sin \alpha}{\alpha}\right)}{(\alpha - \sin \alpha)\left(1 + \frac{i^2}{r_0^2}\right) + 2.88 \frac{r}{r_0} \cdot \frac{i^2}{r_0^2} (\alpha + \sin 2\alpha)} \tag{45a}$$

and for arch with two hinges[16], when the shear is neglected

$$= a \frac{1}{6} \cdot \frac{\sin \alpha (1 - \cos \alpha)}{\sin \alpha + \alpha (1 - 2 \cos \alpha) + \frac{i^2}{r^2} (\alpha - \sin \alpha)} \tag{45b}$$

The coefficient a depends only on the central angle 2α and on the proportion $\frac{i}{r}$; a can be then calculated in advance for different angles α and proportions $\frac{i}{r}$. A numerical table thus computed gives directly a and from (45) M_o can be determined. From this easily follows the force $X = p\,r - H_c$, because M_o is the moment of this force X acting at the centre of gravity of the neutral line for an arch with fixed ends (for an arch with two hinges, the line of action of X joins the two hinges); we further compute H_c and from this force and from M_o the stresses in the crown section of arch. From M_o and H_c, using the formulas previously given, one can determine the bending moment and thrust in the abutment section and thus also the stresses there.

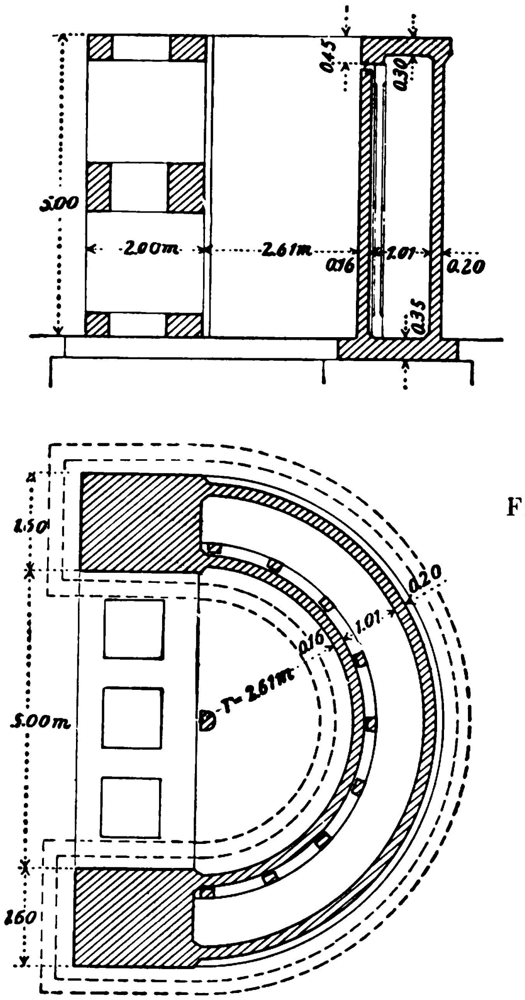

Fig. 6.

If the full action of arch is secured for all cases of loading and temperature changes, that is, if the arch dam is so reinforced and fixed to the rock at the sides of the valley that vertical cracks cannot arise the modulus of elasticity E, can be computed from exact measurements of deflection and temperature. For this it is necessary to determine by analysis of dam (according to *Smith* or *Noetzli*) the radial loading p′ of the horizontal arch; E is then given by equation

(34). It is better to eliminate the temperature change by using the observation of deflections at different depths of water and equal temperature.

The analysis can be controlled by *measurements* on actual dams which were made in a few cases[46]; one can also confirm the theory by systematical *tests on models. Cam. Guidi*[47] tested a model of arch dam in the form of a semi-circular arch with a radius of 2.61 m, constant thickness of 16 cm and height of 5 m; he acted on it with the pressure of water contained between this arch and a greater co-axial arch at a distance of 1.01 m, of a thickness of 20 cm (fig. 16). The two arches were fixed to the bottom and to two great lateral vertical piers; the outer arch was also fixed to a concrete plate on the top, but the inner arch ended under this plate, and between the arch and the plate there was an elastic connection to make possible the examination of the arch under pressure produced by hydraulic jacks. The experiments showed that the elastic line of the horizontal arch did not correspond to a uniform loading with water pressure, but that it corresponds to a loading with $^3/_4$ of water pressure at crown and a pressure growing continuously towards the abutments under full water pressure, as *Guidi* had recommended in his book on the statics of dams[10].

For expermients with models *A. Mesnager* and *J. Veyrier*[48] proposed in 1926 a very good method permitting the same stresses to be obtained on a model of reduced size as in the actual structure. Using instead of water a liquid with specific weight n-times greater (for instance mercury with specific weight $n = 13.6$), one obtains in a model of the same material as the actual dam, reduced in the proportion of $1:n$, at each point the same external pressures and therefore the same stresses as in the corresponding point of the actual dam; the deformations will be similar. Using for the model a material with strength m-times smaller and for loading a liquid with specific weight n, the same effect is obtained with regard to breaking in a model in a scale $1:mn$. In this way *Mesnager* and *Veyrier* tested a model of a graduated arch dam* which they designed for a total height of 70 m (5 degrees of 14 m each) on the river Dordogne at Marège[48]; they used mercury for loading (specific weight 13.6) and made the model of plaster so prepared that its strength was 7,35 times smaller than that of concrete, so that a model on a scale of $1:mn = 1:13.6 \times 7{,}35 = 1:100$ was sufficient. They loaded the model up to breaking and determined the factor of safety (from 3 to 5) of each arch in an actual structure of their design. They also found that the formulas and tables of *Guidi* are good and safe in practice. Both authors proposed to make further experiments on a concrete model on a scale of 1 : 13.6 for loading with mercury.

Tests of arch dams on a large scale were made in U. S. A., where on the suggestion of Mr. *F. A. Noetzli* an experimental arch dam was built in 1926 on Stevenson Creek in California; the experiments were described in detail in the Report (vol. I) published in the Proceedings of A.S.C.E. in May 1928. These american experiments are of the greatest importance for the confirmation of different theories because of their great extent and careful analysis of results. They gave many interesting results and showed clearly which of the theories hitherto used have any competence.

* The first straight graduated dam was designed by *Boulé* in 1894 for the dam on the Nile at Assuan, afterwards in 1912 by *Ruthenberg* in Italy.

Safety of Dams.

Straight dams generally have a small factor of safety, as a rule hardly greater than 1.5. This is proved by the accidents which have occurred to straight dams. As regards the dam of recta, the raising of the supposed water surface by 80 cm sufficed to cause failure[49], so that the factor of safety was here only a little greater than 1. Many other accidents with straight dams were also caused by a raising of the water surface above the highest level considered in planning (because of insufficiency of spillways) and by overflow of dam. The safety of dam increases considerably, without modification of its section, by curving the dam in plan; this can moreover essentially remedy the unfavourable effects of temperature changes and shrinkage of concrete, which can be met in a straight dam by using contraction joints.

The factor of safety of arch dams is, however, considerably greater than that of straight dams. This is also proved by the circumstance that accidents with arch dams are very rare and mostly caused by insufficient foundations (Moyie River Dam[50], Lake Lanier Dam[50], Gleno Dam in Italy[51]). Well constructed arch dams have a considerable degree of safety, much greater than straight dams. A straight dam resists water pressure only by the weight of masonry. If the water pressure increases only a little (by an unforeseen rise of the water level), the largest compressive stress at the down-stream face of the bottom section can be very considerably increased; the ultimate loading which would cause failure of dam, often has to the loading assumed in the design of dam, a proportion (factor of safety) little greater than 1. The curvature of dam increases its safety very considerably; a curved dam is by its form alone secure against tilting. The tests of Mr. *Mesnager* showed that the loading which the dam will safely carry can be raised several times before failure; the factor of safety given by the proportion of ultimate loading to the actual loading is here like that for other engineering structures.

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Summary.

Up to the present time, arch dams have been passing through a long period of development as regards their construction. Also their analysis, the first paper regarding which dates from 1866, developed from plain beginnings to the relative perfection of today. Arch dams are being more and more used and their dimensions are continually increasing; they are of eminent importance for the safety of the whole district situated below them. It is therefore interesting to follow the development of their analysis which clearly shows a more and more complete penetration of human thought into the true meaning and function of this important engineering structure.

At first the arch dam was considered as a system of independent horizontal arches, withstanding water pressure and perhaps also temperature changes and shrinking of concrete. The basis of analysis is here of course the same as for ordinary arches loaded with vertical forces. Nevertheless the nature of loading (radial) requires a somewhat different and especially a more complete analysis than the usual analysis of vertical arches. The analysis of the horizontal arch was gradually improved by considering shear, besides bending moments and thrust; further the theory of thin arches gave way to the theory of thick arches, at first approximate, afterwards exact and based on the general theory of elasticity.

Greater heights of dams led to the necessity of considering the connection of horizontal arches in a vertical direction. This is effected by analysing the arch dam as a system of horizontal arches and vertical cantilevers. The distribution of load in both systems was calculated at first approximately, neglecting the variation of loading in the direction of arch, and supposing the loading of arch to be uniformly distributed. The analysis was gradually perfected: to-day we are able to calculate the exact distribution of forces on arches and on vertical cantilevers, both for symmetrical and for unsymmetrical dams.

The last word in the theory of arch dams is their analysis as elastic shells; this idea was first worked out in practice in the United States of America and also brought to perfection there.

The American engineers working on the theory of arch dams also claim the great merit of having lately carried out large-scale experiments with a great experimental arch dam, and of having compared them with tests on small models. These experiments have cleared up many questions regarding the theory of arch dams and promise to show a safe way for their correct analysis and construction.

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