The ductility of steel: the effect of rapidly imposed and repeating loading

I 4

The Ductility of Steel; the Effect of Rapidly Imposed and Repeated Loading.

Die Zähigkeit des Stahles, die Wirkung der raschen und der wiederholten Beanspruchungen.

La ductilité de l'acier, l'action des efforts rapides et des efforts répétés.

R. L'Hermite,
Directeur adjoint des Laboratoires du Bâtiment et des Travaux Publics, Paris.

Most of the works which have dealt with the ductility of steel and with the study of its consequences on the safety of structures have failed to throw any clear light on the influence of the time factor. There can be no doubt that this factor plays a predominant part, especially where the external effects operate rapidly; that is to say when the rate of the application of load and the velocity of deformation are high. This is true of dynamic loads, for which no equalisation of stresses takes place as a rule.

The relative deformation of two points in a solid body subjected to the action of a force F is the sum of a deformation which disappears more or less rapidly with F (known as the elastic viscous strain) and of a second kind of deformation which is permanent (known as plastic strain). This circumstance has led to the conception of "inherited action", introduced into physics by Volterra. In this special case it may be said that the application of an elementary force d F does not immediately produce the full amplitude of deformation, both elastic and plastic; there is a delay or reactivity in the occurrence of the deformation both when the load is applied and when it is removed, and the consequence of this reactivity is a residual deformation which is multiplied by a "heredity factor" Φ, the latter being a function of such a kind that its value decreases indefinitely with time. Under these conditions the expression for elastic viscous deformation is as follows:

$$x(t) = \int_0^t M[(t-z), F]\, N(F) \cdot \frac{dF}{dz}\, dz$$

The expression for plastic deformation is as follows:

$$x'(t) = \int_0^t \mathfrak{M}[(t-z), F]\, \mathfrak{N}(F) \cdot \frac{dF}{dz}\, dz$$

The first of these expressions applies to all cases of the imposition and removal of load, the second only to the case where $\frac{dF}{dz}$ is positive. In the case, for instance, of repeated loading, the plastic strain arises during the first imposition of load; for a first approximation it does not enter into account except as an initial constant.

The first approximate calculation gives for M (t) the expression $M = 1 - e^{-\lambda t}$, and for N a function which depends on the nature of the solid in question. In the same way we have

$$\mathfrak{M} = \alpha - \beta e^{-\mu t}.$$

We may thus recognise a number of expressions which are in current use:

Plastic flow under constant load:

$$x'(t) = [\alpha t + \beta (1 - e^{-\mu t})] \sigma(F).$$

(This formula agrees exactly with that obtained in experiment by Prof. *Roš*.)

Elastic strain under a load which increases in accordance with a definite law:

$$x(t) = \frac{F(t)}{E} - \frac{1}{E} \int_0^t e^{-\lambda(t-z)} \frac{dF}{dz} \cdot dz$$

In the case of a linear load we have:

$$x(t) = \frac{p}{E}\left(t - \frac{1 - e^{-\lambda t}}{\lambda}\right)$$

The first term represents the total elastic strain and the second represents the delay or elastic hysteresis.

The strain under a sinusoidal load is as follows:

$$x(t) = \frac{p}{E} \cdot \sin \varkappa\eta t - \frac{\varkappa\eta p}{E} \cdot \frac{\lambda \cos \varkappa\eta t + \varkappa\eta \sin \varkappa\eta t}{\lambda^2 + \varkappa^2\eta^2}$$

wherein the second term represents the diminution in amplitude of the strain as a function of the frequency. The coefficient λ may be calculated by comparing this with the experimental results of repeated bending. In a carbon steel which has an ultimate strength of 68 kg per sq. mm we have found $\lambda = 5.25 \times 10^3$.

The total deformation under an increasing load is given by the formula:

$$X(t) = \frac{F(t)}{E} - \frac{1}{E} \int_0^t e^{-\lambda(t-z)} \frac{dF}{dz} dz + \alpha \int_0^t \mathfrak{N}(F)(t-z) \frac{dF}{dz} dz + \beta \int_0^t \mathfrak{N}(F)(1 - e^{-\mu(t-z)}) \frac{dF}{dz} dz$$

A detailed examination of this function shows that for any given total load the plastic strain falls off as the rate of application of load increases. The case of a rapidly increasing load is of very frequent occurrence in engineering work, and it is clear, therefore, that in such work it is not correct to assume the same mode of adaptation and the same laws of plasticity under impact as under a slowly-applied super-load.

The experimental study of these questions has shown, also, that in the case of repeatedly applied loads which obey a harmonic law the modulus of elasticity is variable in time; moreover it has been observed that this variation depended on the amplitude of the load. For a small load the coefficient λ diminishes and tends towards a value λ; in other words the solid adapts itself to the forces imposed upon it. On the other hand when the amplitude and the force exceed a certain value, which is perfectly definite, the coefficient λ tends to be increased. The value of this boundary between the two phenomena is practically the same as the fatigue limit measured independently for the same solid, and this fact provides the missing experimental link between strain and failure under repeated loading. Moreover it agrees with the measurements made of damping capacity, according to which the logarithmic decrements of the oscillations set up by the successive impulses diminish when a point in excess of the fatigue limit is reached, and increase when a point below this limit is attained.

Another series of questions which may be examined on the basis of this theory is that which relates to the propagation of vibrations in solids. It may be sufficient to say that under the high frequency and low amplitude of (for instance) acoustical vibration, plastic phenomena play a subordinate role compared with the conditions of propagation, and only elastic hysteresis may be of some importance. The general equation for the propagation of a vibratory movement, derived from our first equations is as follows:

$$\delta \frac{d^2u}{dt^2} = E \frac{d^2u}{dn^2} + \int_0^t e^{-\lambda(t-r)} \frac{d^3u}{dn^2 dz} dz$$

or

$$\delta \frac{d^2u}{dt^2} = E \frac{d^2u}{dn^2} + \frac{E}{\lambda} \cdot \frac{d^3u}{dn^2dt} - \frac{E}{\lambda^2} \frac{d^4u}{dn^2dt^2} + \dots + (-1)^{n+1} \frac{E}{\lambda^n} \frac{d^{n+2}u}{dn^2dt^2} + \dots$$

Since λ has a high value the equation just quoted may be limited to the first two terms of the second member. It is then exactly similar to the equation for propagation already well known in relation to viscous media, wherein $\frac{E}{\lambda}$ is the coefficient of viscosity.