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Contribution to the Question of Utilising Plasticity in Continuous Girders Subject to Repeated Stresses.

Beitrag zur Frage der Ausnutzbarkeit der Plastizität bei dauerbeanspruchten Durchlaufträgern.

Sur la plasticité dans les poutres continues sollicitées dynamiquement.

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Dr. Hans Bleich,[2] making the assumption of an ideally plastic material, has written as follows: "A statically indeterminate system can be made to withstand an infinitely repeated succession of loadings provided that the indeterminate quantities in the system can be so chosen as to produce a condition of self-stressing characterised by the fact that at any point the sum of this self-stress and the maximum stress calculated from the law of elasticity remains just below the yield point." This statement requires to be checked by fatigue tests, in view of the fact that frequency of loading plays no part in the matter.

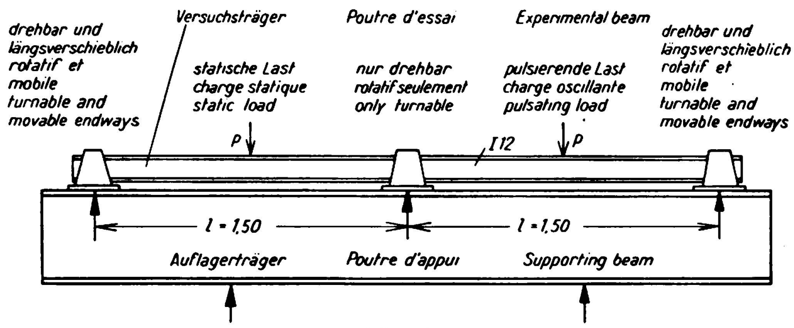

Fig. 1.

Arrangement of experiment.

For this purpose use was made of an unperforated beam (I 12) carried on three supports across spans of 1.50 m each, made of commercial structural steel (Fig. 1). All the bearings were capable of taking either a tensile or compressive load, and in addition the outer bearings had provision for longitudinal movement[3].

[1] Abridged contribution.
[2] Der Bauingenieur, 1932, No. 19/20.
[3] The experiments were carried out in the Materialprüfungsanstalt at Stuttgart (Prof. *Graf*).

Bleich's statement rests on the assumption that a beam which is capable of carrying loads under one particular kind of loading out of many, is also necessarily capable of doing so when the types of loading are altered as many times as is desired.

The loadings applied are shown in Fig. 1. To suit the characteristics of the machine the left hand load was made steady and the right hand load made to pulsate about ten times a minute in excess of an original value of 200 kg. There was, therefore, no intermediate condition of complete freedom from load.

The magnitude of the load P was at first fixed in such a way that according to the theory of elasticity the yield point ($\sigma_F = M : W$) would be reached in the most heavily stressed section. This gave $\sigma_F = 2{,}420$ and $2{,}730$ kg per sq. cm for the respective flanges of the two beams 12 m long, from which the specimens, each about 3 m long, had been prepared.

According to Bleich, if the most favourable condition of internal stress is assumed (Fig. 2) the moment over the support and in the flange would tend to

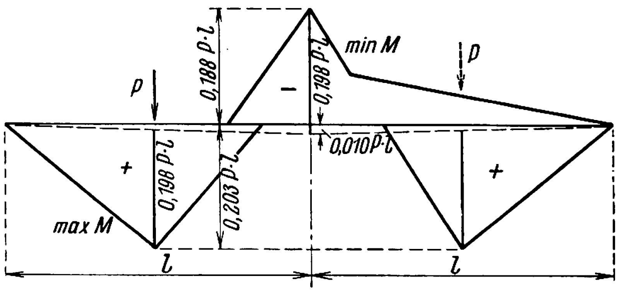

Fig. 2.

equalise, and in the determination of P the yield point would be exceeded only by about 2.5 %, because in both cases of loading an almost "natural compensation of moment" occurs.

The beam withstood 700,000 changes of load with P = 4,210 kg without showing signs of premature breakage through fatigue. The elastic deflection (which could be accurately read on a frame to within $^1/_{100}{}^{\text{th}}$ of a millimetre) corresponded with the calculated values, and the permanent deflections were practically zero. An internal stress effect was therefore ruled out.

The loads P were now increased in the same beam to such an extent that the yield point was exceeded by about 20 %. Under this loading again the beam withstood 630,000 further changes of load. The deflections increased only a little faster, in relation to the load, than had been the case with the first type of loading. The experiment was discontinued since again no fatigue breakage was to be expected. The residual deflections reached only about 15 % of their calculated values which had been arrived at on the following considerations:

In order to obtain an internal stress moment over the central support of 0.01 P · 1 it is necessary that a force of 0.01 P should be applied in the statically determinate system from which the cantilever beam is derived, and this results in a deflection of the supporting edge which is given by

$$f = \frac{0.01 \cdot P \cdot 2\,1^3}{3 \cdot E \cdot J}$$

The same deflection is produced in the statically determinate system if the deflection due to cold working in the central field amounts to $\frac{f}{2}$. Taking $E = 2100$ t/cm² and $I = 328$ cm⁴ we have

$$\frac{f}{2} = \frac{0.01\,P \cdot 150^3}{3 \cdot 2100 \cdot 328} = 0.0165\,P.$$

The cold-deformed girder, considered as a beam on three supports, is further bent elastically in accordance with the area of the moment diagram for internal stress, so that f/2 is diminished by an amount

$$\delta = \frac{0.01 \cdot P \cdot l^3}{16 \cdot EJ} = \frac{0.01 \cdot P \cdot 150^3}{16 \cdot 2100 \cdot 328} = 0.0031\,P.$$

The permanent deflections at the centre of the left hand field corresponding to the conditions of internal stress are given in millimetres by

$$\delta_{i1} = (0.165 - 0.031)\,P = \mathbf{0.134\,P.}$$

The condition of internal stress cannot occur until the yield point σ_F is reached at the centre of the field, and we then obtain

$$P \geq \frac{W \cdot \sigma_F}{0.203 \cdot l}$$

owing to the compensating effect of the internal stress, P may be increased to

$$P^1 = \frac{0.203}{0.198}\,P = \sim 1.025\,P$$

The corresponding permanent deformation is then

$$\delta_{bl} = 0.134\,P^1$$

and this must not be allowed to increase even under loading many times repeated.

The elastic deflections for $P = 1$ tonne at the centre of the left hand field, when the latter alone is loaded (case A), are given by —

$$\delta_{el} = 0.734 \text{ mm}$$

and when both the fields are loaded (case B) —

$$\delta_{el} = 0.446 \text{ mm.}$$

Two of the experiments carried out will be briefly discussed. The loads P amounted to 5.04 and 5.83 tonnes. The yield point was $\sigma_F = 2.420$ kg/cm², and the section modulus $W = 53.1$ cm³. The load at which the yield point was reached had now been exceeded by 1.2 and 1.38 times, and in either case over 500,000 changes of load were made without any appearance of fatigue breakage. The deflections given in Table I were measured at the middle of the left hand field under both conditions of loading, A and B, and also obtained by calculation:

Table I.

Load	Loading	$\delta_{el} + \delta_{bl}$		δ_{bl}	δ_{bl}
		measured	calculated	measured	calculated
5.04 Tonnes	A	3.65 mm	**4.37** mm	0.18 mm	**0.67** mm
	B	2.49 „	**2.92** „		
5.83 „	A	**5.25** „	5.055 „	**1.68** „	0.775 „
	B	**4.75** „	3.375 „		

Under a load of P = 5.04 tonnes the measured values are below those calculated. The actual permanent deflection is very small, so that although the yield point is exceeded by 20 % in the flanges the conditions of internal stress has not yet been called into operation at all. With P = 5.83 tonnes the reverse is true, for in this case the actual deflections are the greater, and the difference is particularly large in the case of permanent deflections. It is apparent here — despite the contrary implication of Bleich's principle — that under loading B additional internal stresses are produced, for in the right hand field permanent deflections occur without any permanent increases. Moreover the moments over the support and in the flange are practically equal. The permanent deflections do not however continue to increase when the experiment is continued. The implication is that structures which are designed to resist bending when calculated in accordance with Bleich's principles possess an additional margin of safety, even under fatigue stresses. This conclusion is attributable to the nonhomogeneous nature of the bending stresses: if the yield point is reached at the edges of the cross section resistance to permanent deformations is still afforded by the remaining portions which are stressed only elastically, and on this account an increase in the strength of 16 % may be expected. This figure is given when W is replaced by 2 S_x (where S_x is the statical moment of one half of the cross section of the beam referred to the x-axis). This effect is further increased by the fact that, as a rule, the yield point in the web is greater than in the flanges, and yet again through phenomena of restraint and through the operation of an upper yield point. Finally, rolling stresses will tend to hinder the development of permanent deformations up to a certain stress which lies above the yield point.

Only the Case A of loading was examined, because this gives a large difference between the moment over the support and that in the span. With P = 6.28 tonnes — a value which would imply that the yield point was exceeded to the extent of 1.3 times ($\sigma_F = 27.3$ kg/mm^2) — the permanent deflection amounted to only 1.6 mm, whereas it would have to be 5.75 mm in order to correspond with a condition of internal stress such that the moments were equalised (moment over support = $^3/_2$ (0.203—0.094) P l = 0.072 P l). The elastic deflection was measured as 4.6 mm, corresponding closely with the calculated amount and being, therefore, smaller than the permanent deflection necessary for equalisation of moments. After more than one million changes of load the girder bent sideways, but no fatigue fracture occured.

The theoretical and practical knowledge now available can be applied with a view to greater economy in the design of continuous beams subject to fatigue stresses: but only on condition that such beams contain no notches — as, for instance, holes or fillet seams — and this is a limitation which to a great extent excludes the use of riveted designs and riveted connection. On the other hand, rolled beams unperforated by holes and having perfect butt welded joints free from surface notches may properly be adopted under conditions of fatigue stress, using this term in the sense of higher stress values in the "carrying capacity" method of design.

The application of that method may, however, be interfered with by premature instability of the beam. Further, it may happen that when the most favourable condition of internal stress is one which implies very large permanent deflections — as is the case in those instances which are the most important from an economic point of view — such condition of stress will be attained before the designed amount of loading has been imposed,[4] and it will then, of course, be impossible to count upon an equalisation of moments.

[4] *Stüssi* and *Kollbrunner:* Bautechnik, 1935, No. 21. *Maier-Leibnitz:* Stahlbau, 1936, No. 20. *Klöppel:* Stahlbau, 1937, No. 14/15.