I 12

The Influence of Ductility of the Steel on the Stability of Structures.

Der Einfluß der Zähigkeit des Stahles auf die Stabilität der Stahlkonstruktionen.

Rôle de la ductilité de l'acier dans la stabilité des constructions.

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Ductility is the property which enables a material to undergo large deformations after the limit of its elastic region has been reached. In structural mild steel the large amount of strain occurring under this condition has no appreciable effect, from a mechanical point of view, on the texture of the metal. The zone of strain which exists, in this way, beyond the elastic region is known as the plastic region.

For a long time it has been recognised that the stability of steel structures is the result of small zones of plastic strain which are produced wherever the elastic strains are great enough to exceed the elastic limit, and the outcome of this is that the maximum stress actually existing in the material is lower than that calculated by the ordinary method based on elasticity. Hence the effect of ductility in the steel is, apparently, to increase the strength by eliminating those zones in which elastic deformation is excessive: a property which, suitably generalized, is now known as adaptation.

Adaptation, however, cannot be counted upon except in systems of stationary loads, wherein such permanent strains as tend to produce fracture are very infrequently repeated. Adaptation cannot be relied on where resistance to alternating loads is expected: it is known that in resisting loads of this kind the apparent extent of the elastic region is considerable, and that for each material there is, so to speak, a true elastic region within the elastic regions commonly so called. The former is known as the endurance region.

Thus the ductility of steel plays no part in stability under alternating loads, but through this phenomenon of adaptation it plays an essential part where stability against stationary or practically stationary loads is concerned.

Owing to the law of adaptation, the ductility of the steel becomes effective in any parts of a construction where, for one reason or another, the elastic limit

has been reached. Consequently the zones of plastic strain are found in the neigh-
bourhood of all points of geometrical or mechanical discontinuity, and these are
very numerous. Zones of plastic strain also occur where the elastic strains are
large. In a well designed structure, however, the zones of plastic strain are
usually of very limited extent. In effect the surplus strength conferred by adap-
tative strains is due to inequality in the distribution of stress, and to the existence
of some zones which are stressed less than others; in a well designed structure
these zones will necessarily be limited.

From these considerations one conclusion may immediately be drawn: while the
ductility of the steel may, indeed, be a phenomenon which plays an essential
part in ensuring the stability of structures, it does nothing to improve the
resistance of a well designed structure, but serves only to correct errors in design
or deficiencies in the homogeneity of material, or compensate for settlement
of the supports. In the author's opinion it would not, therefore, be wise to base
a new method of structural calculation on the exploitation of the property of
plastic deformation.

To say this is not to assert that methods so based must be rejected, and indeed
the author is himself accustomed to make daily use of them. For various reasons,
the forms actually given to structural members are not always those which would
correspond to the most efficient possible use of the material, and it is a natural
procedure to make use of the property of ductility of steel as a means of
partly correcting the mechanical errors which result from faulty shaping of the
material. As an explanatory example, consider the case of an arch, frame,
or continuous gider: the best plan, if it were possible, would be so to design
its constituent members that the elastic limit under a dangerous load would be
attained at all points simultaneously, and the ductility of the material would,
therefore, be of no use for purposes of design; but where the designer is com-
pelled to adopt forms which, from the point of view of mechanical efficiency, are
imperfect, he should avoid falling into the error of calculating them on the
elastic hypothesis under stationary loads, which would merely be adding a second
mistake to the first. The parts should, on the contrary, be calculated from the
hypotheses of plasticity, so as to minimise in this way the loss of efficiency that
attends a faulty choice of shape.

In the author's opinion, then, methods of calculation based on the theory of
plasticity are to be regarded as a last resort, and one which should not be used
except in calculating elements which are mechanically inefficient; nor, of course,
should it be applied in respect of any but fixed loads. From this point of view
it is to be wished that the methods at present in use might be codified with
a view to arriving at simple formulae capable of being applied to the commonest
problems of hyperstatical systems, especially arches and frames. It ought no
longer to be the practice to calculate these common structural forms, when
under fixed loads or only slightly varying loads, by other methods than those
depending on the law of adaptation.

The author, for his part, relies on the following rule in designing any frame-
work subject to loads which are either stationary or may be considered as such:
any system of forces and stresses which will keep in equilibrium a given mechanical
medium admits of realisation if account is taken of the occurrence of adaptation.

If, now, the system depends on a certain number of arbitrary parameters, every attempt should be made so to determine these parameters as to minimise the maximum value of the stresses occuring at the various points in the system. In other words, if an equilibrium is possible from the purely statical point of view, the construction will be stable under fixed loads without the necessity for enquiring whether the system of stresses as calculated is in fact the true system.

The principle which has just been explained has served the author as a guide in all the designs of frames on which he has been engaged, and it has proved itself a particularly useful aid in connection with those structures wherein, unlike the most usual case, the stresses are determined not by the magnitude of the strains but solely by the positions of the loads and the nature of the supports.

This is particularly true of mechanical systems in two dimensions, that is to say, where the stresses are propagated practically over a surface. The properties of such systems are closely related to the mechanical properties of surfaces. Now, when the mechanical phenomena attending the equilibria of surfaces are analysed, one is rapidly led to consider balanced systems wherein a given curvature of surface includes discontinuities of the stresses in parallel elements, such discontinuities entailing sudden variations in the length of the elements. A close investigation will indicate that, owing to the elastic properties of the material, equilibria of this kind are not possible without breakage of the material. Experience shows, however, that system of this kind are in fact perfectly stable, and the explanation of this apparent paradox is to be found once again in the theory of ductility.

Owing to sudden variations in stress the linear element in a surface may suffer large deformations, or the surface may deform itself geometrically in such a way that its linear element undergoes the variation in question. Alternatively, permanent elongations may arise which are the effect of counterbalancing the deformations caused by the mechanical action of the stresses. The author is of opinion that the ductility of the steel, though its precise action may be difficult to ascertain, plays a very important part in these phenomena.

The lines of discontinuity of stress which thus appear in the equilibrium of a surface usually start from points of discontinuity in the perimeter or are merged in the latter. It is easy to eliminate lines of discontinuity due to discontinuities of perimeter, simply by rounding the angles (at any rate for purposes of calculation). Lines of discontinuity which follow the perimeter itself are more difficult to eliminate, and here ductility of material plays an essential part.

Among these lines of discontinuity of stress in a surface the most important are asymptotic lines, where present. If some of these lines are followed, the conditions of equilibrium will lead to the discovery of discontinuities of stress and hence discontinuities of elongation. If the surface is flexible enough to allow it to deform, such deformation will cause asymptotic lines of discontinuity to deviate from the perimeter of the surface by modifying the shape of the latter, and this leads us to the case where asymptotic lines of discontinuity originate at a corner of the perimeter. The ductility of the steel in the neighbourhood of this angle has the effect of overcoming the discontinuity in question and of

7

substituting a fictitious perimeter for the true perimeter, all discontinuity being eliminated.

The study of plastic strains in the steel is perhaps even more important in regard to systems such as those which we have just examined than in regard to ordinary framed structures, because, contrary to what is true of the latter, it would be impossible in the former to construct stable systems without calling into play the power of adaptation of the material, which depends on its ductility. We are thus left with what appears, *a priori*, to be a paradox, namely an isostatic system which is justified by the theory of plasticity.