Permissible concrete stresses in rectangular reinforced concrete sections under eccentric loading

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Permissible Concrete Stresses in Rectangular Reinforced Concrete Sections under Eccentric Loading.

Zulässige Betondruckspannungen in rechteckigen Eisenbetonquerschnitten bei außermittigem Druck.

Contraintes de compression admissibles dans les sections de béton armé rectangulaires sollicitées excentriquement.

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Several investigators have raised objections to the usual method of designing reinforced concrete sections in bending or bending combined with compression, by the method based on the assumption of a straight line relation between stresses and strains in the concrete under compression. Nevertheless, the method is still in general use, and the Building Regulations of nearly all countries are based thereon.

In previous publications[1] and[2] the author has presented a method wherewith the *ultimate moments* or the *ultimate loads* of reinforced concrete members with rectangular cross-section may be computed in fair agreement with the results of actuals tests. On the basis of the ultimate carrying capacity of any rectangular section, determined in this way, the usual method of design may be tried out. Investigation will show how far the method meets the fundamental requirement that the same desired *factor of safety* should be maintained with different grades of concrete, different percentages of reinforcement and different eccentricities of load, and the most suitable working stresses may be determined. The case of simple bending has already been treated[3]; here the case of bending combined with compression will be investigated. Only short members with negligible deflections are considered.

1) Computation of Ultimate Loads.

The usual distinction must be made between *over-reinforced* and *normally reinforced* sections. Failure of the former starts on the compression side of the

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[1] *A. Brandtzaeg:* „Der Bruchspannungszustand und der Sicherheitsgrad von rechteckigen Eisenbetonquerschnitten unter Biegung oder außermittigem Druck." Norges Tekniske Höiskole, Avhandlinger til 25-ars jubileet 1935, F. Bruns Bokhandel, Trondheim, page 677 to 764.

[2] *A. Brandtzaeg:* Det kgl. norske Videnskabers Selskabs Skrifter 1935, Nr. 31, F. Bruns Bokhandel, Trondheim.

[3] *A. Brandtzaeg:* „Die Bruchspannungen und die zulässigen Randspannungen in rechteckigen Eisenbetonbalken." Beton und Eisen, Vol. 35, No. 13, July 5, 1936, pages 219 to 222.

section; no yielding of the tensile reinforcement occurs during failure. With normally reinforced sections the failure starts with yielding of the tensile steel; through opening of the crack of failure the compression area is subsequently reduced and finally crushed. In intermediate cases the two types of failure overlap. While in the case of simple bending the type of failure depends only on the properties of the materials and the percentage of reinforcement, it is, in the case of bending combined with compression, dependent also upon the eccentricity of the load.

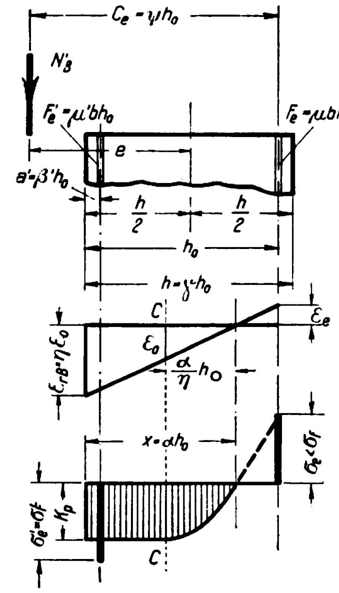

Fig. 1.

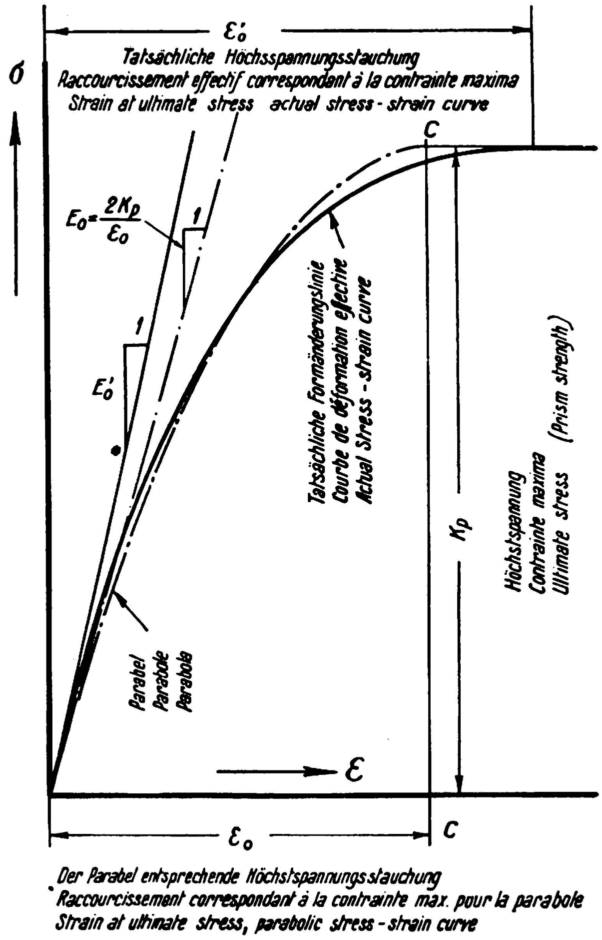

Fig. 2.

a) *Over-reinforced Sections.*

At the failure of a reinforced concrete member in bending or bending and direct compression the ultimate strain on the compression side of the member, ε_{rB}, is very much larger than the strain, ε_o, at which the same concrete under axial compression would reach its ultimate stress, the prism strength, K_P. The size of this *ultimate strain on the compression side* determines to some extent the ultimate carrying capacity of the member. It may be conveniently expressed by means of the *ultimate strain ratio*, $\eta = \frac{\varepsilon_{rB}}{\varepsilon_o}$.

In Figs. 1 and 3 is shown the distribution of stress which is assumed for a section at the stage of failure in bending or bending with compression. Where the compressive strain is smaller than ε_o (to the right of the lines C-C in Figs. 1

and 3) the stresses vary according to the stress-strain curve of the concrete in simple compression (Fig. 2). Where the strain is larger, the stress remains constant equal to the prism strength of the concrete, K_P. The steel stresses also correspond to the strains. Compression steel of mild or intermediate grade generally will have passed its yield point before the stage of failure is reached. No account is taken of tension in the concrete.

The above assumptions are in agreement with the author's own tests (See [1], pages 728 to 735 and [2], pages 54 to 61). *Saliger* has made similar assumptions on the basis of his tests.[4]

For the purpose of the analytical treatment the following equation, proposed by *Talbot,* is substituted for the actual stress strain curve of the concrete:

$$\sigma = E_0 \varepsilon \left(1 - \frac{1}{2}\frac{\varepsilon}{\varepsilon_0}\right) \tag{1}$$

Here σ is the compressive stress and ε the corresponding strain, ε_0 is the abscissa of the vertex of the parabola (Fig. 2) and E_0 defines the slope of the tangent to the parabola at zero stress. By suitable choice of the values of E_0 and ε_0 the parabola is fitted as well as possible to the actual stress-strain curve. Generally E_0 should then be chosen somewhat smaller than the actual modulus of elasticity of the concrete, E'_0, and ε_0 somewhat smaller than the strain, ε'_0, at which the concrete actually reaches its ultimate stress, K_p (Fig. 2). (See [1], pages 738—739 and [2], pages 64—65.)

Other curves, as for instance the one proposed by *von Emperger*,[5] agree somewhat more closely with the actual stress-strain curve. With the curve proposed by Talbot, however, the analysis is simpler, and the curve is sufficiently accurate for the present purpose. In 9 tests made by the author, the error arising from the use of Talbot's curve instead of the actual stress strain diagram of the concrete amounted for the ultimate loads to ÷ 4.6 to + 1.0 per cent, average ÷ 0.48 per cent, and for the ultimate moments to ÷ 0.7 to + 0.7 per cent, average + 0.13 per cent (See [1], page 732 and [2], page 58, Table 8, Columns 13 and 14).

The computation should be made separately for the two cases, Fig. 1 und Fig. 3, with the neutral axis inside and outside the cross-section, respectively.

In the first case the distance to the neutral axis, defined by the ratio $\alpha = \frac{x}{h_0}$, is given by the equation:

$$\left[\frac{1}{2} - \frac{1}{3\eta} + \frac{1}{12\eta^2}\right]\alpha^3 - (1-\psi)\frac{3\eta - 1}{3\eta}\alpha^2 + [2n\eta\psi\mu - (1-\psi-\beta')m'\mu']\alpha - 2n\eta\psi\mu = 0 \tag{2}$$

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[4] *R. Saliger:* „Versuche über zielsichere Betonbildung und an druckbewehrten Balken." Beton und Eisen, Vol. 34, No. 1 and 2, Jan. 5 and 20, 1935, pages 12 to 18 and 26 to 29.

[5] *F. v. Emperger:* „Die Formänderung des Betons unter Druck." International Association for Testing Materials, Congress in Zürich 1931, pages 1149 to 1159. — See also Beton und Eisen, Vol. 35, No. 10, May 20, 1936, page 179.

Here $n = \frac{E_o}{E_e}$ (E_e is the modulus of elasticity of the tensile steel) and $m' = \frac{\sigma'_F}{K_p}$ (σ'_F is the yield point of the compression steel). The other notation is shown in Figs. 1 and 3.

The ultimate load then is:

$$N'_B = \frac{1}{\psi}\left[\alpha\left(1-\frac{\alpha}{2}\right) - \frac{\alpha}{3\eta}\left(1-\alpha+\frac{\alpha}{4\eta}\right) + m'\mu'(1-\beta')\right] bh_0 K_P \qquad (3)$$

The unit stress in the tension reinforcement is found to be:

$$\sigma_e = 2\,n\eta\,\frac{1-\alpha}{\alpha}\,K_P \qquad (4)$$

In the second case, $\alpha > 1$, Fig. 3, we obtain two equations for the ultimate load. Equilibrium of the axial forces requires:

$$N'_B = \left[\frac{3\eta-1}{3\eta}\,\alpha - \frac{\eta}{\alpha}(\alpha-\gamma)^2\left(1-\frac{\eta}{3\alpha}(\alpha-\gamma)\right) + m'\mu' + 2\,n\eta\mu\,\frac{\alpha-1}{\alpha}\right] bh_0 K_P \qquad (5a)$$

and the equilibrium of moments about the center of gravity of the tension reinforcement gives:

$$N'_B = \frac{1}{\psi}\left\{\alpha\left(1-\frac{\alpha}{2}\right) - \frac{\alpha}{3\eta}\left(1-\alpha+\frac{\alpha}{4\eta}\right) + m'\mu'(1-\beta') + \frac{\eta}{\alpha}(\alpha-\gamma)^2\left[\frac{\alpha+2\gamma}{3} - 1 + \frac{\eta}{3}\left(1-\frac{\alpha+2\gamma}{4}\right) - \gamma\frac{\eta}{\alpha}\left(\frac{1}{3}-\frac{\gamma}{4}\right)\right]\right\} bh_0 K_P \qquad (5b)$$

From the equations (5a) and (5b) N'_B may be determined graphically.

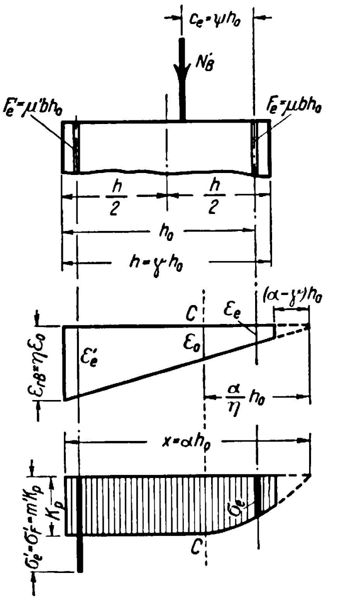

Fig. 3.

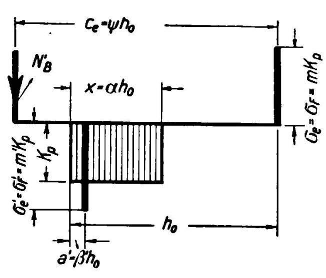

Fig. 4.

b) *Normally Reinforced Sections.*

In the vicinity of the crack that opens up at failure, the compressive stress in the concrete may be taken as constant over the entire compression area of the section (Fig. 4). The resulting error is very small, as is shown in [1], page 698 and [2], page 24. The stress in the tensile steel is assumed to be equal to the yield point, σ_F, as discussed in [3], Sections 4 and 6. After the steel has started to yield, there can be no influence of shrinkage or other tensile stresses in the concrete on the steel stress at the crack of failure.

With these assumptions we have:

$$\alpha = -(\psi - 1) + \sqrt{(\psi - 1)^2 + 2\,m\mu\psi - 2\,m'\mu'(\psi - 1 + \beta')} \tag{6}$$

and

$$N'_B = \frac{1}{\psi}\left[\alpha\left(1 - \frac{\alpha}{2}\right) + m'\mu'(1 - \beta')\right] bh_0 K_P \tag{7}$$

where $m = \frac{\sigma_F}{K_P}$.

With the load acting inside the cross-section, the ultimate load, N'_B, according to Equation (7), is quite large, and it increases very rapidly with decrease of the eccentricity and increase of the percentage of reinforcement. In actual fact, therefore, with $\psi < 1$ nearly all cross-sections conforming to ordinary practice are to be classed as *fully* reinforced, as discussed below, Articles 5 and 6.

2) Values of the Constants K_P, n and η.

By means of the above equations we may compute the ultimate load on any rectangular reinforced concrete member under bending combined with direct compression, provided the constants K_P, n (E_0) and η are known for the particular concrete in question. To make the equations applicable in all cases, the constants should be known as direct functions of some known numerical criterion of the quality of the concrete, as for instance the cube strength, K_W. No such functions, correct under all conditions, are, however, available. The relation of the prism strength, the modulus of elasticity and the ultimate strain ratio to the cube strength varies with a series of conditions, as for instance with the moisture content and the porosity of the concrete, the properties of the cement and the aggregates, etc. Nevertheless it seems possible to state general relations which will be sufficiently accurate for the purpose of a general investigation of the variation of the ultimate load with the quality of the concrete, the percentage of reinforcement and the eccentricity of the load. Better agreement with actual tests in any particular case may, of course, be obtained by determining at least K_P and E_0 experimentally. The following relations are based mainly on the tests described in papers [1] and [2]:

$$K_P = 0{,}77\,K_W \tag{8}$$

$$E_0 = 95\,500 + 390\,K_W \ \text{kg/cm}^2 \tag{9}$$

$$\eta = 1{,}25 + \frac{400}{K_W} - \frac{K_W}{400} \tag{10}$$

These relations have been used in the computations to follow, for concretes with $K_W = 100$ kg/cm² to 300 kg/cm².

Equation (10) represents fairly well the *lowest values of the ultimate strain ratio* found in tests by the author and by *Saliger.*[4] More extensive experiments are, of course, needed to determine what wider field of application the relation may be given. The fact that the ultimate strain ratio decreases with increase of the concrete strength is particularly important (see [3], page 221). One might, perhaps, expect η to decrease with the eccentricity of the load. The tests, however, have shown no regular variation of η with variation of the eccentricity (see [1], page 739 and [2], page 65, Table 9, Column 9).

3) Comparison of Computed with Actual Ultimate Loads.

The tests described in the papers [1] and [2] included the testing of 9 over-reinforced and 4 normally reinforced specimens with eccentrically applied axial loads, with $\psi = 0.661$ to 1.855. The specimens had 0.70 to 4.64 per cent of tensile reinforcement. The concrete used in the tests gave rather unusual values of the ratio K_P/K_W. When the actual values of K_P, as found in the tests, and also the test values of E_0 and η (which, however, are in fair agreement with equations (9) and (10)) are entered in the computations according to equations (2) to (5), ultimate loads are found, which for two of the three groups of over-reinforced specimens agree well with the test results. The greatest deviation is 12 per cent and the average deviation for the 6 specimens of these groups is 5 per cent. On account of differences in the compacting of the concrete in different kinds of specimens, the tests with the third group of over-reinforced specimens gave no basis for such comparison. Also for these specimens, however, the influence of variations in the eccentricity of load and the percentage of reinforcement seems to be well represented by the equations of Section 1 (see [1], page 744 and [2], page 70, Table 10, Column 8).

The actual ultimate loads of the four normally reinforced specimens were on the average 8,8 per cent greater than computed on the basis of the actual values of K_P. When the cube instead of the prism strength is entered in Equations (6) and (7), the actual ultimate loads are on the average 1.7 per cent smaller than the computed ones. It does, in fact, seem probable that during a local failure like that taking place in normally reinforced specimens, the compressive stress in the concrete may well reach a value equal to the cube strength. For the sake of safety, however, the prism strength is used in the computations.

The most complete series of tests of reinforced concrete in bending combined with compression, known to the author, is the one carried out by *Bach* and *Graf.*[6] In Table 1, Column 14, are given the average ultimate loads of the 15 groups of test specimens. The average dimensions, percentages of reinforcement and eccentricities of load are listed in columns 2 to 12, according to the report in paper.[6] The average cube strength of the concrete was $K_W = 225$ kg/cm², consequently $K_P = 0.77\ K_W = 173$ kg/cm², which agrees with the test results for plain specimens in centric compression (see [6], Table 24). According to the equations (9) and (10) we then have $n =$ about 11.5 and $\eta =$ about 2.5. The ratios m and m' have been determined from the values of the yield point of the steel shown in Table 3 of paper [6]. With the constants thus determined, the ultimate loads of the 15 groups of specimens have been

[6] *C. Bach* and *O. Graf:* „Versuche mit bewehrten und unbewehrten Betonkörpern, die durch zentrischen und exzentrischen Druck belastet wurden.“ Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, No. 166 to 169, 1914.

computed, see Table 1, Column 13. As seen from Column 15 the agreement between the computed and the actual ultimate loads is good. For one of the groups of plain concrete specimens the computed load fell 15.3 per cent below the actual value, otherwise the deviations vary between — 3.98 per cent and + 5.15 per cent. The average deviation of the computed from the actual loads for all 15 groups amounts to — 1.13 per cent.

Slater and *Lyse* have tested two plain concrete prisms under eccentric loading.[7] The dimensions of the prisms were $20.3 \times 20.3 \times 30.5$ cm, the prism strength of the concrete was K_P 285 kg/cm^2 and consequently the probable cube strength about $K_W = 370$ kg/cm^2. When η is computed from Equation (10), α from Equation (2) and N'_B from Equation (3) with $\mu = \mu' = 0$, we obtain $N'_B = 74.4$ t. The actual average ultimate load was 70.5 t, that is 5.3 per cent smaller than computed.

In the above cases it is seen that the ultimate loads computed from the equations of Article 1 agree fairly well with the results of tests. It seems, therefore, that the equations may at least be used as the basis of a general investigation of the variation of the ultimate load of eccentrically loaded members with the eccentricity of the load and the percentage of reinforcement.

4) The Factor of Safety.

The permissible or *safe loads* may be computed by dividing the ultimate loads by the *factor of safety*. The proper choice of the factor of safety has been discussed at some length in previous publications (see [1], pages 688 to 693; [2], pages 14 to 19 and [3], pages 221 to 222). If an *actual* factor of safety of 2 is desired, the *nominal* factor of safety for simple compression should be raised to 3.3 or 3.4, on account of the influence of long-time or repeated loads and on account of the difference in strength due to difference in size between ordinary structural members and usual test specimens. *The proposed new Norwegian Building Regulations for Reinforced Concrete,* designated as NS 427, the first part of which was published for discussion in the autumn of 1935,[8] are based on factors of safety in simple compression of 4.13, 3.85, 3.65 and 3.60 respectively for the four Standard Concretes A to D with cube strengths of 290 kg/cm^2, 230 kg/cm^2, 180 kg/cm^2 and 140 kg/cm^2 respectively.

Certain differences in the manner of failure of concrete in simple compression and in bending or bending with compression, make it seem desirable to have a factor of safety 10 % higher for bending and bending with compression than for simple compression. (See [1], pages 751 to 754; [2], pages 77 to 80 and [3], page 222.) The factors of safety for bending and bending with compression to correspond with the above values then should be 4.54, 4.24, 4.02 and 3.96 respectively for the Standard Concretes A to D. These values are used in the computations referred to below.

For the reinforcement there is no such difference between the actual and the

[7] *W. A. Slater* and *Inge Lyse:* „Compressive Strength of Concrete in Flexure as Determined from Tests of Reinforced Beams." Proceedings, American Concrete Institute. Vol. 26. 1930. in particular pages 852 to 859.

[8] „Forslag til Norsk Standard: Regler for utførelse av arbeider i armert betong — NS 427, utarbeidet av Den Norske Ingeniørforening." Supplement to Teknisk Ukeblad No. 38, 1935.

Table I.

Actual and calculated rupture loads for eccentric loading according to tests by Bach and Graf 1914.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Test piece No.	Reinforcement	Eccentricity of load measured from the axis of the test piece	Mean dimensions					Mean percentage of reinforcement		Position of load (fig. 1, 3, 4)		Rupture load			Notes
			b	h	a	h_o	a'	μ	μ'	c_e	$\psi = \frac{c_e}{h_o}$	Calculated N'_B	Mean test result N_B	$\frac{N'_B - N_B}{N_B}$	
		cm	cm	cm	cm	cm	cm	%	%	cm		tons	tons	%	
75, 88, 142	0	10	40.1	40.2	0	40.2	0	0	0	30.1	0.749	138.0	136.0	+ 1.47	without reinforcement
76, 89, 143	0	15	40.1	40.1	0	40.1	0	0	0	35.05	0.874	69.3	81.8	— 15.30	„ „
82, 90, 97	4 ø 16	0	40.1	40.1	3.4	36.7	0	0.559	0	16.65	0.454	277.0	280.3	— 1.18	heavy reinforcement
85, 91, 94	„	20	39.9	40.1	3.6	36.5	0	0.564	0	36.45	0.999	93.6	93.0	+ 0.65	normal „
86, 92, 95	„	30	40.0	40.1	3.6	36.5	0	0.567	0	46.45	1.272	57.9	60.3	— 3.98	„ „
87, 93, 96	„	50	40.0	40.1	3.9	36.2	0	0.570	0	66.15	1.830	28.9	30.0	— 3.67	„ „
107, 108	8 ø 16	10	40.0	40.1	3.7	36.4	3.1	0.558	0.560	26.35	0.724	198.3	202.5	— 2.07	heavy reinforcement
99, 102, 118	„	20	40.1	40.1	3.6	36.5	3.3	0.558	0.556	36.45	0.999	119.3	124.0	— 3.79	normal „
119, 120, 121	„	20	40.1	40.2	3.6	36.6	3.3	0.558	0.555	36.50	0.998	119.0	123.3	— 3.49	„ „
100, 103	„	30	40.1	40.3	3.5	36.8	3.3	0.554	0.552	46.65	1.269	69.3	69.6	— 0.43	„ „
101, 104	„	50	40.2	40.2	3.6	36.6	3.3	0.558	0.552	66.50	1.818	33.3	32.4	+ 2.78	„ „
140, 141	8 ø 22	10	40.0	40.3	3.7	36.6	3.8	1.045	1.043	26.45	0.723	236.6	225.0	+ 5.15	heavy reinforcement
63, 122, 137	„	20	40.1	40.1	3.8	36.3	3.7	1.047	1.050	36.25	0.999	164.8	157.5	+ 4.63	„ „
123, 138	„	30	40.1	40.1	3.7	36.4	3.8	1.044	1.045	46.35	1.272	105.5	105.0	+ 0.48	normal „
65, 124, 139	„	50	40.1	40.1	3.8	36.3	3.7	1.050	1.048	66.25	1.825	55.1	53.5	+ 3.00	„ „

Constants of material: $\eta = 2{,}5$; $n = 11{,}5$. For rounds of 16 mm ø: $\sigma_F = 3773$ kg/cm², $\sigma'_F = 3680$ kg/cm², $K_P = 173$ kg/cm².
For rounds of 22 mm ø: $\sigma_F = 3672$ kg/cm², $\sigma'_F = 3754$ kg/cm².

nominal factor of safety, since for mild and intermediate steel the tensile strength which can be relied upon under such repetition of loading as occurs in most reinforced concrete structures, will come very close to the yield point of the steel, which is the stress used in computing the ultimate loads of normally reinforced members according to Equations (6) and (7). Consequently the nominal factor of safety may be chosen equal to the actual factor of safety desired. In the computations referred to below a factor of safety of 1.8 was used for normally reinforced sections. This should be fully sufficient for a uniform material like steel.

5) Safe Loads and Limiting Points.

In Fig. 5 are shown the safe loads, N_{zul}, computed as described above, for a section with tensile reinforcement only ond for a symmetrically reinforced section. The computation was made for the following case: Position of load 1.5 h_o from the tensile reinforcement (moment arm ratio, $\psi = 1.5$), $\gamma = 1.08$, $\beta = 0.08$ (See Figs. 1 and 3), cube strength of concrete, $K_W = 180$ kg/cm², $\eta = 3.03$, $n = 12.7$ (Standard Concrete C according to NS 427), yield point of steel $\sigma_F = \sigma'_F = 2000$ kg/cm²,[9] $m = m' = 14.4$. With the percentage of reinforcement as abscissa the safe unit loads, $\frac{N_{zul}}{bh_o}$, have been plotted as well for over-reinforced sections [Equations (2) to (5)] as for normally reinforced sections [Equations (6) and (7)]. At any particular value of μ, the lower one of the two corresponding values of N_{zul} does, of course, represent the actual value of the safe load. (Heavily drawn lines in Fig. 5.)

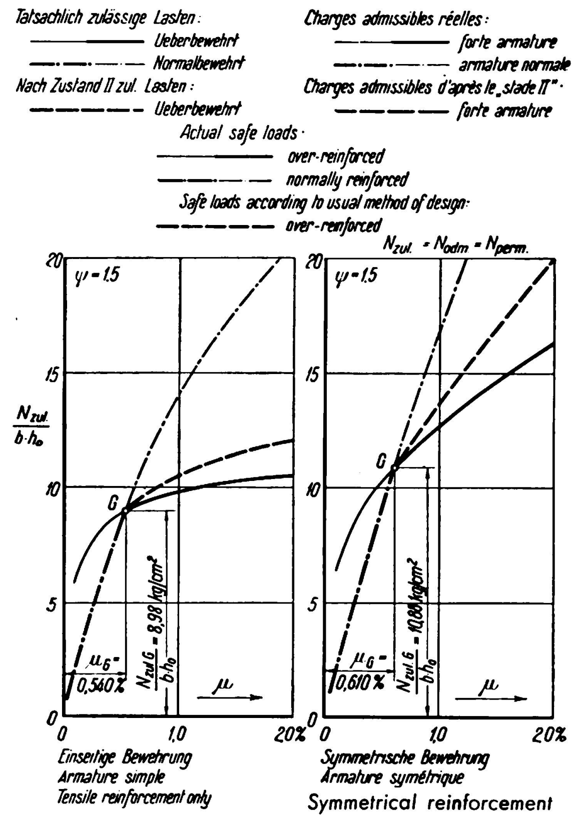

Fig. 5.

Safe loads for concrete C with $\psi = 1.5$ as actually obtained and according to usual method of design.

The point G, where the two lines for N_{zul} intersect, is the *limiting point* separating the two ranges of reinforcement, one range of *partly reinforced* sections, where the reinforcement determines the safe load, and one of fully *reinforced sections,* where the safe load is dependent mainly upon the strength of the concrete.

Lines like those in Fig. 5 might well be used as a means of designing eccentrically loaded rectangular reinforced concrete sections. However, the ordinary method of calculation may as well be used, *provided only that the working*

[9] This is considered as the lower limit for ordinary mild reinforcing steel as used in Norway.

stresses are so chosen that the ordinary calculation will in every case lead to the correct value of the safe load.

For partly reinforced sections, where the reinforcement determines the safe load, this can generally be attained by the use of one single value of the working steel stress for all percentages of reinforcement. For fully reinforced sections, however, the case is different. With one single value of the allowable concrete fibre stress the ordinary method of calculation gives nominal safe loads which increase far more rapidly with increase in the percentage of reinforcement than

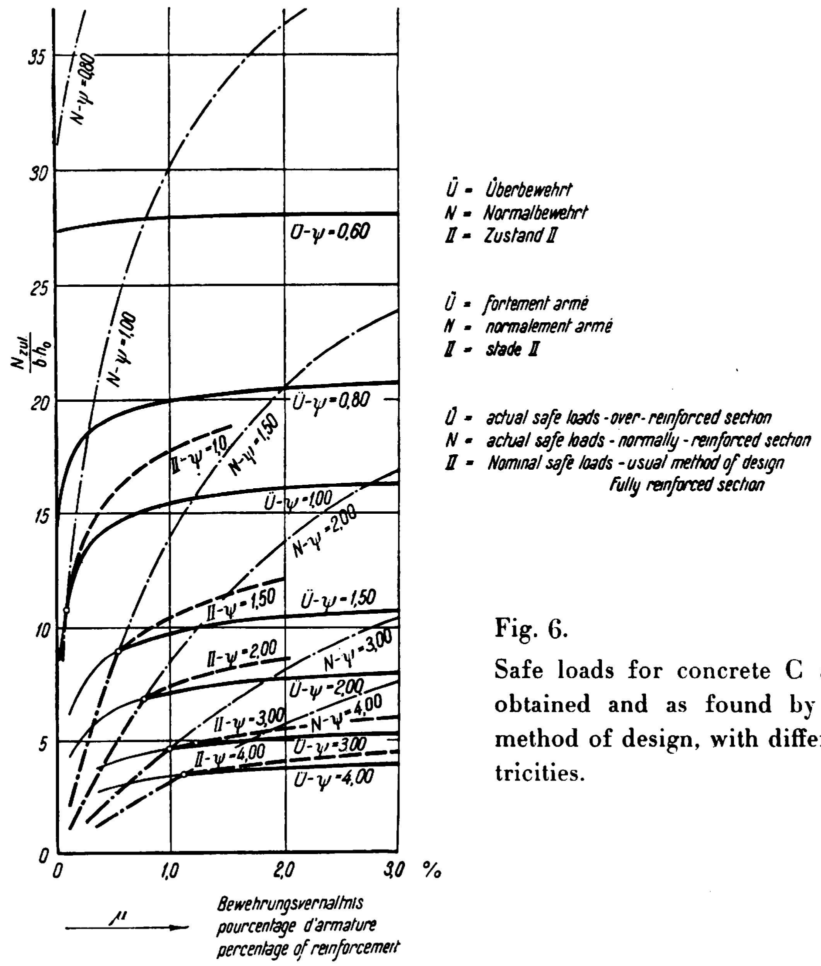

Fig. 6.

Safe loads for concrete C as actually obtained and as found by the usual method of design, with different eccentricities.

actual safe loads, as determined according to the above analysis. This is shown in Fig. 5 and also in Fig. 6, where the actual and nominal safe loads are plotted for several values of ψ, the assumptions being as for Fig. 5. Consequently, one definite factor of safety can only be maintained throughout the range of fully reinforced sections if the working stress for concrete is varied with the percentage of reinforcement.

It has been shown previously that in the case of pure flexure the correct allowable fibre stress in concrete is the stress corresponding to the limiting point, G. (See [1], page 688, [2], page 14 and [3], page 222). The same applies to the case of bending with compression, provided that the eccentricity of load is large. With smaller eccentricities allowable concrete stresses other than those corresponding to the limiting points may be of practical interest. In the first place, when the load is acting inside the cross-section, there hardly is any limiting point to be found, since practically all sections are fully reinforced (See Article 1, b and Fig. 6). In the second place, even with the load acting well outside the cross-section, the percentages of reinforcement corresponding to the limiting points are so small, that in practice very often more reinforcement must be used (Fig. 6).

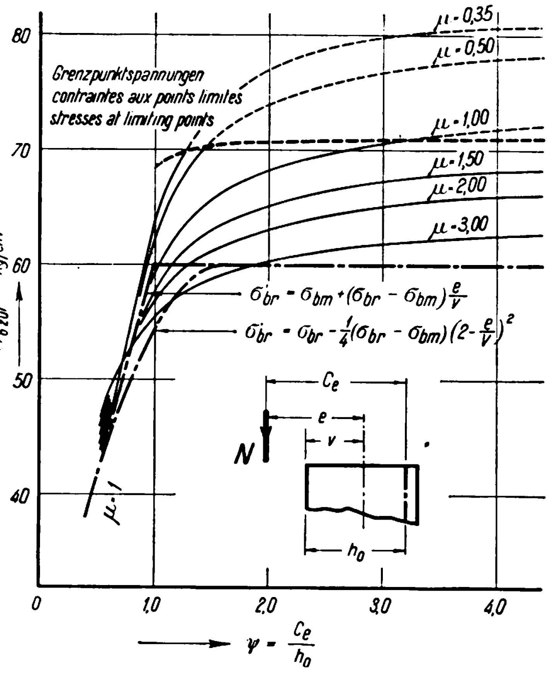

Fig. 7.

Correct permissible stresses for concrete C with different eccentricities, using reinforcement on one side only.

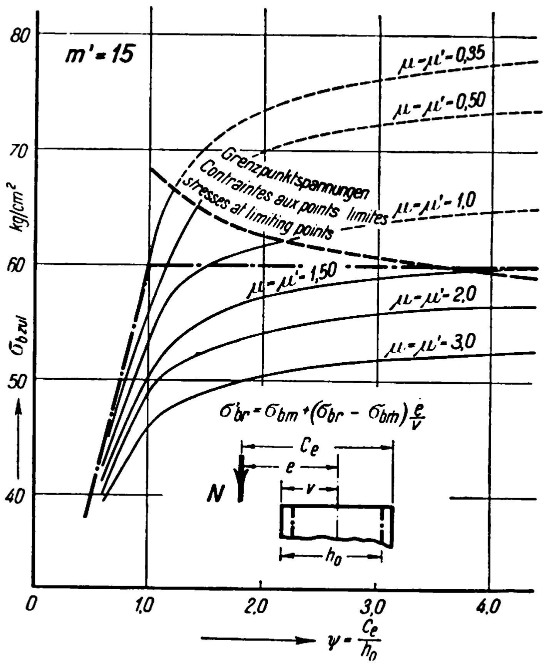

Fig. 8.

Correct permissible stresses for concrete C with different eccentricities, using symmetrical reinforcement (assuming m' = 15).

6) Correct Working Stresses for the Concrete.

From the actual safe loads, determined as described above, the corresponding correct working stresses for concrete, to be used with the ordinary method of calculation, can be computed for different eccentricities of load and different percentages of reinforcement. Working stresses, thus determined for Standard Concrete C with assumptions as in Article 5, have been plotted in Figs. 7 and 8 with $\psi = \frac{C_e}{h_o}$, as a measure of the eccentricity, as abscissa. In addition, the concrete stresses corresponding to the limiting points, discussed in Article 5, have been plotted in the diagrams. Concrete working stresses exceeding the stresses at limiting points, are of no significance, since they correspond to sections for which the steel, not the concrete stress determines the safe load (partly reinforced sections).

As one might expect, the diagrams show that the *correct working stresses for concrete decrease very rapidly with decrease in the eccentricity of the load. As the load approaches the centre of gravity of the cross-section, the correct working stresses approach those valid for simple compression.*

Thus, under the assumptions stated above, the allowable fibre stresses for standard concrete C with tensile reinforcement only should be as follows:

In pure bending, at limiting point $\sigma_{b\,zul\,1} = 71.0$ kg/cm².

In bending with compression, with 1 per cent of reinforcement:

With the load at the edge of the cross-section

($\psi = 1.0$) . . . $\sigma_{b\,zul} = 59.6$ kg/cm² $= 0.84\ \sigma_{b\,zul\,1}$.

With position of load so that the stress at the far edge of the cross-section is zero

($\psi = 0.63$) . . . $\sigma_{b\,zul} = 49.0$ kg/cm² $= 0.69\ \sigma_{b\,zul\,1}$.

With the load acting at a distance of 0,135 h_o from the centre of gravity of the section

($\psi = 0.54$) . . . $\sigma_{b\,zul} = 44.8$ kg/cm² $= 0.63\ \sigma_{b\,zul\,1}$.

It is seen that if the *same* working stress is used in actual design in all these cases, the factor of safety will actually be very much less with small eccentricities of load than in the case of pure flexure.

7) Effectiveness of Compression Steel.

As the Figures 7 and 8 show, the correct working stresses for concrete vary much with the quantity of reinforcement, and in particular with the quantity of compressive reinforcement. With symmetrical reinforcement the correct working stresses are appreciably lower than with tension reinforcement only. The same applies to the case of pure flexure, as the dotted line in Fig. 9 shows. The correct concrete working stress for Standard Concrete C at limiting points is about 21 per cent lower with symmetrical reinforcement than with tension reinforcement only.

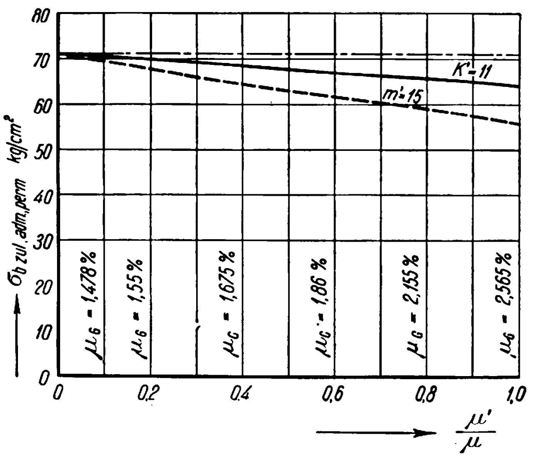

Fig. 9.

Correct permissible stresses for concrete C under pure bending with different amounts of compression reinforcement (calculated partly with $m' = 15$ and partly with $k' = 11$).

The correct concrete working stresses represented in Figures 8 and 9 have been computed from the safe loads by the ordinary method of computation, whereby the stresses in concrete and steel have been assumed to be distributed as indicated in Fig. 10. The stress in the compressive reinforcement has been computed from the equation:

$$\sigma'_e = m'\sigma_{br}\frac{\alpha - \beta'}{\alpha} \tag{11}$$

where σ_{br} is the allowable fibre stress in concrete in the case considered, and m' is equal to $\frac{\sigma'_F}{K_P}$, as defined in Article 1, a. For the concrete assumed here, with

$K_w = 180$ kg/cm^2 and $K_P \cong 138$ kg/cm^2 (cylinder strength at 28 days (f'_c) about 2000 lb. per sq. in.) and for steel with a minimum value of the yield point, $\sigma'_F = 2000$ kg/cm^2 (about 28400 lb. per sq. in.), we have m' approximatively equal to 15, which is the value used in computing the curves of Figures 8 and 9.

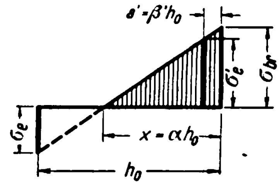

Fig. 10.

Now, while the actual stress in the concrete at failure is equal to K_P (Figures 1 and 3), the nominal stress corresponding to the load at failure according to the stress-distribution of Fig. 10, will be much larger than K_P. Correspondingly, at that load the nominal stress in the compressive reinforcement according to Equation (11) will be much larger than σ'_F. That equation thus leads to an exaggeration of the effect of the compression steel upon the ultimate load. To correct this, a factor k', smaller than m', should be used in Equation (11). The factor should be chosen so as to make the computed stress in the compression steel at failure equal to $m' \cdot K_P = \sigma'_F$. The result should be about the same if k' is taken as given by the equation

$$k' = m' \frac{\sigma_{bm}}{\sigma_{br}} \frac{\alpha}{\alpha - \beta'} \tag{12}$$

where σ_{bm} is the allowable concrete stress in simple compression. At the working load, σ'_e will then be equal to $m'\sigma_{bm}$.

Most building regulations specify the use of the factor n instead of m' in Equation (11). Usually, however, $n = 15$ is used, at least for the grade of concrete considered here, and since that was the value of m' used in computing the curves of Figures 8 and 9, computation according to most building regulations would give the same results as are shown there, with the same exaggeration of the effect of the compression steel.

In that portion of the proposed new Norwegian Regulations, NS 427, which has not yet been published, values of k' approximately in agreement with Equation (12) are specified for use in the cases of bending and bending with direct stress. In simple compression, the ratio between stresses in steel and concrete is taken to be $m' = \frac{\sigma'_F}{K_P}$. For the grade of concrete considered here, $k' = 11$ and $m' = 15$ are the specified values. For the stress in the tensile reinforcement, the ratio $n = 15$ is used in all cases.

The full line in Fig. 9 and the curves in Fig. 11 show the correct concrete working stresses obtained by using $k' = 11$ instead of $m' = 15$ in Equation (11). It is seen that there is still a difference between sections with and without compression steel. This is due mainly to the fact that according to NS 427, σ_{br} for the concrete considered is only 60 kg/cm^2, while according to our computations $\sigma_{br} = 71$ kg/cm^2 would be the correct value. However, much of the difference is eliminated with the use of k' instead of m'.

8) The Working Stresses for Concrete as Specified in Building Regulations.

In the building regulations of most countries very little account is taken of the great influence of the eccentricity of load on the correct concrete working stresses for structural members in bending with compression, which is demonstrated in Figures 7, 8 and 11. According to the regulations of several countries, the full working stress for pure bending may be applied also in the case of bending with compression, provided only that the working stress for simple compression is not exceeded when the load is considered as acting centrally. If, for instance, the allowable fibre stress in flexure is 60 kg/cm^2 and in simple compression 38 kg/cm^2, as specified for Standard Concrete C in NS 427,[8] the full bending stress could in the case treated in Article 6, assuming 1 per cent of tensile reinforcement only, be applied with the load acting only 0.105 h_0 from the center of gravity of the section, that is, with $\psi = 0.508$. The correct working stress would in that case be about 43.5 kg/cm^2, as against 71.0 kg/cm^2 in pure bending. That is, the factor of safety would be about 39 per cent less than in the case of pure bending.

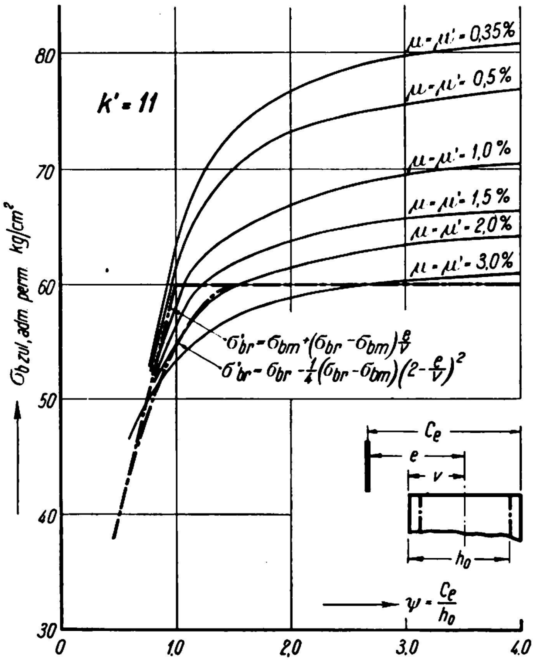

Fig. 11.

Correct permissible stresses for concrete C with different eccentricities, using symmetrical reinforcement (calculated with $k' = 11$ instead of m') (compare Fig. 8).

The latest American regulations[10] provide for an increase in the working stress for eccentrically loaded as compared to centrically loaded columns, trough multiplying the working stress for simple compression with a factor, which for instance with $\psi = 1.0$ and 1 per cent of reinforcement on either side of the cross-section, amounts to about 1.163. In a column without spirals the working stress would then be $0.154\ f'_c \cdot 1.163 \cong 0.18\ f'_c$. ($f'_c$ is the minimum ultimate compressive strength of test cylinders at 28 days, for the concrete considered about 2000 lb per sq. in.) Now, the allowable unit stress in pure flexure is specified to be $0.40\ f'_c$. From Fig. 8 we find the correct concrete working stress with $\psi = 1.0$ and $\mu = \mu' = 1.0$ per cent to be 53.2 kg/cm^2, or about 75 per cent of the correct working stress in pure flexure with no compression steel (71.0 kg/cm^2) which has been determined with the same factor of safety. That means that the factor of safety in the case considered would be the same as in pure flexure, if the working stress were fixed at $0.75 \cdot 0.40\ f'_c = 0.30\ f'_c$. Actually only $0.18\ f'_c$ is allowed, and hence the American Concrete Institute's Regulations provide in this case for a factor of safety which is about 67 per cent greater than the factor of safety actually used in pure flexure.

As seen, the case of bending combined with compression is treated very differently in the building regulations of different countries. According to some

10 *Building Regulations for Reinforced Concrete* (A.C.I. 501—36 T) tentatively adopted, Feb. 25, 1936, Journal American Concrete Institute, March-April 1936, Vol. 7, pages 407—444.

regulations, the factor of safety is much smaller in the case of bending with compression than in the case of pure flexure, according to others, it is larger.

In the proposed new Norwegian Regulations, NS 427[8], an attempt has been made to adapt the working stresses for concrete in bending with compression somewhat better to the correct values. The allowable unit fibre stress for concrete in bending with compression is specified as follows:

a) With the load acting inside the cross-section ($\psi < 1{,}0$):

$$\sigma'_{br} = \sigma_{bm} + (\sigma_{br} - \sigma_{bm}) \frac{e}{v}; \qquad \frac{e}{v} < 1 \tag{13}$$

where: σ_{br} = allowable unit fibre stress in pure flexure,

σ_{bm} = allowable unit fibre stress in simple compression,

e = eccentricity of load, measured from the gravity axis of the equivalent concrete section,

v = distance from gravity axis to extreme fibre in compression.

b)With the load acting outside the cross-section ($\psi \geqq 1$):

$$\sigma'_{br} = \sigma_{br}; \qquad \frac{e}{v} \geqq 1 \tag{14}$$

The allowable unit stresses according to Equations (13) and (14) have been plotted in Figures 7 and 11 for comparison with the correct values. It is seen that although the working stresses specified in the proposed NS 427 do not lead to the same factor of safety in all cases, nevertheless much of the variation implicit in other specifications has been eliminated.

The agreement between correct and specified working stresses would be improved, if the full allowable fibre stress for pure flexure were to be applied only with $\frac{e}{v} > 2$ or $\psi >$ about 1,6, and if a parabolic instead of a linear variation of the working stress for smaller eccentricities were specified, for instance as given by Equation (15):

$$\sigma'_{br} = \sigma_{br} - \frac{1}{4} (\sigma_{br} - \sigma_{bm}) \left(2 - \frac{e}{v}\right)^2; \qquad \frac{e}{v} < 2 \tag{15}$$

The corresponding curves are shown in Figures 7 and 11, they agree quite well with the smaller values of the correct working stresses as here determined.