Questions for discussion on fundamental relationships and principles governing the fatigue strengths of welded connections

IIIa 3

Questions for Discussion on Fundamental Relationships and Principles Governing the Fatigue Strengths of Welded Connections.

Diskussionsfragen über Grundbeziehungen und Begriffsfestsetzungen für die Dauerfestigkeit geschweißter Stabverbindungen.

Thèmes de discussion concernant les relations fondamentales et la détermination des notions se rapportant à la résistance à la fatigue des assemblages soudés.

Dr. Ing. W. Gehler,

Professor an der Technischen Hochschule und Direktor beim Staatlichen Versuchs-
und Materialprüfungsamt, Dresden.

During the last five years numerous experiments on fatigue have been carried out in the German laboratories for testing materials, with a view to the formulation of official regulations for welded connections of structural members. The interpretation of these experiments has given rise to a series of questions, some of which it has been possible to clear up, while others constitute problems regarding which an international exchange of ideas is desirable and which demand further research.

1) *Representation of the fundamental dimensions force, space and time (Fig. 1).*

a) The ordinary force-space plane X—Y (or stress-strain plane) serves to represent the results obtained in the ordinary statical breaking test, which lies at the basis of the theory of strength and elasticity. The effect of the time factor on the breaking test is usually ignored, but may be recognised in the fact that if the experiment is carried out more quickly the usual line 1 in Fig. 1 merges into line 2, or in the case of impact tests may even merge into line 3.

b) If the third axis of coordinates Z is adopted as the time axis, then the Y—Z plane shows the transition to the region of vibration, or to time-strength relationships, and the result of fatigue tests may be represented in it by a fatigue-time line (known as the *Wöhler* line).[1] Here the abscissae z represent the duration of the experiment, though not according to the usual time scale, for it is expressed by the number of alternations of load (for instance 2 million alter-

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[1] *Wöhler:* Zeitschrift für Bauwesen, Berlin 1860, 1863, 1866 and 1870.

nations of 4 seconds each, or 8 million seconds in all). If time is measured in this way, by the number of similar vibrations instead of by the clock, the *Wöhler* line shows how many vibrations can be withstood by the specimen under a given stress. If the experiment continues long enough this line finally becomes parallel to the Z axis at a distance σ_D where it represents the fatigue stress *(Dauerfestigkeit)* that is to say, the amount of stress that can be supported indefinitely.[2]

c) The range of vibration covers a system of coordinates X' Y' Z' which has been displaced parallel to the X Y Z system, the stresses being again indicated by the ordinates y' measured along the direction of the load axis. In the Y' Z' plane (the load-time plane) it is possible to visualise the following three fundamental concepts: I) alternating strength for equal positive and negative stresses *(Wechselfestigkeit* σ_W), II) fatigue strength for stresses pulsating between zero and a maximum *(Ursprungsfestigkeit* σ_U), and III) fatigue range *(Schwellfestigkeit* $\sigma_{D\,max}$), which is the fatigue strength subject to a pre-existing stress σ_m (In reference to I) the limiting values of the stress wave are $\sigma_o = + \sigma_w$ and $\sigma_u = - \sigma_w$

hence $\sigma_m = 0$; in reference to II) they are $\sigma_o = \sigma_U$ and $\sigma_u = 0$, hence $\sigma_m = \frac{1}{2} \cdot \sigma_U$).
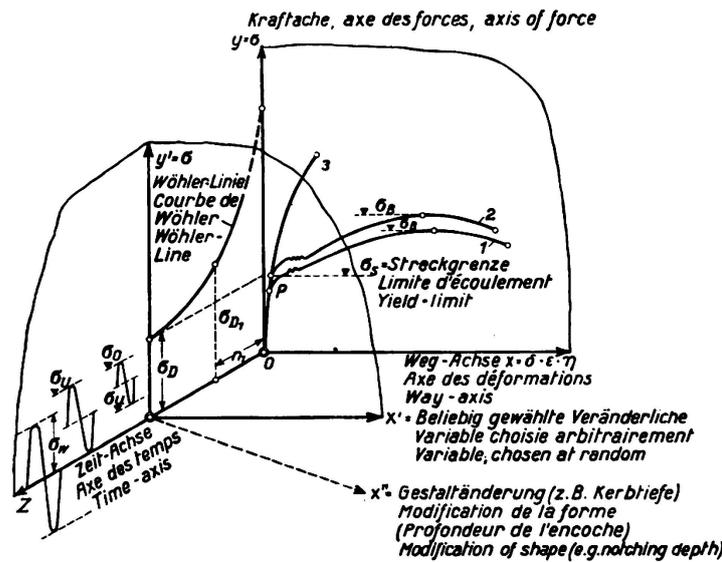

Kraftache, axe des forces, axis of force

y'-δ

y'-δ WöhlerLiniel Courbe del Wöhler. Wöhler- Line

σ_s -Streckgrenze Limite d'écoulement Yield-limit

Weg-Achse x·δ·ε·η Axe des déformations Way-axis

x'- Beliebig gewählte Veränderliche Variable choisie arbitrairement Variable, chosen at random

x"- Gestaltänderung (z.B. Kerbtiefe) Modification de la forme (Profondeur de l'encoche) Modification of shape (e.g. notching depth)

Zeit-Achse temps Axe des Time-axis

Fig. 1.

d) A plane X" of X' rotated about the Y-axis may serve for the representation of any other function of fatigue strength, the stresses σ_D always being measured by the ordinates y'. In this way, for instance, it is possible to indicate the relations of the stresses to the depths of notches in notched bars, or to describe the y — method of the Reichsbahn, wherein x' = σ_{min} : σ_{max} is introduced, or to indicate the relationship to statical pre-stressing *(Haigh*'s construction).

This form of representation (Fig. 1) allows all parts of the theory of strength of materials to be indicated with their symbols.

2) *Permissible stresses and design coefficients for steel bridges.*

A) *Rivetted railway bridges.*[3]

Whereas, under stationary or statical loading, safety is determined by the proportions between the stress σ_s which corresponds to the yield point, and the

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[2] The first two of the questions for discussion enumerated in Section 8 arise here.

[3] The tern "railway bridges" is used to denote bridges which carry railway tracks in contradistinction to roadways.

stress σ_{zul} which arises under live load (so that $v = \sigma_s : \sigma_{zul}$), under conditions of alternating load safety is determined by the *number of alternations*, and this is a statistical problem.

Case 1. Side members of a trussed main girder in a rivetted railway bridge (Fig. 2).

On an old trussed bridge of 39 m span[4] a tensile stress $\sigma_{max} = + 215 \text{ kg/cm}^2$ and a compressive stress $\sigma_{min} = - 70 \text{ kg/cm}^2$ were recorded graphically during the passage of two test vehicles of $4 \times 8 = 32$ tons weight, and the curve of stress was determined graphically as in Fig. 2, corresponding to the total influence line of the test crane. If, during the passage of a locomotive the designed
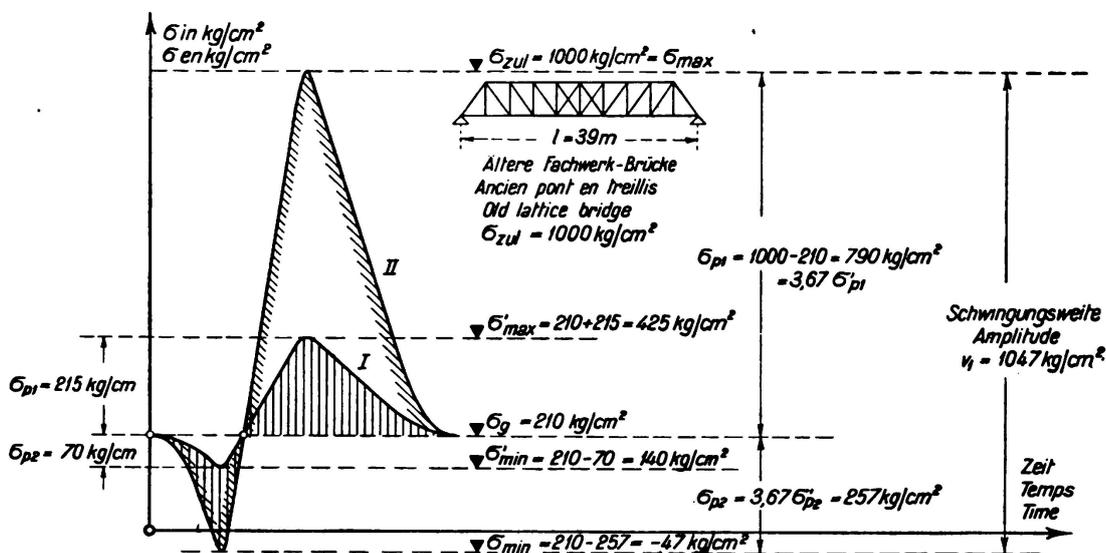

Fig. 2.

Amplitude and static safety of connections of railway bridges.

permissible stress of $\sigma_{zul} = \sigma_{max} = 1000 \text{ kg/cm}^2$ is to be utilised, then the curve of stress corresponding to a uniform permanent stress of $\sigma_g = 210 \text{ kg/cm}^2$ must be magnified in the ratio of $\dfrac{1000 - 210}{215} = 3.67$, giving at the trough of the wave $70 \times 3.67 = 257 \text{ kg/cm}^2$, and $\sigma_{min} = 210 - 257 = - 47 \text{ kg/cm}^2$ (compression). The total amplitude then works out at

$$v_1 = 1000 + 47 = 1047 \text{ kg/cm}^2 = 10.5 \text{ kg/cm}^2. \tag{1}$$

If the problem now arises of determining the fatigue strength σ_{D1} of this member and its connections experimentally, the amplitude must in the same way be fixed at $w_1 = v_1 = 10.5 \text{ kg/cm}^2$ in the experiment. If only a small portion of the stress curve for the tensile member in question falls within the compression zone, the fatigue test may properly be based upon the pulsating strength *(Ur-*

[4] *W. Gehler:* Nebenspannungen eiserner Fachwerkbrücken, p. 67 (Wilh. Ernst & Sohn, Berlin 1910).

sprungsfestigkeit) ($\sigma_{min} = 0$; $\sigma_{max} = 10.5 \text{ kg/cm}^2$). For instance, if the fatigue test shows breakage at $n_{D_1} = 2000000$ changes of load,[5] and if the bridge is traversed daily by $n_1 = 25$ trains, then this number becomes n_{D_1}, and failure need not be apprehended until at the earliest 80000 days or 220 years have elapsed (but under tramway traffic with $n_T = 250$ vehicles per day failure might be expected in only 22 years). Thus the criterion of safety may be expressed in the form of the life of the bridge in days and we have the relationship[6]

$$v_T = n_D : n_T \tag{2}$$

A fatigue experiment of this kind, however, does not yield a true picture of the situation, because it is carried through without interruption, whereas in the actual structure long pauses intervene, especially at night, and in these pauses it is conceivable that a recovery of the material may take place. Even if no influence due to rest pauses has hitherto been disclosed by the experiments as regards the fatigue resistance of the materials themselves, it may well be possible that the conditions in regard to rivetted or welded *connections* are more favourable in this respect.[7]

Case 2. Boom of a continuous solid-webbed or openwebbed main girder.

This example shows the necessity of separating, in the longitudinal section of the girder, the zone in which the calculated limiting stresses σ_{max} and σ_{min} have the same sign from the alternating zone in which they have different signs. Thus according to the ratio

$$\xi = \frac{\min S}{\max S} \quad \text{or} \quad \frac{\min M}{\max M} \tag{3}$$

of the statical load S and moment M in the bars, a vibration coefficient

$$\gamma = \frac{\sigma_{zul}}{\sigma_{D\,zul}} > 1 \tag{4}$$

$\left(\text{corresponding to the buckling coefficient } \omega = \dfrac{\sigma_{zul}}{\sigma_{D\,zul}}\right)$ is to be introduced, because in the fatigue experiments the alternating strength, the pulsating strength and the fatigue range have different values for St. 37 and St. 52. In this way the different $\gamma - \xi$ lines of the Reichsbahn are obtained (B.E., Berechnungsgrundlagen für deutsche Eisenbahnbrücken, Section 36, Table 17). In a similar way to the assumption of $\dfrac{\omega \cdot S}{F} \leqq \sigma_{zul}$ for buckling, the stress is here to be taken as

$$\sigma_I = \frac{\gamma \cdot \max S}{F} \leqq \sigma_{zul} \tag{5}$$

and the calculation may then be performed in exactly the same way as for members under purely statical loading.[8]

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[5] Compare first question for discussion.

[6] Compare fourth question for discussion.

[7] Here the third question for discussion arises.

[8] *Kommerell:* Erläuterungen zu den Vorschriften für geschweißte Stahlbauten, Part II, page 39 (Wilhelm Ernst & Sohn, Berlin 1936).

Case 3. Connections between longitudinal and cross girders.

It is a matter of experience that rivets at these connections easily work loose in service, and in the new regulations of the Reichsbahn (B.E., Section 46) it has been sought to promote safety not only by making the design assumptions more severe (increased thrust max $A' = 1.2$ ($A_g + \varphi A_p$) and increased bending moment in St. 52 compared with St. 37), but also by the requirement of special constructional precautions (such as provision, in every case, of a plate running through on top). The reduced span of the longitudinal girder is also safeguarded by adopting a higher value of the impact coefficient φ, such as for instance $\varphi = 1.6$ under permanent way with sleepers and $l = 5.0$ m instead of $\varphi = 1.4$ for the main girders of medium span. The only way to estimate the true magnitude and effect of the stress variations in this complicated special case, where the distribution of loading is influenced by the superstructure, and to compare it with the results of fatigue experiments on similar types of connection, would be to carry out exact measurements on actual bridges — a problem which still awaits research.

B) Welded railway bridges.

Account is taken of live loading effects in the following ways:

a) By placing stationary train loads in unfavourable positions and plotting influence lines.

b) Impact coefficients of $\varphi \geqq 1$ are adopted (wherein $S = S_g + \varphi \cdot S_p$ or $M = M_g + \varphi M_p$) in order to allow for the effect of impact and vibration through the movement of the loads, by comparison with stationary loads (for instance as a result of driving wheel action, rail joints, etc.). Such effects tend to increase the statical deflection. (This is destined to be a principal problem of bridge investigation in the future).

c) The *vibration coefficient* $\gamma \geqq 1$ is expressed as a function of the calculated statical limits min S and max S in order to allow for the difference in fatigue effects on the structural member under alternating and pulsating loads, and apart from this, different values are used for St. 37 and St. 52; also different values according as the traffic is heavy or light ($n_T = 25$ or $n_1 \geqq 25$ trains per day).

d) The *design reduction coefficient* $a \geqq 1$. Whereas the coefficients γ may be fundamentally the same for rivetted and welded bridges the permissible stresses for welded railway bridges have been still further reduced in accordance with the German fatigue experiments,[9, 10] becoming (see equation 5):

$$\sigma'_I = \frac{\sigma_I}{a} = \frac{\gamma \cdot \max S}{a \cdot F} \leqq \sigma_{zul} \quad \text{or} \quad \frac{\gamma}{a} \cdot \frac{\max M}{W} \leqq \sigma_{zul}, \tag{6}$$

wherein the design factor a is given a different value according to the form of seam (whether a butt weld or a fillet weld) and according to the quality of

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[9] Dauerfestigkeitsversuche mit Schweißverbindungen, 1935, VDI-Verlag, Berlin. Joint report of Staatl. Materialprüfungsamt Berlin-Dahlem and Versuchs- und Materialprüfungsamt Dresden. By *K. Memmler, G. Bierett* and *W. Gehler.*

[10] See footnote 8, *Kommerell*, page 44.

workmanship (e. g., whether the root of the seam has been re-welded or not, and whether the finished seams have been improved by further working). Reduction coefficients of this kind are already in use for welded building frames (DIN 4100, Section 5) wherein, for instance, butt welds may be stressed in tension to $p_{zul} = 0.75\,\sigma_{zul}$, hence $\alpha = 0.75$.

C) *Rivetted and welded road bridges.*

Road bridges are much less frequently exposed to sustained alternating loading than is the case with railway bridges, and since, moreover, the German loading assumptions (DIN 1073) already ensure ample safety as regards weight and
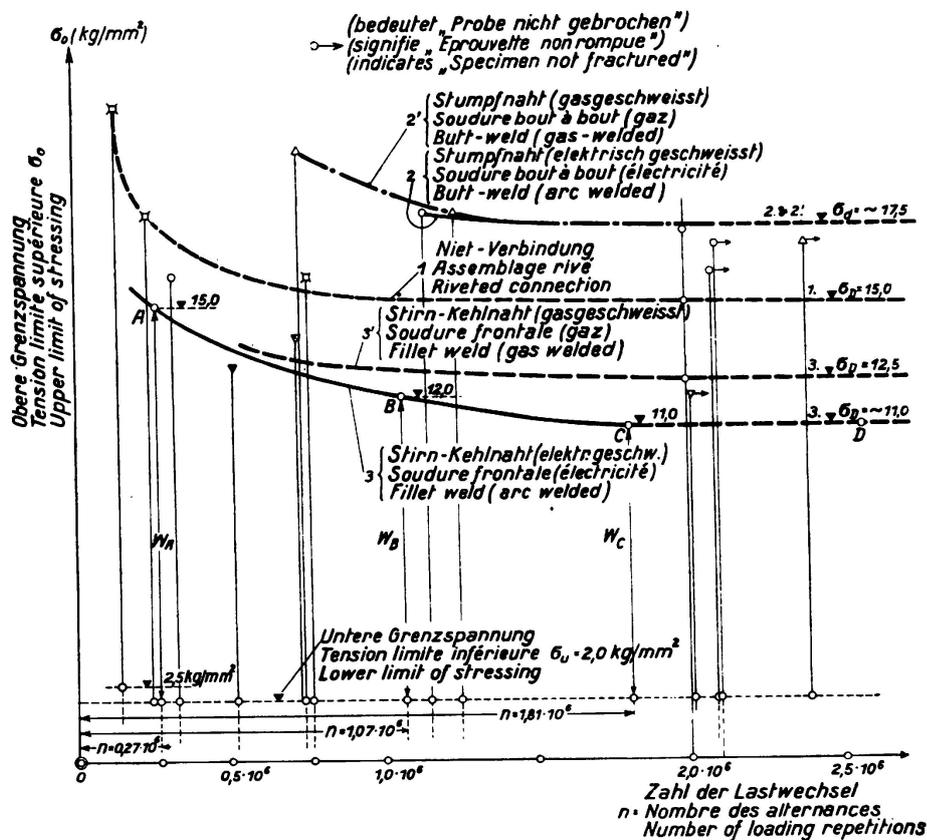

Fig. 3.

Wöhler lines according to Report of Board of Administrators with welded and riveted connections of members stressed in Tension.

traffic density of the loads, these bridges are nearly always regarded as being statically loaded. Consequently the vibration coefficient may be omitted from among the four effects enumerated under B) above and γ may be put equal to unity, while retaining the impact coefficient φ, and also certain design reduction coefficients α in the case of welded road bridges.

3) *The limiting stress-time curve (Wöhler's curve)* (Fig. 3).

Since the fatigue strength σ_D depends on a number of variables (such as n, σ_o, σ_u or σ_m) it is desirable to represent these in different planes with the

axes Z, X' and X" respectively as shown in Fig. 1. The first requirement is the recording of experimental results in the form known as the *Wöhler* curve. Suppose, for instance, that the stress σ_D is to be determined in the pulsator by means of tensile tests for electrically welded side fillet welds[11] (Fig. 3, Line 3). · An upper limiting stress is first fixed arbitrarily at, for instance, $\sigma_o = 15$ kg/mm² with a lower limiting stress of $\sigma_u = 2.0$ kg/mm² and it is found that breakage takes place at n = 270000 changes of load (Point A). On a second attempt
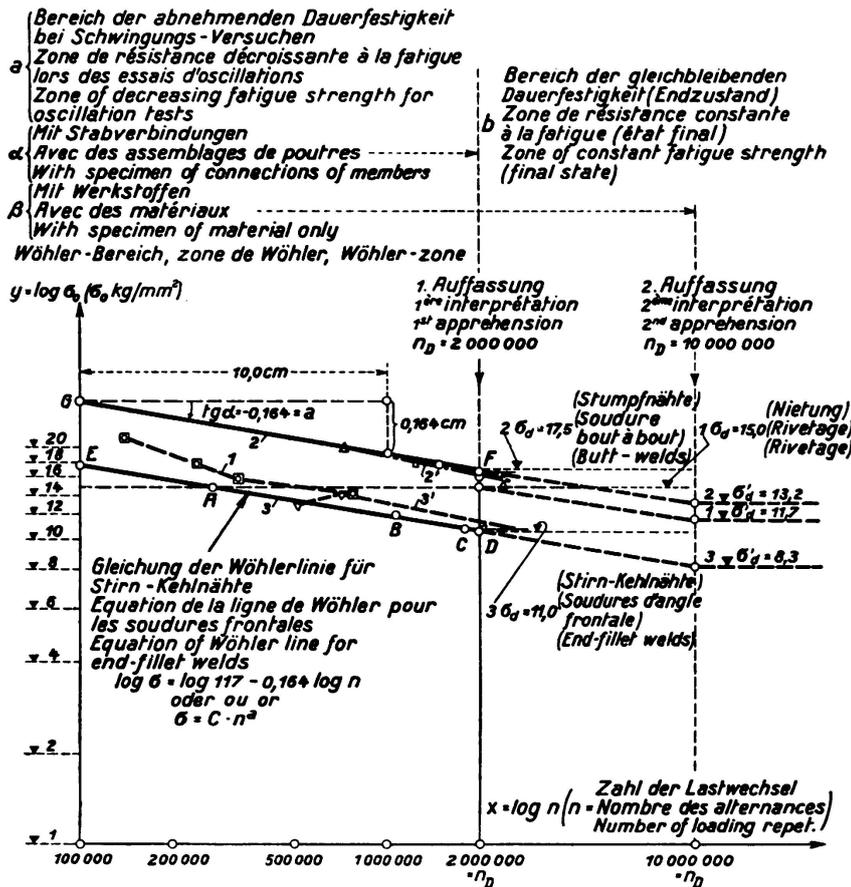

Fig. 4.

Zone of decreasing fatigue strength (Wöhler zone) of the stress-time curve
(plotted logarithmically) for riveted and welded connections.

being made with $\sigma_o = 12$ kg/mm² and the same value of $\sigma_u = 2.0$ kg/mm² there is obtained n = 1.07 million (Point B) and finally in a third experiment with $\sigma_o = 11.0$ kg/mm² there is obtained n = 1.81 million (Point C). Since the portion CD of the line ABC is already approximate horizontal, the final value of the fatigue strength may be assumed at $\sigma_D = \lim \sigma_o = $ approximately 11 kg/mm².[12]

This experiment may now be plotted as in Fig. 4 with $y = \log \sigma_o$ as ordinates and $x = \log n$ as abscisse, the logarithmic scale being adopted along both coordinate axes *(and not only along the X axis, as is commonly done)*. It is then,

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[11] See footnote 9.

[12] First question for discussion.

found that the line ABC approximates closely enough to a straight line ED, which with the parallel line GF indicates the trend of direction of the remaining experimental lines. The equation for the straight line ABC is as follows:

$$\log \sigma = \log 117 - 0.164 \log n \qquad (7a)$$
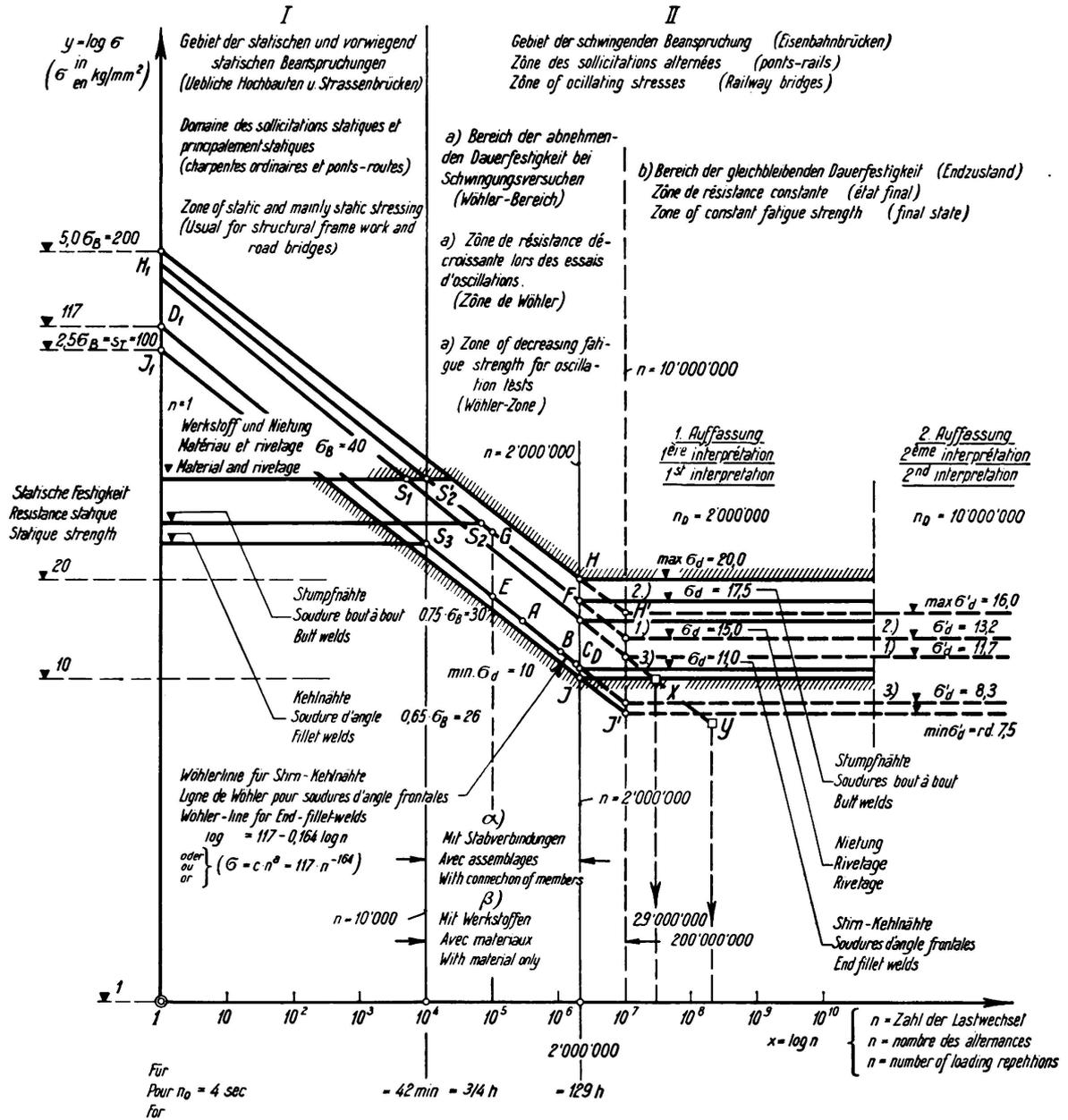

Fig. 5.

The zones of the Stress-time lines and the Wöhler-line for connections of members (in logarithmic scale).

and this corresponds to the exponential curve[13] as represented in Fig. 3:

$$\sigma = C \cdot n^a \qquad (7b)$$

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[13] See also Moore, Am. Soc. Test. Mat. 1922, p. 266 and Basquin, ditto, 1910, p. 625.

wherein $a = \tan \alpha$ 0.164 gives the slope of the lines and $C = 117$ kg/mm² gives the value of σ for $n = 1$.

In Fig. 5 the whole of the limiting stress-time curve is shown in the same logarithmic form. For *the region of falling fatigue strength,* which will here be designated the *Wöhler* region, we have thus set limits at the respective points $n = 10000$ and $n = 2000000$. On the left and right these connect with other horizontal portions, so that the limiting stress-time curve when represented in this logarithmic form consists of straight lines with two bends. The line DE cuts the coordinate axis ($n = 1$) with the ordinate $C = 117$ kg/mm². If, now, a line is drawn through the point J with $\sigma_D = 10$ kg/mm² and $n = 2000000$, and a line through the point J_1 to the axis of coordinates with $\sigma_D = 100$ kg/mm², and if a further line HH_1 is drawn parallel to J_1 with the point H corresponding to the value $\sigma_D = 20$ kg/mm², the result is to enclose a figure which contains practically the same values as are found in fatigue experiments on connections of bars.

The stress-time curve may then be divided up as follows. Firstly there is the region of statical or mainly statical stresses, such as occur in the usual forms of building frames and in road bridges. Secondly there is the region of alternating stresses, such as occur in railway bridges — further divisible into a portion where the fatigue strength is falling away as in vibration experiments (the *Wöhler* region) and a portion where the fatigue strength remains constant and may be looked upon as a final condition (IIa and IIb).

The sub-division into these two portions IIa and IIb is in itself arbitrary, and is the subject of Question 1 for discussion.[14] The knowledge hitherto available from fatigue experiments carried out on *connections* between structural members has led to the adoption of $n_D = 2000000$ (first assumption). If, however, as is usual in testing *materials,* $n_D = 10000000$ is substituted (second assumption), then the rectilinear projection of the lines in the *Wöhler* region (for instance, as far as the point H′ and J′ in Fig. 5) to correspond with rivetted practice, would give $\sigma_D = 11.7$ kg/mm² instead of 15 kg/mm² (see points V and W). The fact that rivetted railway bridges have given good performance under railway traffic when designed with $\sigma_{zul} = 10$ kg/mm² would then be difficult to reconcile with experimental results.

The relation to the fatigue strength of $\sigma_D = 15$ kg/mm² as found for rivetted connections in the fatigue experiments, to the permissible stress $\sigma_{zul} = 14$ kg/mm² in rivetted railway bridges, is

$$v_w = \frac{\sigma_D}{\sigma_{zul}} = \frac{15}{14} = 1.07$$

This affords a further margin of safety which, although small, may be relied upon to compensate for any possible lack of uniformity and quality of the material or other inaccuracies in execution.

Since in the testing of materials for $n > 10000000$ the stress strain line was assumed to be horizontal (Phase IIb) it is surprising that in the experiments carried out at Dresden the fatigue failure of a controlled specimen should have

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[14] See Section 5 (First question for discussion).

taken place after 29000000 changes of load (see point X in Fig. 5) while in the testing machine at Berlin-Dahlem the fatigue failure at a rivet hole of a truss member should not have taken place until after 200000000 changes of load (see point Y). The second point for discussion is whether values of this. order (n > 10000000) have been observed elsewhere, either in experiments or in railway service.[15]

In Fig. 6 the *Wöhler* line is again indicated without distortion of scale at the points V, W, X and Y. Referring to the fatigue test on structural connections, if the line had been terminated at n = 10000000 changes of load (point W) instead of at n = 2000000 (point V), then a fatigue strength approximately 20 % lower would have been obtained. In the exceptionally long-delayed fatigue
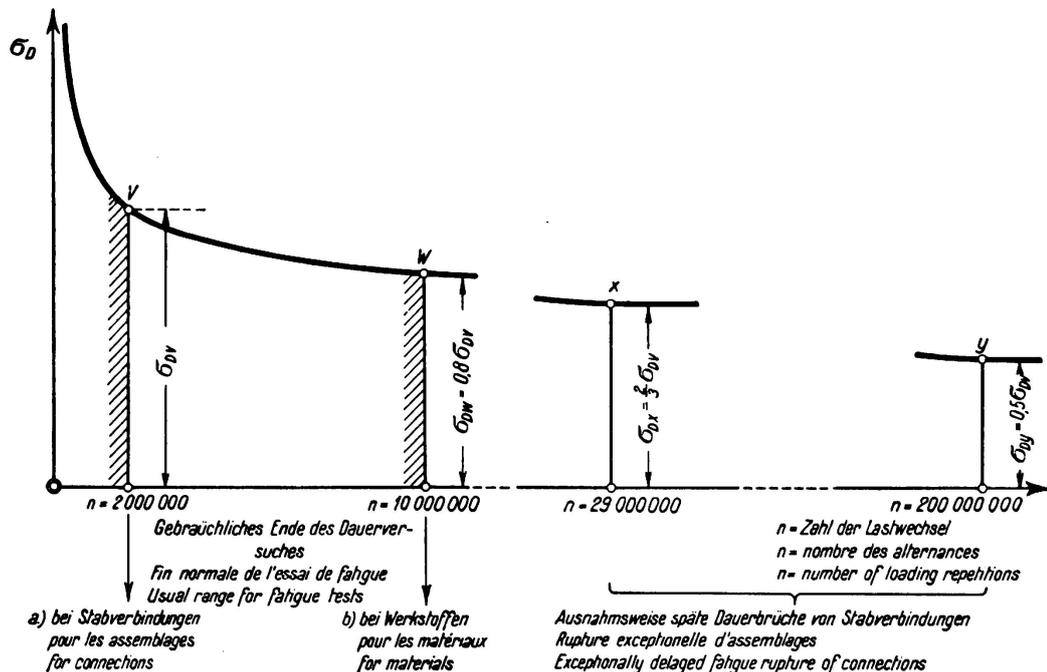

Fig. 6.

Wöhler-line showing the limitation of fatigue tests of connections.

failures indicated by the points X and Y for structural connections, the fatigue strength is to be taken as between $2/3$ and $1/2$ of the value for point V. It follows from this that in the case of structural connections it is a matter of primary importance whether the end condition of the phase of constant fatigue strength (Fig. 5) is reached.

By adopting this logarithmic presentation of the stress-time line, the whole field of statical and fatigue tests results may be incorporated in the stress-time plane. It should also be noticed that the ordinate of the point J_1, belonging to the lower limiting line JJ_1, corresponds approximately to the critical strength

$$s_T = 2,5 \cdot \sigma_B = 2,5 \times 40 = 100 \text{ kg/mm}^2 \tag{8}$$

and that the intersection points S_1 and S_2 also indicate a certain uniformity in their relationship to the horizontal lines of the statical region and the inclined

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[15] See Section 5 (Second question for discussion).

lines of the *Wöhler* region. The test results obtained in the statical and fatigue tests would agree much better if stresses for butt welds were to be taken in future as 1.0 σ_{zul} instead of as 0.75 σ_{zul}; that is to say, if the points S_2 and S'_2 were to be shifted.

Physically, C in Equation (7b) may be regarded as a *coefficient of cohesion* (compare Equation 8). A meaning for the other coefficient a may then be inferred from the equation (7b) as follows:

$$y' = \frac{d\sigma}{dn} = c \cdot a \cdot n^{a-1} = 117 \cdot 0.164 \cdot n^{-0.836} = \sim \frac{19.2}{n}, \qquad (9)$$

which for a first approximation corresponds to a rectangular hyperbola. In the *Wöhler* region the y' line at first falls steeply downwards and finally it runs parallel to the axis of the abscissae (Fig. 3). If the ordinates σ $\left(\frac{kg \cdot mm}{mm^3} = \frac{kg}{mm^2}\right)$ be regardet as representing specific energy, or loading per unit volume of 1 mm^3, then the ordinates give a measure of the efficiency (work/time), and the y' line represents the drop in this efficiency, or the fatigue experienced during the experiment. According to Fig. 5 the trend of this line is similar for the structural connections investigated and for the *Wöhler* region. The value of a may thus be designated as a fatigue coefficient ($a = 0$ in region I and IIb of Fig. 5).

4) *Comparison between the stresses actually arising in the structure according to statical calculations, and the stresses occurring in the test bars.*

Since the experiment has to be arranged as simply as possible it is confined to sinussoidal waves rising and falling between the upper and lower limits of σ_o and σ_u on either side of the average stress σ_m. Actually however — as indicated in Fig. 2 — the range of live load stress above and below the dead load stress σ_g is usually very different; in side members, for instance, being σ_{p1} above $> \sigma_{p2}$ below. In the boom members of girders σ_{p2} may even be equal to zero. Hence the experiment differs from reality not only as regards the shape of the waves, but also as regards their lack of symmetry in the different amplitudes (σ_{p1}) above and (σ_{p2}) below, Unfortunately, again, the effect of this discrepancy has not yet been investigated, and this suggests a further field for research.

5) *Questions for discussion.*

Question 1) Is it expedient to refer fatigue tests on connections of bars to a number of changes of load $n_D = 2000000$ instead of to $n_D = 10000000$ as is customary in the testing of materials? (Compare footnotes 2, 5, 12 and 14; also Figs. 5 and 8, points S and W).

Question 2) Have the exceptionally long delayed fatigue breakages, after 29 and 200 million changes of load, as observed in the German experiments, been confirmed elsewhere either in fatigue experiments on structural connections or in actual railway service? (Compare footnote 2 and 15; also Figs. 5 and 6, points X and Y).

Question 3)   Is there any experimental evidence that a favourable effect on the fatigue strengths of structural connections may be exercised by rest pauses? (Compare footnote 7).

Question 4)   Seeing that rivetted connections, which form the basis for investigating the behaviour of welded connections, give an average fatigue strength of $\sigma_D = 15 \text{ kg/mm}^2$ with a permissible stress (including impact allowance) of $\sigma_{zul} = 14 \text{ kg/mm}^2$, may the criterion for the safety of railway bridges properly be taken as the proportion between the number of changes of stress applied in the experiments and the number of trains per day? Or in other words, is the question answered by stating the life of the bridge in years $v_T = n_D : n_T$ making use of a *statistical* conception of safety? Can any other suitable suggestions be put forward for designating safety?