The determination of lines of principal stress in riveted and welded structures

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The Determination of Lines of Principal Stress in Riveted and Welded Structures.

Darstellung der Hauptspannungslinien an genieteten und geschweißten Konstruktionen.

Détermination des trajectoires des contraintes principales dans les constructions rivées et soudées.

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The technical development of structural engineering, the bridging of wide spans and the consequent need to make the fullest possible use of the materials while conserving adequate safety, requires the making of statical calculations which shall correspond as closely as possible with the stresses actually arising in the structure. Hitherto most calculation has been based upon far-reaching simplifications, and the condition of uno-axial stress which was easy to treat by calculation has formed the basis also of statical experiment. Yet it is only in the rarest cases that this assumption is in fact justified, and the true condition of stress is more often approximately duo-axial or tri-axial. As regards the cross sections of joints and intersections, corners of frames, and similar details of design in steel construction, at least a duo-axial condition of stress must be assumed if the normal and shear stresses are to be more accurately determined.

In view of the great number of marginal conditions it is difficult to determine the stresses by mathematical quantitative methods under this assumption, and consequently it is not possible to arrive in this way at satisfactory agreement between the values of stress as calculated and as arising in practice, in cases where no simple fields of stress can be distinguished. It appears expedient, therefore, to determine the fields of stress and the magnitudes of the stresses in certain sections of particular interest by means of experiments. In many practical cases an accurate knowledge of the direction and magnitude of the stress will probably enable a simple method of calculation, of sufficient accuracy, to be developed mathematically so as to yield results in satisfactory agreement with practice.

Experiments to determine the stresses of the components of duo-axial stress may be carried out by several methods. In one of the older of these the elongations at the corner points of a square net lying in the plane of stress are measured in several directions from which the magnitudes and directions of the principal stresses may be calculated and the lines of principal stress plotted.[1]

In the engineering laboratory of the Technische Hochschule at Darmstadt the determination of the lines of principal stress has been carried out with the aid of a lacquer susceptible to cracking which is coated over the experimental specimens to be examined.[2] When such a specimen is subjected to load the lacquer cracks at right angles to the direction of the maximum elongation and a family of lines of cracking results which corresponds to one system of the lines of principal stress; by making use of the known condition that the two systems of lines of principal stress must intersect one another at 90°, it becomes possible to trace out the second system within the field of stress under examination. When the lines of principal stress have been determined, measurements of elongation are carried out at a number of points along suitable sections in the direction of the lines of cracking disclosed, and the magnitude of the principal stresses at these points is calculated.[3]

The field occupied by the lines of principal stress gives a useful picture of the distribution of stresses and strains in a specimen or structural member. In many cases important conclusions can be drawn directly from the trend and shape of the lines of cracking: thus a notably irregular distribution of the lines of principal stress indicates the position of the highly stressed places independently of any measurements and assists in the choice of cross sections for measurement. The field of stress, as indicated by the lines of principal stress, affords further exact information as to stress conditions. From a knowledge of this the proper assumptions to make as a basis for calculation can be ascertained.

Moreover the lacquer, through its liability to chipping, has the further property of giving a very early indication that phenomena of flow are about to take place, and this enables a good idea to be obtained of the plastic behaviour of the member under examination. Even within the region of the live load as calculated by the ordinary method small isolated zones may be found wherein plastic deformation is taking place, an occurrence which, but for this method, would escape observation. It is still easier to discern the widening of this region of flow that occurs when the load is increased, which can be followed clearly and intelligibly. In the same way, provided that the necessary experimental conditions can be satisfied, the lines of principal stress can be determined in uniplanar or three-dimensional forms even of large dimensions. Thus the starting point for new methods of calculation is afforded by experiments which have carried out on structures built either under practical conditions or (if necessary) in the form of a model to a reduced scale, and in either case the investigations can be made in the same material as is to be used in the actual structure.

In what follows below the lines of principal stress developed in a series of structural members under the experimental load will be described. Fig. 1 shows the system of such lines on simple tensile bars subjected to eccentric loading including a welded butt joint, a butt joint with a cover strap secured by end fillet welds, and finally by a butt joint with vertical plates connected to it by side fillet welds.

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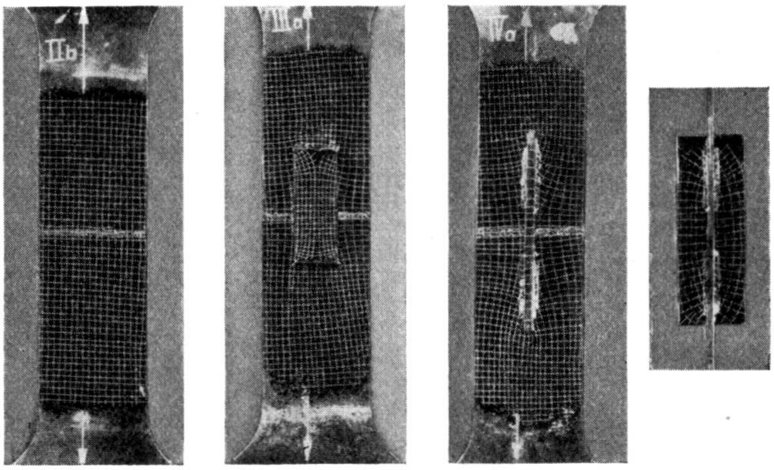
[1] See *Wyss:* Kraftfelder in festen elastischen Körpern.

[2] See Bautechnik, 1936, No. 23.

[3] Elongation line method of the Maybach firm.

In all these test bars the fields of stress were relatively simple. In the first of the bars, that welded with a simple joint, the cracks appeared at right angles to the axis of the bar, and these can easily be determined even assuming the condition of stress to be almost uni-axial. The second of the bars tested already shows, by the shape of the lines of force, the effect of the reinforcements welded on to it, this being recognisable in the deviation of the lines towards the straps. In this respect special attention should be drawn to the trend of the lines of principal stress in the cover strap itself. In the neighbourhood of the end weld the flow of the lines of force becomes irregular, concentrating at the corners of the end welds, while the middle is left free from stress. Moreover an arch shaped plasticity figure occurs in the plate close to the seam, which implies very high stresses in this zone. This local yield, which has begun even within the range of the working load, may well be the reason for the low fatigue strength possessed by connections of this kind, their appearance on fracture under fatigue being similar to this.[4] Virtually the same result was obtained from the third experimental test bar shown in Fig. 1 where once again the strap, in spite of its cross section being partially separated from the bar, participates uniformly in the stress at its middle section — a fact which might also have been inferred from measurements of elongation. The end of this cover strap was tacked while being welded, and in the experiments chipping, originating at the tacking points, occurred in the lacquer, as may be clearly seen in Fig. 2, this again indicates zones in which yield has taken place even though the full working load is not developed. On this test bar being loaded to destruction, giving the result shown in Fig. 3, the yield zone is seen to have extended until fracture took place. The flow figure for the butt welded bar shows a much greater extension of the yield



Load applied

P = 11.40 t P = 15.20 t P = 15.55 t

Fig. 1.

Types of joint. Stress trajectories under working load.

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[4] *Kommerell:* Experience obtained with Structures executed in Germany. IIId, Preliminary Publication of I.A.B.S.E. Congress, Berlin and Munich, 1936.

on one side than on the other, as the result of eccentric loading. The chipping of the laquer running at an angle of 45° is characteristic of the tensile yield point being exceeded, such chipping being caused by the formation of slip planes.

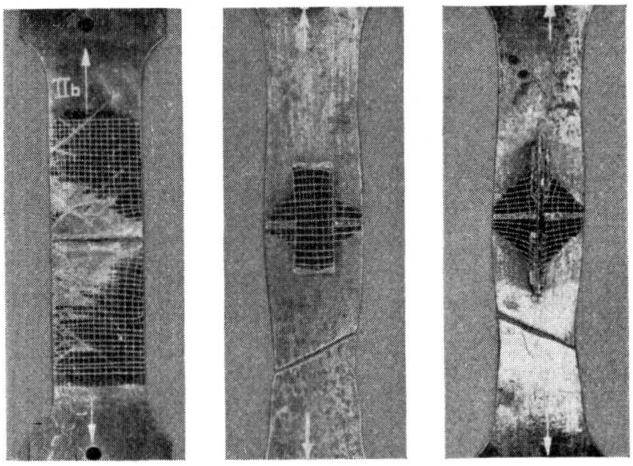
The trend of the lines of principal stress in compression members jointed in two ways is indicated in Fig. 4. In one of these specimens the plate was welded to an I-beam, in a second it was rivetted, and in a third example it was both welded and rivetted. After the lines of principal stress had been developed the fields of stress in the plate were delimited as shown in Fig. 4. In the case of the first and third compression specimens the length of the connecting weld seam is indicated by points.


P = 15.20 t P = 15.55 t

Fig. 2.

Types of joint. Lines of flow in the lacquer under working load.

The welded form of connection gave a very irregular transfer of the forces into the seam and almost all the lines of stress are crowded in to the beginning of the seam, which must, therefore, be subjected to higher stresses. The welded form of bar gave a considerably more equalised field of stress, but a uniform flow of the lines of force was obtained only by the combined use of rivetting and welding, implying that each of these methods of connection shared simultaneously in transferring the force.



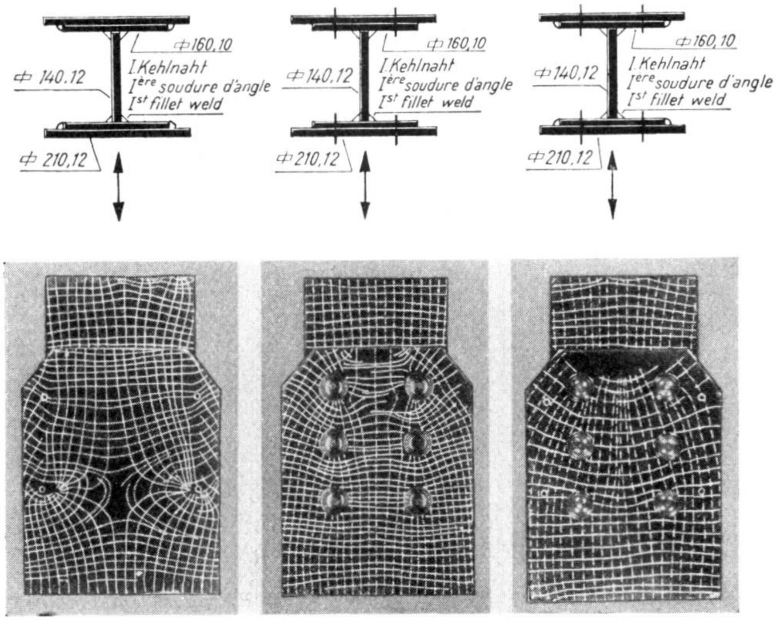
Rupture load

$P_B = 46.80$ t $P_B = 66.78$ t $P_B = 67.53$ t

Fig. 3.

Types of joint. Fracture under static load.

From the available data it was possible to develop the trajectories in a girder with a proportion of h/l = 1/10 subjected to bending. Fig. 5 shows the results of this experiment. The girder was loaded with two isolated loads, 500 mm apart on either side of the centre, the span being 3000 mm. In the middle field there existed, therefore, a constant moment without shear, while in the end fields both these effects were present together. The shape of the lines so developed accurately reflects this statical condition. Whereas calculation gives a zero value of the shear at the middle height of the web plate in the central field, in the end fields the shear should be a maximum. This implies that in the first mentioned the lines should intersect with the axis of the girder at 90°, while in the end fields they should cut it at 45°. Intermediately between the two fields there is a transition zone, and apparently this must always occur in such cases where, statically speaking, a violent transition obtains from one to the other kind of stress. When the load was increased, chipping of the lacquer coating again occurred in the central field, indicating the yielding of the material. Despite the constant moment present throughout the length of this field, so that all cross sections therein should be equally stressed, the yield point was in fact reached earlier in some portions than in others (see Fig. 6). Another notable observation is that in the compression zone the development of the *signs of*



a) welded b) riveted c) welded and riveted

Fig. 4.

Compression test II.

Stress trajectories in welded and compound-jointed pieces. Front view.

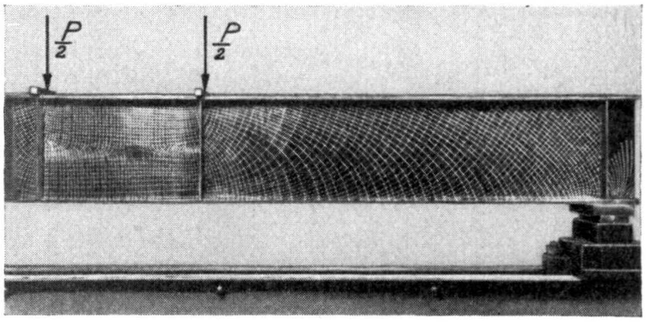
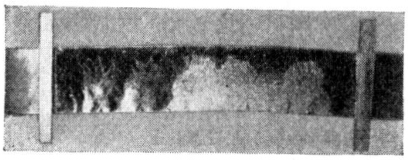


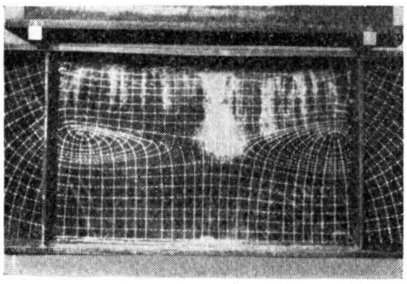
Fig. 5.

Stress trajectories in a girder subjected to bending. $h : l = 1 : 10$.

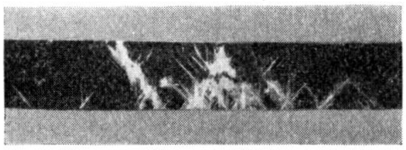
yield occurred at right angles to the direction of pressure, whereas in the tension zone it was at 45° to the direction of tension. Virtually the same observations were made in the compressive and tensile flanges of the middle field. In this instance it appears, therefore, that failure in the tension zone is initiated by the shear strength being exceeded, whereas in the compression zone it is another kind of failure that occurs, presumably through the compressive strength being exceeded. Hence the shear yield point is the determining factor as regards tension, and the compressive yield point as regards compression. It should be emphasised, however, that these observations have so far been carried out only on a small number of specimen parts, and that the results require confirmation by further experiments.



view from top



elevation



under side

Fig. 6.

Yield zones in the middle of a bent beam.

A further illustration, Fig. 7, shows the loss of carrying capacity by shear failure of the web plate in a short girder where the ratio of depth to height is 1 to 4. The lines of principal stress indicated in Fig. 7 were obtained at somewhat below the working load, but even under this loading yield zones appeared in the neighbourhood of the load points and of the end bearings. As the load increased this yielding spread from its origins outward, in the form of slip planes extending further and further into the end fields, these planes being inclined at 45° to direction of the principal stress lines, thus clearly indicating that the girder had failed through shear.

An exhaustive investigation of stress conditions was made in an experimental frame. In the horizontal members of the frame the lines of principal stress can be found in the way already described (see Fig. 8). In this experiment special attention was paid to the flow of lines in the neighbourhood of the corners, and a lack of symmetry of the lines was observed about the diagonal section. In a comparative specimen, made right angled in cross section and subjected to symmetrical loading, regular symmetrical curves were obtained as shown in Fig. 9 and these agree with the curves obtained by optical methods of stress measurement by Cardinal *von Widdern.*[5] At the corners of the frames no displacements of the curves towards the horizontal member are apparent. The reason for this is to be sought in the different methods of loading the vertical and horizontal members, shear and moments being operative in the former and longitudinal forces in the latter. The investigation was amplified by means of numerous measurements on elongation carried out in a series of cross sections, assuming a duo-axial condition of stress, and the results of this were recal-

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[5] *Cardinal von Widdern:* Mitt. Mechan. Techn. Labor., Technische Hochschule, Munich.

culated for normal and shear stress conditions. The evaluation of the experimental results, and collation of the knowledge so obtained, is to be given in a separate publication. In order to afford a comparison of different methods of construction the frame was built with two different designs of corner construction, one side being provided with a strengthened angle gusset opposite to the web of the vertical member and the other side being stiffened. The change in flow of the lines of principal stress is shown in Fig. 10. At the end of the short stiffening there is seen a kink in the line, which suggests a distribution of the stresses over the depth of the web plate and implies a relief of stress in the region of the inner corner. At the same time the development of the lines of stress in the flange at the inner rounded portion shows that good support is afforded to the inner flange at the stiffened corner by contrast with the other corner.

Fig. 11 shows the lines of principal stress on the inside of the inner flange, comparing an unstiffened and a stiffened corner.

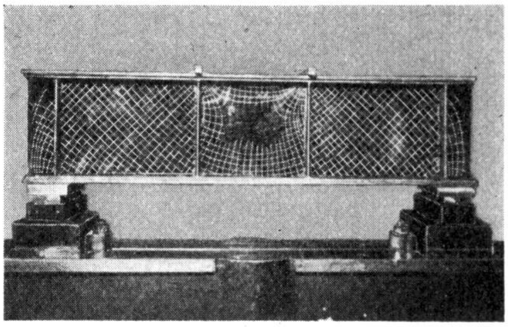


Fig. 7.
Stress trajectories in a girder subjected to bending. h : l = 1 : 4.

To sum up, it may be stated that by tracing out the fields of stress, the yield zones and yield regions, an illuminating picture may be obtained of the behaviour of the structural members and details so examined. The delimitation of the fields of stress makes it possible to gain a further insight into the proportionate magnitudes and distribution of the stresses, especially under the

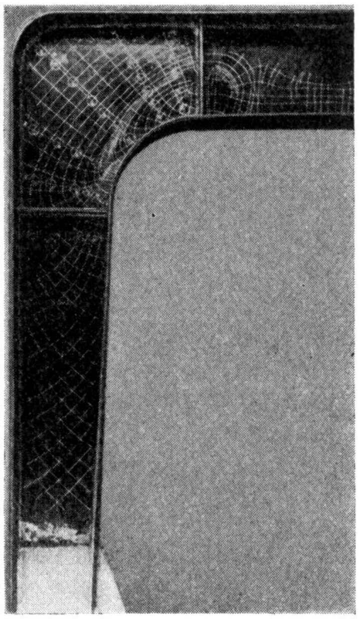


Fig. 8.
Stress trajectories in a frame of I-section.

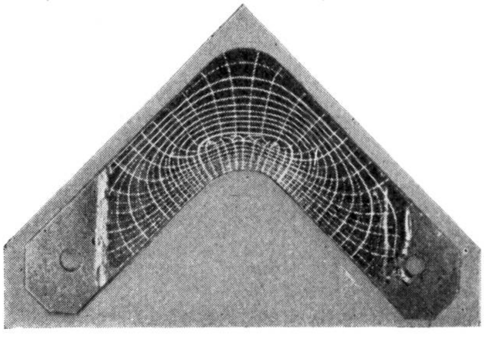
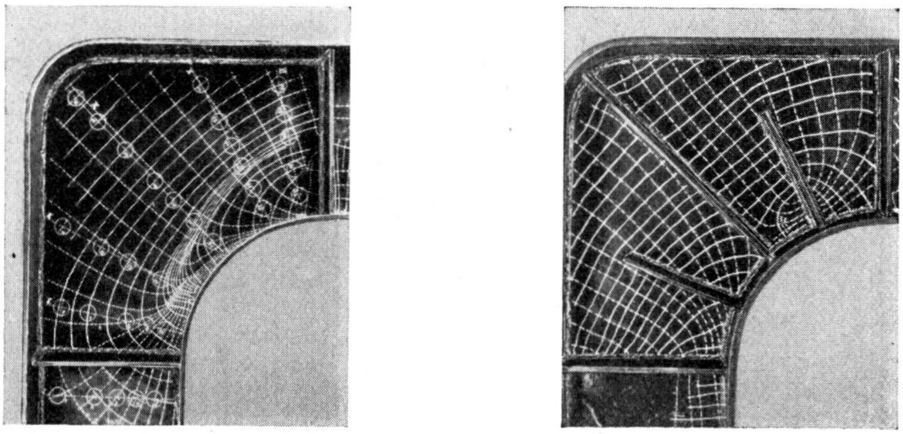


Fig. 9.
Stress trajectories in an angle piece of rectangular cross section.

condition of duo-axial stress. From these experimental results it may be presumed that opportunities will arise for correcting the existing methods of calculation to suit the duo-axial condition of stress, and so obtain agreement with the actual values. In the same way it will become possible for the fundamental assumptions necessary in pre-calculating the various important stress values to be correctly understood.

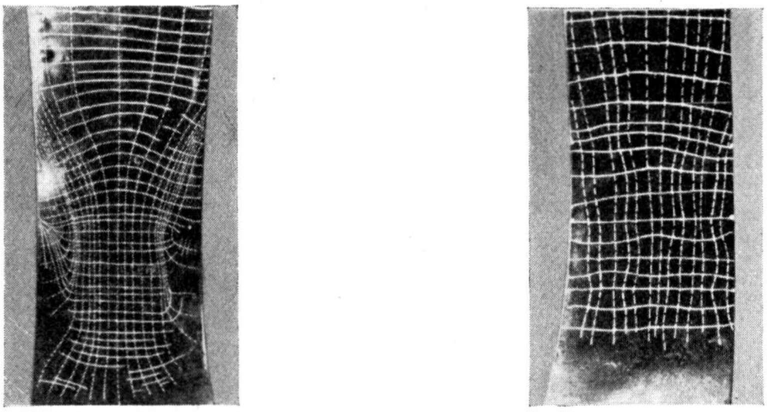


Corner a. Reinforced corner plate.

Corner b. Non-reinforced corner plate but with stiffeners.

Fig. 10.

Comparison of stress trajectories in two corners of frames.



Corner a. Reinforced corner plate.

Corner b. Non-reinforced corner plate but with stiffeners.

Fig. 11.

Stress trajectories in the inside flange of the corner of a portal frame.

The object of these experiments, however, must always be that of arriving at methods of calculation which can be generally applied in practice, show good agreement with the conditions actually arising, and enable the designer to build economical structures with adequate safety.