# Stability of the webs of plate girders taking account of concentrated loads

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## Stability of the Webs of Plate Girders Taking Account of Concentrated Loads.

## Die Stabilität der Stegbleche vollwandiger Träger bei Berücksichtigung örtlicher Lastangriffe.

## La stabilité des âmes de poutres pleines, calculée en tenant compte des charges locales.

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Plate girders are occasionally called upon to carry loads on the compression flange intermediately between stiffeners, as for instance in crane girders, in the rail bearers of railway bridges, and also in the main girders of such bridges when the sleepers rest directly over them.

An examination of the stability of the web-plates of plate girders loaded in this way has been made by the present author firstly in his paper *„Stegblechbeulung unter örtlichem Lastangriff“* [1] and the basis of the treatment there adopted will be outlined here before proceeding to explain a simplified method.

To determine the limit of stability of the web plate accurately in each particular case would involve lengthy calculations. In order to reduce these and to arrive at results possessing general validity a number of approximations had necessarily to be introduced. In the first place the distribution of load along the edge of the web plate owing to the loaded flange being sufficiently rigid to resist bending with the aid of the cross stiffeners is treated separately, and is calculated under simplified assumptions. Such distributions of load p (x) are represented in Figs. 1a and 1b. The former shows an arrangement in which the stiffeners are practically inoperative, representing the case of a plate girder in which the loaded flange has little stiffness to resist bending, or where the stiffeners are placed far apart. Fig. 1b, on the contrary, shows the distribution in a girder with a thick flange and closely spaced stiffeners, the latter considerably reducing the load along the edge of the web plate by transferring a portion of it directly from the flange to parts of the web plate in which there is little risk of buckling. This cooperation of the stiffeners has been dealt with in an approximate way by considering the

[1] Sitzungsberichte der Akademie der Wissenschaften in Wien, math.-nat. Kl., Abt. IIa, Vol. 145, Nos. 1 and 2, 1936.

flange as a bar and the web plate as a separate element and by making use of the following expression for the edge loading p (x) of the latter:

$$p(x) = \sum p_n \cos\frac{n\pi x}{3a} \qquad (n = 1, 3, 5 \ldots) \tag{1}$$

with the coefficient

$$p_n = \frac{2P}{3a}\cdot\frac{1}{1+\frac{2J_0}{t}\left(\frac{\pi}{3a}\right)^3 n^3}\cdot\left\{\frac{3a}{n\pi c}\sin\frac{n\pi c}{3a} - \cos\frac{n\pi}{6}\cdot\frac{\sum\limits_{m=1,3..}\frac{3a}{m\pi c}\sin\frac{m\pi c}{3a}\cdot\cos\frac{m\pi}{6}\cdot\frac{1}{1+\frac{2J_0}{t}\left(\frac{\pi}{3a}\right)^3 m^3}}{\sum\limits_{m=1,3..}\cos^2\frac{m\pi}{6}\cdot\frac{1}{1+\frac{2J_0}{t}\left(\frac{\pi}{3a}\right)^3 m^3}}\right\} \tag{2}$$

Here a denotes the distance between the stiffeners, t the thickness of the web plate, c the half width of the loaded area and $J_0$ the moment of inertia of the loaded flange referred to its horizontal axis. The second bracketted term in Equation (2) represents the effect of the stiffeners, which is often negligibly small. (Equation (2) has been so written that the transition to c = 0 can be immediately effected.) Relations similar to those represented by Equation (2) may also be developed for the case where several loads symmetrical to the y-axis are present in the field under consideration.

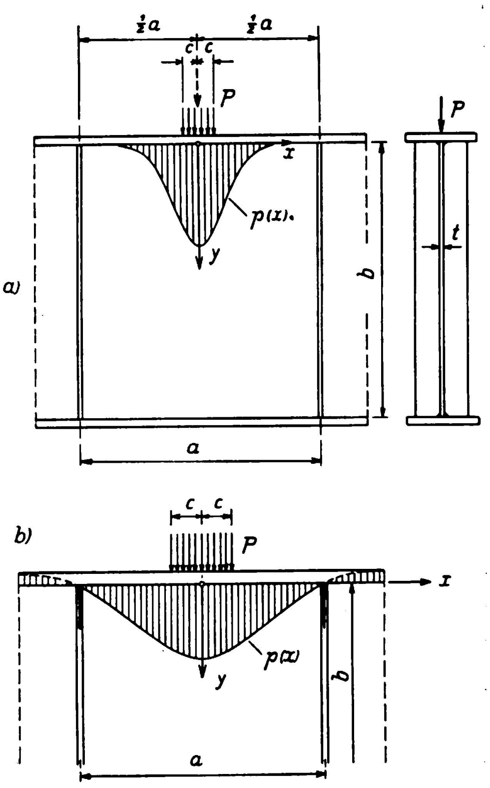

Fig. 1a and b.

With the aid of the distribution p (x) the stresses in the part of the web plate in question may now be calculated. In the earlier paper by the present author, already cited, these stresses have been computed from two constituent parts: namely from the elementary stresses $\sigma_{x1}$ and $\tau_1$ on the one hand, and from the panel stresses $\sigma_{x2}$, $\sigma_y$ and $\tau_2$ which are a consequence of the edge loading p (x), on the other. The shear stresses $\tau_1$ are left out of consideration in what follows below, and the examination of stability is based upon *a condition of stress in the web plate which is symmetrical with regard to the y axis.* The value of the critical stress and load have been determined by reference to the "energy criterion of safety against buckling".

A calculation of the potential energy of the bent web plate showed that part of the energy resulting from the stresses $\sigma_{x2}$ is relieved by the relevant portion of the stresses $\tau_2$. As a first approach it was deemed permissible, therefore, to ignore the stresses $\sigma_{x2}$ and $\tau_2$ in the examination of stability, merely estimating the stress $\sigma_x$ by reference to ordinary bending theory and then calculating the stresses $\sigma_y$ which are not obtainable by elementary means, as a function of the stress in the web plate. It has been found from comparative calculations that the critical stresses determined in this way differ only slightly from those found by taking account of all stresses arising from local effects by using the stricter method.[1]

In what follows below the web stresses will be calculated by this simplified method. Where b represents the depth of the web plate (measured between the rivet lines of the flange angles in the case of rivetted girders) and $\sigma_o$ represents the extreme fibre stress (estimated by elementary means) due to all the loads at the centre ($x = 0$) of the portion of the web plate EF to be examined (Fig. 2a), then the bending stress at this section will be given by

$$\sigma_x = -\sigma_o \left(1 - \frac{2y}{b}\right). \tag{3}$$

The same expression will also be used for the bending stress of all other sections of the web within EF.

For determining the stresses $\sigma_y$ only the portion of the girder CD (Fig. 2b) with the load P is taken into account. The length of this portion of the girder is equal to three times the spacing of the stiffeners a and is therefore large enough to allow of the stresses $\sigma_y$ within EF being determined with sufficient accuracy. Applying Airy's stress function for the parallel strips developed with the edge load p (x) for the length $CD = 3a$ as a half period, we obtain

$$\sigma_y = -\sum_{n=1,3..} \left\{ c_{1,n} \cdot e^{\frac{n\pi y}{3a}} + c_{2,n} \cdot e^{-\frac{n\pi y}{3a}} + c_{3,n} \cdot \frac{n\pi y}{3a} e^{\frac{n\pi y}{3a}} + c_{4,n} \cdot \frac{n\pi y}{3a} e^{-\frac{n\pi y}{3a}} \right\} \cos \frac{n\pi x}{3a}. \tag{4}$$

The constants $c_{1,n} \ldots c_{4,n}$ are to be determined from the edge conditions of the web plate. Since only the stresses $\sigma_y$ are calculated from the stress function, the strict fulfilment of the conditions of transfer (compound action of flange and web) may be ignored, thereby not only shortening the calculations but enabling the results to be expressed in a more general form.

The examination of stability is made over again in relation to panel spaces, and the equations for buckling are introduced under the assumption that the edges of the web plates bounded by the flanges and the stiffeners are "freely supported". Flanges and stiffeners are regarded as stiff so far as bending at right angles to the plane of the plate is concerned, and in examining the field EF (Fig. 2) a bent shape is assumed which forms only a half wave in the direction of the length of the girder. The results are then safely applicable for $a \leqq 0.9b$, for if there were no local loading of the web plate the latter would tend to bend into a half wave under the influence of the bending stresses $\sigma_x$ for to long as $a < 0.9b$, of a local effect enters into play, which tends to create symmetrical buckling, then the buckling will continue to be in the form of a half wave even for greater values of $\frac{a}{b}$ though the half wave may be differently formed. A more accurate examination of these conditions was rendered impossible by excessive amount of development it would have involved, and the limit of $a = 0.9b$ has, therefore, been adopted here.

In order to reduce the number of terms to a minimum the following expression has been applied for the buckling $w\,(x, y)$ of the panel EF:

$$w = \left(A \cdot \sin\frac{\pi y}{b} + B \cdot \sin\frac{2\pi y}{b} + C \cdot \sin\frac{3\pi y}{b}\right)\cos\frac{\pi x}{a} \tag{5}$$

which satisfies the marginal conditions in the form of "Navier's conditions". By means of this equation of three terms the amount of buckling of the plate can be ascertained with approximate accuracy, particularly when the effect of the stresses $\sigma_y$ by comparison with $\sigma_x$ becomes less pronounced (that is to say when the place of greatest buckling of the plate is further removed from the loaded flange). This is in fact true of the more important applications arising in practice.

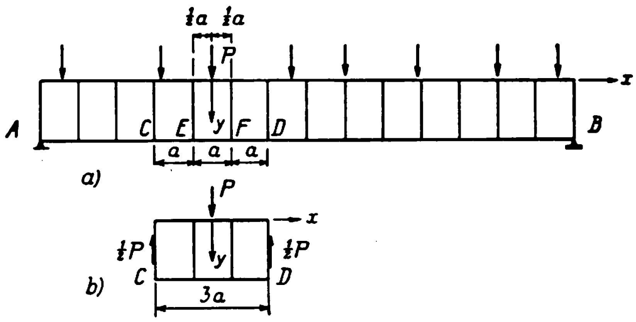

Fig. 2a and b.

The potential energy e of the stretched and bent web plate EF is now calculated[2] with the aid of Equations (3), (4) and (5). For each variation $\delta_w$ in the condition of strain, $\delta_e$ must be equal to zero. If the series of constants A, B and C in Equation (5) is varied in turn, and the corresponding $\delta_e = 0$ is worked out, the following equations for buckling are obtained:

$$\begin{aligned}
&A\left[\frac{1}{k}\cdot\frac{\pi^2}{12\beta}(1+9\beta^2)^2 - 2\varkappa r_1\right] - B\left[\frac{8}{3}\beta + \varkappa r_4\right] - C\varkappa r_5 = 0,\\
&-A\left[\frac{8}{3}\beta + \varkappa r_4\right] + B\left[\frac{1}{k}\cdot\frac{\pi^2}{12\beta}(4+9\beta^2)^2 - 2\varkappa r_2\right] - C\left[\frac{72}{25}\beta + \varkappa r_6\right] = 0,\\
&-A\varkappa r_5 - B\left[\frac{72}{25}\beta + \varkappa r_6\right] + C\left[\frac{1}{k}\cdot\frac{\pi^2}{\beta}\cdot\frac{27}{4}(1+\beta^2)^2 - 2\varkappa r_3\right] = 0.
\end{aligned} \tag{6}$$

Here $\beta = \frac{b}{3a}$, $k = \frac{\sigma_0}{\sigma_e}$, $\sigma_e$ (the Euler stress) $= \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$, $\varkappa = \frac{P}{at}\cdot\frac{1}{\sigma_0}$

and $r_1 \ldots r_6$ denote the summations of the following series:

$$r_1 = \sum_{n=1,3..}\frac{ap_n}{P}\left[\frac{1}{n^2\beta^2} + \frac{n^2\beta^2}{(4+n^2\beta^2)^2}\right]\varphi_n,$$

$$r_2 = 4\sum_{n=1,3..}\frac{ap_n}{P}\left[\frac{1}{n^2\beta^2} + \frac{n^2\beta^2}{(16+n^2\beta^2)^2}\right]\varphi_n,$$

$$r_3 = 9\sum_{n=1,3..}\frac{ap_n}{P}\left[\frac{1}{n^2\beta^2} + \frac{n^2\beta^2}{(36+n^2\beta^2)^2}\right]\varphi_n,$$

[2] *A. Nadai:* Elastische Platten, Berlin 1925.

$$r_4 = 4 \sum_{n=1,3..} \frac{a p_n}{P} \left[ \frac{1}{(1+n^2\beta^2)^2} + \frac{1}{(9+n^2\beta^2)^2} \right] n^2 \beta^2 \psi_n,$$

$$r_5 = 6 \sum_{n=1,3..} \frac{a p_n}{P} \left[ \frac{1}{(4+n^2\beta^2)^2} + \frac{1}{(16+n^2\beta^2)^2} \right] n^2 \beta^2 \varphi_n,$$

$$r_6 = 12 \sum_{n=1,3..} \frac{a p_n}{P} \left[ \frac{1}{(1+n^2\beta^2)^2} + \frac{1}{(25+n^2\beta^2)^2} \right] n^2 \beta^2 \psi_n,$$

with the auxiliary terms

$$\varphi_n = \frac{18}{36 - n^2} \sin \frac{n\pi}{6} \cdot \frac{1 - e^{-n\pi\beta}}{(1 + e^{-n\pi\beta}) + \dfrac{2 n\pi\beta e^{-n\pi\beta}}{1 - e^{-n\pi\beta}}}$$

and

$$\psi_n = \frac{18}{36 - n^2} \sin \frac{n\pi}{6} \cdot \frac{1 + e^{-n\pi\beta}}{(1 - e^{-n\pi\beta}) - \dfrac{2 n\pi\beta e^{-n\pi\beta}}{1 + e^{-n\pi\beta}}}$$

As the constants A, B and C must be different from zero the divisor determinant of Equation (6) must be zero. From this Equation the minimum root $k = k_{min}$ differing from zero can be calculated. The critical stress at the edge of the web plate in the field EF is then $\sigma_{o,\,kr} = k_{min} \cdot \sigma_e$ and the corresponding critical value of the panel load P is $P_{kr} = \varkappa$ at $\sigma_{o,\,kr}$.

To allow of evaluating the critical stresses and loads for the case of inelastic buckling also, the results already obtained for materials possessing unlimited elasticity are recalculated with the aid of the buckling stresses in a bar assumed as a standard of comparison. In determining the maximum stress imposed on the plate, account must now be taken also of that portion of the stress which arises from local effects with regard to $\sigma_x$ and the somewhat larger amount of stress $(\sigma_y)_{x=0,\ y=0}$ may be substituted for this.

In considering these results it must be remembered that only an approximate equation was used for determining the amount of buckling w (x, y), a circumstance which implies excessively high critical stresses. On the other hand various assumptions made in the calculation are too stringent; for instance, it has been assumed that the edges of the plate are capable of rotation, whereas the fixation stresses are always elastic, and the fixation into the flange may considerably increase the stability of the web plate. As regards the inelastic buckling, it is to be observed that the maximum stress is produced in the plate along an edge which is stiffened, and moreover that the incidence of the latter is purely local, so that plastic deformation may bring about a partial compensation of the stress.

It is possible that the *true* buckling load may be considerably higher than the critical value. The resistance to deformation offered by the adjacent panels of the web will tend to hinder its further buckling, and it is possible, moreover that the resistance to bending of the loaded flange may enable further increases in load beyond the stability of the web plate. On the other hand a slender flange will itself require the support afforded by the web plate in order that it may not buckle in the plane of the latter.

The problem here arising will be treated more exhaustively in a later paper which will also include tables for the practical application of the method of calculation developed.