Breaking loads on subsoil below foundations

VIII 8

Breaking Loads on Subsoil Below Foundations.

Bruchlasten des durch Fundamente belasteten Bodens.

Charges de rupture du sol sous les fondations.

M. Buisson,

Ingénieur E.P., Chef du Service de Contrôle des Constructions immobilières au Bureau Veritas, Paris.

What follows has reference to total breakdowns arising in compact soils. In these important special cases the principal stresses existing in the ground can be expressed in a simple manner, the soil being assumed isotropic, elastic, and bounded by a horizontal surface; the reasoning is developed from *Boussinesq's* theory which corresponds to the distribution given by n = 3 of *Fröhlich.* This is true for a strip of finite width and indefinite length uniformly loaded, and the limiting case is that of a semi-plane subject to a uniform load.

The trajectories of the principal stresses are well known, being hyperbolae and rectangular ellipses in the first case, or rectangular parabolae in the second. In the first case the ends of the strips are the foci and in the second case the limit of the loaded portion is the focus of the parabola.

The limit of the plastic zone is defined by the well known condition of breakage

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2} \sin \varphi$$

wherein σ_1 and σ_3 are the maximum principal stresses expressed in terms of the co-ordinates, and φ is the angle of friction. The trajectories of breakage are the isoclines of these hyperbolae and parabolae which intersect these curves at an angle $\frac{\pi}{4} - \frac{\varphi}{2}$. In the case of the hyperbolae the curves in question are spirals and their analysis is difficult, but in the case of the parabolae it is easy to see that the isoclines also are parabolae with their foci being at the end of the loaded portion, the angle between their axes and the verticals being exactly equal to the angle φ (Fig. 1).

This result is important, as it allows the conditions of breakdown for sandy and coherent grounds underneath a quay wall or a retaining wall to be determined. It is, in fact, easy to determine the abscissa of the intersection between the plastic zone and the horizontal which is given by[1]

$$x = \frac{q}{\pi \gamma} \cdot \frac{(1 - \sin \varphi)}{\sin \varphi} \qquad (1)$$

where q is the unit load and γ is the specific weight.

[1] *Fröhlich:* Druckverteilung im Baugrunde. Vienna 1934, J. Springer.

If it be assumed that the wall is in a state of unstable equilibrium when the curves of rupture pass through the centre of the base, and further, that the density (specific weight) of the wall is equal to that of the soil, then the value of the unit breaking load assumed to be uniformly distributed is given by

$$q = \frac{\pi \gamma b (1 + \sin \varphi) \sin \varphi}{(1 - \sin \varphi)^2} \tag{2}$$

where b represents one-half of the width of the wall.

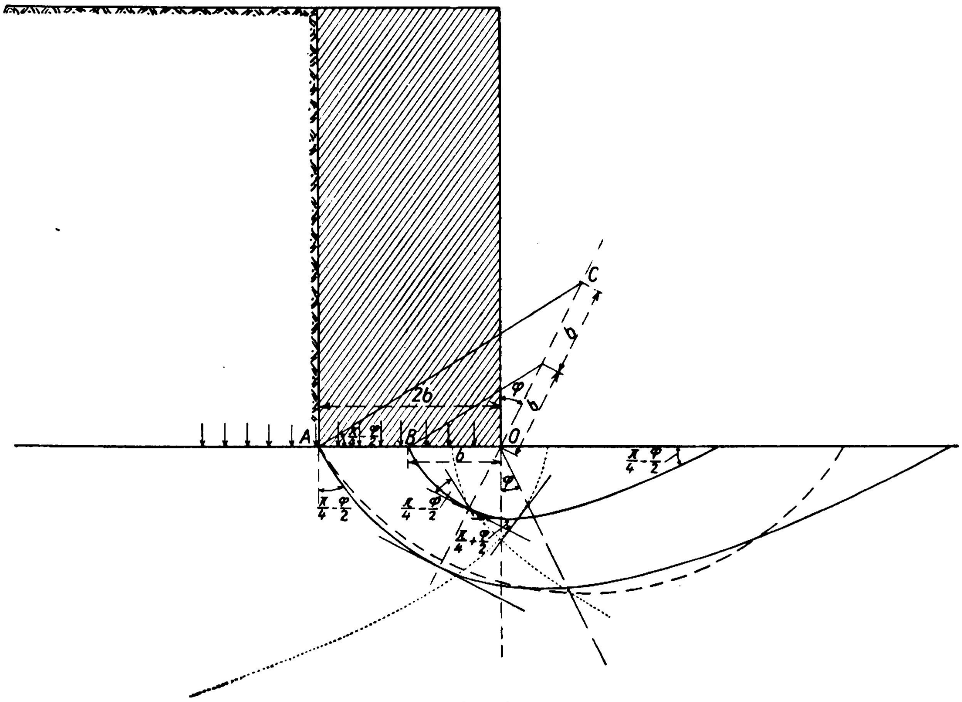

Fig. 1.

In the same way it is possible to determine the minimum length for sheet piling of a quay wall or for a cofferdam.

In the case of clay soil having a shear resistance equal to τ_0 the limiting load is given by

$$q_c = \frac{16 \tau_0}{3} \cdot \frac{1 + \sin \varphi}{\cos \varphi} \cdot \left(1 + \frac{tg^3 \varphi}{\sqrt{2}}\right) \tag{3}$$

applicable to the case of a half plane space under uniform loading, when the condition for the formation of a plastic zone is

$$q = \frac{\pi \tau_0}{\cos \varphi}.$$

These results may be extended by referring them to an experimental fact, as follows. The trajectories of breakage disclosed by tests on a surface such as those described by *Krey*[2] are practically concentric circles, the centre of which can easily be found, being at the intersection of a straight line inclined at an

[2] *Krey:* Erddruck, Erdwiderstand, 4^{th} Edition, p. 269.

angle φ to the vertical passing through the foot of the wall with a straight line passing through the furthermost point of the base, from which may be derived the trajectory of breaking (Fig. 2). This line makes an angle $\frac{\pi}{4} - \frac{\varphi}{2}$ with the horizontal; it is, in fact, perpendicular to the line which makes an angle $\frac{\pi}{4} - \frac{\varphi}{2}$ with the vertical, which in turn is in a tangent to the curve of rupture, for at this point the principal stress is vertical (friction at the base being neglected). It will be seen that this construction follows from the result determined by the parabola above, as is to be expected because close to the foundation the influence of neighbouring points is not felt anywhere but at the points considered, lying within an angle of 50^0 from the vertical. The construction amounts to substituting for the parabola a circle tangential to it at its point of intersection with the horizontal.

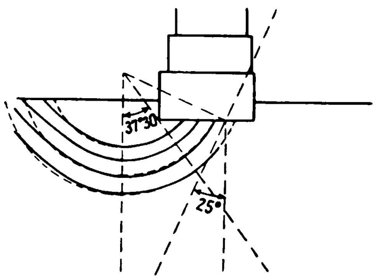

Fig. 2.

The same construction has been checked by the author and has been found to give a good approximation when applied to a small scale model of a quay wall, as in Fig. 3.

In actual fact, for the case of a superficial loading on a finite surface, the extreme portion of the curve differs only slightly from a circle, and comes closer to the tangent. It is found that in practice the angle between the curve and the horizontal is greater than $\frac{\pi}{4} - \frac{\varphi}{2}$ which should, theoretically, be its value. Both in theory

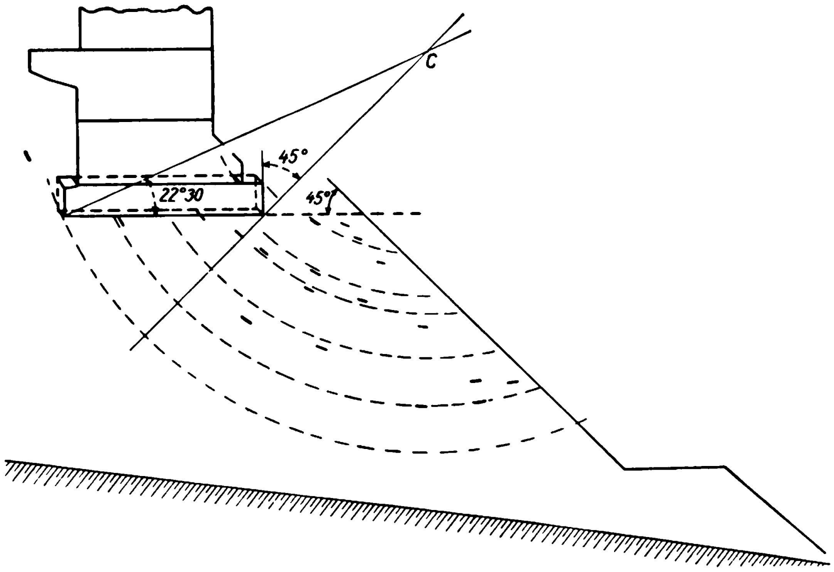

Fig. 3.

and in practice the longest of the curves of rupture are those which originate at the two ends of the plastic zone; hence it may be assumed without serious error (and the error is on the safe side) that the intersection of the circle of rupture (as defined above) with the horizontal coincides with the intersection of the boundary of the plastic zone with the horizontal, given by

$$x = \sqrt{b^2 + \frac{2\,b\,q}{\pi\gamma} \cdot \frac{1 - \sin\varphi}{\sin\varphi}}$$

for the case of an infinite strip. In the circumstances it is certain that tho conditions of rupture are realised over the whole length of the curve. If it be assumed that breakdown occurs when the circular trajectory passes through the centre of the strip, then the abscissa of the intersection of the circle with the horizontal is given by $x = 2b\,(1 + \sin\varphi)$ (Fig. 1).

By equating the two values of x there is obtained in the case of an infinite strip in sandy ground with superficial loading:

$$q_1 = \frac{\pi\gamma b \sin\varphi\,(1 + 2\sin\varphi)\,(3 + 2\sin\varphi)}{2\,(1 - \sin\varphi)}. \tag{4}$$

If the outermost trajectory of rupture is assumed to start from the end, we have

$$q'_1 = \frac{2\,\pi\gamma b \sin\varphi\,(1 + 2\sin\varphi)\,(3 + 2\sin\varphi)}{1 - \sin\varphi} \tag{4'}$$

In the case of a coherent soil offering a shear resistance τ_0 before being loaded, the breaking load caused by sudden loading is given by:

$$q_c = \frac{4\left(\frac{\pi}{2} + \varphi\right)(1 + \sin\varphi)}{1 + 2\sin\varphi}\,\tau_0 \tag{5}$$

for both the first and the second hypothesis, independently of b.

Knowing the breaking load in the infinite strip it is possible to infer approximately the breaking load for a square or circular loaded surface, seeing that the stresses immediately below the vertical through the centre are equal to approximately one half of those which exist under the axis of the strip of the same width. In a very rough way it would be good enough to change $\frac{b}{2}$ to r whereupon the breaking load under the same assumptions as above would be

$$q_2 = \frac{\pi\gamma r\,(1 + 2\sin\varphi)\,(3 + 2\sin\varphi)\sin\varphi}{1 - \sin\varphi} \tag{6}$$

$$q'_2 = \frac{4\,\pi\gamma r\,(1 + 2\sin\varphi)\,(3 + 2\sin\varphi)\sin\varphi}{1 - \sin\varphi} \tag{6'}$$

Using a different method, however, the author has shown that in the case of the square area of loading the breaking load can be expressed approximately in the following way, as a function of a which is the half width of the side of the square:

$$q_3 = \frac{2\,\pi\gamma a \sin\varphi\,(1 + \sin\varphi)\,(1 + 2\sin\varphi)\,(3 + 2\sin\varphi)}{3\,(1 - \sin\varphi)} \tag{7}$$

$$q'_3 = \frac{8\,\pi\gamma a \sin\varphi\,(1 + \sin\varphi)\,(1 + 2\sin\varphi)\,(3 + 2\sin\varphi)}{3\,(1 - \sin\varphi)} \tag{7'}$$

To arrive at this result it is necessary to find an approximate expression for the abscissa of the intersection of the plastic zone with the horizontal plane, for the case of a loaded surface approximating to a square shape, and to equate this to $2b\,(1 + \sin\varphi)$ which represents the abscissa of the intersection between the horizontal plane and the circle of rupture through the centre. It will be noticed that

$$\frac{q_3}{q_1} = \frac{4\,(1 + \sin\varphi)}{3}$$

and with $\varphi = 30^0$ this gives

$$\frac{q_3}{q_1} = 2.$$

For different values of φ the ratio does not vary greatly from this, so that the approximation made above evidently is justified.

The author's method makes it a particularly simple matter to determine the conditions of rupture in cases where present methods are defective or are very laborious to use.

It is distinguished from methods resulting in values for the exponentials in the formulae of rupture by the fact that these are all based on the assumption that the surface of rupture is in the shape of a logarithmic spiral, with its centre at the edge of the loaded surface — an assumption which does not appear to be confirmed by the experiments, for it has already been shown that in the tests carried out by *Krey* the surfaces of rupture are practically concentric circles, with their centres at the position indicated.

Tests and checks are being carried out and will be made the subject of special reports to appear later.

The author is treating the questions briefly summarised here in greater detail in articles about to appear in the journal "Travaux".[3]

[3] Since published in Travaux, N° 46, Oct. 1936; N° 48, Dec. 1936; N° 51, March. 1937.