AI 1

An investigation of the oscillations of suspension bridges in wind

Etude sur les oscillations de ponts suspendus sous l'effet du vent

Eine Untersuchung über die Schwingungen von Hängebrücken infolge Winddruck

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1. INTRODUCTORY

The paper gives a brief review of experiments on the aerodynamic stability of suspension bridges which have been carried out by the National Physical Laboratory on behalf of the Ministry of Transport. The specific purpose of the investigation was to give guidance on the aerodynamic design aspects of a proposed bridge over the River Severn with a centre span of 3,240 ft. (987 m.) The experiments were commenced in 1946 and were concluded early in 1951 by tests which provided a final confirmation of the stability of the preferred design for the bridge.

Although in the time available no fundamental research could be undertaken to elucidate the root causes of the aerodynamic oscillations, much information of a general nature was gained which should be helpful in the design of future suspension bridges. A previous paper* submitted by the author to the 3rd Congress summarised the preliminary stages of the work.

The wind-excited oscillations which have occurred on long-span suspension bridges (notably the original Tacoma Narrows Bridge) have been basically either vertical bending or torsional motions. In vertical oscillations the suspended platform moves up and down and the two cables displace equally and in step. In torsional oscillations the platform twists about a spanwise axis and the cables displace equally but in opposite directions. Both types of motion can occur at various frequencies and in a variety of modes. The instantaneous shape of a spanwise reference line during an oscillation is termed the *wave form* of the oscillation and is either

* C. Scruton, "An Experimental Investigation of the Aerodynamic Stability of Suspension Bridges," *Preliminary Publication for the 3rd Congress I.A.B.S.E.*

"symmetric" or "antisymmetric" with respect to the centre of the bridge according as the displacements of the two half-spans are in the same phase or in anti-phase.

In the investigation to be described the oscillatory behaviour of bridges was studied experimentally by tests of models in wind-tunnels. The two types of model used were similar to those used by Farquharson, Vincent and others* at the University of Washington (U.S.A.) in the extensive investigations which followed the collapse of the Tacoma Bridge in 1940. These were:

(a) *Sectional models*

A sectional model is a short rigid model of a sample length of the suspended structure and is mounted across the wind-tunnel (with its span horizontal and normal to the wind-stream) with freedom to oscillate against spring constraints. In the present investigation the model mountings permitted vertical translatory motions and pitching motions.† These motions, which were the two-dimensional equivalents of the vertical bending and torsional motions of the complete bridge, could take place at the same time for coupled motion tests, or could be isolated for tests with a single freedom.

In general the wind tests of these models involved no more than the observation and measurement of the critical wind speeds and frequencies bounding the ranges over which oscillations were maintained by the wind. Occasionally the damping rates of oscillations in the wind-stream were measured. The tests were made in transverse winds with inclinations varying between ∓15 degrees. It was not considered practicable to test sectional models in horizontally inclined winds.

(b) *Full models*

A full model is a replica of a complete bridge so constructed that its behaviour in a wind-stream is similar to the full-scale bridge. The full model used in this investigation was installed in a large wind-tunnel‡ specially built by the Ministry of Transport for the investigation. The direction of the tunnel wind-stream could not be varied but the effect of inclined winds, both horizontally and vertically, was simulated by inclining the model. Critical wind speeds, frequencies and oscillation modes were recorded.

In the early stages of the investigation it was uncertain whether the stability of a complete bridge could be predicted satisfactorily from experiments on a sectional model alone, since with this method of test the influence of several factors cannot be represented directly. Such factors include, for example, the tower stiffnesses and inertias, the longitudinal camber, the oscillation wave form and the horizontal inclination of the wind. However, it was also clear that full models were unsuitable for routine comparisons between different forms of suspended platform, owing to the length of time required for construction and the high cost. To provide a practical programme it was therefore decided to depend on tests of sectional models for an indication of the most promising structural forms. Whilst the tests were in progress, the design and construction of a full model was also put in hand, with a view to tests of the correlation between the two different experimental methods. This full model was necessarily based on a very early design for the bridge, and it became available for wind-tunnel tests in 1948. The results obtained with these two types of models

* "The Aerodynamic Stability of Suspension Bridges with special reference to the Tacoma Narrows Bridge," *Bulletin No.* 116 *of the University of Washington Engineering Experiment Station.*
  † A few tests were also made with lateral motion (i.e. translation normal to the span and in the plane of the decks).
  ‡ The wind-tunnel is briefly described in Appendix II.

led to the conclusion that sectional model tests were sufficient for reliable full-scale prediction, and accordingly the construction of a further full model based on the design finally preferred for the bridge was considered to be unnecessary. A final verification of the stability of this design was, however, provided by tests of a large-scale sectional model. An increase of the linear scale from 1/100 to 1/32 was considered advisable, because previous tests had shown that the stability was sensitively influenced by details of the structural form which could not be copied with sufficient accuracy on a small-scale model.

2. NOTATION

ρ air density

ν kinematic viscosity of air

B width of bridge between stiffening trusses

g acceleration due to gravity

σ typical material density

E typical elastic modulus

θ angular torsional displacement of suspended platform at any instant of oscillation (radians)

z linear vertical displacement of structure at any instant of oscillation

I_θ torsional moment of inertia per unit spanwise length

I_z mass per unit spanwise length

N_θ and N_z natural frequencies of oscillation in torsional and vertical bending modes respectively

e_θ and e_z elastic stiffnesses corresponding respectively to I_θ, N_θ and I_z, N_z.

δ natural logarithm of the amplitude of successive cycles of oscillation (logarithmic decrement)

δ_θ and δ_z logarithmic decrements for torsional and vertical oscillations in still air respectively

$\delta_{\theta A}$ and δ_{zA} logarithmic decrements due to still air damping

$\delta_{\theta s}$ and δ_{zs} logarithmic decrements due to structural damping

$\delta_{\theta w}$ logarithmic decrement of torsional oscillations due to the wind-stream

V and V_c wind speed and critical wind speed respectively

N and N_c frequencies of oscillation corresponding to V and V_c respectively

$V_r = V/NB$ reduced velocity

$V_s = V/N_\theta B$

α angular inclination of wind to the bridge platform in a vertical plane —upwinds positive (degrees)

β angular inclination of the wind to the bridge in a horizontal plane ($\beta = 0$ and 90 degrees for transverse and longitudinal winds)

3. MODEL AND FULL-SCALE SIMILARITY

(i) Full models

The motions of similar-shaped suspension bridges in wind may be influenced by the bridge size (B), by the density, damping and elasticity of the bridge structure (σ, δ_s, E) and by the viscosity, density and velocity of the air (ν, ρ, V).

By the usual principles of dimensional analysis these parameters can be grouped in the following independent non-dimensional combinations:

$$
\left.
\begin{array}{ll}
\text{(a)} & \sigma/\rho \quad \text{(density)} \\
\text{(b)} & E/\rho V^2 \quad \text{(elasticity)} \\
\text{(c)} & \delta_s \quad \text{(structural damping*)} \\
\text{(d)} & gB/V^2 \quad \text{(gravitational)} \\
\text{(e)} & VB/\nu \quad \text{(viscosity)}
\end{array}
\right\} \quad \ldots \ldots \quad (1)
$$

Full dynamic similarity between model and full-scale is achieved if the model is geometrically similar to full-scale and the above numbers are equal for both systems.

For tests in atmospheric wind tunnels of practicable dimensions and wind speeds the full-scale value of (e) cannot be achieved. The velocity scale would moreover be incompatible with that required by (d). However, there is evidence that the aerodynamic forces acting on bluff bodies such as bridge structures depend mainly on pressure action and only indirectly on viscous action and hence the validity of full-scale prediction by model testing is not seriously impaired by incorrect scaling of the viscous forces.

The design and construction of a model to accord with the similarity numbers (a) to (d) presents considerable difficulties. It will be found generally that materials of construction do not exist which possess the requisite combinations of density and elastic moduli, and that even were such materials available, the exact small-scale reproduction of details might be impracticable. However, it is sufficient if the correct overall inertias and elastic stiffnesses are reproduced in the model. Equivalent similarity numbers to those given in (1) but using inertia and elastic stiffness terms are:

$$
\left.
\begin{array}{ll}
\text{(a)} & I_\theta/\rho B^4, \quad I_z/\rho B^2, \text{ etc.} \\
\text{(b)} & e_\theta/\rho V^2 B^2, \quad e_z/\rho V^2, \text{ etc.} \\
\text{(c)} & \delta_{\theta s}, \quad \delta_{zs}, \text{ etc.} \\
\text{(d)} & gB/V^2
\end{array}
\right\} \quad \ldots \ldots \quad (2)
$$

With a linear scale of $1/n$ the foregoing numbers yield a velocity and a frequency scale of $1/\sqrt{n}$ and $\sqrt{n}$ respectively. The values of V_r and V_s are therefore the same for the full-scale bridge as for the model.

(ii) Sectional models

With strict inertial scaling

True similarity conditions are obviously not observed in the sectional model method of test. The use of the method for full-scale prediction assumes that oscillations of a complete bridge arise from the aerodynamic action on the suspended structure alone and that the other components (e.g. cables) contribute to the dynamic properties only. It also implies that critical values of V_r are not influenced by oscillation wave form.

The inertial coefficients of sectional models represent the total contribution of suspended structure and cables. The model stiffnesses are merely those which provide the required oscillation frequencies and need not, as in the case of full models, be derived from correctly proportioned gravitational and elastic forces. When (d) is

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* It is shown in Appendix I that when the influence of viscosity is negligible values of $\delta_{\theta A}$ are the same for model as for full-scale provided strict inertial scaling is observed. Hence, for the full model tests and for the sectional model tests with strict inertial scaling δ_θ may be used in place of $\delta_{\theta s}$.

omitted and expressions for natural frequencies are substituted in (b), the similarity numbers for sectional models become:

$$\left.\begin{array}{ll} \text{(a)} & I_\theta/\rho B^4, \quad I_z/\rho B^2 \\ \text{(b)} & V/N_\theta B, \quad V/N_z B \\ \text{(c)} & \delta_{\theta s}, \quad \delta_{zs} \end{array}\right\} \quad \cdots \cdots \cdots \quad (3)$$

where I_θ, I_z now represent overall values.

Thus the actual velocity and frequency scale for sectional models are interdependent but one or other may be chosen at convenience.

Inertial scaling not attempted

In the writer's previous paper* tests of sectional models involving isolated motions only were considered. The requirements for similarity were then approached through the solutions of the equations of motion. Similarity conditions equivalent to those given in (3) were, of course, obtained; but in addition, it was shown that when $N_\theta = N_c$ approximately, strict inertial scaling is not essential and the critical values of V_r depend only on the geometric shape of the structure and on the product of (a) and (c) i.e. on $I_\theta . \delta_{\theta s}/\rho B^4$ and $I_z . \delta_{zs}/\rho B^2$ for angular and linear motions respectively.

Procedure for prototype prediction from sectional model test data

Some remarks may be useful on the procedure in the two cases where strict inertial scaling of the model is observed, or is not attempted and $N_c = N_\theta$ approximately.† The critical values of V_r obtained from the models by both these methods are applicable to the prototype, provided the values of $\delta_{\theta s}$ satisfy the stated conditions.‡ Strict inertial scaling is essential for values of V_s (and N_c) to be applicable: critical speeds are then determined from critical values of V_s or V_r by either of the relations $V_c = N_\theta B V_s$ or $V_c = N_c B V_r$. The value of N_c required by the latter relation is not given by the other method, and it is therefore necessary to use the approximation $V_c = N_\theta B V_r$. Experience with various types of bridge sections indicates that this approximation introduces no serious discrepancies within the range of normal winds.

4. THE FULL MODEL EXPERIMENTS

Model construction

A photograph of the 1/100-scale full model is reproduced in fig. 1. The model was designed to give wide scope for modifications both to the elastic properties and to the form of the suspended structure. It represented a truss-stiffened bridge of width 107 ft. (32·6 m.) and of total span 5,040 ft. (1,535 m.); the centre span of length 3,000 ft. (914 m.) had a sag ratio of 1/10. The two roadways of width 39 ft. (11·9 m.) were separated by an 8·3 ft. (2·53 m.) wide reservation.

The required stiffness and inertial properties of the suspended structure were obtained by the use of light rigid components with steel interconnecting springs. These rigid components were mainly of aluminium-balsawood sandwich construction and each component spanned one bay of 60 ft. (18·3 m.). The use of spring interconnections enabled the stiffnesses to be altered and also had the advantage that the structural damping was kept to a sufficiently low value, since the deformations occurred mainly

* *Loc. cit.*
† For convenience, pitching oscillations only are discussed here. The procedure applies equally well to other motions.
‡ See note at foot of page 40.

through flexing of the springs.   Both the elastic and inertial properties of the towers were variable, but it was not considered necessary to reproduce the correct external shape of either the towers or the anchorages.   The model cable consisted of piano wire of diameter 0·024 in. (0·61 mm.), which provided the equivalent of a full-scale cross-sectional area of about 450 in.^{2} (2,900 cm.^{2}) with a 1/100 reduction of Young's
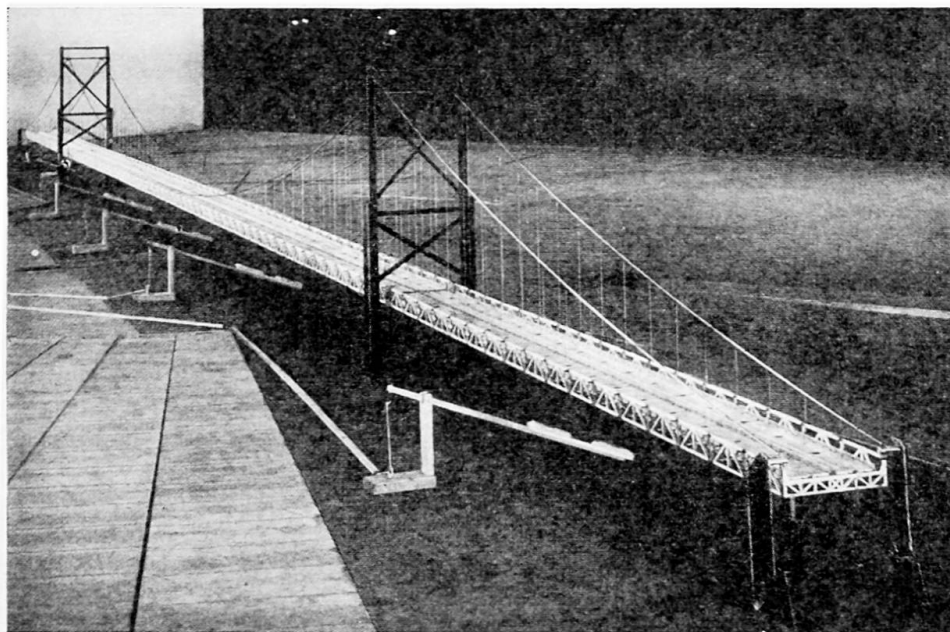

Fig. 1.   The 1/100-scale full model mounted in the wind-tunnel

modulus.   To obtain the correct mass and external shape, hollow brass cylinders were spaced along the wire and fixed to it by a single grub screw.   The model suspenders were made of fishing line which had been prestretched and treated with a beeswax coating to reduce the effect of humidity changes on its length.

The model was mounted on the horizontal turntable which fitted flush with the floor of the wind-tunnel test chamber (see Appendix II and fig. 7).   Changes of the horizontal wind direction were reproduced by rotation of the turntable and the effect of vertically inclined winds was simulated by tilting the whole model about a spanwise axis near the wind-tunnel floor.*   In the second case, the correct representation of the gravitational forces was then restored by attaching suitably angled and spring tensioned cords at several points along the span.   The additional elastic stiffnesses contributed by this arrangement were rendered small by the use of long cords and very weak springs.

Test procedure

In still air tests resonance modes and frequencies were observed by exciting the model through weak springs driven by a reciprocating motion.   The logarithmic decrements of the artificially excited oscillations were measured in the usual way from photographic records.

In wind tests the tunnel speed was gradually increased from zero to a maximum corresponding on full-scale to a little over 200 ft./sec. (61 m./sec.).   The critical wind

* The provision made for tilting the model is not shown on fig. 1.

speeds and frequencies for maintained oscillations in the various modes were noted. In general the modes were observed visually, but cinematograph records were taken of certain typical oscillations.*

Results and conclusions

Tests on the full model were carried out for various vertical wind inclinations α and horizontal wind inclinations β. The effective angle α produced by a given tilting inclination α' of the model depended on the value of β and was given to a close approximation by $\alpha = \alpha' \cos \beta$. Except for longitudinal or near longitudinal winds, the values of vertical inclination attained ranged between ± 15 degrees.

Tests were made on the influence of several design variations such as grade-line camber, tower stiffness and cable loading; and also on the effects of modifications to the external shape of the suspended structure. By covering the stiffening truss panels with paper it was possible to simulate the aerodynamic effect of a plate-girder-stiffened bridge. In this condition the model reproduced many of the modes of oscillation which occurred on the original Tacoma Bridge.

The more important results and conclusions are given below.

(a) Coupling between vertical bending and torsional motions

Each wind-induced oscillation observed corresponded in mode and in frequency to a natural oscillation induced by resonance tests in still air. From this experimental evidence, and from independent visual observations, it was concluded that coupling effects between the vertical bending and torsional motions had little influence on the oscillations. However, it should be noted that the natural frequency ratio N_θ/N_z for corresponding wave forms was approximately 2 and was therefore more appropriate to a bridge with a double rather than a single plane of lateral bracing. There was no means on the full model of substantially reducing this ratio. The effect of a close approach to equality of the natural frequencies in sectional model tests is described later.

(b) Influence of oscillation form

With a given wind inclination and model condition all the different types of torsional oscillation appeared for approximately the same constant value of V_r. A similar conclusion applied for vertical bending oscillations. This indicates that the influence of oscillation wave form is unimportant.

(c) Influence of shape of suspended structure

The stability depended on the shape and arrangement of the components of the suspended structure. The model with plate-girder stiffening exhibited a high degree of instability in both vertical bending and torsional oscillations. No instability in vertical bending motion was found in any test with a truss-stiffened model. Torsional oscillations occurred for certain arrangements of the truss-stiffened model.

The influence of shape is discussed in more detail in the description of the sectional model tests.

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* Some of these records have been incorporated in a short silent film entitled "Oscillations of a Model Suspension Bridge in Wind."

(d) *Influence of elastic stiffnesses and natural frequencies*

Critical values of V_r were not appreciably influenced by variation of the stiffnesses and natural frequencies due to structural modifications which did not involve change of shape of the suspended structure. Such modifications included variation of the tower stiffnesses, unloading the sidespan cables, and fitting a centre tie between truss and cable.

(e) *Influence of wind inclination*

The highest degree of instability was found in transverse winds ($\beta=0$) and the stability characteristics improved progressively with increase of β. Vertical bending oscillations of the plate-girder-stiffened model persisted, but with decreasing amplitude, up to a value of β between 30 and 45 degrees, while weak torsional oscillations were still present at $\beta=60$ degrees. No instability of any type was found in steady longitudinal winds.

The stability was sensitive to the vertical inclination of the wind. The highest degree of instability of the plate-girder-stiffened bridge occurred with slight negative vertical inclination, and that for the truss-stiffened bridges with slight positive inclination.

(f) *Effect of grade-line camber*

The stability was not greatly influenced by variations of the grade-line camber. The indications were that a cambered grade-line yielded very slightly better stability characteristics than a level grade-line.

(g) *Effect of gusty winds*

Some tests were attempted with several types of disturbed airflow, none of which, however, was necessarily representative of natural gusty winds. In longitudinal, as well as in transverse winds, irregular vertical oscillations were set up by the buffeting action of large-scale eddies shed from the gust-making device, but no tendency to torsional motion was observed.

(h) *Correlation between sectional and full model tests*

The full model tests showed that critical values of V_r for specific values of the structural damping were determined by the shape of the suspended platform and were not substantially affected by other structural properties or by the wave form of the oscillation. Also in these tests the highest degree of instability was produced in transverse winds. It was concluded that sectional model tests would be adequate for stability prediction provided they yielded the same critical values of V_r as those given by the corresponding full model.

Table 1 sets out a comparison of the results obtained with the full model and with its sectional model copy. The alignment of the vertical motion in the sectional model tests were not strictly correct except at zero incidence, since the direction of model motion was not altered to correspond to the incidence change. The error in alignment increased with incidence and hence may account for the lack of correlation between the results at $\alpha=\pm10$ degrees for vertical oscillations of the plate-girder-stiffened section. With this exception all the results showed very good agreement between the two methods of test and support the conclusion that reliable predictions of the stability of proposed suspension bridges may be based on sectional model tests only.

TABLE I

Comparative results from sectional and full model tests

The models represented a mid-deck bridge of section A, fig. 3. In its *standard* condition (fig. 1) the model decks were separated by an open reservation and were fitted with paling-type handrailing and truss-type roadway stringers. The values for the structural damping were:

Full Model: $\delta_{zs}=0.035$, $\delta_{\theta s}=0.05$
Sectional Model: $\delta_{zs}=0.06$, $\delta_{\theta s}=0.05$

The full model was tested up to speeds corresponding to values of V_r of 15 and 8 for vertical and torsional oscillations respectively. Higher values were reached in the sectional model tests, but for the purpose of this comparison the above values are taken as the limits for both types of test and a result is given as STABLE when no oscillations occurred up to these limits. Only the lowest critical values are quoted here, since on the full model the critical speeds for the upper limit of the instability range were usually masked by the onset of a further mode of oscillation.

Model Configuration	α degrees	Lowest critical values of V_r			
		Vertical Oscillations		Torsional Oscillations	
		Sectional Model	Full Model	Sectional Model	Full Model
Standard	−15, −10, −5, 0, 5, 10, 15	STABLE	STABLE	STABLE	STABLE
Standard but with all hand-railing removed	10			7·0	6·8
	−15, −10, −5, 0, 5, 15			STABLE	STABLE
Standard but with solid plate handrailing	0			3·4	3·8
	−15, −10, −5, 5, 10, 15			STABLE	STABLE
Standard but with castellated handrailing	−15, −10, −5, 0, 5, 10, 15			STABLE	STABLE
Standard but with the hand-railing on the inner edges of the carriageways removed and a solid cover fitted over of the central reservation	15			5·0	4·5
	10			5·4	4·8
	5			7·2	6·9
	0, −5, −10, −15			STABLE	STABLE
Standard but with the stiffening trusses covered to represent the aerodynamic effect of solid plate-girder stiffening	15	STABLE	STABLE	5·5	STABLE
	10	STABLE	1·8	2·6	2·5
	5	1·7	1·7	2·4	2·7
	0	1·8	1·6	2·4	2·6
	−5	1·8	1·7	2·4	2·5
	−10	STABLE	1·7	2·8	2·4
	−15	STABLE	STABLE	5·8	4·2

5. Experiments with 1/100-scale sectional models

A typical 1/100-scale sectional model is shown by fig. 2. The models were of rigid wooden construction and represented a 340-ft. (103·5 m.) length of the bridge-suspended structure. They were tested in a wind-tunnel with a working section measuring 4 ft. by 3·40 ft. (1·22 m. × 1·035 m.). Two types of mounting, here referred to as the "original" and the "improved," were used.
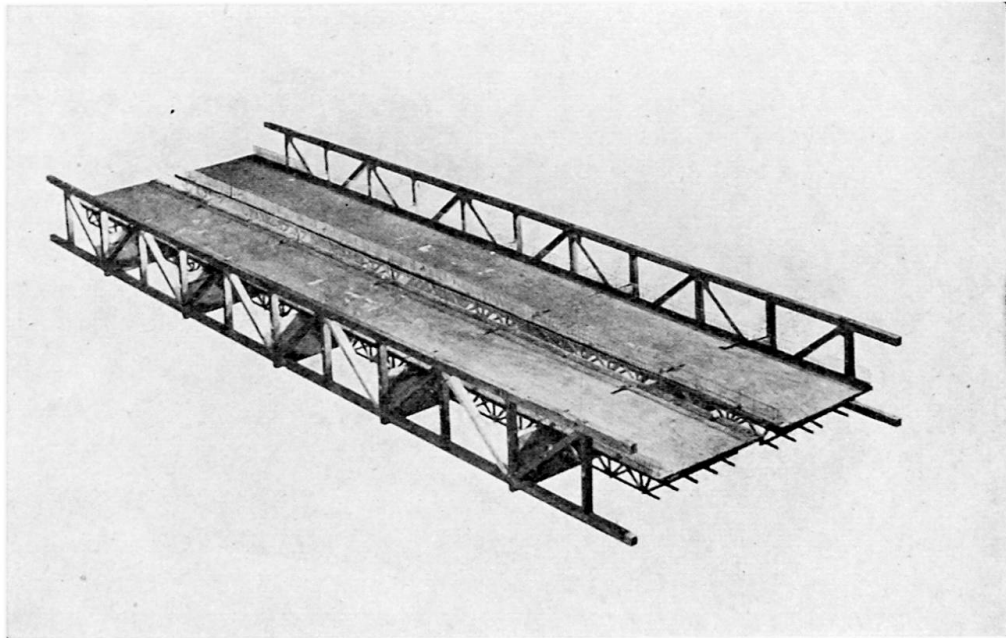

Fig. 2. Typical 1/100-scale sectional model.

The original mounting

This permitted both vertical bending and pitching motions, either singly or together. The apparatus damping was not directly variable, and the inertias of the model on this mounting were very much greater than those required by correct scaling of the prototype values. However, since in the tests $N_c = N_\theta$, the similarity conditions given in paragraph 3 were applicable.

The model was attached at both ends to circular plates which were supported on ball bearings so that pitching motion could take place against the elastic constraint provided by helical springs. Each bearing and spring assembly was carried on a framework which was constrained to move vertically by a steel-strip device; helical springs again providing stiffness. The circular end-plates fitted flush with the walls of the wind-tunnel.

The improved mounting

This was used for pitching motion tests with correct inertial scaling and with the apparatus damping variable from a low initial value. As in the case of the original mounting the model was carried between discs. Each disc was supported on a steel-strip suspension which maintained a fixed axis of oscillation and also provided the required elastic stiffness. The damping due to this suspension was very small. Additional damping was supplied by the action of a thin segment of copper which oscillated with the model and passed between the pole-pieces of an electromagnet.

The damping moment produced by the eddy currents set up in the copper was proportional to the velocity of the motion and could be readily varied by adjusting the current through the coils of the electromagnet.

Discussion of results

The main types of bridge section tested are shown in fig. 3. They were all stiffened by trusses of the single Warren type, but could be readily converted to represent plate-girder-stiffened sections by the attachment of solid covers to the trusses. The models were usually tested with a pitching axis placed approximately centrally with respect to the four stiffening-truss-chords members, but some tests were made with
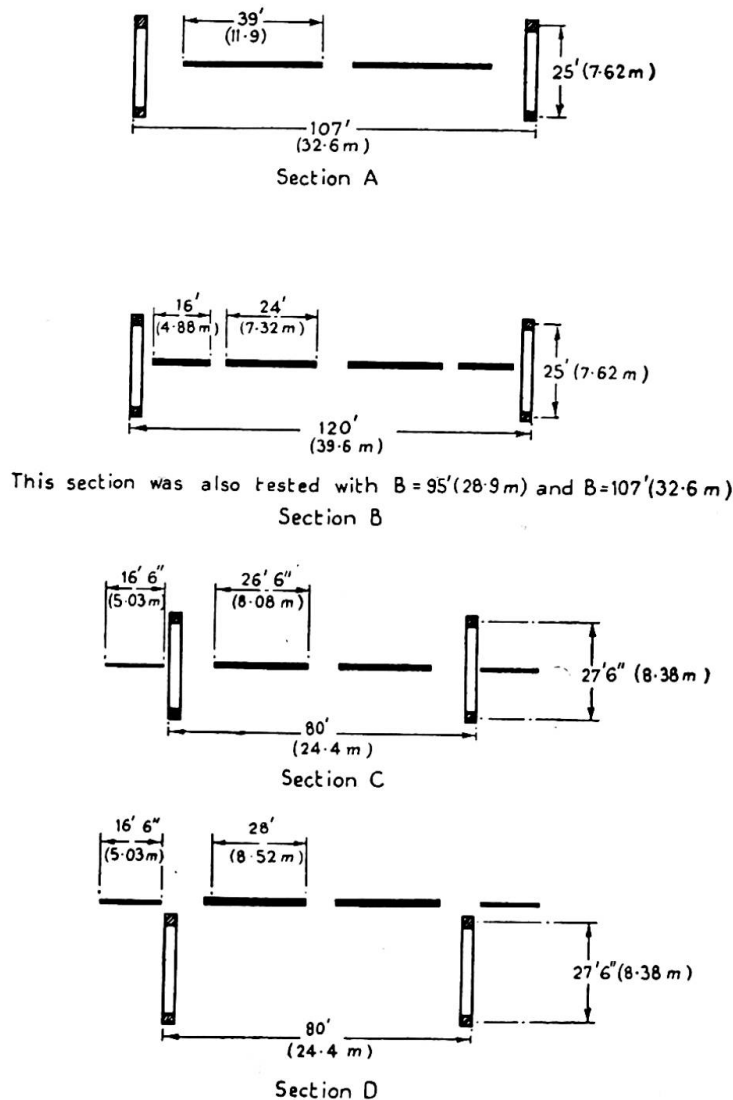

Fig. 3. Main types of bridge sections tested

other axis positions. The maximum of V_r or V_s obtainable in the tests depended on the test conditions, but in all cases corresponded to full-scale wind speeds of well over 100 miles/hr. (161 km./hr.).

Some of the models were tested on both the original and improved mountings.

The good agreement obtained between corresponding sets of results provided experimental verification for the similarity conditions stated in paragraph 3 for the case where strict inertial scaling is not attempted.

The main conclusions derived from the results of the sectional model test are summarised below.

(a) *Influence of structural damping*

A typical diagram showing the influence of structural damping on the stability of a plate-girder-stiffened section is reproduced in fig. 4. Similar tendencies were exhibited by truss-stiffened sections.
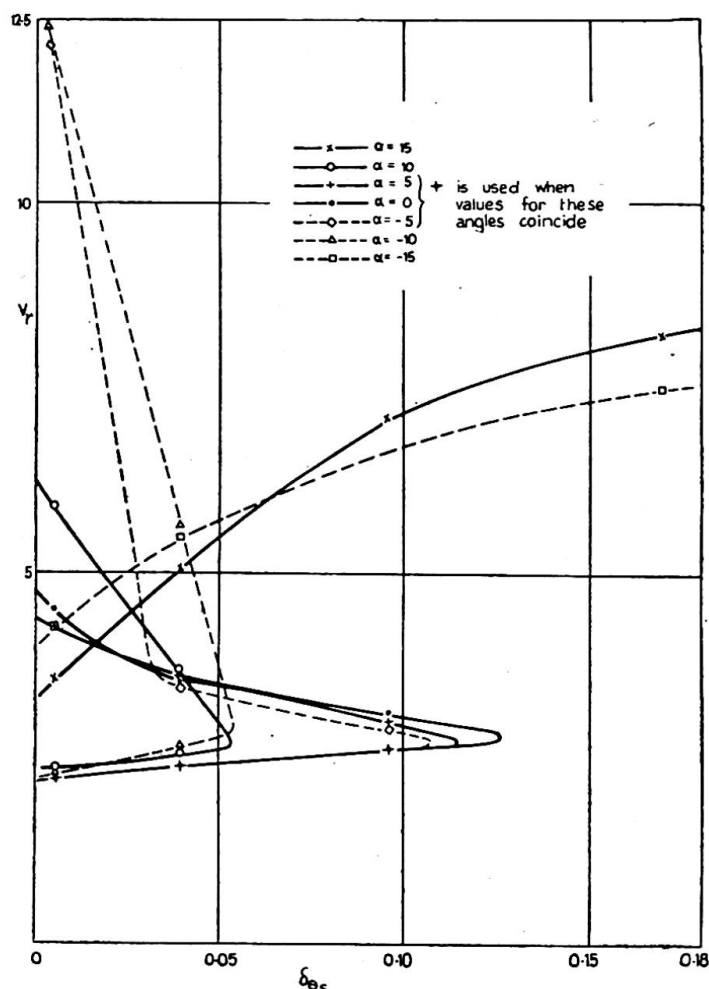

Fig. 4. Influence of damping on the pitching oscillations of the plate girder stiffened section A

Increase of $\delta_{\theta s}$ narrowed the instability range by increasing the critical speeds for the lower boundary and decreasing those for the upper boundary. The magnitude of $\delta_{\theta s}$ necessary to prevent oscillations for all wind speeds provided a qualitative indication of the relative strengths of the instabilities.

(b) *Influence of location of pitching axis*

The vertical position of the pitching axis was varied in tests of the mid-deck section (B, fig. 3) and of the top-deck section (D, fig. 3). For axes lying midway between the stiffening trusses the stability of both these sections was least when the axis was located near the level of the deck.

(c) *Coupling between vertical bending and pitching oscillations*

In sectional model tests with coupled motions particular attention was given to ratios of the natural frequencies near unity, since it was expected that the influence of coupling would then be most marked. Except when the frequency ratio N_θ/N_z closely approached unity, one of the motions was always found to predominate, and the critical frequency and reduced velocity were in fair agreement with those obtained in the corresponding isolated motion test. When $N_\theta/N_z=1$ both motions were present in substantial proportions. In one instance coupled oscillations occurred when $N_\theta/N_z=1$ which were absent when $N_\theta/N_z>1$ or when the motions were isolated. Hence it was concluded that sectional models can be tested satisfactorily with the vertical and pitching freedoms isolated, unless the frequencies for corresponding modes are approximately equal.

(d) *Influence of structural form of suspended platform*

Plate-girder-stiffened sections. These sections were considerably more unstable than truss-stiffened ones. The majority of them were obtained by covering the stiffening trusses of the sections shown in fig. 3 and thus the plate girders represented were rather deeper than is usual in practice. All the plate-girder sections showed instability in both vertical and pitching motions, generally at low wind speeds. For example, the critical values of V_r for the section derived by covering the trusses of A (fig. 3) were about 1·7 and 2·5 for vertical and pitching oscillations respectively (see Table I).

Truss-stiffened sections. No vertical oscillations were excited with any of the truss-stiffened sections. The pitching oscillations were influenced by the form and arrangement of the structural components of the bridge, and were especially sensitive to those of the roadway deck fittings. The results have been discussed in greater detail in the writer's previous paper.* Only those factors which were found to have a corrective influence on aerodynamic instability in pitching oscillations will be listed here:

> Stiffening truss chords of high width/depth ratio;
> Separation of traffic lanes by open slots or gratings;
> Truss-type deck stringers in preference to the plate-type;
> Castellated handrailing, or other types of handrailing designed to break up the
> continuity of the airflow pattern;
> Sidetracks (e.g. footpaths, cycle-tracks, etc.) mounted outboard of the stiffening
> truss.

By the inclusion of a number of these stabilising features in the design, a satisfactory degree of stability was achieved for each of the types of section shown in fig. 3. However, the stability still proved to be sensitive to other factors such as the relative levels of the various roadways and the positioning of the roadway stringers. These effects were only noted and not investigated systematically. In view of the many design features which may possibly influence the stability it is considered that model tests provide the only satisfactory basis for stability prediction.

The sections A to D shown in fig. 3 are lettered in the chronological order of the tests and illustrate successive steps in the evolution of a design with very good stability characteristics. Section A represented a mid-deck design with two roadways separated by an open central reservation. The presence of the gap between the

* *Loc. cit.*

roadways greatly improved the stability. Further improvement was obtained on dividing the two roadways into four, provided adjacent roadways were separated by an open reservation (section B). The two outer roadways, termed "sidetracks," each represented the combination of cycle-track and footpath; the inner ones represented carriage-ways. For economy in the construction of the piers and towers the width between trusses was reduced to 80 ft. (24·4 m.) in section C and the sidetracks were supported outside the trusses. This change also improved the stability. The top-deck section D was the last of this series of 1/100-scale models and it allowed horizontal cross bracing to be incorporated in two widely separated planes. Its stability characteristics were superior to those of the sections tested previously, and this fact, in conjunction with the considerable increase in torsional frequency due to the two planes of bracing, increased the estimated critical speed for torsional oscillations of the full-scale bridge to over 250 miles/hr. (400 km./hr.). Two models of section D were used, the second of which approximately represented the design finally adopted for the Severn Bridge.

6. Confirmatory tests of the stability of the proposed Severn Bridge

Since earlier tests had already shown sectional models to be adequate for the prediction of the stability of a complete bridge, the construction of a full model representing the preferred design for the Severn Bridge was considered to be unneces-
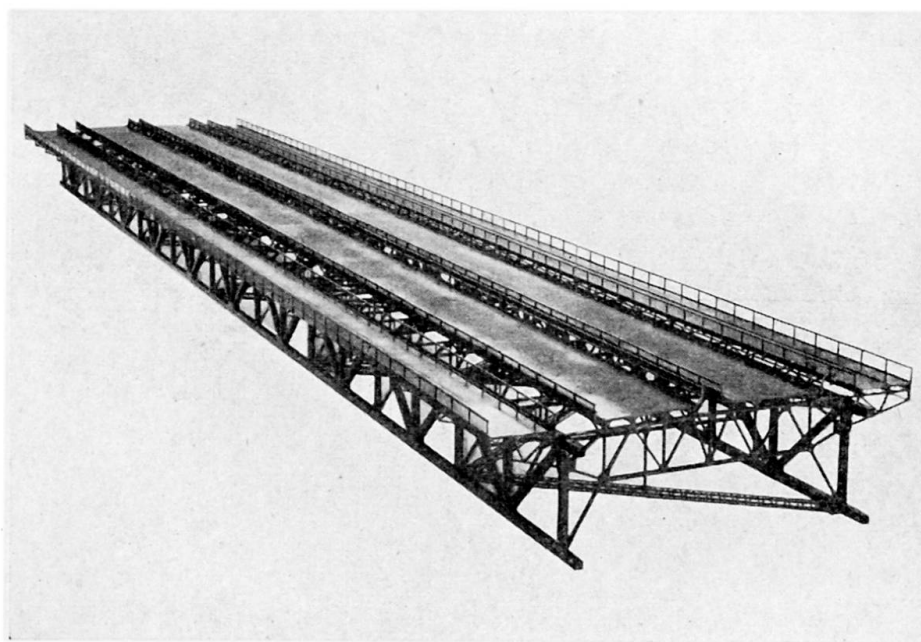

Fig. 5. Sectional model of design proposed for the Severn Bridge (1/32-scale)

sary. However, to provide a final confirmation of the stability, tests were carried out in the large wind-tunnel on a 1/32-scale sectional model. This increase of the linear scale* allowed a more accurate reproduction of fine structural detail.

The model (see fig. 5) represented 600 ft. (183 m.) of the suspended structure and considerable care was taken in its construction to reproduce all the important features

* Preliminary experiments had indicated that the oscillations of a still larger model might be affected by the proximity of the tunnel roof and floor.

of the full-scale design. It was mounted on steel-spring suspensions carried by frames which tilted to give the desired vertical incidence to the wind. The suspension arrangements permitted pitching combined with either vertical translation or lateral translation motions, as well as tests with each of these motions isolated. Tests with the lateral freedom included were necessary, since with the top-deck structure the
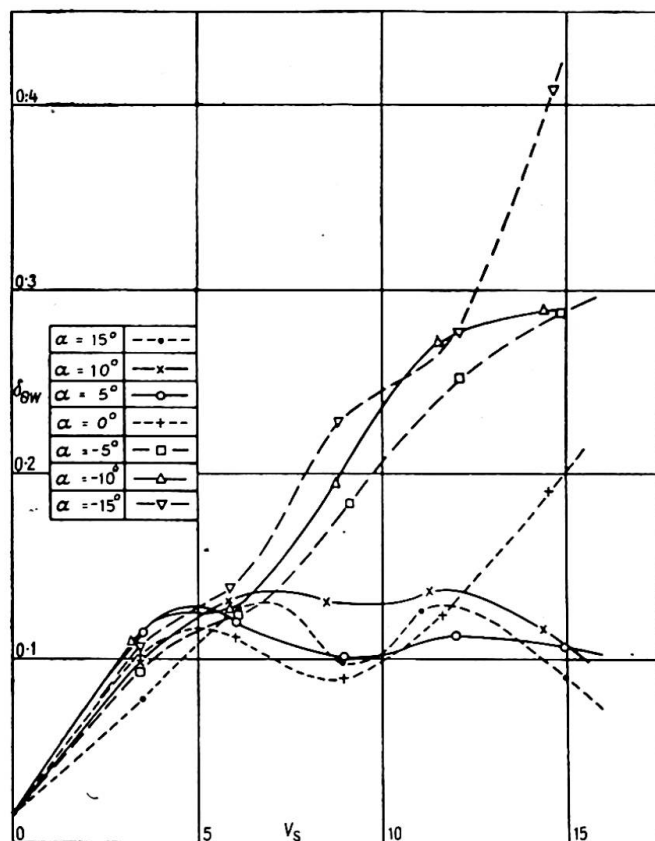

Fig. 6. Damping rates of pitching oscillations due to wind (1/32-scale model of proposed Severn Bridge)

lateral and pitching motions were inertially coupled due to the offset of the centre of mass above the elastic axis of the structure. In all three motions viscous oil dampers were provided to enable the amount of structural damping represented to be varied.

The inertias of the model conformed with the requirements for strict inertial scaling. Initially the frequencies were adjusted to values 3·2 times greater than those calculated for full-scale. This yielded a speed-scale of 1/10 and enabled the tests to be carried out up to wind speeds corresponding to approximately 140 miles/hr. (225 km./hr.). The minimum values of $\delta_{\theta s}$ and δ_{zs} were 0·01 and 0·06 respectively. No instability of any type occurred in these tests.

No further tests involving the lateral and vertical motions were made. With the more important pitching motion* N_θ was reduced to allow tests to be carried out up to wind speeds equivalent on full-scale to about 250 miles/hr. (400 km./hr.). The value of $\delta_{\theta s}$ for these tests was 0·02. No instability was observed for these conditions, but to provide further information the variation of the damping rate due to wind only ($\delta_{\theta w}$) with wind speed was measured. The curves of $\delta_{\theta w}$ against V_s for several wind incidences are reproduced in fig. 6. These show that for negative wind incidences

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* On models and on actual bridges with truss stiffening, instability has been recorded only in torsional (pitching) motions.

the damping rate became increasingly positive. For positive incidences the damping rate increased with the initial increases of V_s and thereafter maintained a substantial positive value for the whole speed range tested. These results verified that the design of suspended structure proposed for the Severn Bridge was satisfactory from the standpoint of aerodynamic stability.

In addition, the model was used to confirm some of the results found on the 1/100-scale models and also to obtain some information on the effect of the width/length ratio of sectional models. Some unstable configurations of the model (e.g. that obtained by covering the central reservation) were tested with model lengths representing 600 ft. (183 m.) and 300 ft. (91·5 m.). The results obtained with these two model lengths showed only small differences.

APPENDIX I

NOTE ON THE DAMPING PROPERTIES OF BRIDGES

The total damping rate (δ) of a structure in still air is made up of the contributions due to the structure only (δ_s) and that due to the surrounding air (δ_A). The application of model test results to prototype prediction requires a knowledge of the values of δ_s for both model and prototype. There is as yet no reliable method for calculating these values for a proposed bridge, and measurements of δ on actual bridges, which might be used for statistical estimates, have only been made on bridges of short span.† The values of δ found for short-span bridges varied from 0·05 to over 0·2. Model test values are, of course, readily obtained by decaying oscillation experiments.

The aerodynamic damping arises from the effects of viscosity and pressure. For oscillations of bridge sections, dimensional analysis yields:

$$\delta_{\theta A} = \frac{\rho B^4}{2I_\theta} f\left[\frac{\nu}{B^2 N_\theta}, \theta_0\right] \quad \cdots \cdots \cdots \quad (1)$$

where θ_0 is the amplitude and $\delta_{\theta A}$ denotes the values of δ_A for pitching oscillations.

The scanty experimental evidence available supports the assumption that the influence of the viscosity parameter is very small and that equation (1) can be written:

$$\delta_{\theta A} = \frac{\rho B^4}{2I_\theta}[a_0 + a_1\theta_0 + a_2\theta_0{}^2 + \ldots] \quad \cdots \cdots \quad (2)$$

where the coefficients a_o, a_1, etc., are approximately constant.

The equivalent expression for linear motions is:

$$\delta_{zA} = \frac{\rho B^2}{2I_z}\left[b_0 + b_1\left(\frac{z_0}{B}\right) + b_2\left(\frac{z_0}{B}\right)^2 + \ldots\right] \quad \cdots \cdots \quad (3)$$

Thus with the same inertial scaling as that required for the wind tests, the values of δ_A are the same for model as for full-scale.

Values of δ_A for complete bridges or their models will be influenced by the oscillation wave form and may be calculated if the sectional values and the wave form are known.

Some values of $\delta_{\theta A}$ were measured during the course of the Severn Bridge investigation. The results obtained for sections A and D shown in fig. 3 gave respectively the relations:

$$\delta_\theta = \rho B^4 / 2 I_\theta \, (0 \cdot 01 + 0 \cdot 29 \, \theta_0)$$

and

$$\delta_{\theta A} = \rho B^4 / 2 I_\theta \, (0 \cdot 05 + 3 \cdot 40 \, \theta_0)$$

APPENDIX II

THE LARGE WIND-TUNNEL

DESIGN AND CONSTRUCTION

The wind-tunnel (see fig. 7) was not required as a permanent structure and hence the main considerations governing the design were that of low cost of construction rather than that of high aerodynamic efficiency. For this reason the tunnel was of the non-return flow type and used only one fan. It was erected in a disused aircraft
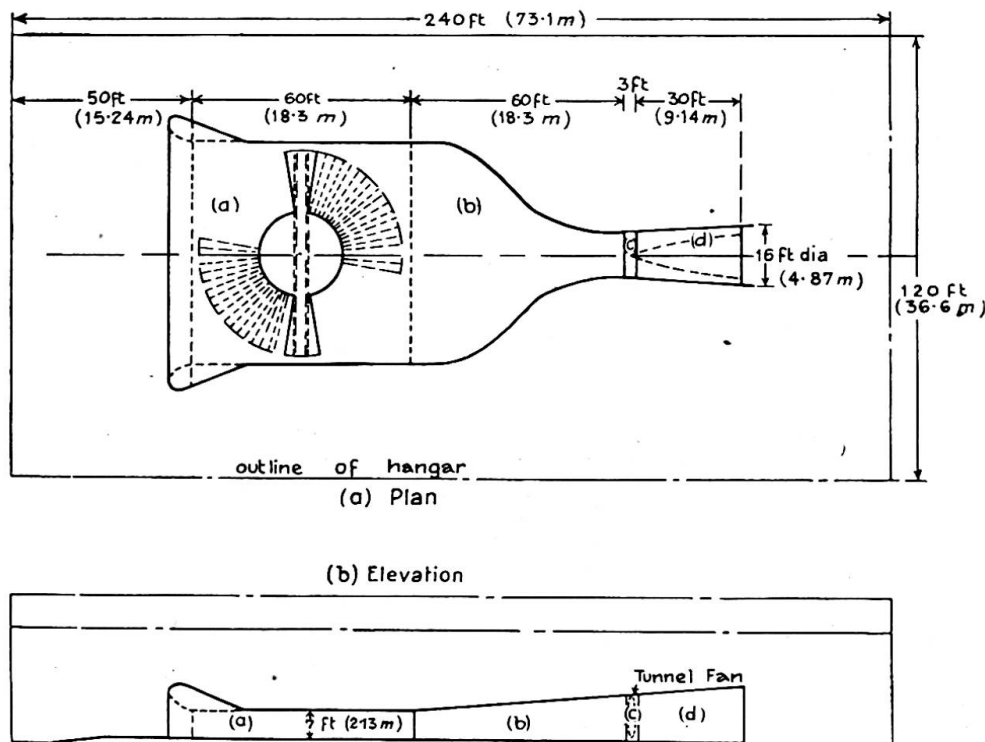

Fig. 7. The wind-tunnel

hangar and was raised from the hangar floor only by the few feet necessary to accommodate a well for the turntable. The fabric of the tunnel consisted mainly of a timber framework lined with wallboard, and the roof was suspended from Bailey bridge girders supported at their ends by vertical concrete pillars. The four main components of the tunnel were:

(i) The test chamber, (a) fig. 7

This had a floor area of 60 ft. by 60 ft. (18·3 m.) and was 7 ft. (2·13 m.) high. The air entered the test chamber through a conventional bell-mouth fairing and a wire-mesh screen for smoothing the airflow. In the centre of the chamber was a 55-ft. (16·75 m.) long turntable contained within a shallow quadrantal pit which allowed a rotation of 90 degrees. The flooring over the central part of the turntable was carried by the turntable itself and the outer annular area was filled in by wedges of 5-degree angles constructed of boarded trestles, which were lifted successively when the model had to be rotated relative to the wind direction.

(ii) The contraction chamber, (b) fig. 7

The contraction chamber was 60 ft. long (18·3 m.) and in this distance the cross-section developed smoothly from a 60-ft. rectangle to the 12-ft. (3·65 m.) diameter circle of the adjoining fan annulus. A wire-mesh screen at the front of the contraction chamber helped to ensure even airflow in the test chamber.

(iii) The fan and fan annulus, (c) fig. 7

The fan annulus, 12 ft. (3·65 m.) in diameter and 3 ft. (0·91 m.) in length, was of all-timber construction and stiffened to ensure that the small clearance between it and the two-bladed fan was maintained. The fan was driven by a concentric 130-h.p. motor with fine speed control.

(iv) The diffuser, (d) fig. 7

This was of circular section, 30 ft. (9·14 m.) in length, and expanded from the fan annulus to a diameter of 16 ft. (4·87 m.) at the discharge.

Performance

Tests of the aerodynamic characteristics were made initially on a 1/12-scale model of the tunnel and hangar.

In the actual wind-tunnel extensive measurements of the distribution of airflow within the test chamber showed that at all speeds up to the maximum of 22 ft./sec. (6·7 m./sec.) the velocity variations both along the length of the chamber and vertically were less than 3%. In the horizontal direction across the test chamber the variation of wind speed of nearly 10% was recorded at a distance of 5 ft. (1·52 m.) from the sides. A variation of this order was predicted by the model tunnel tests and was not considered important in view of the clearance of nearly 5 ft. between the full-model anchorages and the sides of the chamber.

Summary

The paper presents a general review of the experiments carried out in an investigation of the aerodynamic stability of suspension bridges undertaken by the National Physical Laboratory of the Department of Scientific and Industrial Research on behalf of the Ministry of Transport. The specific purpose of the investigation was to assist the designers of the proposed Severn Bridge, but much of the information gained is applicable to suspension bridges generally.

Wind-tunnel tests using both sectional and full models are described and the limitations of these two experimental techniques are discussed. The reliability of the use of data obtained from sectional model tests alone for the prediction of the

behaviour of full-scale bridges is verified by comparisons of the results obtained by both methods. A comparison is made of the stability of various bridge sections and the design features favourable to the promotion of stability are indicated. The sectional models used for these tests illustrate the evolution of the design of the suspended platform for the proposed Severn Bridge and the results show that a satisfactory degree of stability can be attained by attention to the structural shape and arrangement of the details of the suspended platform.

Résumé

Cette communication constitue un bref exposé de recherches relatives à la stabilité aérodynamique des ponts suspendus. Ces recherches ont été entreprises par le National Physical Laboratory du Department of Scientific and Industrial Research, à l'instigation du Ministère des Transports. Le but spécifique était de fournir des informations aux dessinateurs chargés de l'établissement du projet de pont sur la Severn. Toutefois, un grand nombre de renseignements ainsi obtenus s'appliquent également aux ponts suspendus en général.

L'auteur expose également les essais qui ont été effectués en soufflerie, tant sur modèles complets que sur modèles partiels; il étudie les avantages et les inconvénients de chacune des deux méthodes. La valeur des résultats obtenus exclusivement sur modèles partiels, du point de vue de la prévision du comportement des ponts réels, a été confirmée par la comparaison entre les deux méthodes.

Différents profils de ponts font l'objet de comparaisons du point de vue de la stabilité et l'auteur indique les dispositions qui permettent d'accroître la stabilité.

Les modèles partiels qui ont été utilisés pour ces essais mettent en évidence le développement de la conception du tablier; les résultats montrent que l'étude minutieuse de la forme et des caractéristiques de détail du tablier permet d'obtenir une stabilité suffisante.

Zusammenfassung

Die vorstehende Arbeit gibt einen kurzen Ueberblick über Versuche, die für das Ministry of Transport im National Physical Laboratory des Department of Scientific and Industrial Research im Zusammenhang mit einer Untersuchung der aerodynamischen Stabilität von Hängebrücken ausgeführt wurden.

Die Untersuchung wurde ursprünglich für den Konstrukteur der geplanten Severn-Brücke ausgeführt, aber die Ergebnisse erscheinen von allgemeinem Interesse für die Konstruktion von Hängebrücken.

Windkanalversuche an Teilmodellen sowohl als vollständigen Modellen werden beschrieben, und die Vor- und Nachteile der beiden Methoden besprochen. Die Zuverlässigkeit von ausschliesslich an Teilmodellen erhaltenen Ergebnissen für die Voraussage des Verhaltens von Brücken in natürlicher Grösse wurde bestätigt durch den Vergleich von mit den beiden Methoden erhaltenen Ergebnissen.

Die Stabilität verschiedener Brückenprofile wird verglichen, und Konstruktionen werden vorgeschlagen, die die Stabilität erhöhen.

Die in den Versuchen benutzten Teilmodelle zeigen die Entwicklung der Konstruktion der Fahrbahnplatte der geplanten Brücke; und die Ergebnisse zeigen, dass durch geeignete Form und sorgfältig ausgearbeitete Einzelheiten der Fahrbahnplatte ausreichende Stabilität erzielt werden kann.

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