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General review of the present status of the experimental method of structural design

Aperçu de l'état actuel de la méthode expérimentale de calcul des ouvrages

Ueberblick über den heutigen Entwicklungsstand der experimentellen Verfahren zur Bemessung der Tragwerke

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EXPERIMENTAL AND ANALYTICAL METHODS OF DESIGN

When considering the experimental method of structural design, the problem arises of knowing its position in relation to the analytical methods of the Theory of Elasticity and Strength of Materials. These methods sum up the knowledge on the behaviour of solid bodies subject to loadings which could be interpreted and expressed quantitatively, that is, dealt with theoretically.

The analytical methods of design, like all physical theories, have the great advantage of providing knowledge of all the phenomena in a given domain. A theory fills the gaps existing in the knowledge of the isolated cases which led to its creation; it even permits the observed phenomena to be surpassed to an extent which reveals the audacity of the theory.

Thus, the bending theory of Strength of Materials, which has been of so great a service to mankind, both in relation to safety of structures as well as to economy of materials, has allowed the prediction of the behaviour of a very large number of structural members which had never been observed before, as regards either materials, shape, dimensions or loading.

In contrast with the analytical methods, the experimental methods provide knowledge about isolated cases, since each structure to be studied requires the construction and observation of a model. This does not strictly hold, since there is always, at least qualitatively, an application of theory to the phenomena which permits the behaviour of structures not very different from others previously studied to be foreseen.

With regard to analytical methods, the question which arises is as follows: do they permit, in their present status, the behaviour of structures to be foreseen with the accuracy demanded in practical engineering?

The analytical methods give results applicable to solids of given shapes and submitted to certain loadings. Besides this, except in a very few cases, they are established on the assumption that the materials are homogeneous, isotropic, and obey Hooke's law. Since they are theories, they are open to the possibility of being applied beyond the field for which they were established, which will result in a loss of accuracy, and extremely unreliable results may even be obtained.

Thus, with regard to the shape of structures, which are of an infinite variety, the designer constantly applies theories to solids of shapes very different from those for which they were established. Besides this, he divides the structure in parts whose reciprocal reactions he at times ignores and at other times fixes arbitrarily, considering them as hinged, built in, etc.

With regard to loadings, it is also very often necessary to make considerable simplifications so as to convert the real loadings into others the effect of which can be calculated.

It was mentioned that the analytical methods are in general developed in the hypothesis that the materials obey Hooke's law. In the concept of safety which is generally followed today, by which, for given loadings, known as working loadings, the stresses developed should not exceed the safety stresses, this hypothesis has not, as a general rule, an important effect on the results of the calculations compared with that derived from the simplification of the shapes and of the loadings. This is because with common building materials the curvature of the stress–strain diagrams, up to values of stresses generally adopted as safe stresses, is small. Also up to these stresses the creep of the materials does not often influence the stress distribution to a degree which need be taken into consideration.

However, either in the application of the probabilistic concept of safety, at present awakening great interest, [1, 2] or of the concept of safety in relation to failure, which is already frequently applied, the hypothesis of the materials following Hooke's law takes all the value from nearly all the existing analytical methods of design.

In fact, within the probabilistic concept it is necessary to predict the behaviour of structures for all possible intensities of loading, even for those which are not very probable, for which the structures may suffer deformations which go far beyond the elastic range or even suffer failures. The dimensions to be chosen for a structure are those which minimise the sum of the initial cost of the structure and the cost of maintenance; in the latter there should be included the repair expenses due to the action of loadings of great magnitude, and also the expenses due to any damage, such as excessive deformations, personal accidents, etc.

For the application of the concept of safety with regard to failure it is only necessary to determine the magnitude of the loadings which cause failure.

It can safely be said that the possibilities of the analytical methods are very limited in relation to the behaviour of structures for great deformations. This results from the great analytical difficulties which arise when non-linear relations between strain and stress have to be considered; the situation is made worse by the need to consider simultaneously the dependence of the phenomena on time.

It was just the difficulty of establishing non-linear theories associated with the fact that the structures suffer, in general, deformations too great for their use when the elastic range is well passed, which led to the deficient concept of safety based on the consideration of working loads.

[1] For references see end of paper.

As the designer, up to a few decades ago, besides the knowledge of the behaviour of similar structures and his intuition, only had at his disposition analytical methods, he had to establish the necessary hypotheses, however extraordinary they may have been, so that the problems he had to solve fell within the theories at his disposal, having at times to choose, not the most convenient solutions, but those which could be handled by those methods.

This situation, with the difficulty of comparing the predictions of the analytical methods, especially with regard to the values of strains and stresses, and the real behaviour of the structures, has led to an excessive confidence in the precision of those methods, and even to a certain conventionalism in their application.

The progressive improvement of the techniques for measuring strains and stresses and the appearance of new materials suitable for building models have led to a great development of the experimental method of structural design, especially in the last decade.

When the analytical methods are not satisfactory, it is in general possible to predict with the necessary accuracy and within reasonable time and expense the behaviour of structures by the use of models.[3]

In the following paragraphs the similarity conditions which the models should satisfy are presented briefly.

MECHANICAL SIMILARITY

(a) *Models made from the same materials as the prototype*

Let us consider a prototype (fig. 1) made from any materials, homogeneous or heterogeneous, isotropic or non-isotropic, which, for the loadings applied, do not obey Hooke's law. Suppose that the prototype is in static equilibrium under the action of surface forces F'_p, F''_p, . . . (generally represented by F_p), and of the reactions of supports, fixed or movable, R'_p, R''_p, . . . (generally represented by R_p).

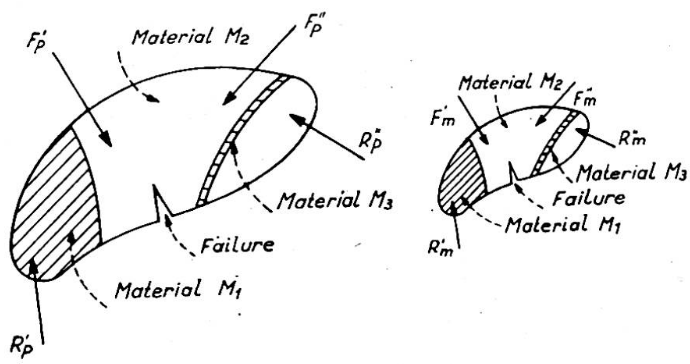

Fig. 1

Let us build a model geometrically similar to the scale of $1/\lambda$, made from the same materials as the prototype, bound in the same way, and supported by homologous supports of the same type. Subject it to homologous forces, F_m, to a scale of $1/\lambda^2$, $F_m = F_p/\lambda^2$, so that the surface stresses, f_m, equal the homologous stresses of the prototype, f_p, $f_m = f_p$.

It can be shown that the displacements of homologous points of the prototype and of the model, δ_p and δ_m, the strains of homologous segments, ϵ_p and ϵ_m, and the stresses in homologous elemental surfaces, t_p and t_m, are related by

$$\left.\begin{aligned} \delta_m &= \frac{1}{\lambda}\delta_p \\ \epsilon_m &= \epsilon_p \\ t_m &= t_p \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

whatever may be the deformation, even if failures take place, either for stable or unstable equilibriums. The reactions of the supports of the model are given by $R_m = R_p/\lambda^2$, that is, the homologous reaction stresses, r_p and r_m, are equal, $r_m = r_p$.

It has been said that the supports would have to be of the same type; that is, for fixed supports, either hinged or built in, there would have to correspond fixed supports of the same type, and for supports which suffer displacement there would have to correspond supports such that their displacements, under the loading $R_m = R_p/\lambda^2$, or $r_m = r_p$, would be $1/\lambda$ of the displacements suffered by the supports of the prototype when submitted to the action of R_p or r_p.

It is obvious that the similarity condition presented demands that the initial states of strain and stress of the model be the same as in the prototype.

As for body forces, such as the weight, similarity does not exist unless steps be taken to convert the homologous body forces to a scale of $1/\lambda^2$, that is, in the case of the weight, the equivalent of multiplying the specific weights of the materials of the model by λ. For this purpose appropriate forces may be applied to the model or it may be subject to a rotation which produces convenient centrifugal forces.

Also in the case of dynamical equilibriums there is not similarity even when the surface forces only are considered.

As to the effects of loads which tend to produce change in volume, such as temperature or contraction in the case of concrete, the relations (1) hold as long as the unit volume change is the same, which implies, in the case of temperature, subjecting the model to variations of temperature equal to those suffered by the homologous points of the prototype.

The similarity conditions presented so far demand that it be assumed that if the elements of volume of a model are subject to the same state of stress (in general varying with time) as the homologous elements of the prototype, the state of strain will also be the same even for strains in the neighbourhood of failure. The state of stress of the model being the same as that of the prototype, two homologous points are immersed in media whose states of stress are analogous but where the stress gradient, in any direction, is λ times greater in the model.

Hence the conclusions presented were derived on the assumption that the relation between strain and stress of an element of volume does not depend on the stress gradient which exists around this element. In the case of solids in elastic deformation, the Theory of Elasticity even admits the hypothesis, which has been amply verified, that the relation between strain and stress of an element of volume does not depend on the state of stress around the element.

However, it is conceivable when leaving the elastic range, especially when dealing with ductile materials, that that relation depends on the state of stress which exists around the element of volume, and that it may vary even when only the gradient of the state of stress varies.

The experimental verification of the influence of stress gradient has frequently led to results which do not agree. In the case of steel, which has been the material most studied, the results which show the existence of this influence are more numerous.[4, 5]

It should be noted, however, that the influence of the stress gradient on the similarity relations will only be important in the case of very large strains and of scale values under certain limits, the equilibrium studied and the degree of accuracy required for the model study having to be taken into account.

Another objection to the conclusions presented results from the consideration of the influence of the volume on the probability of failure,[6] which has been observed in brittle materials [7] and in the brittle rupture of ductile materials [8] subject to tensile stresses. The mean tensile strength varies with the volume of the piece, a reduction occurring when volume increases.

Hence when wishing to study models in which failures are produced by tension it may be necessary to take this effect into consideration, especially as there exists the possibility of the results not being on the side of safety. But, as in the majority of cases the structures built from brittle materials are designed in such a way that tensile failures do not expose them to risk, the objection which has just been raised is not of great significance.

In any case, to verify if there does exist any influence due to the scale and what the influence would be, observations can be made on models of different scales and comparison of the results made by means of the expressions (1).

Except for special cases, we think that those influences of the stress gradient and volume do not limit the conclusions arrived at with regard to similarity to the point of having practical interest.

(b) *Models made from materials different from those of the prototype* [3]

It often happens, as will be seen later, that it is not possible or even convenient to make the models from the same materials as the prototype.

Consider the general case of the prototype of fig. 1 built from any materials. Let ϵ_p be the extensions undergone by an elemental parallelepiped of any of the materials when subject at its surface to the stresses t_p in equilibrium.

In a geometrically similar model, in order to observe displacements, strains and stresses proportional to the homologous ones of the prototype, it is necessary, in the first place, that the materials of the model be such that when an elemental parallelepiped is subject to stresses $t_m = t_p/\alpha$, the strains developed be $\epsilon_m = \epsilon_p/\beta$, α and β being constants. When the creep of the materials has to be taken into consideration, if the stresses t_p be reached at the time θ_p, the stresses t_m will have to be reached at the time $\theta_m = \theta_p/\tau$, τ being a constant. Therefore, for the materials of the model there will have to be scales for stresses $1/\alpha$, for strains $1/\beta$, and for time $1/\tau$.

The condition which we have just stated implies that, for any of the materials of the model, the uni-axial loading σ (tension, compression) curve as a function of the strain ϵ (fig. 2) be obtained from the curve of the homologous material of the prototype by multiplying the ordinates and abscissae, respectively, by $1/\alpha$ and $1/\beta$, that is to say, by a change of scales of the axes. When it is necessary to take the creep of the materials into consideration this relation between the diagrams has to be verified whichever way the stresses applied to the prototype material change with time; as was seen, the stresses of the model material can be applied according to a certain scale of time.

The above-mentioned relation between the uni-axial loading diagrams is not sufficient to verify the general condition stated before, which refers to any loading. However, it is sufficient that in the majority of cases this relation holds to allow us to assume, with sufficient accuracy, that the materials of the model satisfy the general condition. Besides this, it should be noted that in the case where it is not demanded that the relation holds up to failure, it is sufficient that the development of the curves be similar to be able to determine the factors $1/\alpha$ and $1/\beta$ with reasonable accuracy.

If it is desired to foresee the behaviour of the prototype even after failures have appeared, the materials of the model should satisfy the condition stated, even for

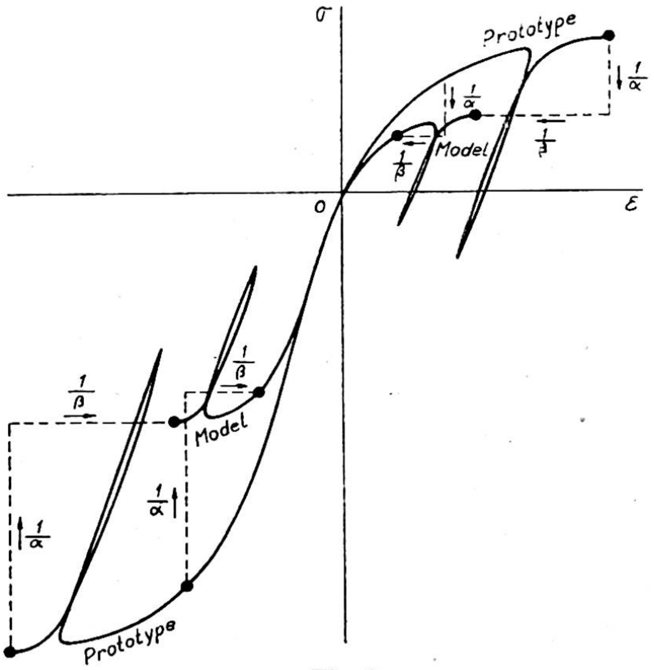

Fig. 2

stresses which bring about failure of the parallelepiped. It will be necessary therefore that in the curves of fig. 2 the ultimate strengths of homologous materials be in the relation of $1/\alpha$ and that they correspond to strains in the relation of $1/\beta$.

Conditions have so far been considered which should be satisfied by the materials of the model. In the case of the prototype being submitted to surface forces F_p in static equilibrium, if homologous forces $F_m = F_p/\lambda^2\alpha$, that is, stresses $f_m = f_p/\alpha$, be applied to the model of scale $1/\lambda$ at homologous times, the relations

$$\left.\begin{aligned} \delta_m &= \frac{1}{\lambda\beta}\delta_p \\ \epsilon_m &= \frac{1}{\beta}\epsilon_p \\ t_m &= \frac{1}{\alpha}t_p \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (2)$$

are verified in homologous times provided that the displacements are small.

The model has to be supported on homologous supports of the same type. To the supports of the prototype with displacement there will have to correspond supports such that under the action of forces $R_m = R_p/\lambda^2\alpha$ or $r_m = r_p/\alpha$, they will undergo displacements $1/\lambda\beta$ of those undergone by the homologous supports of the prototype when subject to R_p or r_p.

In the case of $1/\beta = 1$, that is, of a parallelepiped of any of the materials of the model having the same strains as a parallelepiped of the homologous materials of the prototype for the loading $t_m = t_p/\alpha$, the relations (2) hold even for large displacements. It is possible then to study by models equilibriums in which phenomena of instability appear, the scale of $1/\lambda^2\alpha$ being that of critical homologous loads.

In the general case of the prototype in dynamical equilibrium under the action of surface forces and body forces, especially the weight, in order that the relations (2) should hold for homologous times, it is necessary that, besides the surface forces satisfying the relation $F_m = F_p/\lambda^2\alpha$, or $f_m = f_p/\alpha$, the following relations should hold:

$$\frac{1}{\rho} = \frac{\lambda}{\alpha} \quad \ldots \ldots \ldots \ldots \quad (3)$$

$$\frac{1}{\tau} = \sqrt{\frac{1}{\lambda\beta}} \quad \ldots \ldots \ldots \ldots \quad (4)$$

where $1/\rho = d_m/d_p$, d_m and d_p being the specific weights of the materials of the model and prototype respectively. In general it is not possible to satisfy all these conditions.

When dealing with vibrations, the scale of the homologous periods of vibration is given by (4).

In the particular case where the effects of weight are negligible, it will only be necessary to verify relation (4). If, besides this, it is not necessary to take the effects of creep into consideration, then the material does not demand a time scale, from which it results that for each value of the scale of the model there is one value of the time scale.

In the case of static equilibriums in which the effect of weight has to be considered, it will only be necessary to verify the condition (3). For the current values of the scales this condition demands that the model materials have high specific weight and high deformability.

With regard to the effects of the temperature and other loads which tend to produce changes in volume, the relations (2) hold provided that the model is subject to temperature changes Δ_m given by

$$\Delta_m = \frac{\chi}{\beta}\Delta_p$$

where Δ_p is the temperature change at the homologous point of the prototype and $1/\chi$ is the scale of the coefficients of thermal expansion.

All the conclusions presented are obviously subject to the same objections presented in section (a).

(c) *Prototype under elastic deformation*

Consider a prototype made up of various elastic materials with moduli of elasticity E'_p, E''_p, . . ., and Poisson's ratios ν'_p, ν''_p, . . . From the results presented in (b) it is concluded that for similarity to exist it is necessary that a geometrically similar model be made of elastic materials whose homologous constants E'_m, E''_m, . . ., and ν'_m, ν''_m, . . ., satisfy the relations

$$\frac{E'_m}{E'_p} = \frac{E''_m}{E''_p} = \ldots = \frac{1}{\mu}$$

and

$$\nu'_m = \nu'_p, \quad \nu''_m = \nu''_p, \ \ldots$$

where $1/\mu$ is the scale of the moduli of elasticity. Since the influence of Poisson's ratio on the states of stress and strain is often negligible, the conditions of equality for these ratios may often be ignored. In this case if the prototype be made of only one material it is sufficient that the model material be elastic.

When the prototype is only submitted to the action of surface forces, the scale of these forces $1/\phi$, may be given any value, provided that the elastic limit is not surpassed, the relations (2) taking the form

$$\left.\begin{aligned} \delta_m &= \mu\frac{\lambda}{\phi}\delta_p \\ \epsilon_m &= \mu\frac{\lambda^2}{\phi}\epsilon_p \\ t_m &= \frac{\lambda^2}{\phi}t_p \end{aligned}\right\} \quad (5)$$

on the condition that the displacements be small and the model be supported in a way analogous to that of the prototype. To the supports with displacement have to correspond supports which undergo displacements to the scale $\mu\lambda/\phi$, when subject to reactions $R_m=R_p/\phi$ or $r_m=\lambda^2 r_p/\phi$.

When it is assumed that the materials of the prototype and of the model follow Hooke's law up to failure, in order to be able to study the effects of loads which produce failures, it is necessary that the ultimate stresses, σ_m and σ_p, satisfy the conditions $(\sigma_m/\sigma_p)_{tension}=(\sigma_m/\sigma_p)_{compression}=\lambda^2/\phi$ which fix the value of the scale of forces. In the case of studies in which failures occur, since the superposition of the effects of loads does not hold, it is generally necessary to apply all the loads simultaneously.

In the case of large displacements, the conclusions arrived at in this section hold as long as the scale of forces be

$$F_m=\frac{1}{\lambda^2\mu}F_p \text{ or } f_m=\frac{1}{\mu}f_p$$

and the relations (5) will take the form

$$\begin{aligned} \delta_m &= \frac{1}{\lambda}\delta_p \\ \epsilon_m &= \epsilon_p \\ t_m &= \frac{1}{\mu}t_p \end{aligned}$$

We thus see that the model has to remain geometrically similar to the prototype after deformation.

Phenomena of instability can then be studied on models, the critical homologous loads being to the scale of $1/\lambda^2\mu$.

In the more general case of the prototype being in dynamic equilibrium under the action of surface and body forces, especially the weight, it is necessary that the materials of the model satisfy the conditions stated and also that

$$\left.\begin{aligned} \frac{1}{\phi} &= \frac{1}{\lambda^3\rho} \\ \frac{1}{\tau} &= \frac{1}{\lambda}\sqrt{\frac{\mu}{\rho}} \end{aligned}\right\} \quad (6)$$

That is, once the scale $1/\lambda$ and the materials of the prototype and the model have been defined, the values of the force and time scales are fixed; the homologous forces have then to be applied at times to the scale of $1/\tau$. The relations (5) will hold when the model, supported in a manner similar to the prototype, starts from a position in which the displacements are to the scale of $\mu\lambda/\phi$ and the velocities to the scale of $\mu\lambda\tau/\phi$.

When dealing with vibrations, $1/\tau$ is the scale of the homologous periods of vibration.

In the particular case when it is not necessary to consider the weight, only the time scale will be fixed. Once the force scale has been fixed, the first of the relations (5) fixes the scale of displacements, and hence also the initial position of the model, from which the velocities, to the scale already referred to, have to be applied.

In the case of static equilibriums where the effect of surface forces and weight have to be taken into consideration simultaneously, the first relation of (6) will have to be verified.

For the common scales of the models it is often difficult to study the effect of weight due to the low value of the strains. Hence at times recourse is taken to the methods already mentioned, equivalent to increasing the specific weight.

(d) *Elastic equilibrium in two dimensions and equilibrium of structures consisting of bars*

The Theory of Elasticity shows that in a homogeneous plate in two-dimensional elastic equilibrium, the state of stress does not depend on Poisson's ratio, unless the plate has holes, and that in the boundary of each hole or in the outer boundary of the plate, forces act whose resultant is not equivalent to zero or to a couple.

Hence the conditions referred to in the section (c) for the model material are simplified in the present case; for the determination of the state of stress it is sufficient that the material of the model be elastic.

When the materials of the prototype and the model have different values for Poisson's ratio, the homologous strains and displacements are not proportional. Therefore, when there are statically indeterminate supports, even the proportionality of the stresses ceases to hold.

For the same reason if the plate be made of different elastic materials there will only be similarity when the homologous Poisson's ratios are equal.

If a plate is subject to body forces acting in its plane, the state of stress is still in general independent of Poisson's ratio when the body forces are of constant intensity, which condition is satisfied by the weight.

In two-dimensional equilibriums it is easy, in view of the small thickness of the plate, to apply to the model complementary forces equivalent to the increase of specific weight.

By the use of Biot's analogy it is possible to determine the effect of weight and in general the effect of body forces, substituting these forces for forces acting in the boundary of the plate.[9]

If the plate is subject to variations of temperature or other causes of change in volume, as it is necessary to introduce conditions relative to the strains, in order to have similarity it is necessary that $\nu_m = \nu_p$.

It should be noticed that in the cases mentioned in which the state of stress depends on Poisson's ratio, the influence of this ratio is generally small and in the majority of cases may be ignored.

In solids subject to plane strain the determination of stresses can be easily made from a plate in two-dimensional equilibrium, which frequently has a considerable practical interest.

Finally, consider the case of structures consisting of straight or curved bars existing, or not, in a plane.

Within the simplifying hypotheses of the Strength of Materials it is generally possible to analyse these structures on models in which the cross-sections of the bars are not geometrically similar to those of the prototype.[3] This possibility has great practical interest, as it permits the substitution of the shapes of these sections, often very complex, for others easier to reproduce in the models.

When models whose sections are not geometrically similar are used, proportionality can only hold between homologous shearing forces, normal forces, and bending moments.

In the particular case of plane structures consisting of bars in static equilibrium under the action of forces acting in their plane, for such a proportionality to exist it is in general sufficient that, along all the bars, the moments of inertia I_p and I_m of the homologous cross-sections of the prototype and of the model be proportional, $I_m/I_p = 1/C$.

This permits the construction of models with rectangular sections of constant thickness, which greatly simplifies the construction of models.

The forces may be applied at any scale, $1/\phi$, which will be the scale of the shearing and normal forces developed; $1/\lambda\phi$ will be the scale of the moments, denoting now by $1/\lambda$ the scale of the axes of the bars.

In the dynamic equilibriums under the actions of surface forces and of the weight it is in general sufficient that, besides the mentioned proportionality between the moments of inertia, the areas S_p and S_m of the homologous cross-sections be proportional, $S_m/S_p = 1/C_1$, the constant C_1 being of any value. The scale of the applied forces and of the time must have the values :

$$\frac{1}{\phi} = \frac{1}{\lambda\rho C_1}$$

$$\frac{1}{\tau} = \sqrt{\frac{C\mu}{\lambda^3\phi}}$$

The model should start from a position in which the displacements are to the scale $C\mu/\lambda^3\phi$ and the velocities of the scale $\tau C\mu/\lambda^3\phi$. When the weight can be neglected the forces scale can assume any value.

Construction of the models

Mechanical similarity, as has just been seen, requires certain conditions in the models with regard to shape, materials and loadings. Let us see what are the possibilities to fulfil these conditions.

(a) *Scales*

Except in special cases similarity demands that the models be geometrically similar to the prototype, but without fixing the scale value.

A scale near unity has the advantage of permitting the reproduction in the model of the characteristics of the prototype, such as shapes, joints between parts, residual stresses, etc.

However, in the case of large structures, which are the most common in civil engineering, such a scale cannot generally be adopted, both for economic reasons and the time needed for the construction of the models. Furthermore the application of loads in large models demands very expensive equipment, and the observations, besides taking a lot of time, are more difficult and less accurate, especially if they have to be made in the open air.

The reduction of the scale is accompanied in general by economy, rapidity and ease of model studies. In the majority of cases these factors vary greatly with the change in scale.

On the other hand, the smaller the scale the greater is the difficulty of reproducing the shapes. As a rule, however, it is possible to simplify the shapes considerably, either by omitting some details or by replacing parts for others of a convenient deformability, without prejudicing the precision of the results. In the case of structures of large dimensions with simple shapes, at times scales of about 1/500 are adopted.

In fixing the minimum possible scale it is necessary to bear in mind:

the smallest parts to be reproduced in the model, which should not be so small as to make their construction and observation difficult;

the accuracy with which it is possible to set up the equipment for applying forces and other loading;

the accuracy, dimensions and way of placing the measuring apparatus, especially the magnitude of the bases of the extensometers in view of the gradients of strains which are anticipated.

(b) *Materials*

The materials chosen for construction of models should, in a general way, obey the following conditions:

have the mechanical properties demanded by similarity which should not be appreciably affected by the common ambient variations of temperature and humidity;

be easily worked and joined;

have such deformability that, under the action of easily obtainable loading intensities, the accuracy demanded for the measurement of displacements and strains be reached;

allow the measuring apparatus to be easily mounted either on the surface as inside;

be economical.

When it is wished to study by a model the behaviour of a prototype in which complex mechanical properties have to be taken into consideration, such as non-linear relations between stresses and strains, non-reversible strains and creep, it should be seen in the first place if it is possible to build the model with the same materials as the prototype.

This is at times difficult even for scales that are not very small. Thus in the case of models of structures of reinforced concrete the difficulty often arises of the aggregate being too large; when using the same concrete for the model it may also be necessary to take into consideration the variation of wall effect and rate of drying. In metallic and in reinforced-concrete models, it is difficult to find on the market sections, plates and bars with the necessary dimensions and with the same properties as those of the steels used in the construction. For this reason it is necessary at times to make the sections specially from plates laminated to the appropriate thickness (fig. 3). In reinforced-concrete models it is, in general, possible to substitute a single bar for groups of bars and thus use commercial sizes.

The plates and bars of small dimensions which exist on the market are often annealed, but it is as a general rule possible to give them properties analogous to those of the steels of construction by stretching them.

It is, however, possible to use materials in the models different from those of the prototype. Thus for concrete structures it is easy to find mortars satisfying the

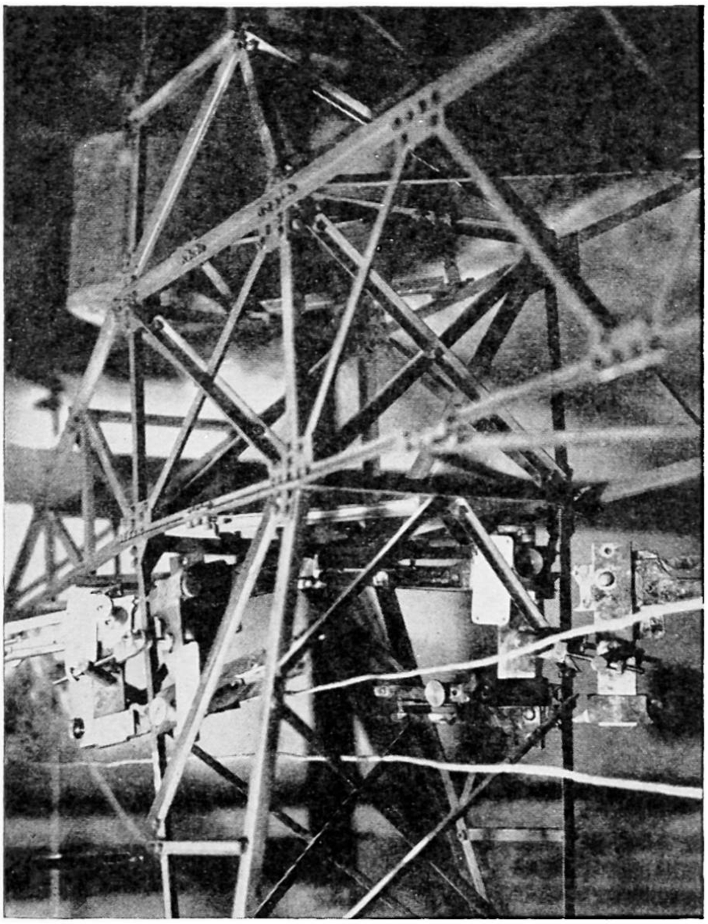

Fig. 3. Part of a steel model to a scale of 1/6 of a high-voltage steel mast 33 m. high. Huggenberger extensometers were used for strain measurements

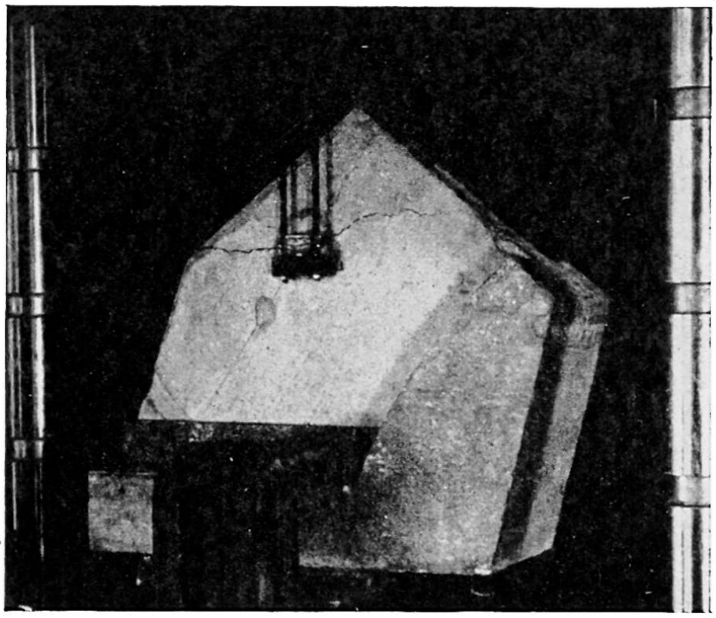

Fig. 4. Reinforced-mortar model to a scale of 1/50 of a guide wall of a spillway dam. Used for studying up to failure the forces exerted by the gates

conditions stated (fig. 4); it is advisable that $1/\alpha$ be small and $1/\beta$ be great so as to obtain, for small magnitudes of loading, deformations measurable with accuracy. In the case of reinforced-concrete structures the steel should be replaced by a material for which $1/\alpha$ and $1/\beta$ have the same values as for the mortar.

When choosing a material for a model different from that of the prototype, it is sufficient, in general, to verify if the similarity condition stated is satisfied in uni-axial loading. Tests may also be carried out on pieces geometrically similar, made from the materials of the prototype and the model, which are submitted to homologous loadings to scale in order to determine if the relations (2) are satisfied. It is convenient, as is obvious, that the shapes of the pieces and the loadings be chosen to obtain equilibriums analogous to those to be studied.

When the prototype is in elastic equilibrium there are many materials available for the construction of models, among which can be mentioned celluloid, plastics, plaster of Paris, metals and cork agglomerates.

In the choice of the material for a given case consideration should be given in the first place to facility of construction. In the case of complex and curved shapes it is convenient as a rule to make use of mouldable materials, such as plaster of Paris or some plastics.

In the second place attention should be paid to the advantage of the material having a high proportional limit and a low modulus of elasticity, to measure strains accurately when applying small forces. The materials with these properties have, in the majority of cases, an appreciable creep; however, in general, it can be assumed,

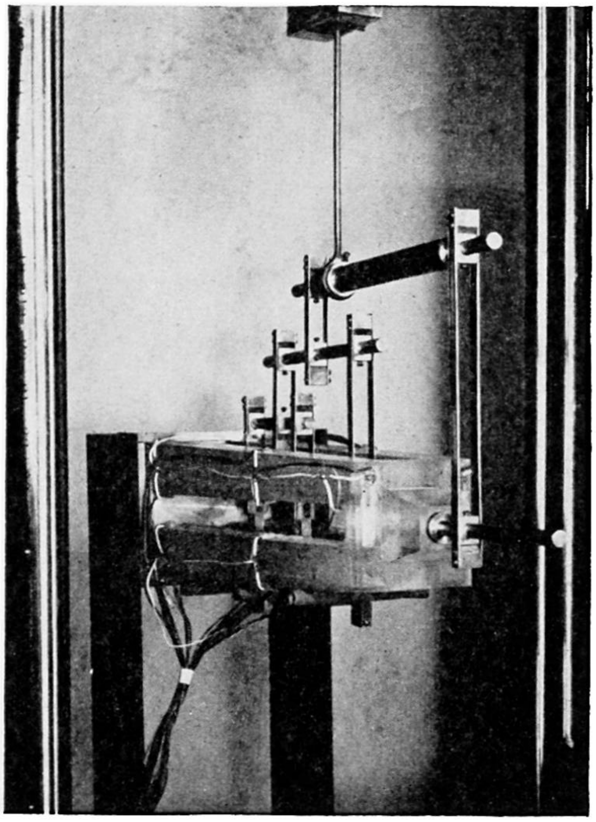

Fig. 5. Perspex model (laid horizontally), to the scale of 1/200, of a monument about 100 m. high to be built in concrete. Electric strain gauges were used for both static and dynamic strain measurements

without affecting the accuracy, that the materials referred to above have a modulus of elasticity which is a function of time when under the action of constant load; thus the relations of similarity established for elastic materials will hold.

Of the materials mentioned the most used at present are plastics which together with celluloid have the advantage of a high proportional limit, generally above 1 %, both in tension and compression. They are, also, easily workable.

Celluloid and the majority of plastics in use today, such as those known by the trade names of perspex, plexiglas and lucite, which consist of polymethyl methacrylate, and those known as bakelite, marblette and trolon, which are phenolformaldehydes, have moduli of elasticity ranging from 15,000 to 45,000 kg./cm.^{2} Poisson's ratio varies between 0·30 and 0·40.

Celluloid and the three plastics first mentioned have the great advantage over the other plastics of being easily glued (fig. 5).

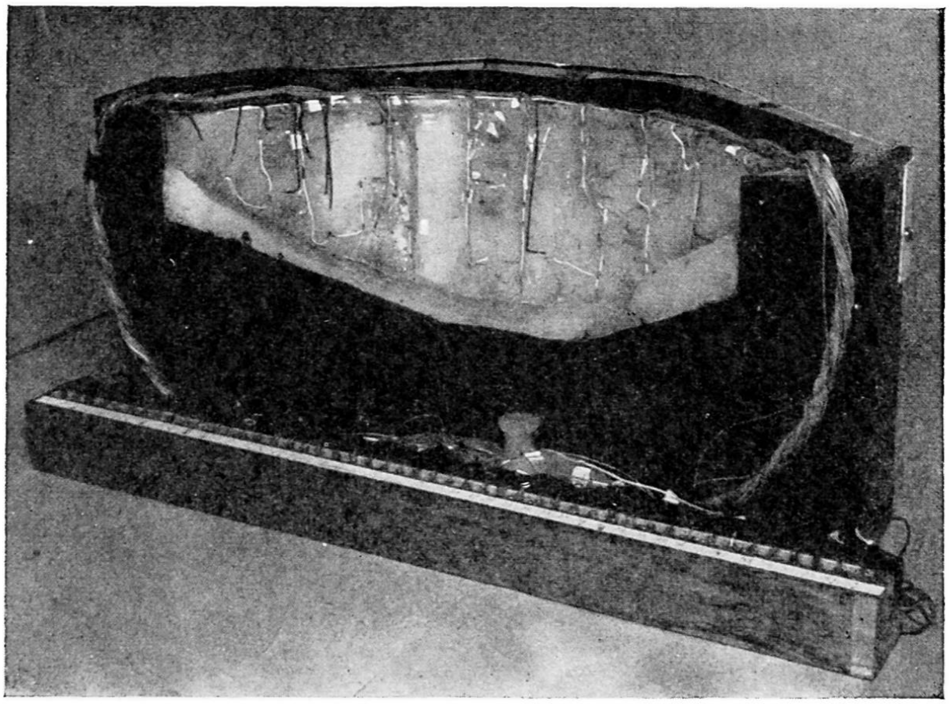

Fig. 6. Alkathene model to a scale of 1/200 of a 35 m. high arch dam. The model was subjected to mercury hydro-static pressure and the strains were measured by means of electric strain gauges specially built for use on alkathene

Another plastic now used in the Laboratorio de Engenharia Civil (Lisbon) is alkathene, a commercial name for a polythene. It has a very low modulus of elasticity, about 2,000 kg./cm.^{2}, and can be moulded at about 140° C. (fig. 6). This plastic cannot be glued but the surfaces to be joined can be welded. This is done in a way similar to the welding of metals, using a bar of alkathene and a jet of hot air.

The fact that alkathene can be welded, together with the great facility with which it can be cut, even with wood working tools, permits the shapes of the models to be modified at will in the search for the most convenient forms for the structure being studied.

Another material mentioned, plaster of Paris, with which diatomite is often mixed, has the advantage of being easily moulded and very economical (fig. 7).[10, 11] It has, however, the grave inconvenience of being brittle and often develops invisible cracks which can completely upset the field of stresses. Its mechanical properties vary between wide limits with the water content and its humidity at the time of use. Its modulus of elasticity may vary between 5,000 and 80,000 kg./cm.^{2}, the lowest values

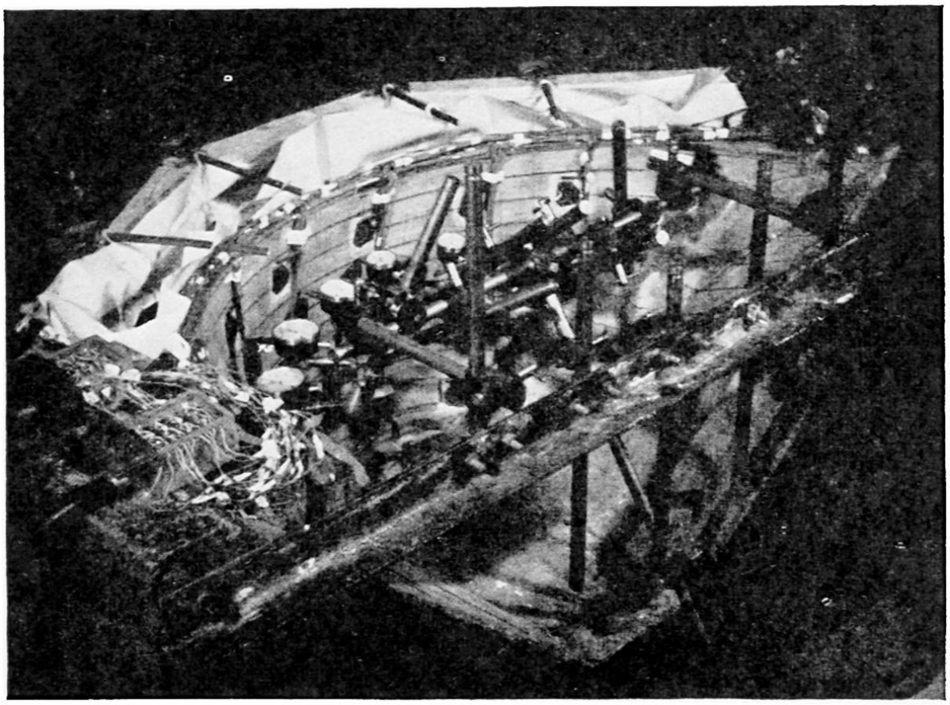

Fig. 7. Model to a scale of 1/300 of a 130 m. high arch dam and its foundations. It was built from plaster diatomite mix and electric strain gauges were used for strain measurements

being obtained with the addition of diatomite. The strains at the proportional limit, however, vary very little, having values of approximately 0·1 %, which at times upsets the accuracy of the measurements of the deformations. Poisson's ratio varies between 0·15 and 0·25.

(c) *Application of the loads*

Concentrated loads are easily applied to models by means of weights, jacks or springs. As the values of the forces to be applied to produce the same deformation diminish as the square of the scale, it is very convenient to use the lowest scale, since the equipment for the application of the forces can become much more simple and economical.

The distributed loads are at times substituted by concentrated forces, more or less near each other according to the precision required and the space needed to be free for observing the loaded surface.

When the distributed forces act normally to the loaded surface they can be applied by means of fluids. When the intensity of these forces is very high, use can be made of flexible cushions into which the fluid is introduced under the necessary pressure.

Referring to the determination of the effects of weight in models, it was mentioned that in general it is necessary to use complementary forces or subject the model to a rotation. The application of complementary forces does not present any difficulties when, as is common, dealing with structures with small thickness, since these forces can be substituted by surface forces. However, when the forces have to be applied to the interior of the models, the arrangements needed become very complicated;[12] the use of centrifugal forces is also not easy.

With alkathene it is already frequently possible to determine the effect of weight on models of moderate dimensions.

The study on models of the effects of temperature presents two difficulties, the application of given temperatures and the influence of these temperatures on the measuring apparatus. For this reason very few studies have been made on this aspect.[13]

Observations on the models

To predict the behaviour of a structure by means of models implies, in a general way, the determination of displacements, strains and stresses.

Following the common concept of safety, it is particularly important to determine the stresses developed under the action of working loads, as it is from these stresses that the structures are designed.

In the design in relation to failure it is only of essential interest to determine the intensity of the loadings which produce failure.

In design by the probabilistic concept there will be above all the need to measure the displacements and the characteristics of the failures caused by the action of various loadings with all possible intensities. From these measurements it will be possible to evaluate the damage, such as that resulting from excessive deformation, the need of repair, etc., which will occur in the prototype.

(a) *Measurement of displacements, strains and stresses* [14]

The measurement of displacements in the models is carried out by means of deflectometers with a sensibility of 1/10 and 1/100 mm. and, rarely, of 1/1,000 mm.

The measurement of strains is in the majority of cases the most important determination, as this permits the determination of the stresses once the relation between strain and stress is known. For materials in elastic deformation it is sufficient to know the modulus of elasticity and Poisson's ratio.

The measurement of strains in models is made almost exclusively at points on the surface. Measurements in the interior present besides the difficulties inherent in such measurements, those originating in the reduced size of the models. However, as the greatest strains and stresses appear in general at the surface, such difficulties are as a rule of little importance.

Among the extensometers used in the measurements of strains on models, we can mention the Huggenberger and Johansson mechanical extensometers. These extensometers have a satisfactory accuracy on short bases, which, in general, have to be used on models. The Johansson extensometers can be applied on a base of 3 mm. Like all mechanical extensometers they only permit measurements at the surface and they have the drawback of requiring, together with the accessories, an excessive space; besides this they often require considerable time to mount.

The vibrating wire extensometer is also sometimes used.[11] The minimum length of the wires is about 2 cm., which at times is excessive; besides, the placing and observation of the wires is a prolonged operation. They permit, however, being read at a distance, which is an advantage when there are inaccessible parts in the model or when the model is large. The wire extensometer is the most reliable for observations over long periods.

Finally, electrical resistance extensometers[14] are, without doubt, the most appropriate for measurements on models and are almost exclusively used today. In fact, they occupy least space and are the lightest, they are easily mounted without requiring any accessories, and can be observed at a distance. The measuring bases can be of any value above a few millimetres and their precision is satisfactory. Above all, when, as is usual, it is necessary to determine a large number of strains, the electrical gauges give results most rapidly and economically. The only inconvenience of the electrical extensometers is their instability with time, though there are already some types in which this inconvenience is reduced.

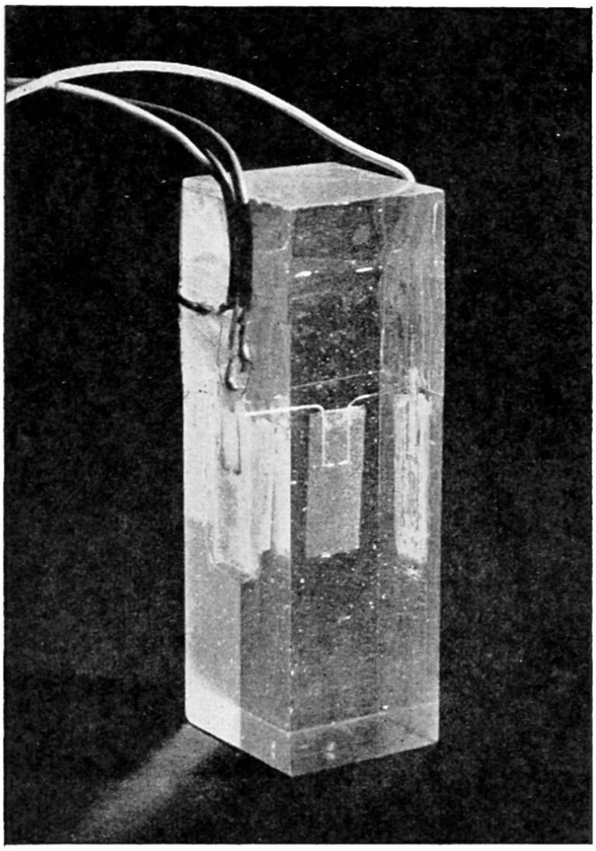

Fig. 8. Electric strain gauge inside a prism of a plastic. In a compressive test the values given by this strain gauge were in full agreement with those placed on the surface

The electrical extensometers, due to their small dimensions, lend themselves to the measurement of strains in the interior of models. In the case of mouldable materials they can be placed in position at the time of moulding (fig. 8), and conveniently protected against humidity if necessary. With the appearance of the electrical gauges it can be said that the difficulties in measuring strains in models have almost ceased to exist.

The accuracy with which the extensometers measure the strains depends largely, as is obvious, on the magnitude of the strains to be measured and on the experimental conditions. All the extensometers referred to permit, as a general rule, measurements to be made to within an error of $\Delta\epsilon = 10 \times 10^{-6}$.

Assuming this value, the table below gives the approximate values of relative errors within which the strains and stresses can be measured in materials most commonly used in model construction when they are strained to the proportional limit. The influence of the error in the modulus of elasticity on the error in the stresses is not considered, as in general it has no importance.

Materials	Strains assumed ϵ (%)	Relative error of the strains and stresses $\Delta\epsilon/\epsilon$ (%)
Celluloid and plastics . .	1	0·1
Plaster of Paris . . .	0·05	2
Mortars and concretes .	0·02	5
Metals	0·2	0·5

It can be seen that the strains and stresses can be obtained with an entirely satisfactory accuracy.

The determination of the isostatics, that is, of the principal directions on the surface of models of celluloid, plastics and metals, can be made very easily, in view of the great deformability of these materials, by the use of brittle coatings [14] (fig. 9). It is possible to obtain the appearance of cracks for strains of about 10^{-4}. The method is particularly advisable when dealing with models of complex shapes; it can be applied in dynamic equilibriums. The knowledge of the isostatics has the great advantage of permitting a reduction in the number of observations to be made with the extensometers for determining the states of strain.

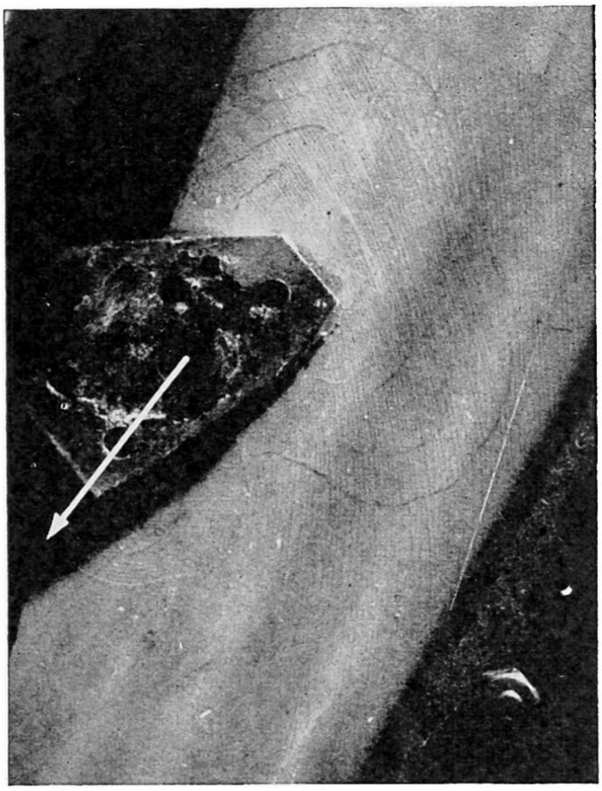

Fig. 9. Application of the brittle coating method to the determination of the isostatics in a spillway guide wall

Techniques for the application of the brittle coating method for the measurement of the magnitude of the strains and stresses are being developed, and have already reached some interesting results. The development of methods which may give results over an area is of great interest as it avoids readings having to be taken at various points, which is necessarily a prolonged operation, and the probability of making errors is reduced.

When the relation between strain and stress is not linear and the creep has to be considered, it is not generally possible to determine the stresses from the measurement of strains.

Recently a property was brought to light [15] which permits the direct determination of the stresses. This property is the following: if at a point in a solid made of any material, an elastic solid of small dimensions be introduced and intimately joined to that solid, the stresses developed in the elastic solid only depend on the state of stress in its neighbourhood as long as its modulus of elasticity be sufficiently small in relation to that corresponding to the deformations of the surrounding solid. Thus by measuring the stresses set up in the elastic solid, for example by means of its deformation (fig. 10), it is possible to determine the state of stress in the solid made of any material.

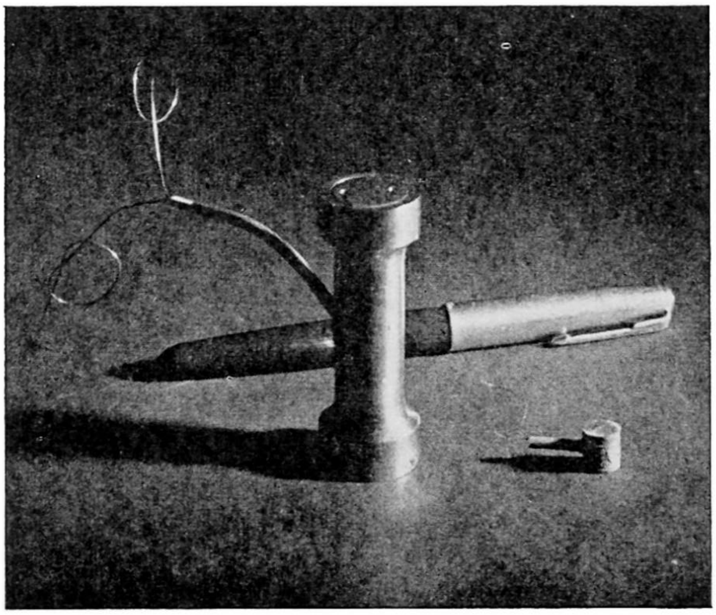

Fig. 10. Small magneto-striction cells to be left inside models for direct stress measurements

(b) *Photoelastic method*

The determination of the stresses in two-dimensional elastic equilibriums can be done by photoelasticity.[16, 17] Compared with the general method of determining the stresses from the measurements with extensometers, the photoelastic method has the advantage of being more rapid and economical, and also reaching, in general, greater accuracy. The fact that models with greatly reduced dimensions can be used appreciably contributes to this economy. In the case of the study of high stress concentrations, this fact makes the use of this method very convenient, as the use of extensometers in this case requires the use of large models.

The photoelastic method has the advantage of making observations all over an area. The attempts to apply photoelasticity to three-dimensional equilibriums have not yet reached results of practical value. At present it is preferable to study such equilibriums by leaving extensometers in the interior of mouldable models.

(c) *Models of structures consisting of bars*

The models of structures consisting of bars, which we will call briefly linear structures, may be studied by using the general methods just referred to.

Before the appearance of electrical strain-gauges the measurement of strains was made difficult by the extensometers and their accessories having excessive dimensions and weight compared with the dimensions and rigidity which it is convenient to give to the models of linear structures.

When the sections of the model are not geometrically similar to those of the prototype, the measurement of the strains permits the determination of the shearing and normal forces and of the bending moments in the model, which can be transferred to the prototype.

For the study of linear structures many special methods have been developed. The methods most used are those which permit the determination of the influence lines of the statically indeterminate forces (exterior and interior) from the reciprocity theorem of Maxwell–Betti.[18, 19] Obtaining the influence lines by this way has the great advantage of avoiding the application of forces to the models, which is particularly important in the case of structures having a large number of members. In spite of this determination of the influence lines being in principle possible for any linear structure, the experimental difficulties have limited its application to structures in plane equilibrium.

The various methods based on that theorem differ from each other in the magnitude of displacements imposed on the model, in the technique of applying these displacement and in the technique of the measurement of the displacements corresponding to the forces whose effect it is desired to determine.

The methods which use large displacements have the advantage of making it possible to observe directly the functioning of the structures, and to measure the corresponding displacements easily. They have, however, the grave disadvantage of the results being affected by the redistribution of stresses due to the large displacements imposed; for this reason the methods today have little more than pedagogic value.

However, at times the inconvenience referred to is not important; thus in the case of continuous beams, for the determination of the influence lines of the reactions of the supports, these can be displaced even to one-fifth of the spans without errors of more than a few per cent resulting. It is in such a case a method to be recommended.

Of the methods based on the theorem of reciprocity, the one most employed is that of Beggs,[20] in which small displacements are imposed by means of a special device and the measurement of corresponding displacements is made by means of microscopes.

The application of this method is only advisable for the determination of the influence lines corresponding to external indeterminate forces. In fact, for the determinations corresponding to interior indeterminate forces in complex structures, which are those requiring experimental study, there is not, in the majority of cases, room enough to mount the device for imposing the displacements. Besides this they cannot be imposed to the edges of the section but only at a distance which is often excessive. On the other hand, time taken for mounting is prolonged and awkward and, frequently, the rigidity of the model does not permit the imposition of sufficiently large displacements.

The measurements of the displacements, either small or large, are made very conveniently by the photographic method.[21] In this method the model is photographed on the same plate before and after a displacement is imposed; the displacements can be measured with a microscope or on a screen on which the plate is projected (fig. 11). This method permits rapid readings to be made and its accuracy even for small displacements is the same as that obtained by direct readings on the model with a microscope.

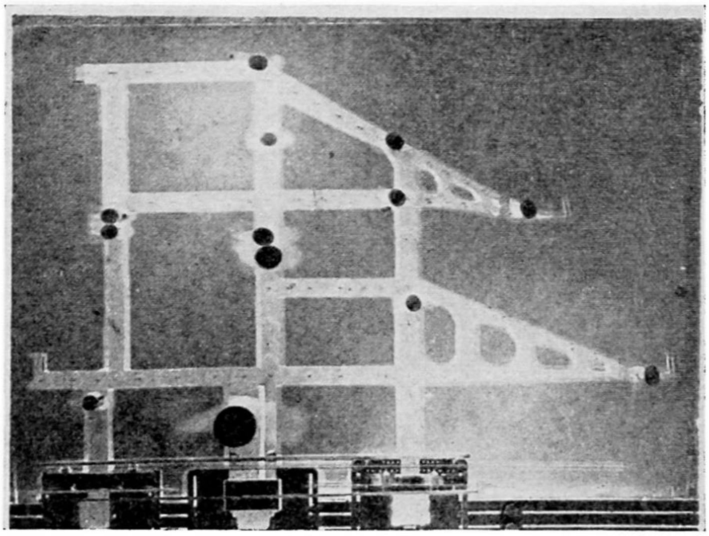

Fig. 11. Photograph obtained in studying a linear structure by the photographic method

The photographic method supplies a record of the results of the test and of the conditions under which it was carried out. It can reveal certain causes of error, such as deficient design of the model, accidental movements of built-in members, deficient working of the device for imposing the displacements, etc.

In brief, in the experimental study of linear structures, it is advisable to apply the general method, measuring the strains by means of electrical strain-gauges, except in the determination of the influence lines corresponding to external indeterminate forces when the structure is in plane equilibrium. For this determination it is advisable to use the Beggs method and measure the displacements by the photographic method.

CONCLUSIONS

The essential aspects of the problem of experimental design of structures have been presented in this paper.

The conclusion is reached that the choice of shapes and determination of the dimensions of any structure can be made, as a general rule, from observations on models, even when it is wished to take into consideration its behaviour beyond the elastic range. Models also lend themselves to the determination of the influence of the variation of the properties of the materials throughout a structure.

At present it is in the choice of materials for models and in their construction that difficulties are at times met with, whilst previously, before the appearance of electrical strain-gauges, it was in the observation of the models that the greatest difficulties were met, and which were frequently insoluble.

It can be truly said that a model, even on a very reduced scale, is in general a much more faithful image of the prototype than the hypotheses adopted by analytical methods, either from the point of view of the shape, or the material or even of the loading. This does not, of course, minimise the value of analytical methods, which have the great advantage of being, except in very special cases, more rapidly and economically applied, of not requiring equipment and also of furnishing results which are easily checked.

These advantages indicate the use of the analytical methods in the primary design of a structure, in which phase it is necessary to obtain a rough estimate of the possible solutions, which, as a general rule, are numerous. For the final design of small and medium structures the analytical methods are also generally the most adequate.

It is in the design of important structures, with, say, a value of over £10,000, that the studies on models, whose cost is in the region of some hundreds of pounds, is recommended, unless completely reliable analytical methods are available.

The analytical and experimental methods should not be put in opposition, as at times is the tendency, but rather be considered as tools to be wisely used in the safe and economical resolution of structural design problems.

It should be emphasised that to obtain results in periods compatible with those usually required for the elaboration of plans and to win the confidence of the authorities interested in the plan, it is necessary to have specially equipped and organised laboratories. For the laboratories to work economically they need to have an important volume of permanent work.

The use on a large scale of the experimental method as a routine method of design gives valuable opportunities for perfecting the knowledge and formulating theories of the behaviour of structures. It often happens that when studying a model certain effects which had not been considered are found to be the most important. The difficulty and high cost of the observation of the prototypes is a further reason which weighs in favour of a wider use of models as a research instrument.

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Summary

The aim of the paper is to give a general view of the present status of the experimental method of structural analysis within both the elastic and non-elastic ranges.

The requirements of mechanical similarity to be met for model shape, materials and loading, for static or dynamic equilibriums are presented and the actual possibilities are then indicated for such requirements being fulfilled.

Finally, the possibilities and the exigencies of the experimental method of structural analysis are mentioned.

Résumé

Le présent rapport a pour but de donner un aperçu de l'état actuel de la méthode expérimentale de calcul des ouvrages, soit dans le domaine élastique, soit au delà de ce domaine.

A cet effet, l'auteur commence par présenter les conditions auxquelles doivent répondre les formes, les matériaux et les sollicitations des modèles, en équilibre statique ou dynamique; il expose ensuite les possibilités actuelles d'observation de ces conditions.

En conclusion, il mentionne les possibilités et les exigences de la méthode expérimentale de calcul des ouvrages.

Zusammenfassung

Mit vorliegendem Bericht wird versucht, einen Ueberblick über den heutigen Stand der experimentellen Methoden zur Tragwerksuntersuchung, sowohl innerhalb wie auch ausserhalb des elastischen Bereiches zu geben.

Dafür wird zunächst auf die Bedingungen mechanischer Aehnlichkeit hingewiesen, denen die Durchbildung, Baustoffe und Beanspruchungen der Modelle bei statischem bezw. dynamischem Gleichgewicht genügen müssen. Im weiteren werden die heutigen Möglichkeiten, solche Bedingungen zu schaffen, dargelegt.

Zum Schluss wird auf die Möglichkeiten und Anforderungen der experimentellen Methode eingegangen.

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