AII 3

Photoelasticity applied to structural design

La photoélasticimétrie appliquée au calcul des ouvrages

Spannungsoptische Bemessung von Tragwerken

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Introduction

This paper presents some experimental studies for the design of structures, carried out in the Laboratório de Engenharia Civil (Ministério das Obras Públicas), Lisbon, in which the photoelastic method was used.

As is known, it is possible, in general, to reproduce the real behaviour of structures in models even when very reduced dimensions are chosen. Once the model is built, the general test method consists of the application of loads and the measurement of displacements, stresses and strains.

In order to measure the stresses, extensometers or the photoelastic method are commonly used.

The advantage of the photoelastic method is the ease, rapidity and economy with which it permits the determination of the fields of stress. The fact that photoelasticity supplies images in relation to the complete field of stress, besides avoiding errors, allows the rapid localisation of the regions of important stresses. The small scale to which the models can be built is one of its principal advantages; in fact, the construction of models is simplified and the forces to be applied are small.

On the other hand photoelasticity requires the use of transparent materials and it is only practicable to study plane states of stresses. The numerous attempts which have been made to extend this method to the study of three-dimensional states of stress have not reached a degree of real practical interest; in such cases the authors think it advisable to use extensometers, left in the interior of mouldable models.

The restrictions mentioned considerably limit the field of application of photoelasticity. Besides, photoelasticity only serves to determine the state of stress within the elastic limit.

The application of photoelasticity, like other experimental methods, is only advisable when there are no analytical methods which furnish results with the desired accuracy, or when their application is less economical.

The authors believe that the studies which follow show well how photoelasticity can be used to advantage in solving problems of structural design.

Study of the influence of the deformability of the foundations on the behaviour of an aqueduct

The problem of studying the stress distribution in a concrete aqueduct for different mechanical properties of the soil appeared in the study of the new Lisbon water supply.

The greater part of the aqueduct will be built in a trench.

Fig. 1 shows the shape of the cross-section initially proposed for the conduit, together with some modifications which were tested. To carry out the tests the loads were taken as those obtained from the usual design theories for the thrust of earth fills.

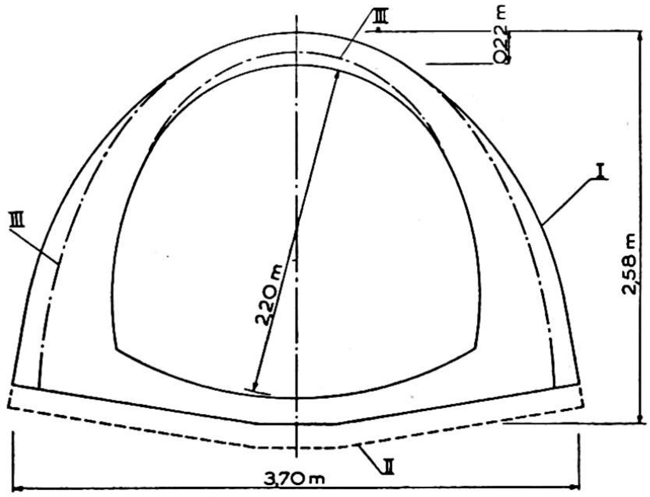

Fig. 1. Cross-section of the aqueduct
I. Section initially proposed
II. Section with base of double thickness
III. Modified section

A bakelite plane model was built to the scale of 1/20 with a thickness of 1·0 cm. The distributed loads applied to the prototype were replaced by adequate concentrated loads, which were applied by jacks as shown on fig. 2. In order to maintain a constant load, the oil-pressure tube, common to all jacks, was connected to another jack the piston of which was loaded by a weight.

Loadings corresponding to the following hypotheses were considered:

(*a*) full conduit, earth filling with an angle of friction of 35° and 4 m. thick at the crown of the aqueduct;

(*b*) empty conduit, earth filling 8 m. thick with an angle of friction of 25°.

These hypotheses had led to the highest stresses in analytical calculations, considering the upper part of the conduit as a built-in arch. An asymetric loading was also considered, which corresponded to loading half the arch. Successive tests were made on the model supported by foundations with different mechanical properties.

For studying the hypothesis of the aqueduct and the foundation having the same mechanical properties, the soil was reproduced from the same bakelite from which the model was made. Afterwards, the model was supported on bases of cork agglomerate and rubber, materials which reproduce foundations respectively 30 and 300 times more deformable than the material of the conduit.

To reproduce soil of much greater deformability than that of the structure, the model was supported on a tube so as to obtain a uniform pressure on its base. For this a rubber tube of 1·8 cm. external diameter was used, filled with water and closed at the ends.

In order to compare with the analytical calculation previously made, a test was carried out in which the arch was built-in by placing the model between two roughened steel plates tightly joined together by bolts.

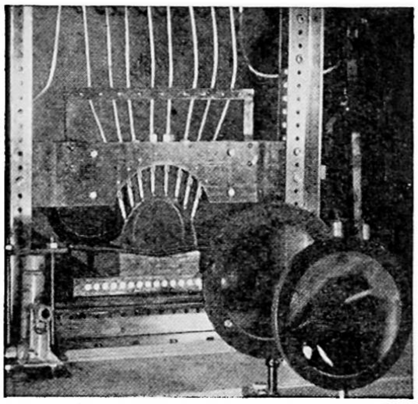

Fig. 2. Test arrangement

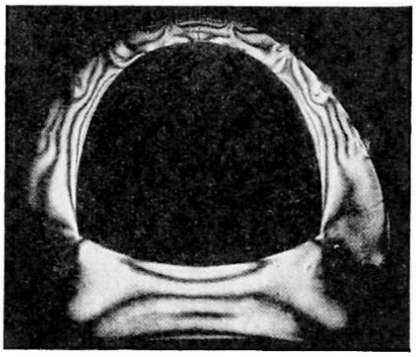

Fig. 3. Isochromatics

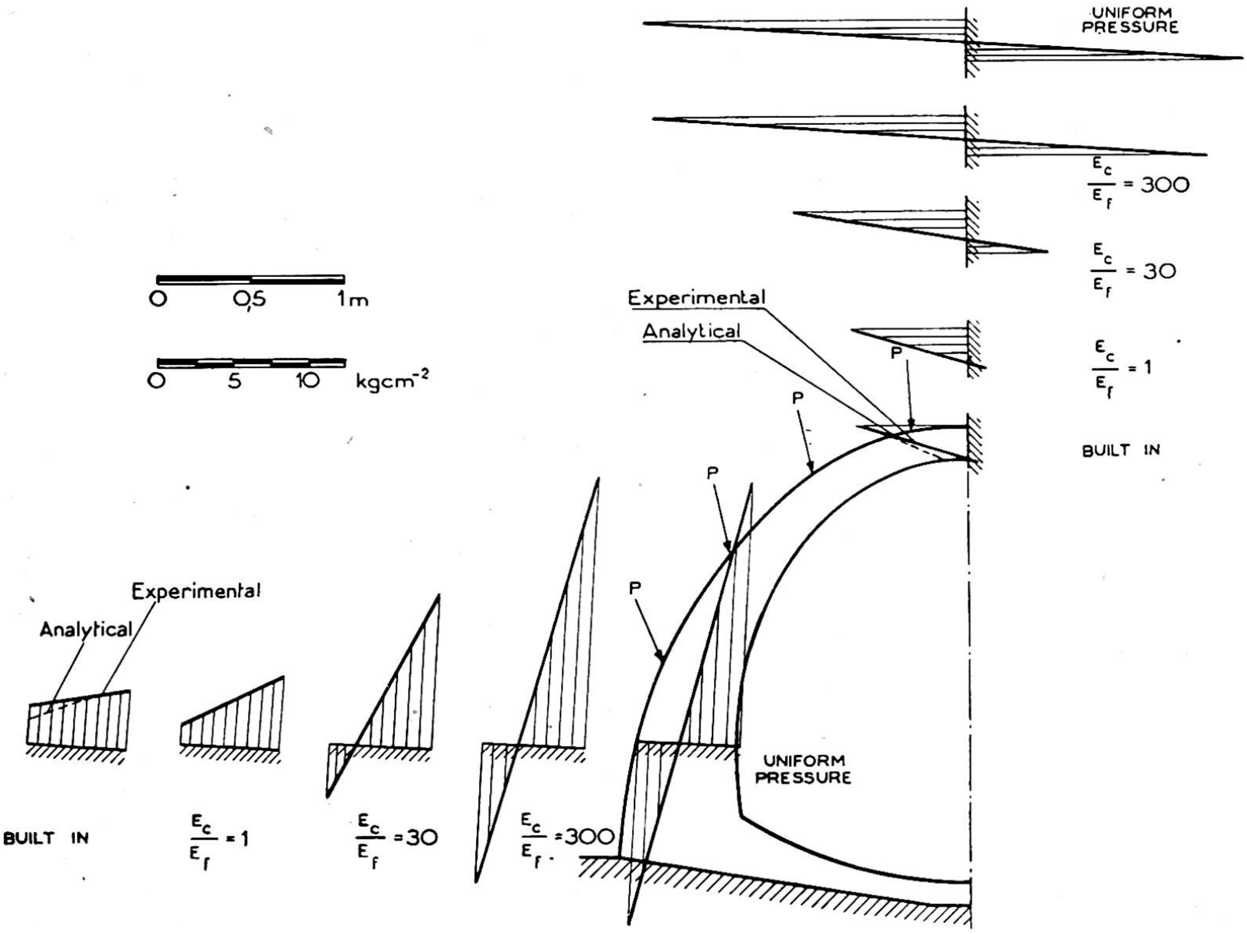

Fig. 4. Stresses in the prototype for different values of the foundation deformability

For the structure dealt with it is sufficient to know the stresses at the boundaries. These stresses were determined from the order of the isochromatics, one of which is shown in fig. 3.

The stresses at the crown and at sections near the springing points for model I (fig. 1), when subject to loading *a*, are shown in fig. 4. Thus it will be seen that the analytical results obtained for the built-in arch agree with those obtained experimentally for the same condition.

As the deformability of the ground increases, the absolute values of the stresses increase. Thus, at the crown, when the modulus of elasticity of the foundation material, E_f, is $\frac{1}{300}$ of that of the structure, E_c, the compressive stresses rise from 6 kg./cm.^{2}, the value obtained in the case of the built-in arch, to about 20 kg./cm^2; at the same time tensile stresses of approximately 16 kg./cm.^{2} develop at the internal face. In the section near the springing, for the same conditions, the compressive stresses at the internal face increase from 4 to 17 kg./cm.^{2} and tensile stresses of about 10 kg./cm.^{2} appear at the external face.

The increase of the deformability of the foundation beyond that mentioned above does not lead to any appreciable variation in the maximum stresses.

For loading *b* the influence of the deformability of the foundation is similar. The maximum stresses observed are not very different.

It should be noted that for common soils and particularly for those crossed by the aqueduct, the stresses developed in the structure should correspond to the relation E_c/E_f of several hundreds.

With regard to the base of the aqueduct the increase of stresses in the middle of its upper face is particularly important as the deformability of the foundation increases.

For the relation $E_c/E_f = 300$ the tensile stress reaches 25 kg./cm.^{2} for loading *a* and 31 kg./cm.^{2} for loading *b*, stresses that would require considerable reinforcement in the base.

A model was tested in which the base had double the thickness (fig. 1). The solution of increasing the thickness of the base, though giving a reduction in the tensile stresses when the foundation deformability is large, is not economical. In order to decrease the stresses at the base the authors also studied the solution of leaving the central zone free (fig. 5) by means of a channel beneath the central part of the conduit,

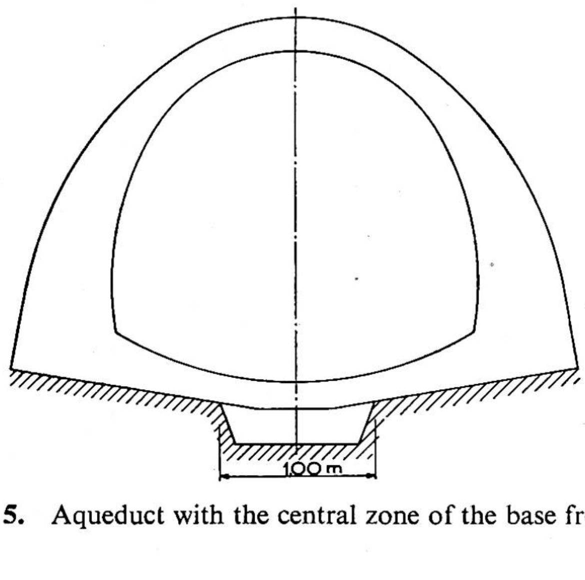

Fig. 5. Aqueduct with the central zone of the base free

which could be also used for drainage. Thus the uplift pressures would be avoided, which otherwise might induce cracking at the base.

For a width of the channel of 1 m. the tensile stresses, which, as already mentioned, were about 30 kg./cm.^{2} for the hypothesis of $E_c/E_f=300$, become nearly nil. The stresses in the arch were also reduced due to the channel.

The distribution of stresses observed in the tests carried out led to a modification of the cross-section as shown in fig. 1. Tests similar to those already described were carried out on this new cross-section.

In spite of this solution corresponding to a reduction of 20% in the volume of concrete, the maximum stresses developed did not suffer any appreciable change. The opening of a channel under the central part of the base reduced the stresses as in the previous case.

It is of interest to mention that some years ago the authors carried out some photoelastic tests on another conduit, in which the deformability of the foundation was also taken in account. The results of these tests, which also showed a large influence of the deformability of the foundation, were later fully confirmed by the behaviour of the structure.

Study of stress distribution around the spillway openings of an arch dam

When designing the reinforcement to be placed around the flood-discharge openings of Castelo do Bode Dam (fig. 6) it was found impossible to calculate the reinforcement.

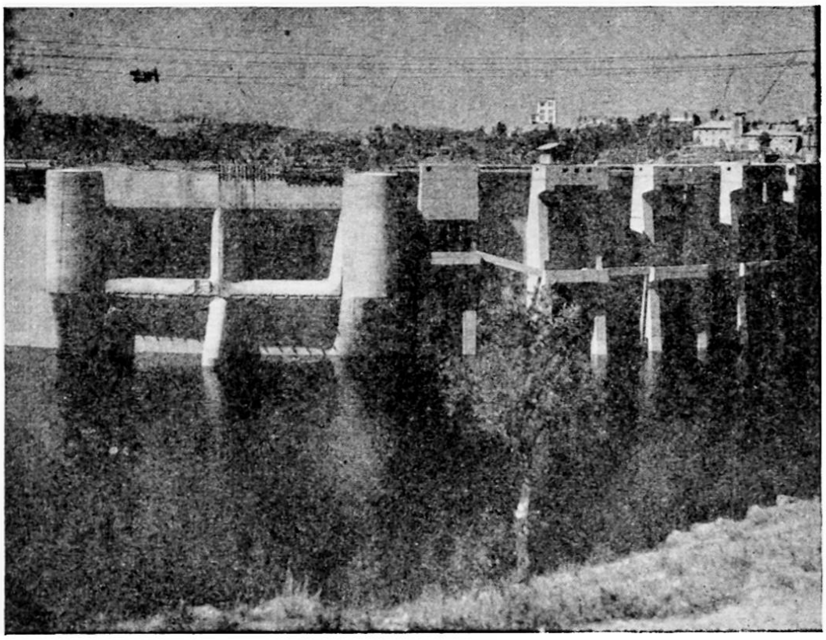

Fig. 6. Upstream view of the spillway openings of the Castelo do Bode Dam

To determine the stresses developed the experimental method was used. Measurements were taken on three-dimensional plaster of Paris models, which faithfully reproduced the dam and the rock of foundation.* These models were used not only to study the stresses around the spillways but also those developed in the entire dam.

* "Note on the Studies of Dam Problems carried out in the Laboratório de Engenharia Civil," Publication No. 13, Laboratório de Engenharia Civil, Lisbon, 1950.

Fig. 7 shows the diagram of the normal stresses acting at the edges of the horizontal section which passes at middle height of the openings, when the dam is subject to the full hydrostatic pressure.

For the interpretation of these diagrams that are far from simple, photoelastic tests were carried out.

Plane models to a scale of 1/500 of constant and variable thickness were used (fig. 8), by which it was possible to study the influence of the thickness change on the distribution of stresses around the spillway openings.

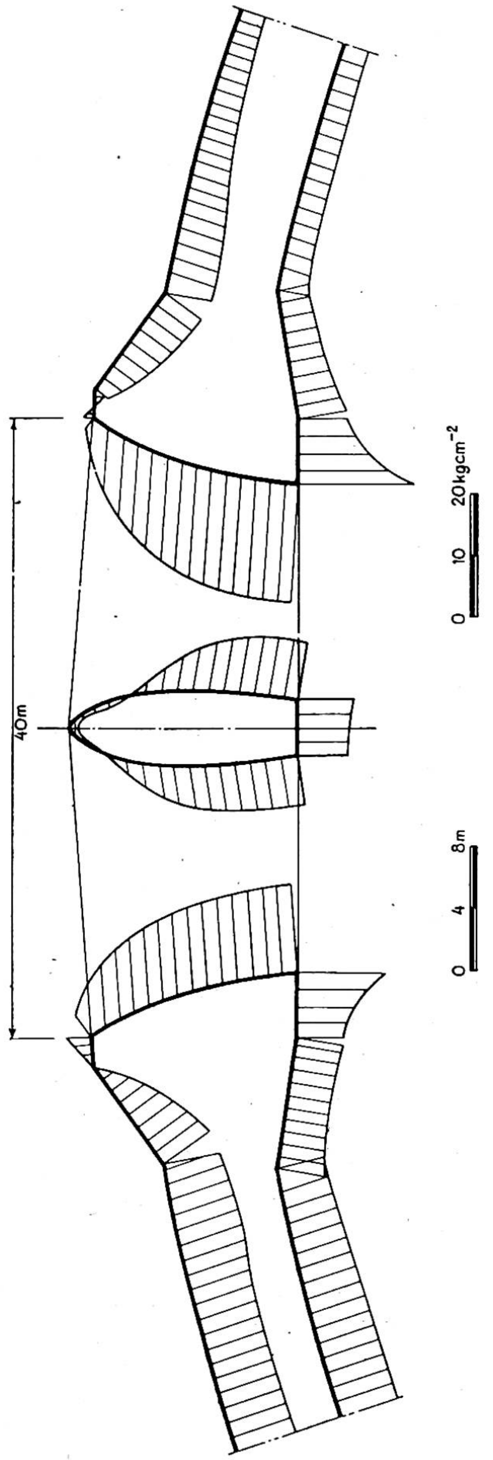

Fig. 7. Normal stresses in horizontal mean section of the spillway openings

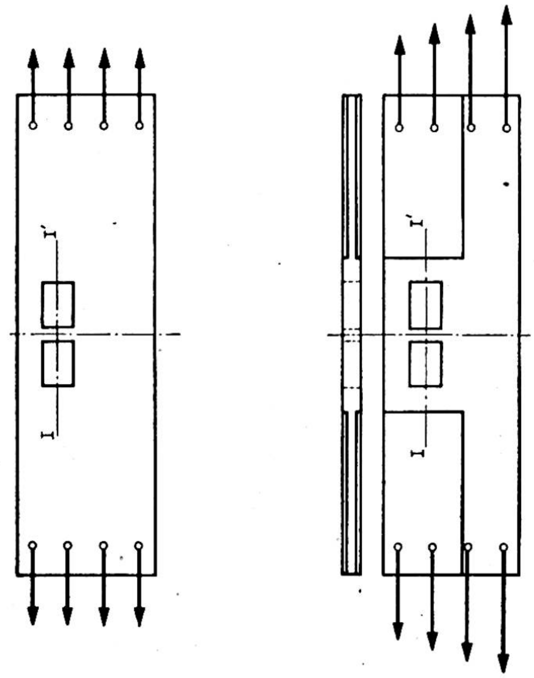

Fig. 8. Photoelastic models

Forces which reproduced the mean compressive stresses in the arches of the dam were applied to these models. The values of these mean stresses were determined by by the tests on the three-dimensional models.

Two models of constant thickness were made, one of bakelite to determine the isochromatics, and another of celluloid to determine the isoclinics. Forces were applied to these models to produce a uniform stress field in the region not affected by the spillway openings.

The model of variable thickness was made by cementing together sheets of celluloid so as to obtain steps of thickness corresponding in a simplified way to the shape of the spillway and reproducing the increase of sectional area around the openings. Forces were applied to this model which were proportional to the normal forces in the arches at different levels and which also corresponded to an approximately uniform distribution of stresses in the area not affected by the spillway openings.

It was desired to determine, above all, the normal stresses along section I–I′ (fig. 8).

The difference of the principal stresses was obtained from the isochromatics and from the readings taken with a Babinet–Soleil compensator. To confirm the values of the stresses at the faces of the spillways openings, measurements were carried out with Johansson strain-gauges of a 0·3 cm. base.

Knowing the isoclinics and the difference of principal stresses along section I–I′, the normal stresses were calculated by integration along the section concerned. As this section may be regarded as symmetrical the calculation was quite easy.

The diagrams of the normal stresses along the section I–I′ for the models of constant and variable thickness are shown in fig. 9. These stresses were calculated on the assumption that the mean compressive stress developed in the arches of the dam is 21 kg./cm.^{2}

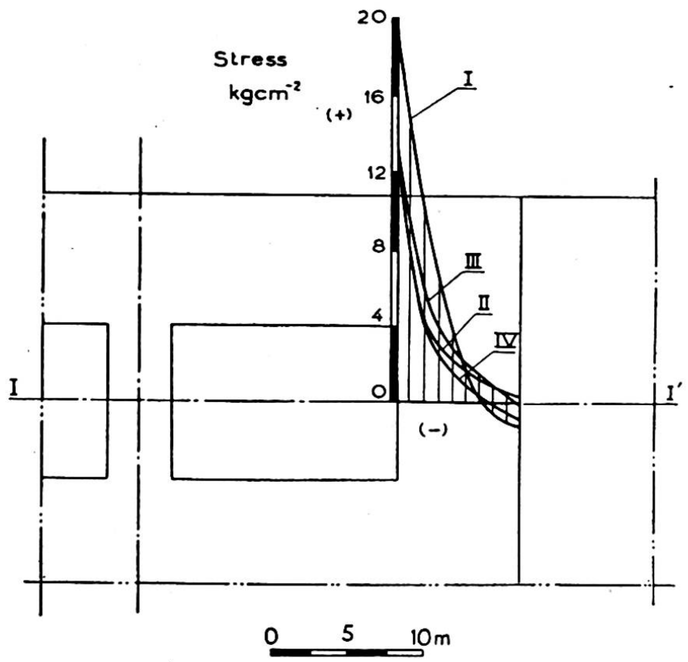

Fig. 9. Normal stresses along the section I–I′, transferred to the prototype

I. Determined from the constant thickness model
II. Determined from the variable thickness model
III. Determined from the three-dimensional model (mean values of the stresses at corresponding points of the upstream and downstream face). (Left bank)
IV. Determined from the three-dimensional model (mean values of the stresses at corresponding points of the upstream and downstream face). (Right bank)

The value of 18 kg./cm.² for the tensile stress at the face of the spillway openings obtained from the constant-thickness model is, as was to be expected, greater than the value of 12·5 kg./cm.² obtained from the variable-thickness model.

It is interesting to note that the maximum stress obtained from the variable-thickness model agrees with the mean stress developed along the face of the spillway opening measured on the three-dimensional models. It should be emphasised that this mean stress is not far below the maximum stress developed at the face of the spillway opening. This stress is in turn the maximum tensile stresses in the spillway area.

In fig. 9 are also presented diagrams of the mean values of the stresses at corresponding points of the upstream and downstream faces obtained in the three-dimensional models.

The agreement between these diagrams and that obtained from the variable-thickness model is quite satisfactory. The photoelastic variable-thickness model, of course, does not take into consideration the bending and other effects which were determined by the three-dimensional model. However, the photoelastic method was of good service to solve the proposed problem.

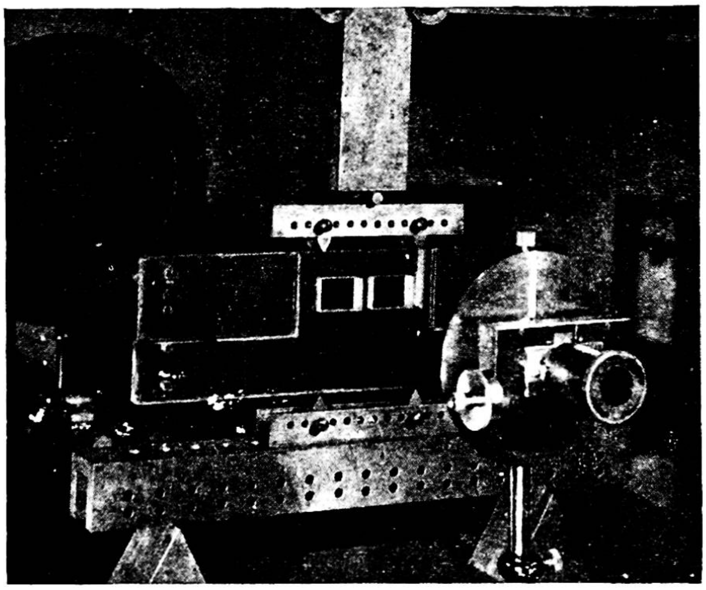

Fig. 10. Test arrangement

In order to eliminate the tensile stresses and the resulting cracks, which were inconvenient specially due to the high velocity of the water at the spillways openings, the use of prestressed concrete in this area was tried.

The distribution of the stresses due to the prestressing was studied on the variable-thickness model using the test arrangement shown in fig. 10. It was also easy to determine the stresses due to the weight and to the hydrostatic pressure on the upper face of the openings. Fig. 11 shows the diagrams of the stresses thus obtained.

It is interesting to note that, at section I–I′, the effects of the prestressing and of the weight of part of the dam over the spillways openings are distributed through a large area, and so the vanishing of the tensile stresses is not attained.

Due to this fact it was thought advisable to limit the stress distribution area by creating a vertical joint located about 3 m. from the openings (fig. 12).

Experimental tests made accordingly showed that the weight alone was enough

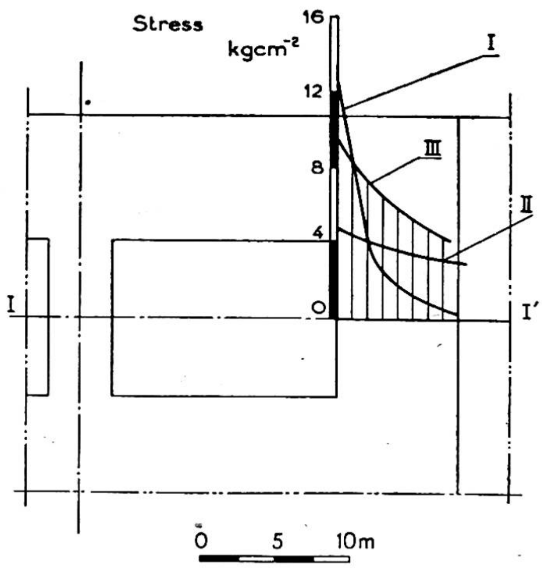

Fig. 11. Normal stresses along section I–I′ transferred to the prototype
I. Tensile stresses due to normal stresses in the arches
II. Compressive stresses due to the weight
III. Total compressive stresses due to the weight and a prestress of 4,000 tons

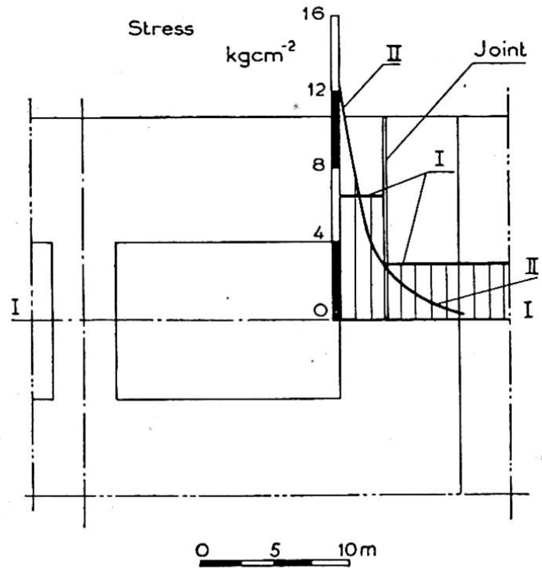

Fig. 12. Normal stresses along the section I–I′, transferred to the prototype, on the hypothesis of leaving a joint during construction
I. Compressive stresses due to weight (open joint)
II. Tensile stresses due to normal stresses in the arches

to produce a compression stress of 6·5 kg./cm.^{2} (fig. 12). Therefore, after grouting the joint the maximum tensile stress in service will be 6 kg./cm.^{2}; to absorb the tensile stresses, which develop only in a small area, normal reinforcement was used. So it was possible to achieve a considerable economy.

STUDY OF THE REINFORCEMENT OF THE GUIDE WALLS OF DAM SPILLWAYS TO SUPPORT THE FORCES TRANSMITTED BY THE GATES

In Castelo do Bode dam the flood discharge called for two gates (fig. 13), each one having to support a maximum thrust of about 4,000 tons. It was therefore necessary to provide the guide walls with reinforcements capable of transmitting this thrust to the body of the dam.

For designing this reinforcement a photoelastic test was carried out on a bakelite

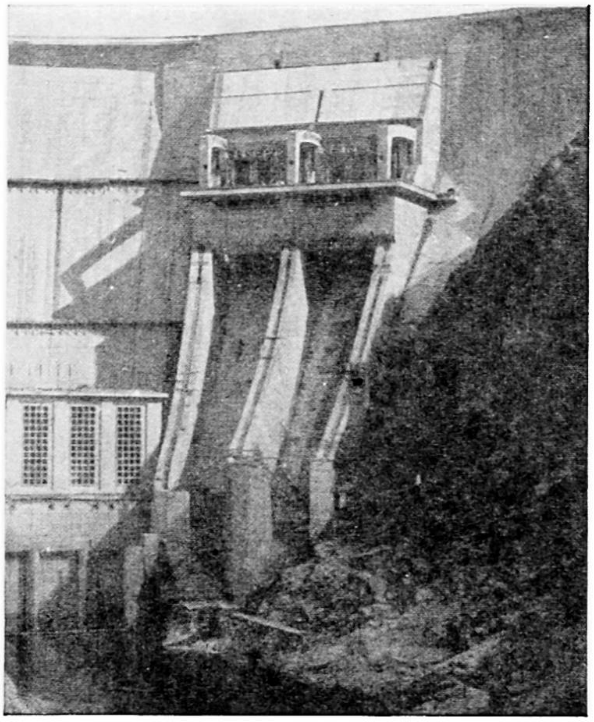

Fig. 13. Spillway of Castelo do Bode Dam

model to a scale of 1/200 (fig. 14). A force which reproduced the thrust was applied to the model.

In fig. 15 are shown the isochromatics obtained.

The isostatics plotted from the isoclinics are shown in fig. 16.

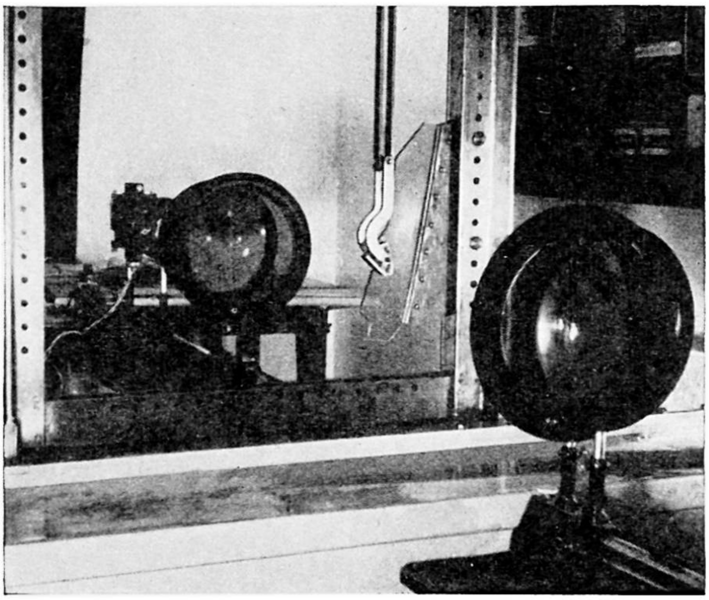

Fig. 14. Test arrangement

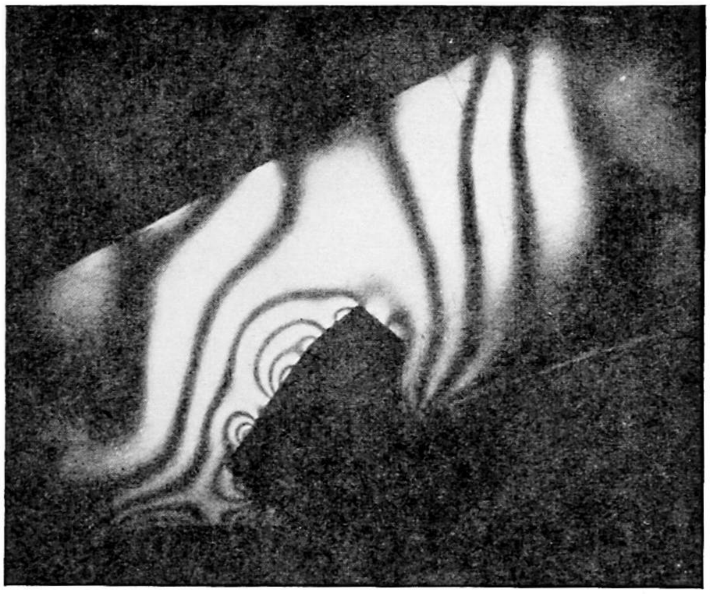

Fig. 15. Isochromatics

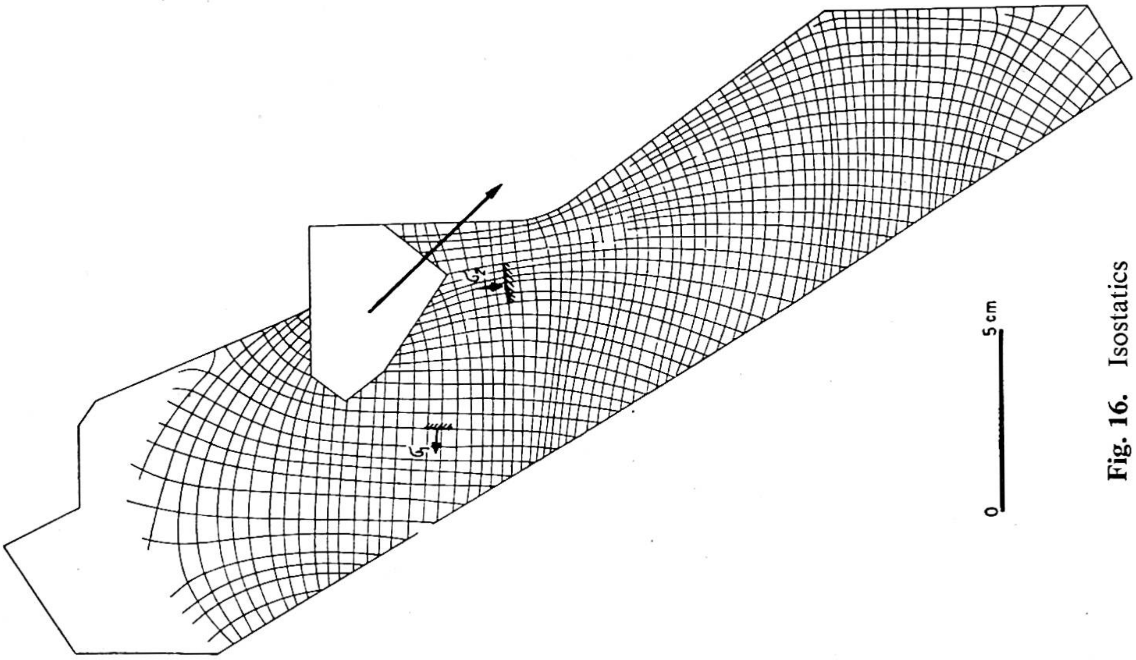

Fig. 16. Isostatics

The stresses were calculated using the method of integration along straight sections. In the fig. 17 are shown the values of these stresses transferred to the prototype.

The static equilibrium of several sections of the model was satisfied to within errors of 3 %, which are fully satisfactory.

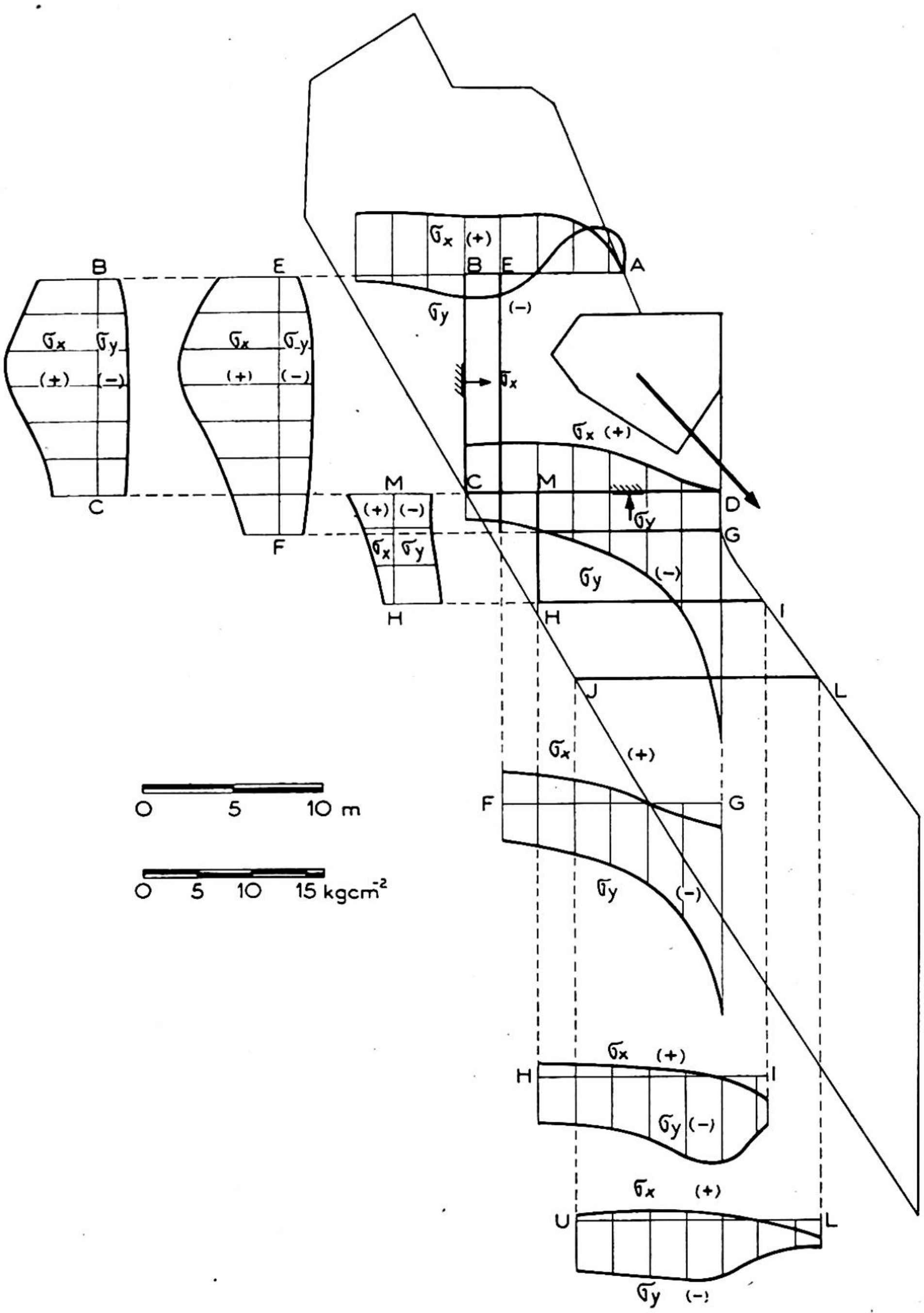

Fig. 17. Normal stress transferred to the prototype

The reinforcements were placed following the isostatics and the area of their cross-sections was established according to the stresses given by the model.

A similar problem arose in the Mabubas Dam (Portuguese West Africa), whose guide walls are shown in fig. 18. The thrust of the gates is transmitted to the guide walls by means of cantilevers and the maximum thrust in each wall is 1,200 tons.

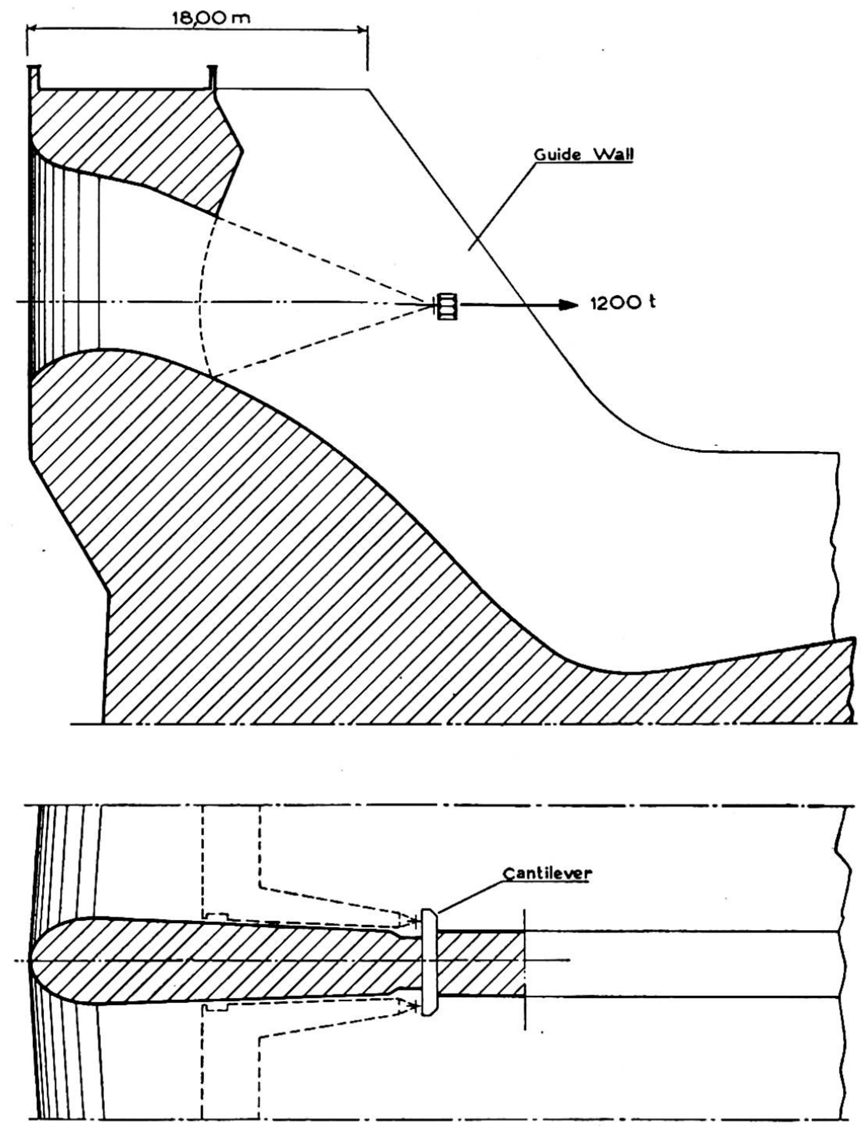

Fig. 18. Guide walls of Mabubas Dam

As in the previous case a bakelite model was made to a scale of 1/200. To determine the principal stresses in the wall a graphic integration was made along the isostatics indicated in fig. 19.

Based on the results obtained the walls were reinforced as shown in fig. 20.

In order to study the local effect of the loads transmitted by the cantilevers to the guide walls, a reinforced-concrete model was built to a scale of 1/10. Fig. 21 shows a view of the test.

Stresses were measured on this model not only near the beam but also at some points where the stresses had been determined by the photoelastic model. In fig. 22 are compared, along one of the isostatics, the stresses obtained in the photoelastic test with those obtained on the concrete model when working in the elastic range. As was expected, the stresses agree closely.

The test on the concrete model was carried beyond the elastic range and gave valuable information about the behaviour in the neighbourhood of the failure. The first cracks, which were detected for a load equal to twice the working load, appeared

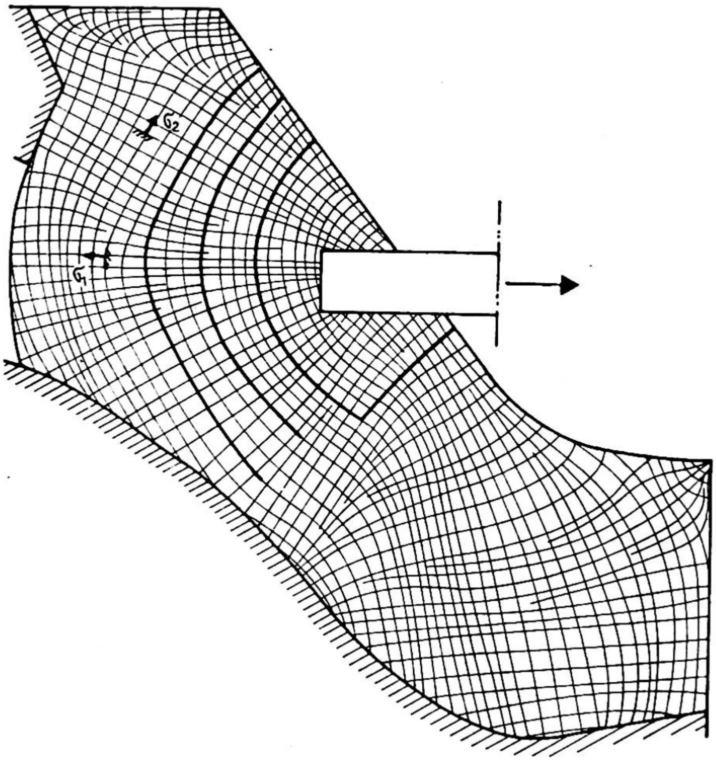

Fig. 19. Isostatics

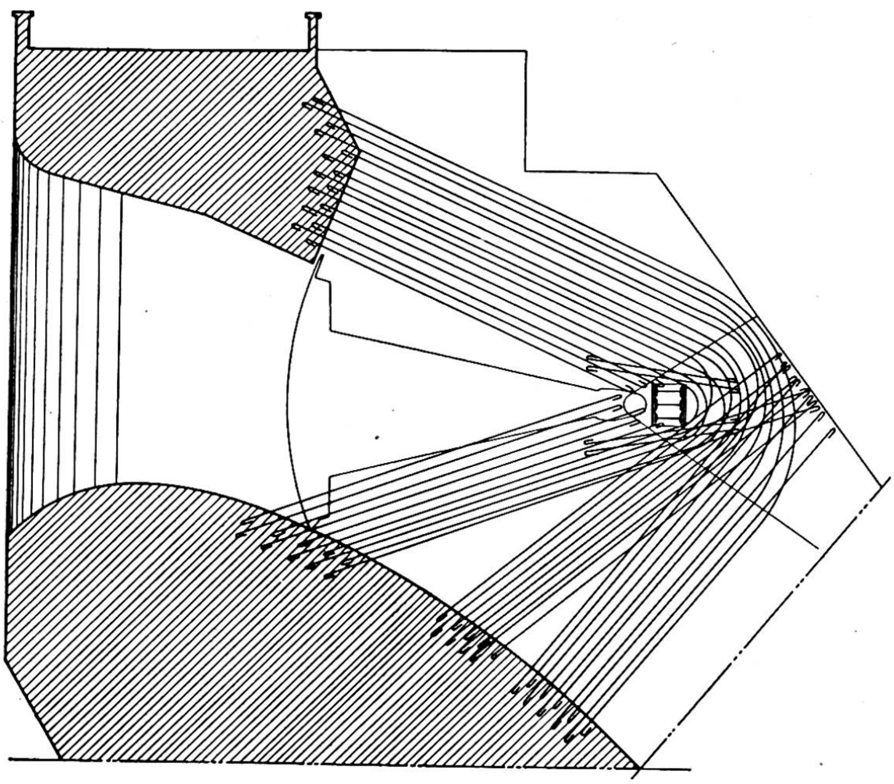

Fig. 20. Reinforcement in the guide wall designed from the photoelastic test

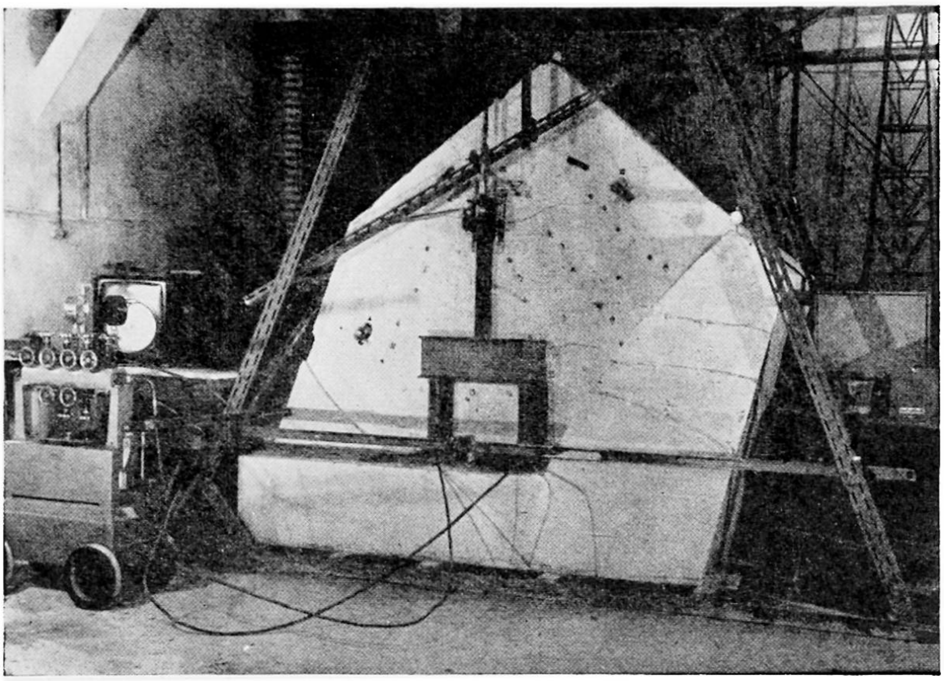

Fig. 21. Reinforced-concrete model to a scale of 1/10

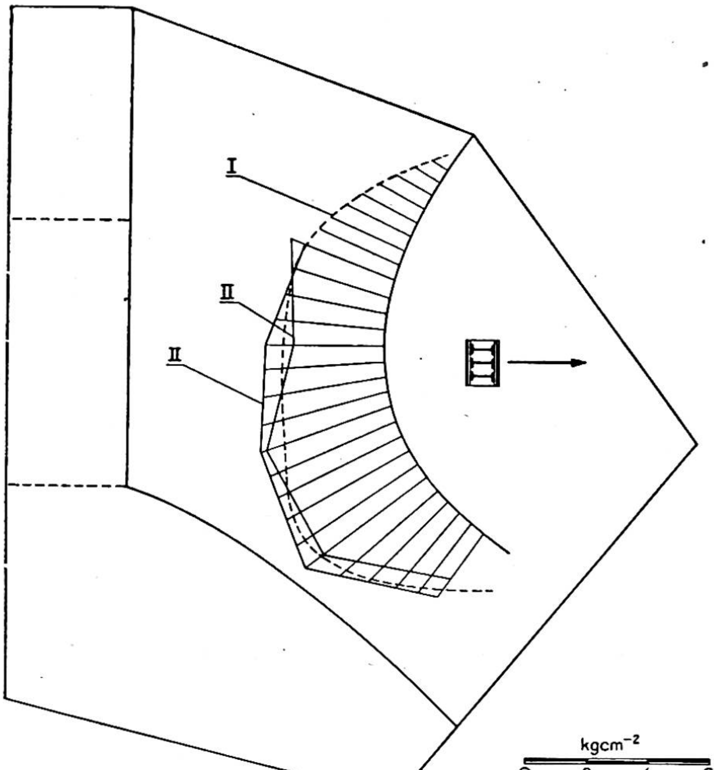

Fig. 22. Stresses along one isostatic
I. Determined from the photoelastic model
II. Determined from the concrete model

near the upstream flange of the cantilever. These cracks later spread through the whole wall and led to the failure.

The results of this test suggested the need to strengthen the reinforcement near the cantilever, as shown in fig. 20.

In order to study the legitimacy of undertaking tests until failure on small reinforced models, another model was built to a scale of 1/50 (fig. 23). In both models the development of the failure was absolutely identical.

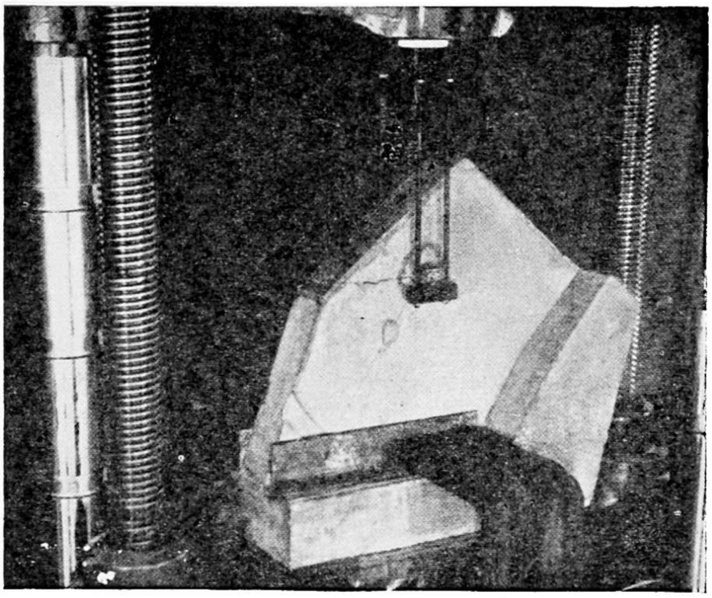

Fig. 23. Concrete model to a scale of 1/50

Conclusions

The studies presented show well how advantage can be taken of photoelasticity in spite of its only being applicable to plane elastic states of stress.

As was seen, it permits not only the choice of the best shapes but also, in the case of reinforced concrete, to define the directions of the reinforcement from the isostatics and its sectional area from the tensile stresses observed.

However, as to the design of reinforced concrete from homogeneous and elastic models there are two objections.

In the first place it should be noted that for the reinforcement to function under stresses for which it is commonly designed, it is necessary for the concrete to crack; from these cracks there will result a redistribution of stresses.

A second objection, and as a general rule a more important one, is that an elastic behaviour analysis is being considered; that is, the behaviour of the structure for loadings which cause large deformations or even ruptures are not taken into consideration.

These same objections arise, however, in relation to the usual design of reinforced-concrete structures from the results of the Theory of Elasticity and Strength of Materials, obtained on the hypothesis of the materials being homogeneous and elastic.

To reproduce perfectly the behaviour of reinforced-concrete structures it is advisable to use reinforced mortar or concrete models. In one of the studies mentioned, models of this type were additionally used.

Summary

The paper deals with some studies carried out at the Laboratório de Engenharia Civil, Ministério das Obras Públicas, Lisbon, in which use was made of the photoelastic method for model stress analysis.

The following studies are reported:

Influence of the deformability of the foundations on the behaviour of an aqueduct.

Stress distribution around the spillway openings of an arch dam.

Reinforcement of the guide walls of dam spillways to support the forces transmitted by the gates.

In each case the solution for construction resulted from the conclusions drawn from the experiments.

Reference is also made to the position of the photoelastic method in relation to the other methods of experimental stress analysis.

Résumé

Les auteurs exposent quelques études exécutées au Laboratório de Engenharia Civil, Ministério das Obras Públicas, Lisbonne, dans lesquelles la méthode photoélastique a été utilisée pour la détermination des contraintes sur des modèles d'ouvrages.

Les études exposées sont les suivantes:

Influence de la déformabilité des fondations sur le comportement élastique d'un aqueduc.

Distribution des contraintes autour des ouvertures du déversoir d'un barrage-voûte.

Ancrage des vannes aux guideaux des déversoirs de barrages.

Dans chaque cas, la solution constructive a été choisie d'après les conclusions des essais.

Les auteurs étudient également la position de la méthode photoélastique, par rapport aux autres méthodes expérimentales de détermination des contraintes.

Zusammenfassung

In der vorliegenden Arbeit werden einige Untersuchungen beschrieben, bei denen das spannungsoptische Verfahren zur Spannungsermittlung bei Modellen gebraucht wurde.

Die erwähnten Studien, die im Laboratório de Engenharia Civil, Ministério das Obras Públicas, Lisboa, durchgeführt wurden, betreffen:

Den Einfluss der Nachgiebigkeit des Baugrundes auf das elastische Verhalten einer Wasserleitung.

Den Spannungszustand um die Oeffnung des Ueberfalls einer Bogenstaumauer.

Die Verankerung der Schützen an den Leitmauern des Ueberfalls einer Bogenstaumauer.

Die konstruktive Ausbildung wurde in allen Fällen auf Grund der Versuchsergebnisse gewählt.

Es wird auch auf den heutigen Stand der spannungsoptischen Verfahren gegenüber anderen experimentellen Methoden eingegangen.

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