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Dynamic behaviour of a Gerber beam

Das dynamische Verhalten von Gerber-Trägern

Comportamento dinâmico das vigas Gerber

Comportement dynamique des poutres Gerber

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This paper deals with the problem of the free and forced vibrations of a three span Gerber beam which are studied by means of the fundamental differential equation, the results being extended to the behaviour of a Gerber bridge submitted to vibration.

In order to obtain the vibration displacement of the three span Gerber beam shown in Fig. 1 (a), a different function is usually used for each of the five elementary parts. Thus, as the number of boundary conditions increases, the frequency equations become very difficult to solve. In order to eliminate this difficulty the following method is used.

A simple beam A' F' (Fig. 1 (b)) is considered, the span, the cross section and the material of which are the same as the Gerber beam AF. Concentrated external forces equal to the periodic reactions R_B, R_E, which occur on the beam at the intermediate supports B, E of the Gerber beam during the vibration, are applied at the corresponding points B', E' of the simple beam A' F'. Next, pair moments M_C, M_D are applied at points C', D' on the simple beam A' F' corresponding to hinges C, D of the Gerber beam. When considered this way, the vibration characteristics of the Gerber beam in question become clear if the vibration of the simple beam is solved.

I. *Free vibration of a three span Gerber beam*

As stated above, the free vibration of the uniform section Gerber beam AF shown in Fig. 1 (a) can be substituted by that of the simple beam A' F' (Fig. 1 (b)) subjected to periodic support reactions R_B, R_E, and pair moments M_C, M_D. The fundamental differential equation of this simple beam can be simplified as follows

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}=g(x,t) \tag{1}$$

where: E: Young's modulus of beam material
I: Geometric moment of inertia of beam
ρ: density of material
A: sectional area of beam

$$\begin{aligned} g(x,t)=&R_B\,U_i(x-l_1)+h\,M_C\,U_m(x-l_2)+\\ &+h\,M_D\,U_m(x-l_3)+R_E\,U_i(x-l_4) \end{aligned} \tag{2}$$

where: $U_i\ (x-l)$: Unit Impulse Function

$$U_m(x-l_m)=$$
$$=\lim_{\varepsilon\to 0}\{2U(x-l_m-\varepsilon)-U(x-l_m-2\varepsilon)+U(x-l_m+2\varepsilon)-2U(x-l_m+\varepsilon)\}/\varepsilon^3 \tag{3}$$

This U_m is called the pair moment function. U: unit step function.
The general solution of this equation can be expressed as the product of the normal function and time function.

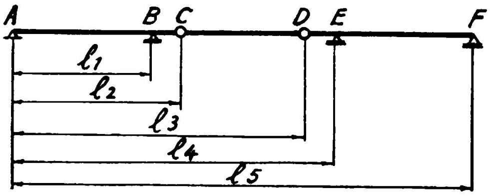

FIG. 1 (a)

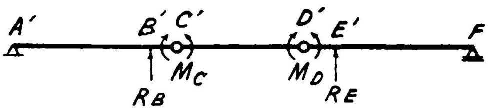

FIG. 1 (b)

$$y(x,t)-q(t)\,Y(x)$$

Using this $q(t)$, R and M become

$$R(t)=q(t)\,R_0$$
$$M(t)=q(t)\,M_0$$

where R_0, M_0 are constants corresponding to the amplitude of R, M.

Therefore the form of the normal function can be transformed into the following expression.

$$\begin{aligned} Y(x)=&\frac{Y'_{(0)}}{2\lambda}(\sinh\lambda x+\sin\lambda x)+\frac{Y'''_{(0)}}{2\lambda^3}(\sinh\lambda x-\sin\lambda x)\\ &+\frac{R_{0B}}{2EI\lambda^3}[\sinh\lambda(x-l_1)-\sin\lambda(x-l_1)].\ U(x-l_1)\\ &+\frac{h\,M_{0C}}{EI\lambda}[\sinh\lambda(x-l_2)+\sin\lambda(x-l_2)].\ U(x-l_2)\\ &+\frac{h\,M_{0D}}{EI\lambda}[\sinh\lambda(x-l_3)+\sin\lambda(x-l_3)].\ U(x-l_3)\\ &+\frac{R_{0E}}{2EI\lambda^3}[\sinh\lambda(x-l_4)-\sin\lambda(x-l_4)].\ U(x-l_4) \end{aligned} \tag{4}$$

Adopting the boundary conditions at supports B, E, F and hinges C, D, six equations are established from which the ratios of the coefficients can be determined by leaving one unknown coefficient. Thus the normal function Y (x) is determined. Next, the following frequency equation is derived by eliminating all unknown coefficients.

$$\begin{vmatrix} \sinh\lambda l_1 & -\sin\lambda l_1 & 0 & 0 & 0 & 0 \\ \sinh\lambda l_2 & \sin\lambda l_2 & \sinh\lambda(l_2-l_1)+\sin\lambda(l_2-l_1) & 0 & 0 & 0 \\ \sinh\lambda l_3 & \sin\lambda l_3 & \sinh\lambda(l_3-l_1)+\sin\lambda(l_3-l_1) & \sinh\lambda(l_3-l_2)-\sin\lambda(l_3-l_2) & 0 & 0 \\ \sinh\lambda l_4 & -\sin\lambda l_4 & \sinh\lambda(l_4-l_1)-\sin\lambda(l_4-l_1) & \sinh\lambda(l_4-l_2)+\sin\lambda(l_4-l_2) & \sinh\lambda(l_4-l_3)+\sin\lambda(l_4-l_3) & 0 \\ 0 & -\sin\lambda l_5 & -\sin\lambda(l_5-l_1) & \sin\lambda(l_5-l_2) & \sin\lambda(l_5-l_3) & -\sin\lambda(l_5-l_4) \\ \sinh\lambda l_5 & 0 & \sinh\lambda(l_5-l_1) & \sinh\lambda(l_5-l_2) & \sinh\lambda(l_5-l_3) & \sinh\lambda(l_5-l_4) \end{vmatrix} = 0 \qquad (5)$$

On the other hand, the following relation is obtained with constants A_j, ε_j.

$$q_j(t) = A_j \sin(\alpha_j t + \varepsilon_j) \qquad (6)$$

where

$$\alpha_j = \sqrt{\frac{EI}{\rho A}}\,\lambda_j^2 \qquad (7)$$

and λ_j is the j^{th} root of eq. (5). If λ_j is introduced into the normal function eq. $Y(x)$, function $Y_j(x)$ of the vibration of the j^{th} order including any one constant becomes determined.

II. *Forced Vibration with Damping*

(1) *Free periodic force*

The case when a periodic exciting force acts at a point $x = \xi$ on the beam will be considered. This corresponds to the case when a vibrator is installed at a certain point on the bridge slab. Eq. (8) will be considered as the fundamental differential equation.

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} + 2\mathscr{K}\rho A\frac{\partial y}{\partial t} = g(x,t) + F_0 e^{i\omega t} U_i(x-\xi) \qquad (8)$$

Using the j^{th} order normal function $Y_j(x)$ of the free vibration, the general vibration displacement y is developed into the following series.

Also for

$$\left.\begin{aligned} y(x,t) &= \sum_{j=1}^{\infty} q_j(t)\, Y_j(x) \\ g(x,t) &= \sum_{j=1}^{\infty} q_j(t)\, g_{oj}(x) \end{aligned}\right\} \qquad (9)$$

where g_{oj} is the value of g_o corresponding to the j^{th} order normal function.
The solution of this equation for $F = F_o \sin \omega t$ is

$$q_j(t) = \frac{F_o\, Y_j(\xi)}{\rho A M_j \sqrt{(\omega^2 - \alpha_j^2)^2 + 4\,\mathcal{K}^2 \omega^2}} \left[-\frac{\omega}{\omega_f} e^{-\mathcal{K} t} \sin(\omega_f t + \delta_1) + \sin(\omega t + \delta_2) \right] \quad (10)$$

where

$$\left.\begin{aligned} \omega_f &= \sqrt{\alpha_j^2 - \mathcal{K}^2} \\ \delta_1 &= \tan^{-1} \frac{2\,\omega_f\, \mathcal{K}}{\omega^2 - \alpha_j^2 + 2\,\mathcal{K}^2}, \quad \delta_2 = \tan^{-1} \frac{2\,\omega\, \mathcal{K}}{\omega^2 - \alpha_j^2} \end{aligned}\right. \qquad (11)$$

(2) *A constant moving exciting force*

The fundamental differential equation when a constant force moves to the right, from the left support at a constant velocity is

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} + 2\,\mathcal{K} \rho A \frac{\partial y}{\partial t} = g(x,t) + F_o\, U_i(x - vt). \qquad (12)$$

By solving this eq., the time function $q_j(t)$ was obtained

(3) *Moving periodic force.*

The fundamental differential equation for the case when a periodic force moves to the right, from the left end A of the beam at a constant velocity is

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} + 2\,\mathcal{K} \rho A \frac{\partial y}{\partial t} = g(x,t) + F_o\, e^{i\omega t}\, U_i(x - vt) \qquad (13)$$

This equation was also solved by using Laplace transformation, but the solution will not be explained in this paper as it requires too much space.

III. *Applications for special cases*

(1) *Beam of Varying Section.*

With actual Gerber bridges, the cross sections is variable.

In what follows a consideration is made on the difference between the free vibration of a beam of varying section and that of a beam of uniform section. The fundamental differential equation in this case is

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right)+\rho A\frac{\partial^2 y}{\partial t^2}=g(x,t) \tag{14}$$

As an approximate solution, Ritz's method, which is applied widely, is used. As normal function Y(x) obtained for the beam of uniform section clearly satisfies the boundary conditions, this function is used and the approximate solution of eq. (14) is expressed as

$$Y(x)=\sum_{j=1}^{m} C_j Y_j(x) \tag{15}$$

(2) *Symmetric Gerber Beams of Uniform Section*

A three span Gerber beam is normally used in the symmetric form. For this special case, the normal function can be readily simplified.

IV. *Example of Numerical Calculation*

The authors calculated the free vibration period of a Gerber beam bridge at Kyokawa with three spans as shown in Fig. 2.

First the values for a uniform section beam of equal section are obtained by eq. (5) as follows by putting $\beta_j=\lambda_j l_5$

$$\beta_1=8.75,\quad \beta_2=9.87,\quad \beta_3=12.19,\quad \beta_4=15.50,\quad \beta_5=18.38$$

The normal mode using the calculated values is as shown in Fig. 3. Next the normal modes of vibration Y(x) of beam of varying cross section are,

$$\left.\begin{array}{ll}\text{odd mode} & Y=C_1Y_1+C_3Y_3\\ \text{even mode} & Y=C_2Y_2+C_4Y_4\end{array}\right\} \tag{16}$$

The periods obtained are

$$\left.\begin{array}{ll}T_1=0.203\text{ sec.} & (T_{1u}=0.208\text{ sec.})\\ T_3=0.101\text{ sec.} & (T_{3u}=0.107\text{ sec.})\end{array}\right\} \tag{17}$$

T_{1u}, T_{3u} are respectively the periods of the free vibration of the 1st and 3rd orders of the uniform section beam.

Likewise for the case of skew symmetry

$$T_2 = 0.156 \text{ sec.} \qquad (T_{2u} = 0.165 \text{ sec.})$$

From equation (17)

$$100 \, (T_1 - T_{1u})/T_1 = 2.5$$

Thus, concerning the free vibration period of the 1st order, the error is only 2.5 % even if calculated as a uniform section.

In the above, only the mass ρA and rigidity EI of the main beam of the bridge were considered, but in actual bridges, the slab and other

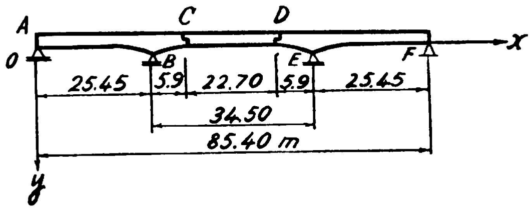

FIG. 2. Dimensions of Kyokawa Bridge

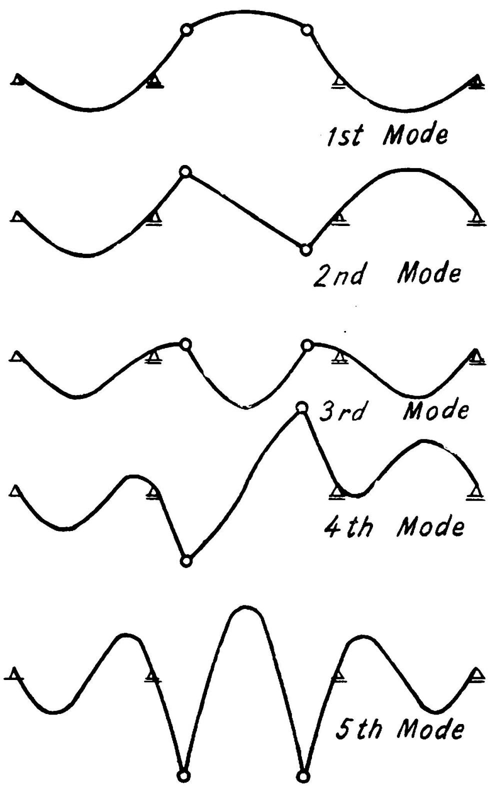

FIG. 3. Normal Modes

members cannot be neglected. Concerning the rigidity, it is considered that the slab and main beam work together as one body and resist against the bending moment when in the low stress state which occurs during the vibration. So the sectional moment of inertia I of the bridge can be calculated by the composite beam I_v . I_v was calculated by taking $n = E_s/E_c$ as 6, 7 and 8.

n was taken small owing to the fact that the value of E_c is comparatively large in the low stress state. As a result the relation between the free vibration period and n becomes as shown in Fig. 4. As expected, n = 6 shows a shorter period than n = 8, but it is recognized that the change in T due to the difference of n is very small. Assuming a composite beam with a uniform moment of inertia of mean section and taking n = 8, the free vibration period $(T_{1u})_{n=8}$ of the lst order becomes $(T_{1u})_{n=8} = 0.367$ sec. The calculated value of $(T_1)_{n=8}$ for the beam of varying section is as shown in the figure $(T_1)_{n=8} = 0.356$ sec. Thus the difference of $(T_1)_{n=8}$ and $(T_{1u})_{n=8}$ is less than 3 %. Comparing the value of eq. (17) and these values, it was found that the difference between the value calculated as a varying section with that of a uniform section with mean section was less than 5 %.

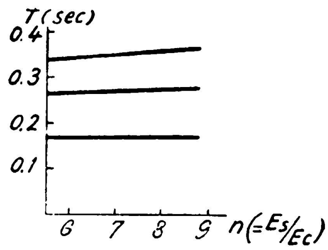

FIG. 4. T — n Curves

A vibration test was made on the Kyokawa Bridge, a Gerber deck plate girder bridge, on which numerical calculation by the above method was done and the measured value of the free period was $T_{1t} = 0.285$ sec. Comparing this with $(T_1)_{n=6} = 0.240$ sec. calculated for n = 6, the ratio of T_{1t}/T_1 is 0.84, or 84 %. Almost the same value was obtained when the difference between measured stress and calculated stress was compared. So this difference is believed to be due to some other cause.

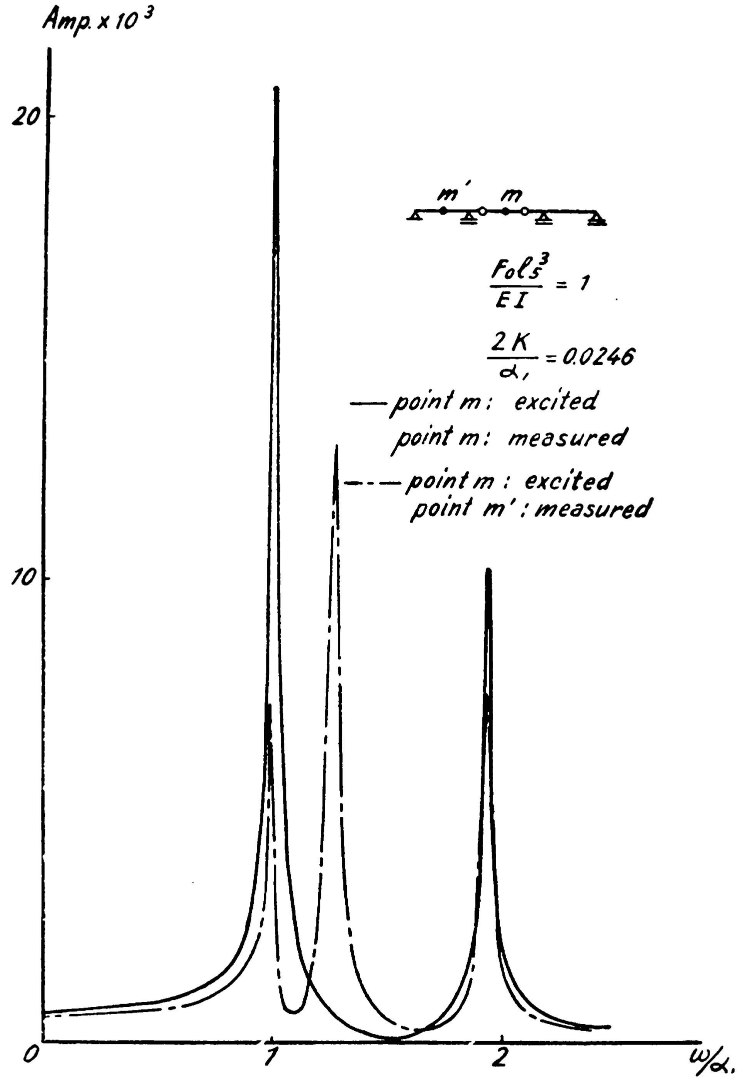

FIG. 5. Resonance Curves

Next, the resonance occuring when a periodic force acts at a certain position as in the case of the vibration excited by a vibrator is studied by the resonance curve.

If the state of the amplitude is illustrated as the function of ω/α_1, for the case of $2\mathcal{K}/\alpha_1 = 0.0246$, the resonance curve in Fig. 5 is obtained. The resonance curve by the measured values of

Kyokawa Bridge is as shown in Fig. 6. The similarity is clear when both are compared. The common characteristics which can be clearly seen from these curves is that the peak of the resonance curve appears distinctly not only for the free vibration of the 1st order, but also for the 2nd and 3rd orders.

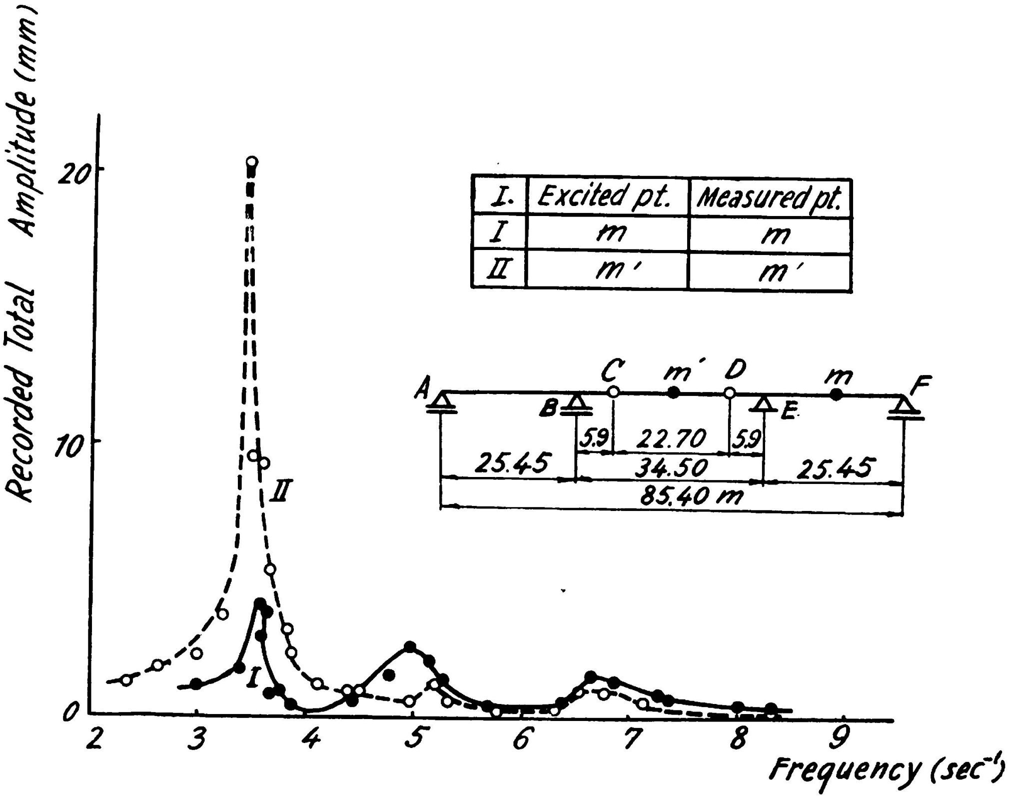

FIG. 6. Resonance Curves Kyokawa Bridge

This is the great difference with the simple beam in which the resonance of the free vibration period of the 2nd order is hardly recognizable. Thus it is presumed that the Gerber beam has a greater probability of producing resonance.

The form of the vibration of the Gerber beam subjected to a fixed periodic force was also calculated. When the anchoring beam is vibrated, the amplitude of the opposite anchoring beam is slightly smaller because of the effect of skew symmetric mode of vibration.

It must be noted that when the center of the suspended beam is vibrated, the amplitude at this point is large, from which it is readily understood that the suspended beam vibrates greatly due to the hinge.

SUMMARY

1. The form of the frequency equation of a symmetric type Gerber beam was obtained. The normal mode was shown by examples.

2. The effect of the varying section on the vibration period is very small. In the calculation mentionned in the paper, it is about 5 %.

3. If it is assumed that the slab and beam vibrate in a composite state, calculated values almost equal to the measured values are obtained. Thus the slab should be taken into consideration in the vibration of the plate girder bridge with concrete slab. The change in the period due to the change of coefficient «n» is very small.

4. The resonance curve and form of vibration caused by a periodic force were shown. From this it can be said that a Gerber beam vibrates more readily than a simple beam. It is noticed that:

a) Resonance is liable to occur not only in the 1st order, but also in the 2nd and 3rd orders.

b) Amplitude of the resonance at the center of the suspended beam is comparatively large.

Also, as compared with the continuous beam, the free vibration period is longer.

ZUSAMMENFASSUNG

1. Es wird die Form der Schwingungsgleichung eines symmetrischen Gerber-Balkens abgeleitet. An Beispielen wird die allgemeine Ableitung gezeigt.

2. Der Einfluss des veränderlichen Querschnittes auf die Schwingungsperiode ist sehr klein; im Berechnungsbeispiel dieses Aufsatzes beträgt er ca. 5 %.

3. Unter der Voraussetzung, dass Platte und Balken zusammenwirkend schwingen, sind die gerechneten Werte fast gleich den gemessenen. Daher sollte bei der Berechnung Platte und Träger zusammenwirkend gerechnet werden. Der Einfluss des «n»-Koeffizienten auf die Schwingungsperiode ist sehr klein. Es werden die Resonanzkurven und die Art der Schwingung gezeigt, die durch eine periodisch wirkende Kraft erzeugt werden. Auf Grund dieser Darstellung kann gesagt werden, dass ein Geber-Balken leichter in Schwingung gerät als ein einfacher Balken, und zwar aus folgenden Gründen:

a) Die Resonanz kann nicht nur in erster Ordnung, sondern auch in zweiter und dritter Ordnung auftreten.

b) Die Resonanzamplitude in der Mitte des eingehängten Trägers ist verhältnismässig gross.

Im Vergleiche mit dem durchlaufenden Träger ist die Periode der freien Schwingung grösser.

RESUMO

1. Estabelece-se a forma da expressão da frequência de vibração de uma viga simétrica do tipo Gerber e ilustra-se a sua forma corrente por meio de exemplos.

2. A influência da secção variável sobre o período de vibração é muito reduzida. Para o cálculo que se apresenta é de cerca de 5 %.

3. Supondo que o tabuleiro e a viga vibram em conjunto, os valores calculados e os valores medidos diferem pouco. O tabuleiro deve portanto ser tomado em consideração no estudo da vibração de uma ponte de tramos rectos com lage de betão. A influência da variação do coeficiente «n» sobre o período de vibração é muito fraca.

4. Estabelece-se a curva de ressonância e a forma de vibração devidas à acção de uma solicitação periódica. Depreende-se que uma viga do tipo Gerber entra mais fàcilmente em vibração do que uma viga simples.

Verifica-se que:

a) Fenómenos de ressonância podem ocorrer não só na 1.ª mas também nas 2.ª e 3.ª ordens.
b) A amplitude da ressonância no centro da viga suspensa é relativamente importante.

Em relação a uma viga contínua, o período de vibração livre é maior.

RÉSUMÉ

1. Les auteurs établissent la forme de l'expression donnant la fréquence de vibration d'une poutre Gerber symétrique et montrent au moyen d'exemples sa forme courante.

2. L'influence de la section variable sur la période de vibration est très faible. Dans le calcul indiqué elle est de 5 % environ.

3. Si l'on suppose que le tablier et la poutre vibrent ensemble, les valeurs calculées et les valeurs mesurées sont peu différentes. Le tablier doit donc être pris en considération dans l'étude de la vibration d'un pont-poutre à dalle en béton. L'influence de la variation du coefficient «n» sur la période de vibration est très rèduite.

4. Les auteurs établissent la courbe de résonance et la forme de la vibration dues à l'action d'une sollicitation périodique. Il en résulte qu'une poutre Gerber vibre plus facilement qu'une poutre simple. Il est est à remarquer que:

a) Des prénomènes de résonance ont lieu non seulement en 1ère phase mais aussi en 2ème et 3ème phase.
b) L'amplitude de la résonance au centre de la poutre suspendue est relativement élevée.

Par rapport à une poutre continue, la période de la vibration libre est plus longue.