Remarks about the design of reinforced concrete beams

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Objekttyp: **Article**

Zeitschrift: **IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht**

Band (Jahr): **5 (1956)**

PDF erstellt am: **22.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-6129

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Remarks about the design of reinforced concrete beams

Zur Bemessung von Stahlbetonbalken

Observações acerca do dimensionamento de vigas de betão armado

Remarques sur le calcul de poutres en béton armé

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1. *Introduction*

The remarks in this paper are the result of preliminary studies carried out with a view to revising the Portuguese reinforced concrete code and introducing into it clauses which make it possible to take full advantage of the strength of the steels available at present.

As the ruin of reinforced concrete beams may be caused by excessive cracking, deformation or rupture, some considerations about the behaviour of reinforced concrete in relation to these three features are presented.

Mention is also made of the results obtained by tests on some concrete beams reinforced with mild steel and plain twisted steel (1). These tests were carried out with a view to permitting the manufacture in Portugal of steels hardened by twisting. By means of tests the indispensable experimental basis was sought for a better understanding of the existing bibliography.

Simple bending tests were carried out on 14 beams with a span of 2.8 m. The beams were 20 cm wide and 30 cm deep. The percentages of the reinforcement were 0.47 % (8 ϕ ¼″), 0.70 % (3 ϕ ½″) and 1.59 % (3 ϕ ¾″). The loads were applied at a quarter span.

Half the beams were reinforced with mild steel (minimum yield stress of 24 kg mm^{-2}) and the other half with twisted steel (minimum conventional stress of proportionality at 0.2 ‰ of 42 kg mm^{-2}).

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(1) Mr. Augusto Tavares de Castro, Assistant Engineer of the L. N. E. C., was in charge of the tests.

Fig. 1 shows one view of the arrangement for testing the beams.

The Laboratório Nacional de Engenharia Civil has obtained steels from various origins and with different mechanical properties, on which tests are to be carried out with a view to confirming the conclusions presented.

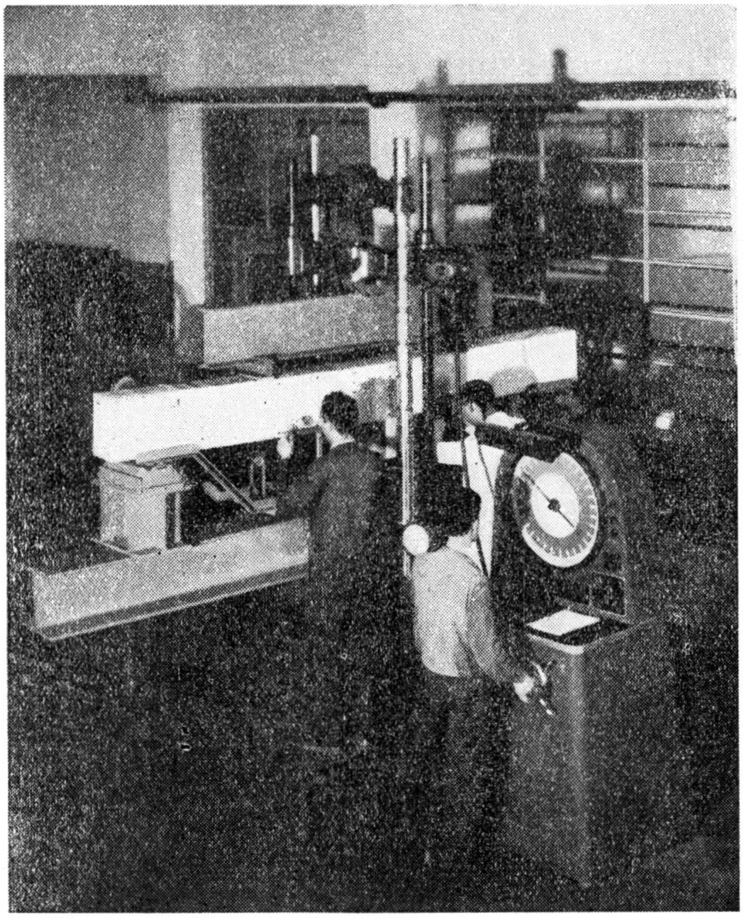

FIG. 1. Test of a beam

2. *Cracking*

Figs. 2 and 3 show some results in relation to cracking of beams observed during these tests. Fig. 2 gives the widest crack observed as a function of the stresses in the steel, calculated in accordance with the classic design method for reinforced concrete ($m = 15$). As an analysis of this figure shows, in the mild steel beams, the width of the widest cracks remained below 0.25 mm up to stresses in the neighbourhood of the yield stresses for the steel.

In the twisted steel beams, the width of the cracks of 0.25 mm corresponds to stresses in the steel lying between 30 and 40 kg mm^{-2}.

In fig. 3 the width of the cracks was plotted against the percentages of the ultimate bending moment. The analysis of this figure also shows that at 50 percent of the bending moments for twisted steel the widest cracks are 0.25 mm whilst those for mild steel beams are about half this value. When the comparison is made in this way, the cracks in the twisted steel beams are seen to be greater than those in the mild steel ones, but it must be remembered that for the same percentage of the

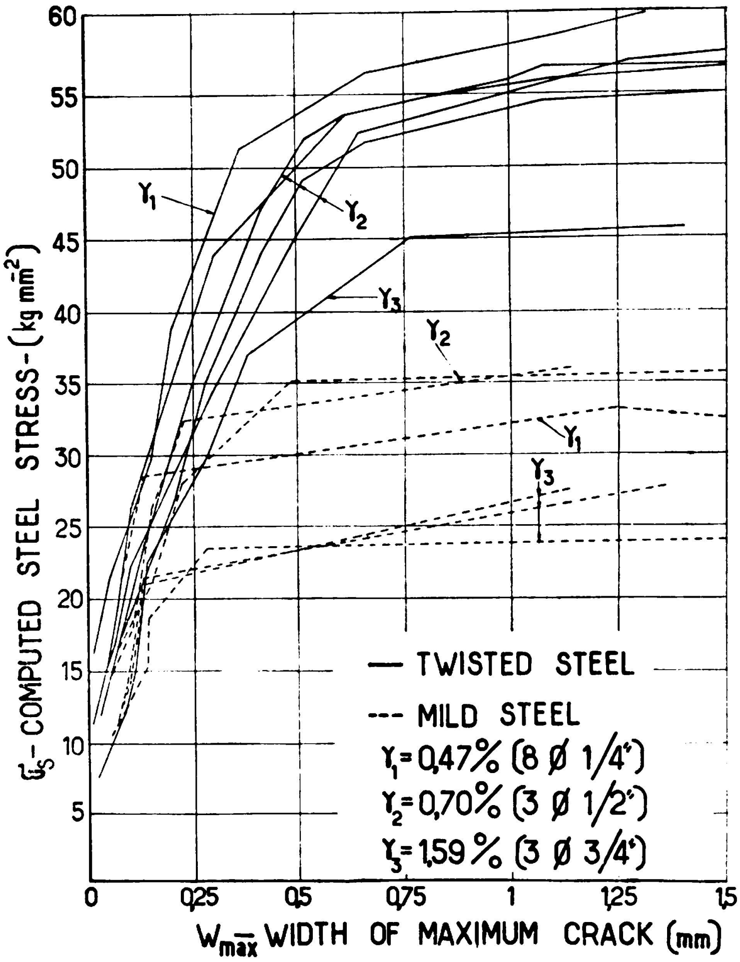

FIG. 2. Width of maximum crack in function of the computed steel stress

ultimate bending moments the stresses in the twisted steel are much greater than in the mild steel.

To interpret the results obtained in relation to cracking the method given by Rüsch (1) was followed.

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(1) Rüsch, H. — Der Zusammenhang zwischen Rissbildung und Haftfestigkeit unter besonderer Berücksichtigung der Anwendung hoher Stahlspannungen-Preliminary Publication, Fifth Congress — International Association for Bridge and Structural Engineering, Lisbon, 1956.

Fig. 4 shows a diagram in which the mean distances between the cracks, a, are taken as ordinates and as absissae the values of a parameter H defined by the expression:

$$H = \frac{\tau_{0.1}}{0.1\sigma_{cc}} \quad \frac{u}{A_{tc}}$$

where

$\tau_{0.1}$ = the mean adherence stress obtained in a pull-out test, as described in the publication mentioned and for a displacement of 0.1 mm at the free end of the bar.

$0.1\sigma_{cc}$ = the estimate of the tensile strength of concrete, which is equal to one tenth of the cube strength.

u = perimeter of the cross section of the reinforcement.

A_{tc} = the area of the tensile zone of the cross section of the beam, calculated by the classical method of design of reinforced concrete.

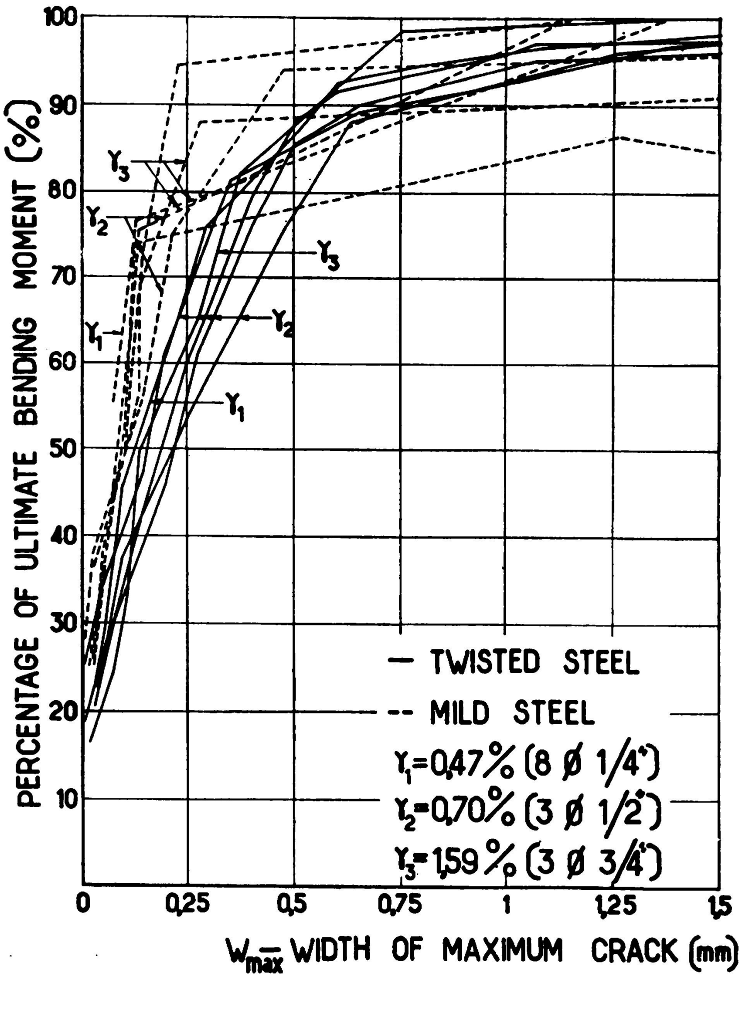

FIG. 3. Width of maximum crack in function of the percentage of ultimate bending moment

As the same quality concrete was used in all the beams, σ_{cc} was given a mean value of 250 kg cm^{-2}. For $\tau_{0.1}$ also a mean value of 25 kg cm^{-2} was taken for both mild and twisted steel. The tests showed that the adherence for twisted steel was greater than that of the mild steel for

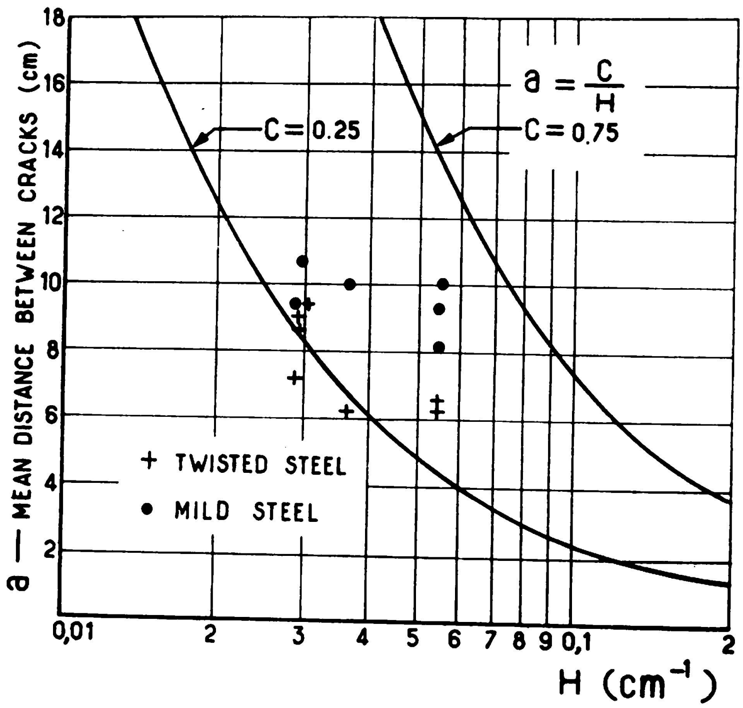

FIG. 4. Interpretation of cracking results (Rüsch's analysis)

high values of slip (inspite of the twisted steel not having fillets). For a slip of 0.1 mm the difference between the two steels was not appreciable and therefore the same value was adopted for the two types.

The analysis of fig. 4 shows that the results come out approximately between the curves given by Rüsch, the twisted steel beams showing less spacing between cracks than the mild steel ones, which is proof of greater adherence. On the other hand smaller distances between cracks do not correspond to greater values of H which is not surprising, for, as the author referred to says, for high values of H the expression deduced by Kuuskoski, $a = \frac{c}{H}$, no longer has significant value. It can also be seen in Rüsch's diagram that for values of $H > 0.05$, a no longer decreases.

To check if the results confirm the hypothesis that the maximum width of the crack, W max, is practically proportional to the mean distance, a, and to the stress in the steel, σ_s, the diagram of fig. 5 was plotted, taking as ordinates the values $K = \frac{W_{max}\, E}{a\, \sigma_s}$ and as absissae the values of the stresses calculated in the steel.

The analysis of this diagram shows that roughly for stresses at which the number of cracks is stable, K is found to be between 1 and 2, which agrees with Rüsch's results. It is to be noted, however, that the scatter of the values is relatively high and cannot be only considered as random, as the greater ratios between the areas and the perimeters of the reinforcements correspond to great values of K. This fact is also seen in fig. 2, where for the same stresses wider cracks were observed in beams having larger diameter bars. This variation, however, is small for the beams tested.

The analysis of the relation between the greatest width and the mean width of the cracks shows that the relation is approximately cons-

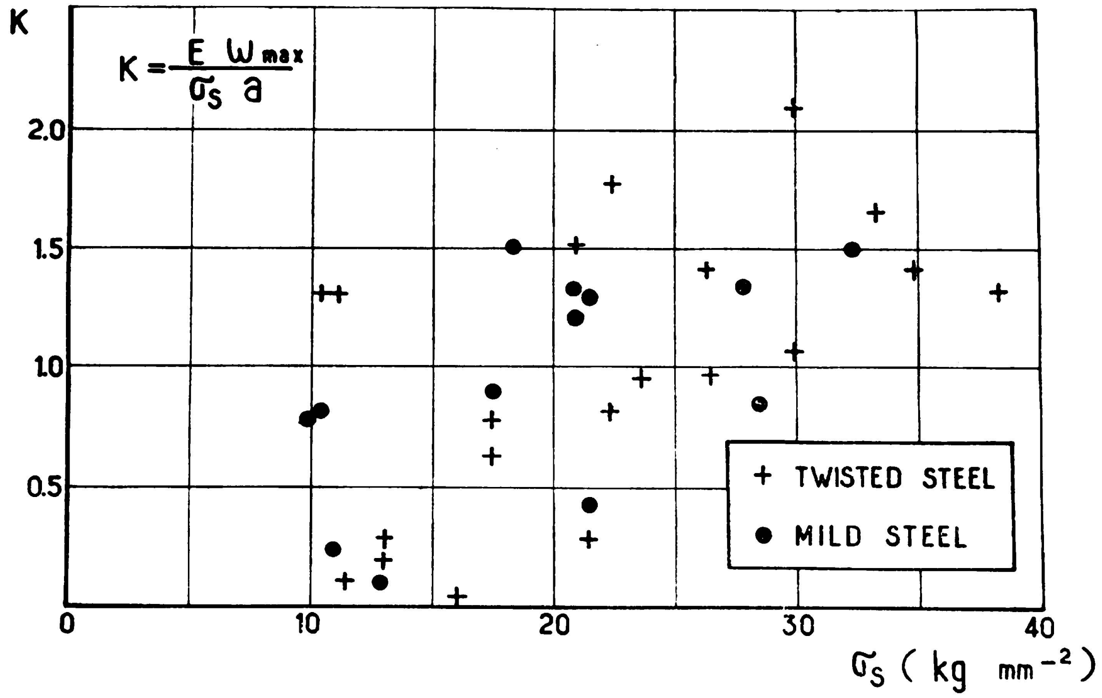

FIG. 5. Relation between spacing and maximum width of cracks

tant for the range of stresses in the steels concerned and for the different types of beams. The values obtained lie between 1.5 and 2.0.

Rüsch's diagram for the relation between a and H shows that for low values of H, H $<$ 0.05, the relation given by Kuuskoski holds and therefore the spacing between cracks increases as H decreases.

The conclusion, in accordance with the above remarks, is that for a given maximum steel stress, σ_s, in order to limit the maximum width of cracks, there must be a maximum permissible spacing of the cracks. It was also seen that for low values of H the spacing was influenced by the quocients $\frac{\tau_{0.1}}{\sigma_{cc}}$ and $\frac{u}{A_{tc}}$. Hence the limitation of spacing should be made for the different values of $\frac{\tau_{0.1}}{\sigma_{cc}}$ and by the limitation of the quocient $\frac{u}{A_{tc}}$.

For practical purposes it would be helpful to substitute a more easily calculable value for the relation $\frac{u}{A_{tc}}$. It can be shown that the expression given above $a = \frac{C}{H}$ with $C = 0.50$ is equivalent to the following $a = 0.0075 \frac{\sigma_{cc}}{\tau_{0.1}} \frac{d}{\gamma}$

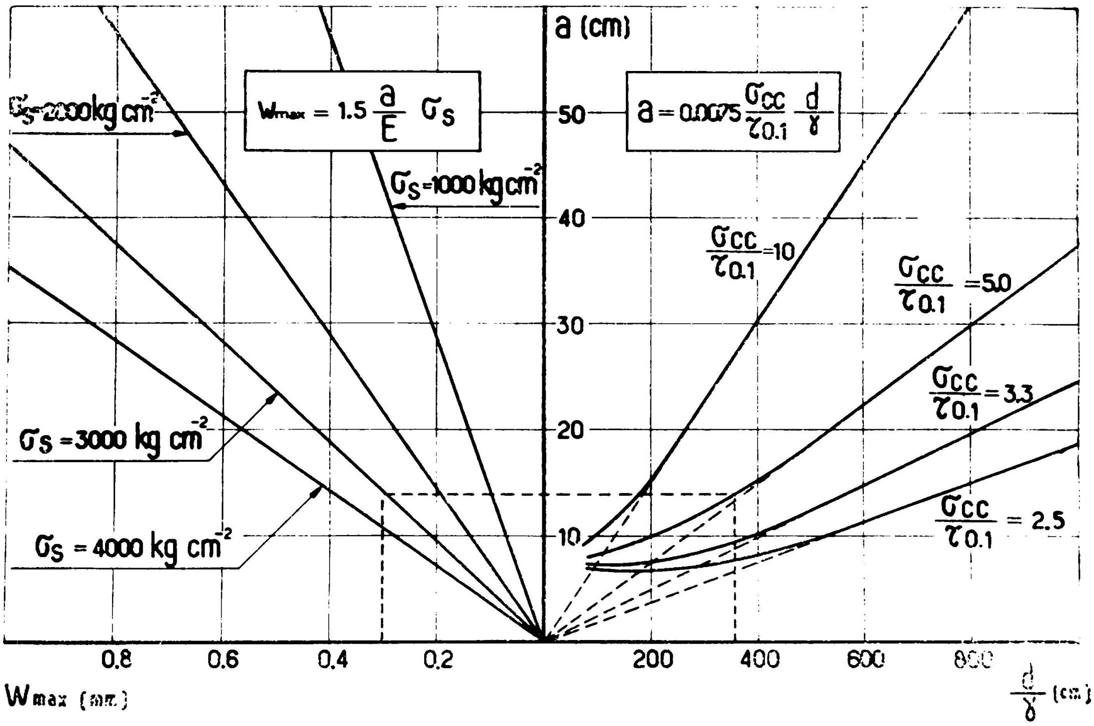

FIG. 6. Diagram connecting maximum width of cracks with $\frac{d}{\gamma}$

where

d = diameter of reinforcing bars
γ = percentage of reinforcement in relation to the area b h
(b = width of the web of the beam, h = effective depth).

When establishing the last expression it was assumed that A_{tc} could be approximately calculated by the expression, $A_{tc} = 0.6$ b h.

For the spacing to be less than a given limit it is necessary for the values of $\frac{d}{\gamma}$ (quocient of the diameter of the bars and the percentage of the reinforcement) not to exceed certain limits.

A diagram in which the values of W_{max}, a and $\frac{d}{\gamma}$ respectively are marked is plotted in fig. 6.

The lines plotted correspond to different values of σ_s and $\frac{\sigma_{cc}}{\tau_{0.1}}$. It is easy to determine by this diagram the value of $\frac{d}{\gamma}$ which should not be exceeded in order to limit the maximum crack width. Taking, for example, $W_{max} = 0.3$ mm and $\sigma_s = 3000$ kg cm^{-2}, a becomes 14 cm and for $\frac{\sigma_{cc}}{\tau_{0.1}} = 5.0$, $\frac{d}{\gamma}$ becomes approximately 400 cm.

The limitation of the values of $\frac{d}{\gamma}$ does not, from the cons-

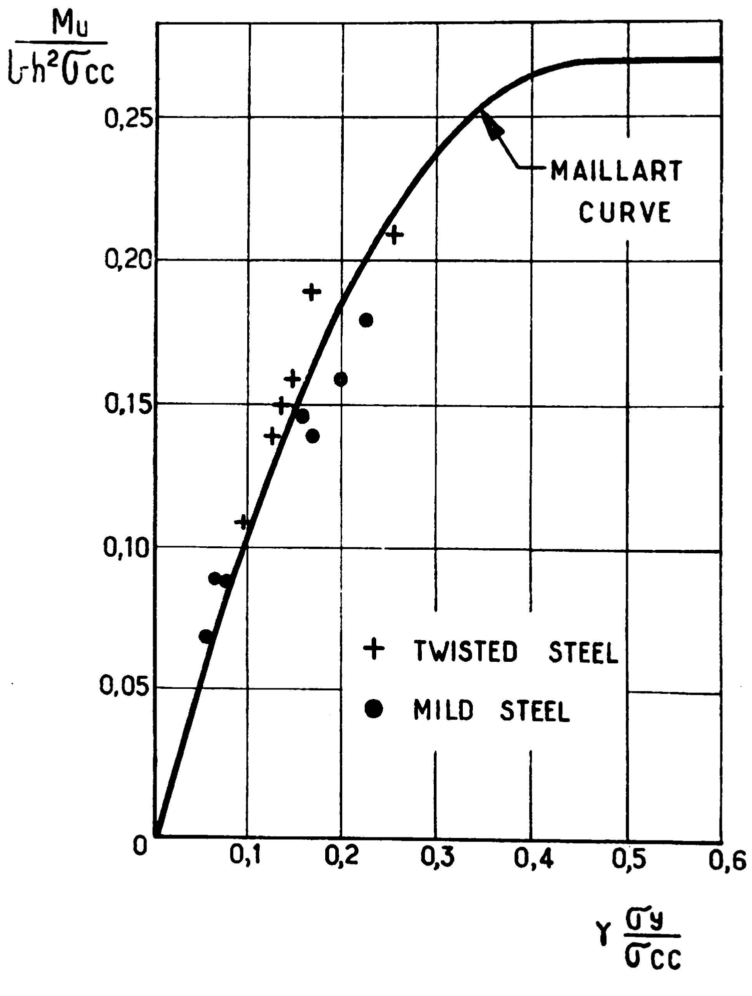

FIG. 7. Analysis of the ultimate bending moments

truction point of view, impose large restrictions and it would be a simple and efficient method to limit the width of cracks, especially when high stresses are adopted for the steel.

3. *Deformability and ultimate strength*

The analysis of the results obtained showed that the ultimate bending moments, Mu, reached were in agreement with those derived by present day theories.

In fig. 7 the values of $\frac{Mu}{b\,h^2\,\sigma_{cc}}$ were taken as ordinates and those of $\gamma \frac{\sigma_y}{\sigma_{cc}}$ as absissae where σ_y is the yield stress of the steel. As was to be expected, agreement was found between the experimental results and the curve established by Maillart, the variations being explainable by the randomness of the mechanical properties of the materials.

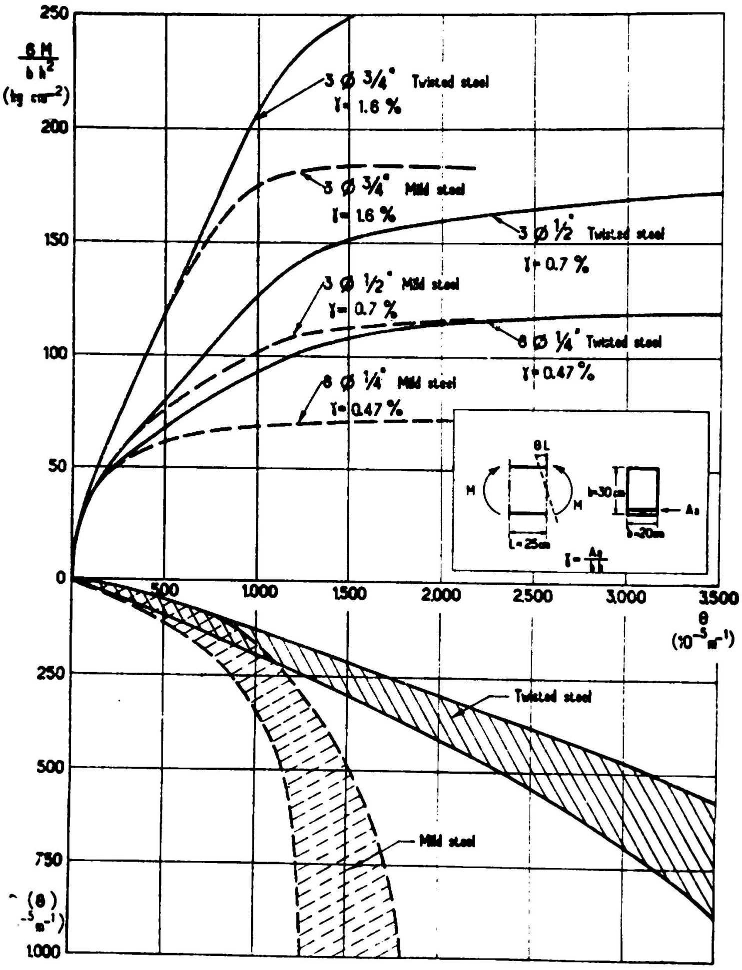

FIG. 8. Analysis of the deformability of the beams

The diagram of fig. 8 was plotted to analyse the deformability of the beams, taking as ordinates the values of $\frac{6\,M}{b\,h^2}$ (M = applied bending moment) and as absissae the mean values of the rotation per unit of length. These rotations were measured in each beam between 6 cross sections, 25 cm apart from each other.

As in each beam various measurements of rotations were made and as various beams were tested it was possible to calculate the standard deviations of the rotations and plot them against their mean values, as can be seen in the bottom part of fig. 8. The analysis of this diagram shows that for small deformations the rotations have a coefficient of variation which is practically constant and equal for beams with mild steel and twisted steel. As the rotations increase, a considerable increase in their scatter is seen for the mild steel beams, whilst the coefficient of variation for the twisted steel remained constant even for high values of rotations.

The scatter of the deformability depends to a great degree on the beam dimensions, a factor which cannot be ignored when studying this scatter.

The results given show that for reinforced structures with mild steel it would be difficult to foresee the deformation for advanced stages, near the rupture, because the values are very scattered. This does not happen in the case of high tensile steel, as the coefficient of variation remains constant for large deformations and these can consequently be foreseen with satisfactory precision.

4. *Conclusions*

The considerations presented are an attempt to demonstrate that with current cases it is possible to make full use of the capacity of the available steels without being limited by phenomena of cracking.

To avoid excessive cracking limits have to be given to the relation $\frac{d}{\gamma}$ between the diameter of the bars and the percentage of reinforcement. The limits will have to be estalished principally as a function of the working stresses adopted for the steel and the degree of adherence between steel and concrete.

In structures not needing to be watertight or which do not have to stand severe corrosive ambient conditions, the limitations to be established for $\frac{d}{\gamma}$ do not restrict the planning of structures very much. It seems to be advisable to introduce clauses relative to the limitations of these values into the codes.

As far as cracking is concerned it would be of value to proceed with experiments principally with a view to studying the influence of adherence for high values of $\frac{d}{\gamma}$.

With regard to ultimate strength, the existing theories are perfectly satisfactory and it is not considered necessary to seek further experimental confirmation. It seems however that there would be advantages in studies on the relation between the randomness of the mechanical properties and that of the behaviour of the beams. These studies would contribute towards improving the probabilistic criteria for defining the safety of reinforced concrete structures and would be of interest not only in relation to ultimate strength but particularly in relation to deformation.

SUMMARY

Some results of bending tests of concrete beams reinforced with normal and twisted steel are presented. Interpretation is made of cracking, deformability and rupture.

As for cracking it is verified that, in order to prevent too wide cracks, it is necessary to limit the ratio between the diameter of the bars and the percentage of reinforcement.

Concerning deformability, the analysis of dispersions that it was possible to make, has shown that the dispersions were considerably different for the beams reinforced with mild and twisted steel.

Finally, regarding rupture, the results obtained agree with those foreseen by the existing theories.

It seems of interest, nevertheless, to carry out studies that, taking into account the randomness of the mechanical properties of the materials, allow to foresee the statistic behaviour, not only in relation to rupture but particularly in relation to deformation.

RESUMO

Os resultados obtidos pelo ensaio à flexão de algumas vigas de betão, armadas com aço normal e aço torcido liso, são interpretados em relação à fendilhação, deformabilidade e rotura.

Em relação à fendilhação verifica-se que, para evitar largura excessiva das fendas, se torna necessário limitar a relação entre o diâmetro dos varões e a percentagem de armadura.

Quanto à deformabilidade, a análise que foi possível fazer relativamente a dispersões mostrou que estas eram bastantes diferentes para as vigas com aço macio e aço torcido.

Finalmente, no que se refere à rotura, os resultados obtidos concordaram com os previstos por teorias existentes.

Considera-se, no entanto, que haveria interesse em realizar estudos que, entrando em consideração com a aleatoriedade das propriedades mecânicas dos materiais, permitissem prever probabilìsticamente o comportamento não só em relação à rotura mas particularmente em relação à deformação.

RÉSUMÉ

On présente les résultats obtenus lors de l'essai à la flexion de quelques poutres en béton, armées avec de l'acier normal et de l'acier torsadé, et l'on fait leur interprétation du point de vue de la fissuration, de la déformabilité et de la rupture.

En ce qui concerne la fissuration, on constate que, pour éviter une largeur excessive des fissures, il faut limiter le rapport du diamètre des ronds au pourcentage de renforcement.

Pour la déformabilité, l'analyse que l'on a pu faire par rapport aux dispersions a montré que celles-ci sont bien différentes pour les poutres armées d'acier normal et d'acier torsadé.

Finalement, pour la rupture, les résultats des essais sont en accord avec les résultats fournis par les théories existantes.

On considère que, tenant compte du caractère aléatoire des propriétés mécaniques des matériaux, il serait toutefois intéressant de faire des études pour la prévision probabiliste du comportement, en ce qui concerne non seulement la rupture mais surtout aussi la déformabilité.

ZUSAMMENFASSUNG

In der vorliegenden Arbeit werden die Ergebnisse der Biegeversuche einiger Stahlbetonbalken (mit Normal- und gedrilltem Stahl bewehrt) in bezug auf Rissbildung, Verformung und Bruch dargestellt.

Zur Frage der Rissbildung ist festzustellen, dass der Quotient zwischen Durchmesser der Rundstäbe und Bewehrungsgehalt begrenzt werden muss, um eine unzulässige Breite der Risse zu vermeiden.

Soweit es in der Verformungsuntersuchung möglich war, eine Analyse der Streuungen durchzuführen, zeigte sich, dass diese Streuungen für Normal- und gedrillten Stahl ziemlich unterschiedlich sind.

Was den Bruch anbetrifft, stimmten die Ergebnisse der Versuche mit den auf Grund der laufenden Theorien zu erwartenden Werte überein.

Es wäre jedoch interessant, weitere Versuche durchzuführen, welche, unter Berücksichtigung der Streuung mechanischer Eigenschaften der Baustoffe, eine Wahrscheinlichkeitsvoraussage des Verhaltens der Balken in bezug auf Bruch und ganz besonders auf Verformung ermöglichen.