Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 6 (1960)

Artikel: Fatigue design and endurance of metal stuctures

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DOI: https://doi.org/10.5169/seals-7017

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Fatigue Design and Endurance of Metal Structures

Calcul de la résistance à la fatigue des ouvrages métalliques

Ermüdungsberechnung und Dauerfestigkeit von Metallbauten

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In his General Report Professor Stüssi discusses the problem of the reduction of the endurance limit under random sequences of variable stress-amplitudes of a stress-spectrum including amplitudes below as well as above the conventional constant-amplitude endurance limit. On the basis of preliminary test-results performed to check the conclusion reached in my paper in the "Preliminary Publication" [1] that the stress-amplitudes below the conventional endurance limit produce significant damage because of their interaction with the stress-amplitudes above this limit, he attempts to show that this conclusion is not confirmed. He also suggests that the strain-hardening effect produced by high stress-amplitudes might be responsible for the specific and, as he says, "unexpected" test results reported in my paper. This latter suggestion is not convincing, as according to all evidence strain-hardening should be expected to raise the endurance limit rather than to lower it.

However, before discussing the points raised, particularly the discrepancy between Professor Stüssi's and my own test results, I should like to present new results of random-fatigue tests performed on a much more widely used metal than the SAE 4340 high-strength steel used in the first test series. The purpose of these tests was the same as that of the previously reported tests: to demonstrate that stress-amplitudes below the conventional endurance limit produce significant damage if mixed with a small number of stress-amplitudes above this limit, so that the design-significance of the endurance limit obtained in constant-amplitude tests is, at least, problematic.

The material used was ASTM-A-285 weldable mild carbon steel with nominal ultimate tensile strength $\sigma_u = 53{,}000$ psi, estimated conventional endurance limit in bending $S_E = 28{,}000$ psi $= 0.53\ \sigma_u$ and yield limit in tension of 34,000 psi. The testing procedure is similar to that described in my paper in the "Preliminary Publication"; the stress-spectrum applied is exponential with respectively 0, 1, and 2 stress-amplitudes below the conventional endurance limit. Table I shows the actual test results in ascending order for the three stress-spectra applied; S_1 denotes the lowest, S_5, S_6 and S_7 the highest stress-amplitudes in terms of σ_u of the spectrum, p indicates the ratio of the lowest and of the highest stress-amplitude cycles in the total number of cycles applied. It should be noted that the nominal ultimate strength σ_u and the yield stress refer to uni-axial tension tests while the fatigue tests were performed in rotating bending; thus the maximum stress-amplitude applied of $0.95\ \sigma_u$ is, in fact, only slightly above the expected yield stress in bending which for round specimens is at least 40 percent above the yield stress in tension.

Table I. Number of Cycles to Failure in Thousands for ASTM-A-285 Steel Specimens under Randomized Exponential Load Distributions of Slope $h = 17.3$

	Spectrum No. 1	p	2	p	3	p
Spec.	$S_1 = 0.55$	0.82218	$S_1 = 0.45$	0.82200	$S_1 = 0.35$	0.822000
No.	$S_5 = 0.95$	0.00100	$S_6 = 0.95$	0.00018	$S_7 = 0.95$	0.000026
1	622.5		2,408.5		2,068.7	
2	646.3		2,743.0		3,879.7	
3	696.4		3,166.2		5,937.1	
4	816.5		4,356.9		6,497.4	
5	877.6		5,145.7		6,914.4	
6	971.4		5,747.0		7,076.0	
7	1,453.0		6,625.1		7,230.3	
8	1,705.4		7,606.4		8,055.1	
9	1,966.6		8,615.8		9,020.3	
10	2,019.5				9,342.3	
11					9,648.4	
12					9,871.9	
13					11,411.7	
14					20,111.0	
15					22,257.8	
V'_{OR}	1,294.0		5,827.7		10,561.0	
N'_{OR}	350.0		1,000.0		1,000.0	

The applied load spectra have identical slopes of 17.3 and identical highest stress-amplitudes $0.95\ \sigma_u$. They are therefore practically identical with Spectrum A (most severe) applied in the tests on SAE 4340 steel. They differ by their relation of the lowest stress amplitude S_1 to the conventional endurance limit $S_e = 0.53\ \sigma_u$. V'_{OR} and N'_{OR} denote, respectively, the "characteristic" life (probability level of failure $P = 1 - 1/e$) and the "minimum" life ($P = 0$) obtained by extreme value theory interpretation of test results [2].

The test results are evaluated in Table II which, in the last column, shows the effect, on the random fatigue life under the given stress-amplitude spectrum, of including or of not including stress-amplitudes below the conventional endurance limit. Thus the inclusion of two stress-levels below this limit reduces the fatigue life by a factor of almost four, which is of the same order of magnitude as that observed for the same spectrum on SAE 4340 steel.

Table II. Compensated Fatigue Life for ASTM-A-285 Steel Specimens for Tests with and without Inclusion of Stress Levels below the Endurance Limit $S_E = 0.53\ \sigma_u$

Spectrum No.	h	No. of stress levels below S_E	S_1	V'_R (mode) in thousands	Compensating factor	Compensated life (mode) in thousands
1	17.3	0	$0.55\ \sigma_u$	1,294	$1/(1-p_1-p_2)$ $=30.36$	39,932
2	17.3	1	0.45	5,828	$1/(1-p_1)=5.62$	32,757
3	17.3	2	0.35	10,561	1.00	10,561

The results on ASTM-A-285 Steel therefore confirm the conclusions reached previously for SAE 4340 Steel: application of stress-levels below the conventional endurance limit produces significant fatigue damage. The damage is the more pronounced the larger the proportion of stress-amplitudes below this limit.

The key to the discrepancy between Professor Stüssi's and my own test-results is in this last conclusion, which confirms the trend established by the results on SAE 4340 Steel (there is a misplaced decimal point in the last figure of Spectrum C 1; the number should be 330.0 instead of 33.0). Damage at stress-amplitudes below the conventional endurance limit resulting from interaction with high stress-amplitudes becomes pronounced only when the damage directly produced by the latter is very small. When the variable-amplitude fatigue life is essentially determined by stress-amplitudes above the endurance limit, interaction effects become insignificant.

None of Professor Stüssi's test programs contains stress-amplitudes below the conventional endurance limit; the lowest amplitude of program I of $0.546\ t/cm^2$ is practically *at* rather than *below* the endurance limit ($0.55\ t/cm^2$). Thus the tests are not designed to discover possible damage *below* the endurance limit and are, in this respect, not really comparable to my own test. With respect to damage *at* the endurance limit by stress-amplitudes exceeding it, comparison of the mean values Δn for programs I and II seems to support Professor Stüssi's implied conclusion that no damage is produced *at* this limit: the sum of Δn for program I is $1684{,}1 \times 10^3$, while the sum of Δn for program II plus the (non-applied) number of cycles at $0.546\ t/cm^2$ would be 1630.9×10^3 and thus clearly within the scatter-range of program I.

It should, however, be noted that in Professor Stüssi's tests the percentage of high stress-amplitudes is very high in comparison with my own tests: almost 18 percent of the stress-cycles are at the highest two amplitudes, producing directly roughly 86 percent of the total damage according to the linear damage law, compared to much less than 3 percent of stress cycles at these two amplitudes producing directly less than 2 to 5 percent of the total damage in my tests. Whatever damaging interaction effects between the high stress-amplitudes and the endurance limit might exist, they can hardly be noticeable when the fatigue life is essentially determined by the highest two stress-levels alone. The fact that the linear damage law is applicable in the interpretation of Professor Stüssi's tests shows, in fact, that stress-interaction effects are unobservable; this does not necessarily mean that they are non-existent, but only that Professor Stüssi's test programs have not been designed to bring these effects out. In all random fatigue tests performed at Columbia University in recent years it could be clearly shown that the linear damage law $\Sigma(\Delta n_i/n_i)=1$ is approximately valid only when all stress-amplitudes are relatively high and fatigue lives relatively short ($\leqq 10^6$ cycles); the wider the range between the highest and lowest stress amplitudes and the smaller the percentage of the former, the larger the deviation from the linear law and the stronger the stress-interaction effect [3].

Professor Stüssi's statement in his General Report that my test results are not confirmed by his preliminary tests could therefore only be understood to mean that his specific test programs are not quite relevant to the purpose of my tests, and that therefore our results are not comparable. His results show as clearly that there are conditions under which the stress-interaction effect in fatigue is irrelevant, as mine show that there are other conditions under which this effect is highly significant. Our results are thus neither incompatible, nor does any difference between them prove anything beyond the fact that test-conditions have been sufficiently different to produce different results.

With respect to the test conditions it appears, however, that the exponential stress-amplitude spectra underlying my tests with their very small percentages of high amplitude stress cycles are closer to real conditions of structures under variable loads than the stress-programs selected by Professor Stüssi. In fact they have been derived from load records of airplane wings in operational flight. Therefore my conclusion that the constant-amplitude endurance limit is a fatigue design and performance characteristic of dubious value is not affected by the results of Professor Stüssi's tests.

References

1. A. M. Freudenthal, Prelim. Publication, Sixth Congress IABSE, Stockholm, 1960, p. 27—33.
2. A. M. Freudenthal and E. J. Gumbel, Advances in Applied Mechanics, vol. 4, p. 117, Academic Press, New York 1956.
3. A. M. Freudenthal and R. A. Heller, Journal Aeron. Sciences, vol. 26 (1959), p. 431—442.

Summary

On the basis of preliminary test-results performed to check the conclusion reached in the author's paper in the "Preliminary Publication" that the stress amplitudes below the conventional endurance limit produce significant damage because of their interaction with the stress-amplitudes above this limit, Professor Stüssi attempts to show that this conclusion is not confirmed. New results of random-fatigue tests performed on ASTM-A-285 weldable mild carbon steel confirm the conclusions reached previously for SAE 4340 Steel: Application of stress-levels below the conventional endurance limit produces significant fatigue damage.

The author states that his own tests and those of Professor Stüssi based on different specific test programs are not comparable.

Résumé

Se fondant sur les résultats d'essais préliminaires, effectués dans le but de contrôler les conclusions que l'auteur a avancées dans la «Publication Préliminaire» (conclusions indiquant que des contraintes d'amplitude inférieure à la résistance classique à la fatigue peuvent causer d'importants dommages, à cause de leur interaction avec des contraintes d'amplitude supérieure à cette limite) le Prof. Stüssi essaie de prouver que ces conclusions ne sont pas confirmées. Des nouveaux essais effectués sur l'acier doux, soudable ASTM-A-285 confirment les conclusions tirées des résultats obtenus pour l'acier SAE 4340 et qui sont: l'application de contraintes d'amplitude inférieure à la résistance classique à la fatigue cause d'importantes dégradations par fatigue.

L'auteur constate que ses résultats d'essais et ceux du professeur Stüssi ne peuvent pas être comparés par ce qu'ils se fondent sur des programmes spécifiques différents.

Zusammenfassung

Auf Grund von ersten Versuchsergebnissen zur Überprüfung der Schluß-folgerung, zu der der Autor in seinem Beitrag im «Vorbericht» gekommen ist,

daß nämlich Spannungsamplituden unterhalb der herkömmlichen Ermüdungsgrenze beträchtlichen Schaden anrichten wegen ihrer Wechselwirkung mit den Spannungsamplituden über dieser Grenze, versucht Prof. Stüßi zu zeigen, daß diese Schlußfolgerung nicht bestätigt wird. Neue Ergebnisse von Ermüdungsversuchen unter veränderlichen Spannungswerten, ausgeführt an schweißbarem, normalem Baustahl ASTM-A-285, sollen aber die früher gezogenen Schlußfolgerungen für SAE 4340-Stahl bestätigen: Anwendung von Spannungsstufen unter der konventionellen Dauerfestigkeitsgrenze ergeben einen bemerkenswerten Ermüdungsschaden.

Der Autor stellt fest, daß seine Versuchsresultate und diejenigen von Prof. Stüßi, die von verschiedenen spezifischen Versuchsprogrammen ausgehen, nicht vergleichbar sind.