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Elastic-Plastic and Limit Analysis of Non-Homogeneous Arched Bridge Structures

Etats élasto-plastiques et état limite des constructions de ponts non-homogènes, en particulier de ponts voûtés

Elastisch-plastische Zustände und Grenztragvermögen von nichthomogenen Brückenkonstruktionen, insbesondere Bogenscheibenbrücken

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1. In the theory of bridges, arched bridge structures have often found practical application (Fig. 1). In the present paper, a method is proposed which enables the states of stress and strain in such structures to be determined. The possibility of taking certain types of non-homogeneity of the material into consideration is also provided. The non-homogeneity may be due, e.g., to a variable amount of reinforcement.

The method consists in the application of an appropriate conformal mapping. By means of a suitable mapping function, the system under consideration is transformed into a circular ring segment (Fig. 2) and the problem is analysed in this auxiliary system. This is comparatively easy with the boundary

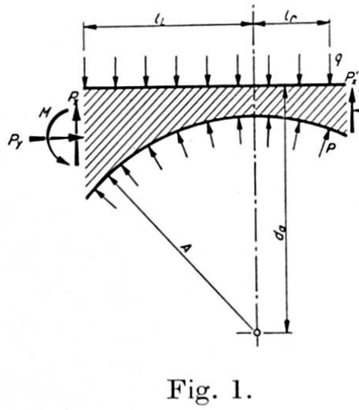

Fig. 1.

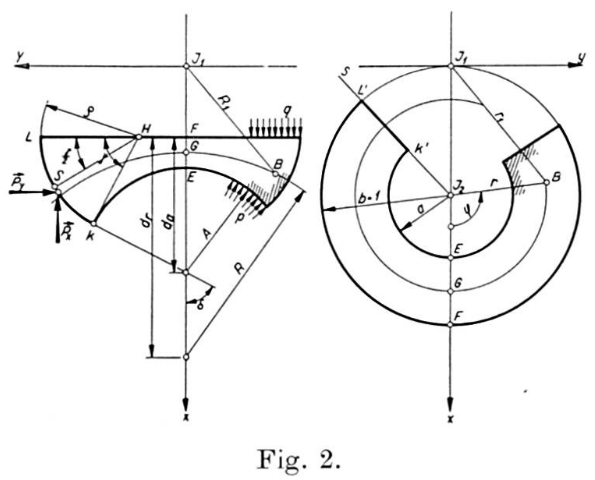

Fig. 2.

conditions — for the inner and outer edges — assumed, these being particularly simple. The solution is then retransformed into the original system (Fig. 1).

In this way both the purely elastic and the elastic-plastic states may be analysed. It is also possible to determine the ultimate load-carrying capacity of such structures.

Fig. 2 shows the system under consideration. The bridge is assumed to be asymmetric; the symmetric form can, of course, be readily obtained as a particular, simpler case of the more general, asymmetric form. Such symmetric systems, which are theoretically somewhat easier to handle, are generally met with in practice.

The load is assumed to be uniformly distributed over the upper and lower edges. The load p on the lower edge may, of course, be assumed to be zero, the only load being the load q acting on the upper surface (roadway). This results in a further simplification of the computation procedure.

2. The analytic function

$$Z = X + i\,Y = f(z) = \frac{C^2}{\bar{z} \pm h} \tag{2.1}$$

transforms the original plane O into the inverted plane I. In this transformation, circles in the O-plane are mapped into circles in the I-plane. The following simple geometrical relations are valid

$$\begin{aligned} \Phi_1 &= \varphi_1, & r_1 &= \frac{1}{R_1}, \\ \frac{1}{R_1} &= \sqrt{r^2 + 2\,h\,r\cos\varphi + h^2}, & \varphi_1 &= \operatorname{arc\,tg}\frac{r\sin\varphi}{r\cos\varphi + h}. \end{aligned} \tag{2.2}$$

The stress function Ω in the original system O corresponds to the stress function ω in the inverted system I. These functions are related in the following simple manner:

$$\omega = \frac{1}{R_1^2}\,\Omega\,(R_1, \Phi_1) = r_1^2\,\Omega\left(\frac{1}{r_1}, \varphi\right). \tag{2.3}$$

From this equation, we know the relation between the stress field in the original system O and that in the inverted system I.

This relation is particularly simple if the stresses in the original system are expressed in a curvilinear system of coordinates (R, Φ) consisting of two families of orthogonal circles (Fig. 1). Then we have

$$\begin{aligned} \sigma_R &= \sigma_r\,(r^2 + 2\,h\,r\cos\varphi + h^2) + 2\,M', \\ \sigma_T &= \sigma_t\,(r^2 + 2\,h\,r\cos\varphi + h^2) + 2\,M', \\ \tau_{RT} &= -\tau_{rt}\,(r^2 + 2\,h\,r\cos\varphi + h^2), \end{aligned} \tag{2.4}$$

where
$$M' = \omega - \frac{\partial\omega}{\partial r}[r + h\cos\varphi] + \frac{\partial\omega}{\partial\varphi}\,h\,\frac{1}{r}\sin\varphi,$$

with the notations of Figs. 1 and 2.

3. The elastic problem is solved in a simple manner. The stress function assumes, in the inverted system I, under the conditions described, the form

$$\omega = a_0 + b_0 \ln r + d_0 r^2 \ln r. \tag{3.1}$$

Hence, the stresses in the original system O are readily obtained as

$$\begin{aligned} \sigma_R &= 2a_0 + b_0(2\ln r - 1 + r^{-2}) + d_0(2\ln r + 1 - r^2), \\ \sigma_T &= 2a_0 + b_0(2\ln r - 3 - 4r^{-1}\cos\varphi - r^{-2}) + d_0(2\ln r + 3 + 4v\cos\varphi + r^2), \end{aligned} \tag{3.2a}$$

$$\tau_{RT} = 0. \tag{3.2b}$$

There are three unknown constants in these expressions; these constants should be so chosen as to satisfy the boundary conditions in question. The structure is considered to be elastically clamped along the lateral edges. This type of support is characterized by a clamping moment M and a reaction force P (Fig. 1). It is evident that the curvilinear net of coordinates coincides with the principal stress trajectories.

4. The problem of ultimate load-carrying capacity. The set of equilibrium equations and boundary conditions has to be completed with the yield condition

$$(\sigma_R - \sigma_T)^2 + 4\tau_{RT}^2 = 4[K(R_1, \Phi_1)]^2. \tag{4.1}$$

The material of the bridge may be homogeneous or non-homogeneous. In the general case of non-homogeneity, the solution cannot be obtained in a closed form and should be sought for by means of one of the numerical methods (the method of characteristics, for instance).

There exists, however, the possibility of making use of a certain circumstance, which was already pointed out in some of our previous papers. This consists in the following:

We can introduce into the analysis a particular type of non-homogeneity K_{I}, enabling us to find the corresponding solution in a closed form.

It is evident that such a non-homogeneity does not necessarily reflect the actual conditions; however, it can be shown that there may exist more of such types of non-homogeneity which lead to simple closed-form solutions.

Let us denote them by the symbols $K_{\mathrm{I}}, K_{\mathrm{II}}, K_{\mathrm{III}}, \ldots$

If one of these types of non-homogeneity represented the actual mechanical properties of the system considered, the problem could be considered to be solved. However, such a case will, in general, be only exceptional.

Another approach is then possible. Since the curvilinear net of coordinates coincides with the principal stress trajectories [cf. Eq. (3.2b)], the yield condition (4.1), in such a particular case, is seen to be linear. Then a linear combination of the possible types of non-homogeneity

$$K(r,\varphi) = \sum_i \lambda_i K_i(r,\varphi), \qquad i = \mathrm{I}, \mathrm{II}, \ldots \tag{4.2}$$

may be considered. The parameters $\lambda_{\mathrm{I}}, \lambda_{\mathrm{II}}, \ldots,$ can now be chosen in such a manner as to approach the actual conditions to the best possible extent. These conditions are, for example, those of a specific type of non-homogeneity or those of homogeneous properties of the structure, depending upon the manner in which the original problem was stated.

Thus, for the particular problem studied in the present paper, solutions were found for four different types of non-homogeneity: $K_{\mathrm{I}}, K_{\mathrm{II}}, K_{\mathrm{III}}, K_{\mathrm{IV}}$. It follows that a linear combination may be used which can be expressed in the following symbolic manner:

$$K_{\mathrm{V}} = \pm \kappa^2 K_{\mathrm{I}} \pm \lambda^2 K_{\mathrm{II}} \pm \mu^2 K_{\mathrm{III}} \pm \nu^2 K_{\mathrm{IV}}. \tag{4.3}$$

The above method was used to solve the problem under consideration. The state of stress $(\sigma_R, \sigma_T, \tau_{RT})$ was found at every point of the structure in such a manner as to satisfy the boundary conditions required.

The moment M characterizing the elastic clamping and the reaction force P were also found.

The critical load intensity for which the load-carrying capacity of the structure is exhausted was also determined.

5. The question now arises what type of non-homogeneity best describes the actual conditions. In reinforced concrete bridges, the amount of reinforcement will — in general — increase when the crown is approached (the maximum being attained at the crown itself). Then the mechanical properties will exhibit a corresponding increase of elastic and plastic moduli.

This corresponds, in a relatively satisfactory manner, to one of the types of non-homogeneity considered in the present paper (K_{III}).

The other type corresponds, approximately, to homogeneous structures (K_{II}).

Between these two limiting types, other types may be introduced by selecting appropriate values for the parameters κ, λ, μ, ν.

This choice of solutions enables the non-homogeneity function K_{V} to be adapted to various possible practical cases, in a relatively satisfactory manner.

A method for the determination of the upper and lower bounds of the solutions thus obtained will be demonstrated in a separate paper. This is of considerable importance as a means for estimating the accuracy of the solution and formulating variational problems.

It seems that the method proposed may be useful for solving actual practical problems. It should be mentioned that it may also be extended to other problems of elastic-plastic equilibrium and plastic flow.

References

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Summary

In the theory of bridges, two-dimensional vaulted systems are often analysed, arched bridge structures being one of the possible and frequently used practical applications. In this contribution, a method of analysis is suggested which enables the states of stress and strain of such structures to be assessed, and at the same time it is demonstrated that certain types of non-homogeneity of the material may be taken into account.

The method consists in transforming the system under consideration into a concentric ring by means of a simple mapping function. The solution is first found in this auxiliary system and then retransformed into the original system.

The analysis deals both with purely elastic and with elastic-plastic states. In addition, a method is indicated for determining the ultimate load-carrying capacity of such structures.

Résumé

Dans la théorie des ponts on étudie souvent des systèmes voûtés bidimensionnels, les ponts en arc étant une des applications les plus usuelles. Cette contribution propose une méthode de calcul qui permet de déterminer l'état de tension et l'état de déformation de tels ouvrages, tout en offrant la possibilité de tenir compte de la non-homogénéité du matériau du système.

Cette méthode consiste à transformer le système considéré en un segment circulaire concentrique, à l'aide d'une simple fonction de transformation conforme. La solution est recherchée dans ce système auxiliaire.

L'étude traite aussi bien d'états purement élastiques que d'états élasto-plastiques. De plus, on peut déterminer l'état limite et la capacité portante de telles constructions.

Zusammenfassung

In der Brückenstatik werden oft scheibenartige Tragsysteme untersucht, wobei insbesondere Bogenscheibenbrücken schon vielfach praktische Anwendung gefunden haben. Es wird eine Methode vorgeschlagen, die es erlaubt,

den Spannungs- und Formänderungszustand derartiger Tragkonstruktionen zu ermitteln, wobei gleichzeitig die Möglichkeit gegeben wird, deren Nichthomogenität in Betracht zu ziehen.

Die Methode besteht darin, durch Einführung einer einfachen Abbildungsfunktion das untersuchte System auf ein konzentrisches Kreissegment konform abzubilden und in diesem Hilfssystem die Lösung zu suchen.

Es werden sowohl rein elastische als auch elastisch-plastische Zustände untersucht. Außerdem wird gezeigt, wie das Grenztragvermögen (die Grenzlast) derartiger Konstruktionen ermittelt wird.