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Autor: Dbrowski, Ryszard

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On the Problem of Aerodynamic Stability of Suspension Bridges

Contribution au problème de la stabilité aérodynamique des ponts suspendus

Zum Problem der aerodynamischen Stabilität der Hängebrücken

RYSZARD DĄBROWSKI
Dr. Ing., Gdańsk

In connection with the paper by Mr. DELCAMP I take this opportunity to draw your attention once more to the much discussed problem of aerodynamic stability of suspension bridges. This should be done with particular reference to earlier works by HIRAI, presented at the IABSE Congress in Lisbon [1], and by VLASOV, prepared for the Congress of Applied Mechanics in Brussels and republished in the second edition of his well known book [2].

The differential equations of the problem — after differentiation with respect to time and omission of some unimportant terms — can be written according to [2] as follows

$$E J_x \eta^{IV} - H \eta'' - \frac{\gamma F}{g} \omega^2 \eta'' + (M_y \varphi)'' = 0, \tag{1}$$

$$M_y \eta'' + E J_\varphi \varphi^{IV} - \left(G J_d + H \frac{b^2}{4}\right) \varphi'' - \frac{\gamma F r^2}{g} \omega^2 \varphi + k u b^2 \frac{v^2}{2g} \varphi = 0, \tag{2}$$

where η, φ are vertical displacement and rotation of the bridge section respectively.

Eqs. (1), (2) represent equilibrium conditions of vertical forces and torsion moments respectively. Stiffness terms and inertia forces (containing angular frequency ω) are well known, H denotes here total force in two cables. Aerodynamic forces are taken into account in the last term of Eq. (2) according to the negative slope theory:

$$m = C_T(\varphi) u b^2 \frac{v^2}{2g}, \qquad C_T = k \varphi,$$

where denotes: C_T = torsion coefficient, u = air specific weight, b = width of the bridge section, v = wind velocity, g = gravitational acceleration and k = negative slope constant.

Corresponding equations in [1] are in essentially similar — there are only introduced to Eq. (2): one term due to structural damping and a hypothetical alternating torsion moment corresponding to KÁRMÁN trails action.

In comparison with previous works by BLEICH [3] and STEINMAN [4] the aerodynamic forces in both discussed papers are taken into account in a simplified form. The main new feature of HIRAI-VLASOV equations is the presence of terms containing bending moment about vertical bridge axis, M_y, caused by static action of horizontal wind forces q (Fig. 1). This moment in combination with vertical displacement and rotation gives rise to additional vertical forces and torsion moments — last term in Eq. (1) and first term in Eq. (2) respectively.

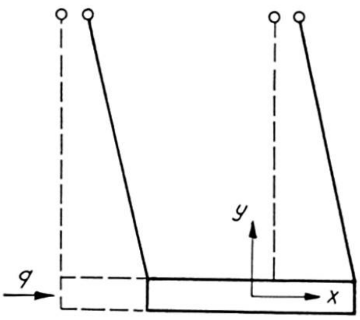

Fig. 1.

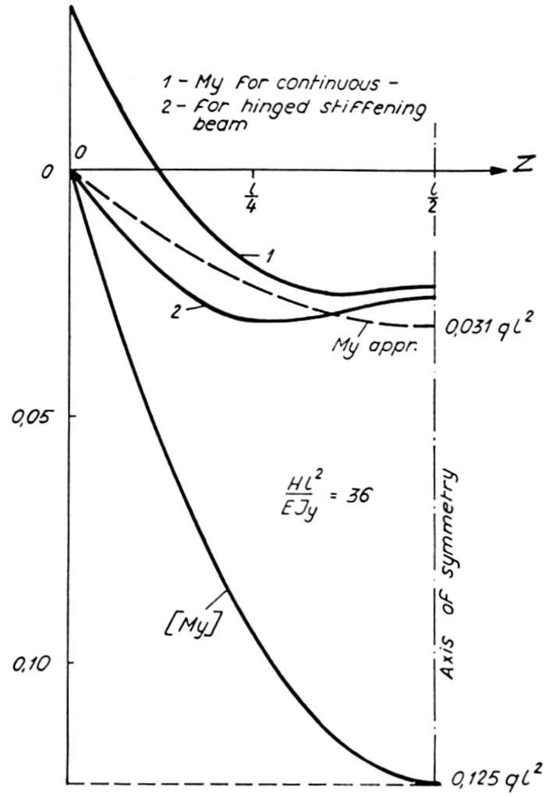

Fig. 2.

It is remarkable that both authors take for M_y moment in a simply supported beam under uniform load, denoted here as $[M_y]$:

$$[M_y] = \frac{q}{2} z (l - z)$$

with l as central span length and z = longitudinal axis.

No attention is paid there to following effect (Fig. 1): Horizontal displacement due to static wind action gives rise to horizontal components of hanger forces, which are reducing both displacements and moments. This effect can be negligible in bridges with a low span to width ratio, as was the case in model tests conducted by Hirai and in many known suspension bridges. The reduction of bending moments is, however, of substantial importance in case of a high span to width ratio (e.g. Tacoma bridge) and in general should not be disregarded [1]).

As stated, the bending moments in Eqs. (1), (2) should be reduced due to the action of restoring forces. The proper evaluation of reduced bending moments M_y is a seperate problem. In this connection publications by SELBERG, THEIMER and TOPALOFF — see [6] — should be mentioned. For the purpose of these remarks, however, a simple relation as a rough approximation for M_y can be given:

$$M_{y\,appr} = [M_y] \frac{1}{1+0{,}083\,H\,l^2/E\,J_y}, \tag{3}$$

where $E\,J_y$ is the horizontal stiffness of the bridge section. This equation was obtained on the assumption that restoring forces are distributed according to a sine curve with maximum value in $l/2$ equal to $8\,H\,\xi/l^2$, combined with following simplified relation: $M_{y\,appr} = [M_y]\,\xi/\bar{\xi}$; $\bar{\xi}$ is horizontal deflection in $l/2$ due to $[M_y]$ and ξ corresponding value due to wind and restoring forces acting together.

The Eq. (3) stresses the importance of the parameter $H\,l^2/E\,J_y$. For great values of this parameter (i.e. for high span to width ratios) the reduction of bending moments increases and reduced values are nearly inversely proportional to this parameter. Accurate distribution of M_y may differ remarkably from that of $M_{y\,appr}$ — maximum values of M_y occurs in general nearer to the quarter points of span and not in $l/2$. Fig. 2 shows actual distribution of M_y, computed in [6] [2]), and approximate parabolic distribution according to Eq. (3) for one parameter $H\,l^2/E\,J_y = 36$. Maximum values of bending moments are, however, in fair agreement. Table 1 gives some corresponding maximum values for comparison [3]).

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[1]) The computation of the critical wind velocity for the Tacoma bridge was one of the objects in papers under discussion. In this connection it is worthwhile to remember that no precise mesured dates are available for comparison in this case because of some faults in mesuring devices, see [5].

[2]) Values computed in [6] apply exactly to one type of suspension bridges, characterised by a span to cable sag ratio equal 8 and wind acting on cable amounting to 0,1 of the total wind acting on the bridge.

[3]) Values given in table 1 apply to a stiffening beam simply supported in horizontal direction.

The effect of differences in moment distribution can be ascertained on the basis of the method of virtual displacements, applied in [2] for the solution of Eqs. (1), (2). Bending moments appear there in the term

$$\int_0^l M_y\,\eta''\,\varphi\,dz$$

With corrected terms in Eqs. (1), (2) further procedure is as outlined in papers [1], [2].

Table 1

$\frac{H l^2}{E J_y}$	Maximum values of	
	$M_{y\,appr.}$ acc. Eq. (3)	M_y acc. to [6]
0	$1{,}0 \cdot ql^2/8$	$1{,}0 \cdot ql^2/8$
36	$0{,}251 \cdot ql^2/8$	$0{,}240 \cdot ql^2/8$
72	$0{,}143 \cdot ql^2/8$	$0{,}146 \cdot ql^2/8$
144	$0{,}077 \cdot ql^2/8$	$0{,}084 \cdot ql^2/8$
288	$0{,}040 \cdot ql^2/8$	$0{,}046 \cdot ql^2/8$

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Summary

Attention is drawn to the necessity of taking into account the effect of horizontal components of hanger forces, caused by horizontal displacement of bridge sections, in general equations of aerodynamic vibrations of suspen-

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indicating that for the most important case of vibrations with two sine half waves of displacement and rotation in central span the values of M_y in $l/4$ add more to total value of this integral, and thus to total effect of bending moments, than those in centre of span.

In many cases, the stiffening beam is acting in horizontal direction as a continuous beam and therefore bending moments M_y are to be below the values of $M_{y\,appr}$ (see Fig. 2). It seems thus justified to use Eq. (3) without further refinements — at least for preliminary calculations.

sion bridges.

An approximate relation for the reduction of bending moments about the vertical bridge axis, comprised in these equations, is given.

Résumé

L'auteur attire l'attention du lecteur sur la nécessité de tenir compte, dans les équations générales relatives aux vibrations aérodynamiques des ponts suspendus, de l'influence des composantes horizontales des efforts dans les suspentes, composantes dues aux déplacements horizontaux du pont.

Il donne une relation approchée quant à la réduction des moments de flexion par rapport à l'axe vertical du pont, moments qui interviennent dans ces équations.

Zusammenfassung

Es wird auf die Notwendigkeit hingewiesen, den Einfluß der von der horizontalen Verschiebung der Brückenquerschnitte herrührenden Horizontalkomponenten von Hängekräften in allgemeinen Gleichungen der aerodynamischen Schwingungen von Hängebrücken zu berücksichtigen.

Eine angenäherte Beziehung für die Reduktion der in diesen Gleichungen enthaltenen Biegungsmomente bezüglich der vertikalen Brückenachse ist angegeben.