Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 8 (1968)

Artikel: The effect of torsion on the ultimate strength of reinforced concrete spans in bending

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DOI: https://doi.org/10.5169/seals-8813

DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

The Effect of Torsion on the Ultimate Strength of Reinforced Concrete Spans in Bending

Effet de la torsion sur la résistance à la rupture de poutres en béton armé sous flexion

Die Wirkung der Torsion auf die Traglast des unter Biegung stehenden Stahlbetonträgers

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Previous authors have shown that the ultimate bending capacity of rectangular reinforced concrete beams subjected to a torsion moment can be determined by equating the external and internal moments across the assumed failure surface. Starting with an assumed stress-strain relationship for the material, an expression can be derived for the ultimate bending moment as a function of the geometry of the section, the material properties, the ratio (ϕ) of applied bending to torsion moment, the angle of crack (α) up the vertical faces of the beam and also the angle of inclination (β) of the compression fulcrum about which the ultimate rotation takes place. This expression was simplified by Evans and Sarkar[1] who assumed a constant value for α and derived a relationship for β in terms of the principal stresses.

In a more recent work,[4] the authors have modified the equation given by Evans and Sarkar by considering three ranges of ϕ values corresponding to the cases of predominant torsion, combined bending and torsion, and predominant bending. On this basis the following relationships were established:-

For $\phi < 2$ $\quad \cot\alpha = 0.63/\sqrt{\phi}$ (Predominant Torsion)

For $2 \leq \phi \leq 8$ $\quad \cot\alpha = 0.8/\phi$ (Combined Bending and Torsion)

For $\phi > 8$ $\quad \cot\alpha = 0.1$ (Predominant Bending)

These/

These equations apply to both solid and hollow sections for, within the range of accuracy normally worked to in reinforced concrete design, the difference between α_{solid} and α_{hollow} is small. For a range of sections considered by the authors the largest difference for α values was less than 1%.

A geometrical relationship

$$\cot\beta = \cot\alpha(2jk + 1)$$

was also substituted in the equation so that there is a considerable simplification in the final equations which now become:-

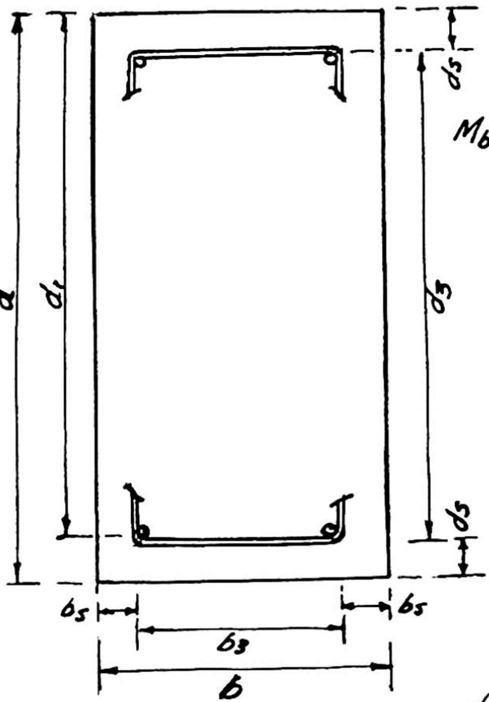


FIG. 1.

$\underline{\phi < 2}$

$$M_{bu} = \frac{A\phi + 0.2B^2Cn^2 + 0.4BTb_3(d - d_s - n) + 0.4Tdd_3}{\phi + \frac{0.63B}{\sqrt{\phi}}} \quad \text{---- (1)}$$

$$\text{where } n = \frac{L\phi + 0.4BTb_3}{(\phi + 0.4B^2)C}$$

$\underline{2 \leq \phi \leq 8}$

$$M_{bu} = \frac{A\phi^2 + 0.32B^2Cn^2 + 0.64BTb_3(d - d_s - n) + 0.64Tdd_3}{\phi^2 + 0.8B} \quad \text{---- (2)}$$

$$\text{where } n = \frac{L\phi^2 + 0.64BTb_3}{(\phi^2 + 0.64B^2)C}$$

$\underline{\phi > 8}$

$$M_{bu} = \frac{\phi\left[\frac{(1 + 0.01B^2)Cn^2}{2} + L(d_1 - n) + 0.01BTb_3(d - d_s - n) + 0.01Tdd_3\right]}{\phi + 0.1B} \quad \text{----- (3)}$$

$$\text{where } n = \frac{L + 0.01BTb_3}{(1 + 0.01B^2)C}$$

and $A = \frac{1}{2}Cn^2 + L(d_1 - n)$; $L = A_L f_L$; $C = f_c'b$; $T = \frac{f_T A_T}{s}$; $B = (2k+1)$.

Example

To illustrate the use of these equations consider a reinforced concrete section 10 ins. wide subjected to a bending moment of 902,500 lb.in. and a torsion moment of $\frac{902,500}{6}$ lb.in.

Assuming/

Assuming permissible stresses of 1,000 - 20,000 lb/in^2 in steel and concrete the procedure is as follows:-

1. Using the applied bending moment only, the overall depth and required steel area are calculated.

2. Taking load factors of 2 and 3 for steel and concrete respectively so that $f'_c = 2{,}000\ lb/in^2$, $f_T = 40{,}000\ lb/in^2$, and assuming $A_T = \frac{1}{30}$ of 1.7% of bd, S = 15 in. and b_3 = 8 in. the constants L, C, T, and B can be calculated for this section (K = 2) and moment ratio (ø = 6).

3. Using the above constants the values of n (4.397 in), A(2,046,700) and M_{bu} (1,921,000 lb.in.) can be determined.

4. The true load factor is thus $\frac{1{,}921{,}000}{902{,}500}$ = 2.13 and this can be compared with the load factor in pure bending of 2.23.

If the applied torsion was changed then A would have to be re-calculated and new values for n and M_{bu} determined. For example for:-

ø = 2, M_{bu} = 1,121,000 lb.in. and the load factor = 1.24
ø = 10, M_{bu} = 1,990,000 lb.in. and the load factor = 2.21

In order to test the validity of the assumptions used in deriving the equations a number of beams were tested and good correlation was obtained between the experimental and theoretical results (Table 1.)

Table 1

Beam No	ø	Ultimate Moment, M_{bu}, lbs. ins. x 10^3			Remarks
		Experimental	Theoretical (Equations 1,2,3.)	Theoretical (Equation 4)	
1	10.9	4.4	5.7	-	Plain concrete
2	11.2	9.2	9.6	9.4	No transverse reinforce-ment
3	9.0	13.9	14.3	14.1	
4	6.0	11.6	12.6	11.3	
5	8.0	9.2	9.8	7.8	
6	3.6	8.9	12.2	10.4	
7	6.4	9.9	14.4	15.0	Bond failure
8	3.0	5.3	12.0	15.0	" "
9	6.0	15.0	14.5	15.3	
10	7.1	13.8	15.1	15.1	
11	4.7	14.4	14.0	14.0	
12	8.1	14.2	14.8	14.7	

Note:

1. Yield strength of longitudinal reinforcement in beams 2 and 5 = 30,000 lbf/in^2. All other beams, yield strength = 50,000 lbf/in^2.

2. Beams 5 were tested over a reduced span.

Although the results obtained using equations 1, 2 or 3 show reasonable agreement with the practical results, the division of the problem into three separate cases is cumbersome and some advantage would be gained if one equation could be found to cover the complete range of ϕ values. A further disadvantage with the equation results from the fact that any change in the applied loading requires a new set of calculations even for the same section and steel area.

Applying equations 1, 2 and 3 to a given section and given ultimate material properties, the value of the ultimate moment can be calculated for the complete range of ϕ values from 0 (pure torsion) to (pure bending). These values of ultimate moment can be plotted on a Torsion Moment - Bending Moment diagram and a single equation which is independent of ϕ can be found to approximate to the resulting plot.

Following this procedure, for a particular section, it can be shown (Fig.2) that the ellipse, which has a semi major axis corresponding to the pure bending case and a semi minor axis corresponding to the case of pure torsion, is reasonably close to the torsion - bending plot.

The equation of the ellipse is not the only equation which could be found to approximate to the curve and the accuracy of the approximation can be increased by determining a polynomial expression which fits the curve - this would, however, increase the complexity of the design equation.

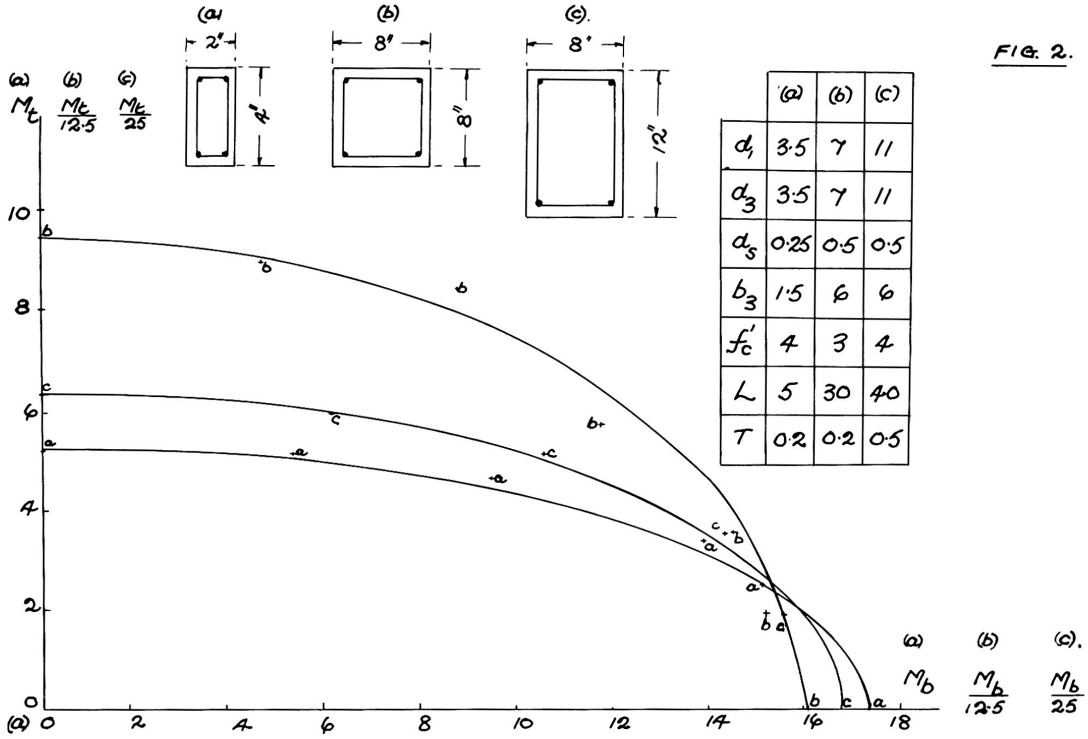
The ellipse equation can be expressed in the form:

$$M_{bu} = M_u \sqrt{1 - \left(\frac{M_t}{T_u}\right)^2} \qquad \text{................(4)}$$

where M_u = semi major axis

= $Ld_1 - (L^2/2C)$

and/



	(a)	(b)	(c)
d_1	3·5	7	11
d_3	3·5	7	11
d_s	0·25	0·5	0·5
b_3	1·5	6	6
f_c'	4	3	4
L	5	30	40
T	0·2	0·2	0·5

FIG. 2.

and T_u = semi minor axis

$$= {}^1/_B \sqrt{\tfrac{1}{2}(1 + B^2)Cn^2 + L(d_1 - n) + BTb_3(d - d_s - n) + Tdd_3}$$

where $n = \dfrac{L + BTb_3}{(1 + B^2)C}$

The ultimate moments calculated using equation 4 are included for comparison in Table 1. It can be seen that the results compare favourably with both the experimental values and the theoretical values calculated using equation 1, 2 and 3.

Example

To illustrate the use of equation 4 consider the same example as before. The procedure is now:-

1. Calculate the constants L, C, T and B, as before.
2. Determine the values of M_u (pure bending) and T_u (Pure Torsion) from the equations above.

   For this example M_u = 2,009,500 lb. ins.

   T_u = 508,800 lb. ins.
3. Apply equation 4 taking M_T = 902,500/6 lb ins.

   so that M_{bu} = 1,919,000 lb ins.
4. The true load factor = $\dfrac{1,919,000}{902,500}$ = 2.13 as before.

If the applied torsion was changed then equation 4 can be used directly since the values of M_u and T_u remain the same.

For ϕ = 2, M_{bu} = 927,000 lb in. and the load factor = 1.03

ϕ = 10, M_{bu} =1,977,000 lb in. and the load factor = 2.19

The meaning of the moment and torsion symbols and also ϕ is shown in Fig.3. The assumption made in deriving equation 4 is that given the working, bending and torsion, moments the value of γ (and hence ϕ) is fixed and that the path to ultimate moment is along AB – the bending moment at B being M_{bu}.

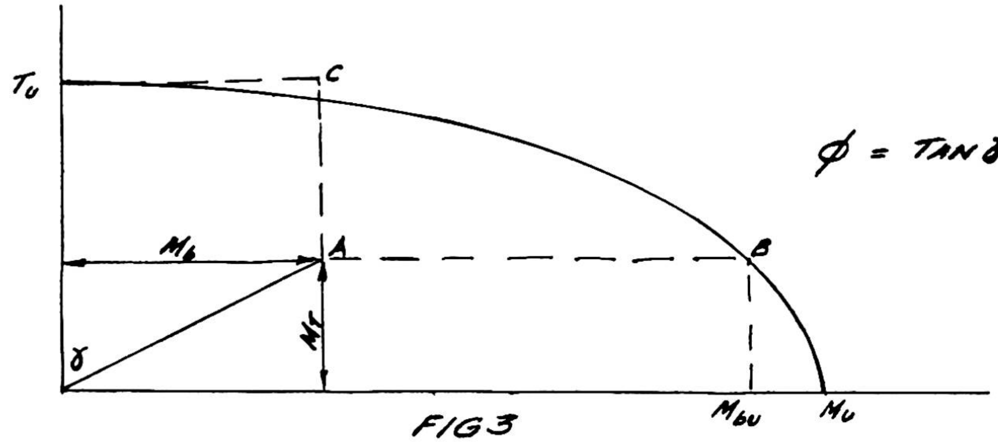


FIG 3

There is, of course, a lower limit to the value of ϕ which is imposed by the condition that the applied torsion moment (M_T) cannot exceed the value of the pure torsion (T_u) which can be carried by the particular section, i.e. M_T cannot increase beyond C.

For the example considered $\phi > \frac{M_b}{T_u} = \frac{902,500}{508,800} = 1.77$

and the load factor at this limiting value of $\phi = 0$. (Fig.4).

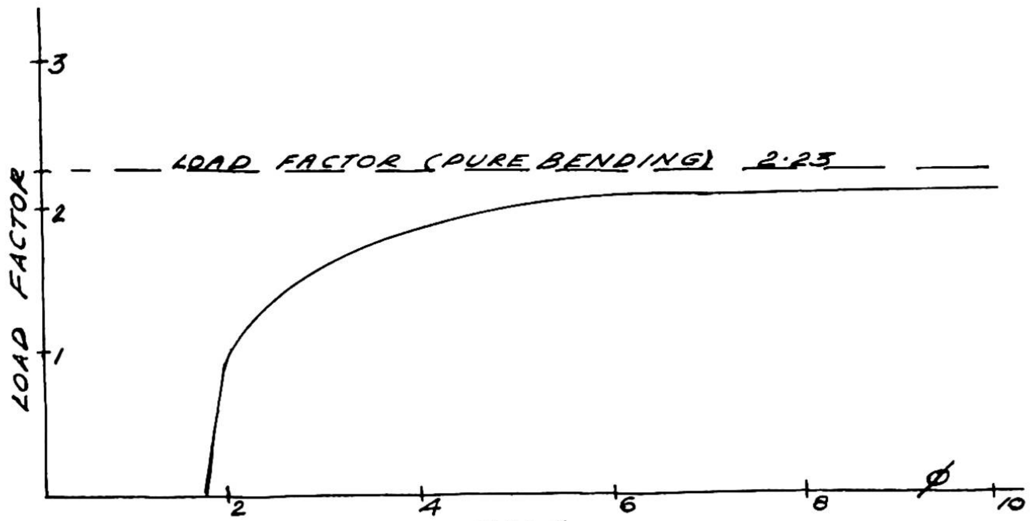


FIG 4

By a similar procedure, given the working strength of a section in torsion, it can be shown that there would be un upper limit to the value of ϕ.

Notation

- j = neutral axis constant
- k = d/b
- b, b_3, d, d_1, d_3, ds – as shown in Fig.1.
- A_T = area of one leg of stirrup.
- A_L = area of bottom longitudinal reinforcement.
- S = spacing of stirrups.
- α = angle of crack in vertical face.

β = angle of inclination of compression fulcrum.
f_L = yield stress of longitudinal reinforcement.
f_T = yield stress of transverse reinforcement.
f_c = ultimate compressive concrete stress.
M_b = applied bending moment.
M_T = applied torsion moment.
M_{bu} = ultimate bending moment.
M_{Tu} = ultimate torsion moment.
M_u = pure bending moment.
T_u = pure torsion moment
ϕ = M_b/M_T

Acknowledgements

This research was carried out in the Department of Civil Engineering and Building Science at Edinburgh University and the authors wish to acknowledge the help and encouragement of Professor A.W. Hendry.

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SUMMARY

The determination of the ultimate bending capacity of rectangular reinforced concrete beams subjected to a torsion moment is considered using two methods. The first method, using equations derived by equating the external and internal moments across the assumed failure surface, is modified into the second method which is much simpler to apply without much loss in accuracy. The results obtained using the two sets of equations are compared with experimental results.

RÉSUMÉ

On considère deux méthodes pour déterminer la résistance à la rupture d'une poutre rectangulaire en béton armé soumise à la flexion et à la torsion. La première méthode égalise les moments extérieurs et intérieurs dans la section de rupture. Les équations ainsi obtenues sont modifiées et fournissent la deuxième méthode, qui sans perte de précision notable, est cependant d'emploi beaucoup plus facile. Une comparaison est faite entre les résultats obtenus par ces deux méthodes et des résultats expérimentaux.

ZUSAMMENFASSUNG

Für die Berechnung der "Biegetraglast" eines einem Drehmoment unterworfenen Stahlbetonträgers werden zwei Verfahren in Betracht gezogen. Die erste Methode benützt Gleichungen, die man dann erhält, wenn man die inneren und äusseren Momente über die vermutete Bruchfläche ausgleicht, und wird in die zweite übergeführt, welche viel einfacher in der Anwendung ist ohne grosse Genauigkeitsverluste. Die durch den Gebrauch der beiden Gleichungssätze erhaltenen Ergebnisse werden mit den Versuchen verglichen.

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