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DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

Dynamic Behaviour of Structures and Dynamic Modeling

Le comportement dynamique des constructions et la simulation dynamique

Das dynamische Verhalten von Bauwerken und dynamische Simulation

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The constructions are due to the corpuscular nature of matter mechanical systems with large degree of freedom. The equations of motions can be derived from the equilibrium of the forces: in case of linear systems this would lead - however only theoretically - to the very compact matrix-formulated set of the differential-equationsystem

$$\underline{M} \ddot{\underline{x}} + \underline{C} \dot{\underline{x}} + \underline{K} \underline{x} = \underline{G} \quad /1/$$

with N simultaneous equations [1],[2].

To overcome the difficulties caused by the very large, but nevertheless finite number  $1 \ll N < \infty$ , there are two different ways possible. The infinite increase of the degree of freedom results models of continuously distributed parameters, dealt mathematically by partial differential equations. Conversely the decrease of the degree is equivalent with the concentrating of the properties to  $1 \ll n < N$  discret points: models with concentrated parameters. In praxis only the first dominating particular solutions of Eq./1/ are of interest, being characteristic for the total dynamic behaviour

of the structure. From this point of view both type of models can be adapted equivalently; to that method is given preference, that optimally secures suitable results for the adapter.

The dynamical behaviour of a system  $[\underline{M}, \underline{C}, \underline{K}]$  with excitation  $\underline{G}$ , namely  $\underline{F}$  force- and/or  $\underline{x}$  displacement-excitation is characterized by the response  $\underline{x}, \dot{\underline{x}}, \ddot{\underline{x}}$  /Fig.1/. This means mathematically the transform of Eq./1/ to its explicite form. The modern high-speed electronic compu-

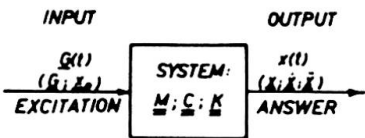


Fig.1.

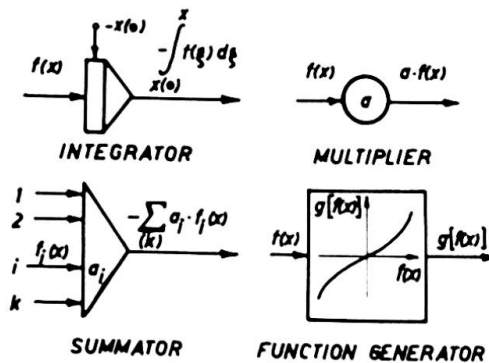


Fig.2.

ters offer due to the rapid flow of information an easy possibility for solving Eq./1/. In digital computers the simultaneous integration is produced by stepwise iteration. In possession of the elements of the analogue computers /Fig.2./ the mathematical Eq./1/ can be simulated electrically by a "laboratory model"; the several differential-equations represent in the logic block-diagram product-sums based on integrating chains and are easily realizable by means of electric circuits.

The internal mass, damping- and spring-forces of a nonlinear vibrator of unique degree of freedom are in equilibrium with the external forces of excitation:

$$F_m + F_c + F_k = m \ddot{x} + F_c \dot{x} + F_k x = F_G = G \quad /2'/$$

or after mathematical rearrangement - without altering the physical information-content -

$$\ddot{x}/t/ = \frac{1}{m} \left[ c \left( - \int_0^t \dot{x}/\tau/d\tau - k \int_0^t x/\tau/d\tau + G/t/ \right) \right] \quad /2''/$$

The logic block-structure of Eq./2/, Fig.3, is in addition the principal programming plan and switching graph of the equivalent simulating model, the analogue computer.

One of the simplest mind-model of a springed vehicle with one degree of freedom is to be seen on Fig.4. /by ignoring the pitching component of the movement/. Jumps of the two axes at a velocity  $v_0$  can be represented in the model by twin pulses.

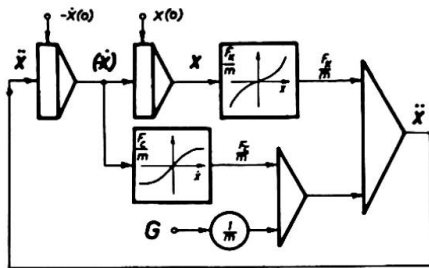


Fig.3.

In case of linear system the matrices of Eq./1/ have the actual form:

$$\underline{M} = \begin{bmatrix} m & 0 \\ 0 & 0 \end{bmatrix}; \quad \underline{C} = \begin{bmatrix} c & -c \\ -c & c \end{bmatrix}; \quad \underline{K} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix};$$

and  $\underline{G} = \begin{bmatrix} mg \\ 0 \end{bmatrix} \quad /3'/$

if time-dependent force-excitation is missing. Expressing the derivatives of the highest order by the other terms:

$$x = \frac{c}{m} (-\dot{x}) - \frac{c}{m} (-\dot{x}_0) - \frac{k}{m} (-x_0) + g \quad /3''/$$

$$-\dot{x}_0 = (-\dot{x}) - \frac{k}{c} x - \frac{k}{c} (-x_0)$$

Fig.9.a illustrates the logic block-diagram of Eq./3/.

Constructions loaded by space- and time-variable moving loads get their new, deformed shape of equilibrium only after the decay of transient oscillations; the final shape however can be derived by well-known statical treatments

too. As mentioned above, in praxis the interesting modes of maximal amplitudes /that of the lowest natural frequencies/ dominate and are characteristic for the dynamical behaviour of the structure. The reduction of the degree of freedom is to be carried out in such a manner, that these modes of technical interest should be included by the selected model.

For illustration let us take a special archstiffened sing-

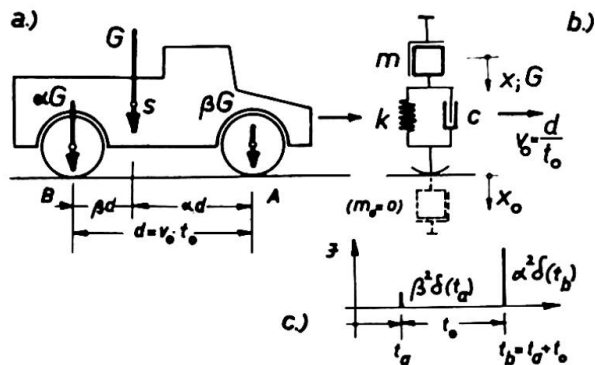


Fig. 4.

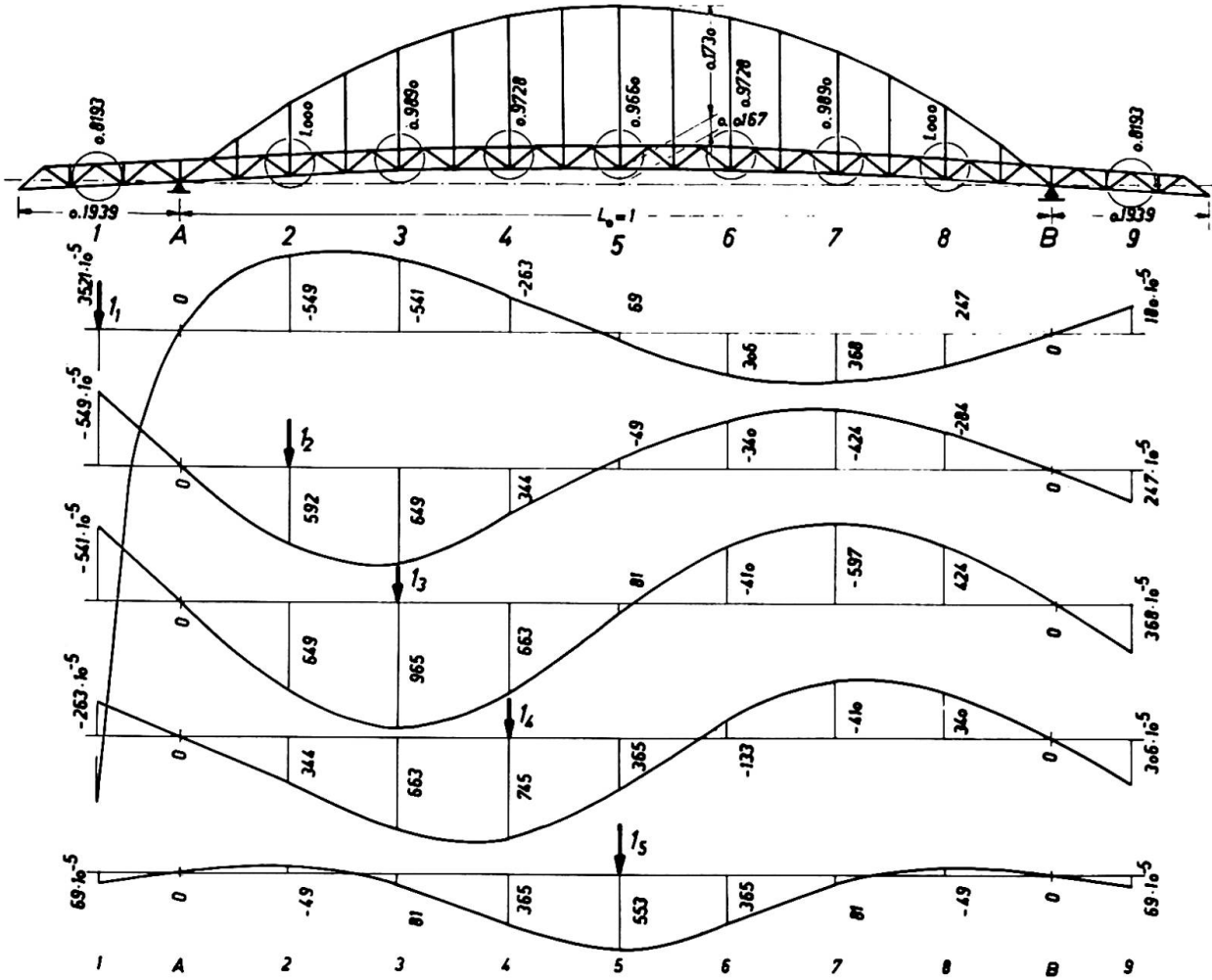


Fig. 5.

le span bridge construction: a Gerber-truss /Fig.5/, the two side-extensions of which being cantilevers; only the last secondary longitudinal girders of the deck-plate are on both sides pin-connected with the bridge and the abutments, respectively. The total statical informations are involved in the matrix of the displacement influence-line ordinates; for dynamical behaviour the mass- and the damping-distributions are needed too.

The derivation of the mass-matrix  $\underline{\underline{M}}$  implies principally no difficulties. For the actual damping conditions are hardly to be seized exactly, consequently for simpler numerical algorithms it is assumed, that the damping-matrix  $\underline{\underline{C}}$  is diagonal /physically: presence of only grounded dampers/. The direct determination of the spring-matrix  $\underline{\underline{K}}$  is often quite troublesome indeed; but in statics convenient methods are available for the computation of the displacement-influence-lines:  $\underline{\underline{M}}_{[kl]} = \underline{\underline{H}}$ , the inversion of  $\underline{\underline{H}}$  being a submatrix of  $\underline{\underline{K}}$ :

$$\underline{\underline{H}}^{-1} = \underline{\underline{K}}_R \quad /4/$$

The missing elements are to be determined by the reciprocity-law of Maxwell  $k_{ij} = k_{ji}$ , the equilibrium of the forces embodied in  $\underline{\underline{k}}_j$ , fundamental relation:

$$\sum_{i=1}^n \underline{\underline{M}}_{ij} k_{ij} = 0, \text{ respectively.}$$

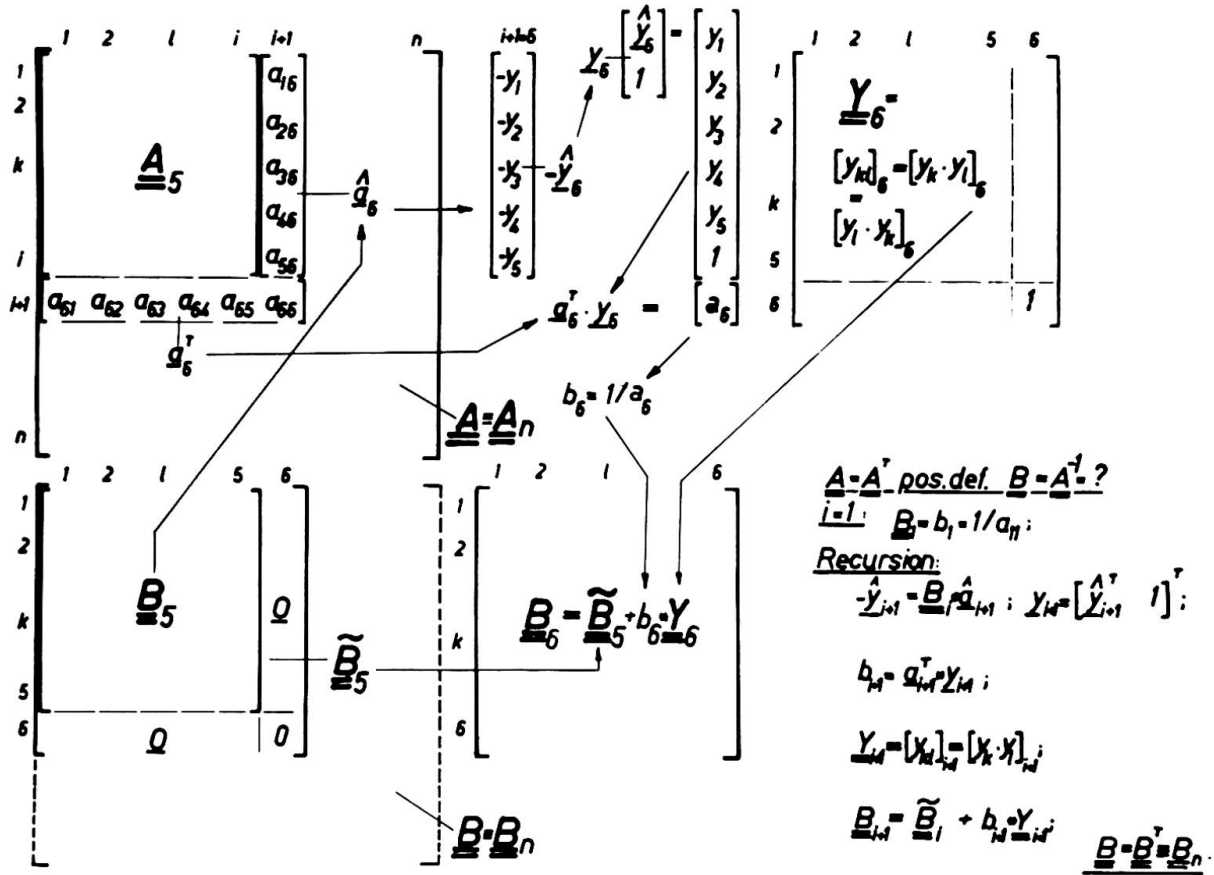


Fig. 6.

Fig.6.gives a recursive procedure for matrix-inversion like Eq./4/ for symmetrical and positively definite matrices, common in linear structural engineering. Among others, the given method has the advantage, that all mathematical steps can be interpreted mechanically too[3].

For symmetrical constructions all asymmetrical effects can be resolved into symmetrical and antimetrical components; applying this to Eq./1/ :

$$\begin{aligned}
 \underline{M}_S \ddot{Y} + \underline{C}_S \dot{Y} + \underline{K}_S Y &= \underline{F} = \underline{E} Y + \underline{K}_{RS} Y_R & / 5' / \\
 \underline{M}_A \ddot{Z} + \underline{C}_A \dot{Z} + \underline{K}_A Z &= \underline{F} = \underline{E} Z + \underline{K}_{RA} Z_R
 \end{aligned}$$

where

$\underline{F} = \underline{Y} + \underline{Z}$  means the force-excitation,  
 $\underline{X}_R = \underline{Y}_R + \underline{Z}_R$  the displacement-excitation,  
 $\underline{x} = \underline{Y} + \underline{Z}$  the output displacement-vector, while

$$\begin{aligned}
 \underline{A}_S &= [a_{ik}]_S = a_{ik} + a_{n-(i-1),k} & / 5'' / \\
 \underline{A}_A &= [a_{ik}]_A = a_{ik} - a_{n-(i-1),k}
 \end{aligned}$$

are the new matrices of the symmetric and antimetric components. Let us express the accelerations in terms of the other elements:

$$\begin{aligned}
 \underline{M}_S \ddot{Y} &= \underline{M}_S^{-1} \left[ \underline{C}(-\dot{Y}) - \underline{K}_S Y + \underline{E} Y + \underline{K}_{RS} Y_R \right] & / 5''' / \\
 \underline{M}_A \ddot{Z} &= \underline{M}_A^{-1} \left[ \underline{C}(-\dot{Z}) - \underline{K}_A Z + \underline{E} Z + \underline{K}_{RA} Z_R \right]
 \end{aligned}$$



even if the separate systems are linear. The several components of the excitation are on one hand the dead-load weights and the eventual centrifugal forces caused by the rotating excentricities of the motor, the forced movement of the rolling load along the vertical trace of the deckplate. On the other hand there may be dead loads or vibrators located at fixed points of the construction; wind- and earthquake-forces are of this kind.

The difficulties caused by the continuous space-variation of the load in a model of discrete concentration of parameters can be removed by the adaptation of the well-known lever-law approximately, explained in Fig.8.

Should all effects of possible excitations taken simultaneously into consideration, then the model is to be build up by the principles shown in Fig.9 a. and b. The variable, nonlinear feedback-systems are based on the application of the lever-rule, mentioned above and can be easily realised electrically by means of sliding potentiometers. The sliding contacts are to be moved with such a relative velocity, that the actual vehicle under consideration may have; research studies on motor accelerations, as well as brakings can be made without difficulties. If the model of the moving vehicle has several degrees of freedom, the feedback-systems are of multi-channel type, of course.

In practice the several effects can be studied naturally separately too, but it is always to be kept in mind, that the systems with space-variable loads are nonlinear and the linear law of superposition is not yet valid.

For practical purposes often only the eigenvalues of the unloaded structure are of technical interest, being characteristic parameters of the total dynamical behaviour. The spring-matrix may be generally given in form of displacement-influenceline ordinates. In this case even the matrix-inversion of

$\underline{H}^{-1} = \underline{K}_R$  is redundant. Pre-multiplying Eq./1/ by  $\underline{K}^{-1} = \underline{H}$ , we get another mathematical form of the Eq./1/:

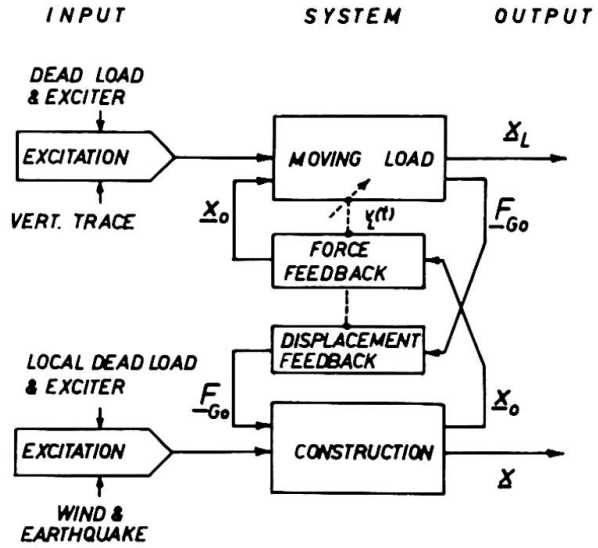


Fig.7.

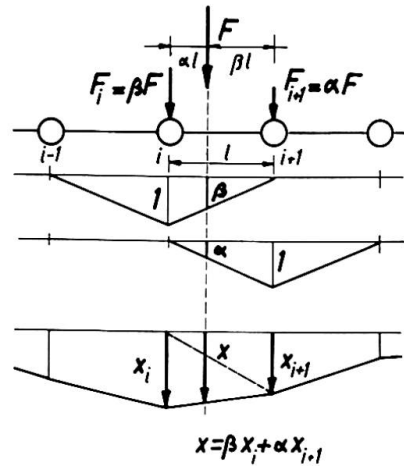


Fig.8.

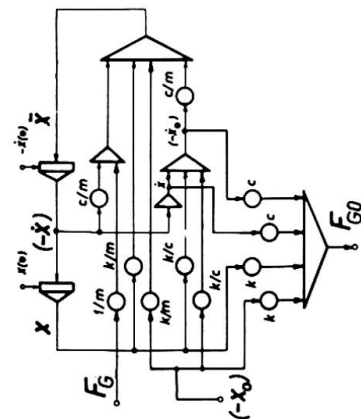
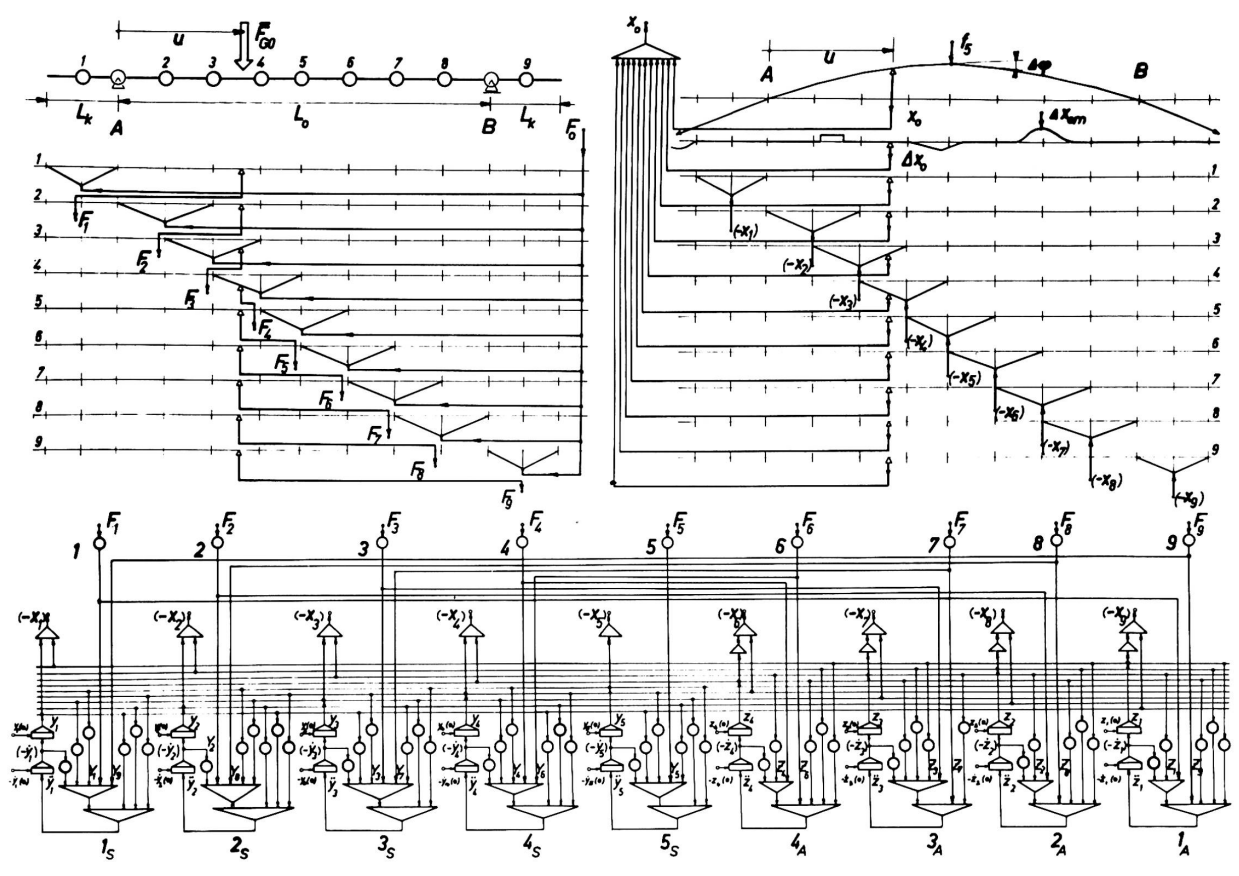


Fig. 9 a.

Fig. 9. b.





$$\underline{\underline{H}} \underline{\underline{M}} \ddot{\underline{x}} + \underline{\underline{K}}_R^{-1} \underline{\underline{C}} \dot{\underline{x}} + \underline{\underline{E}} \underline{x} = \underline{\underline{A}} \ddot{\underline{x}} + \underline{\underline{D}} \dot{\underline{x}} + \underline{\underline{E}} \underline{x} = \underline{0} \quad /6'/$$

Developing the accelerations to the main diagonal  $\langle \underline{\underline{A}}_{Tr} \rangle$  of  $\underline{\underline{A}}$  :

$$\underline{\underline{E}} \ddot{\underline{x}} = \langle \underline{\underline{A}}_{Tr} \rangle^{-1} \left[ - \left( \underline{\underline{A}} - \langle \underline{\underline{A}}_{Tr} \rangle \right) \ddot{\underline{x}} - \underline{\underline{D}} \dot{\underline{x}} - \underline{\underline{E}} \underline{x} \right] \quad /6''/$$

only grounded dampings are assumed again.

Resolving Eq./6/ again into symmetric and antimetric components, the logical block-diagram decomposes into two independent separate parts; for the illustrative problem shown in Fig.10.

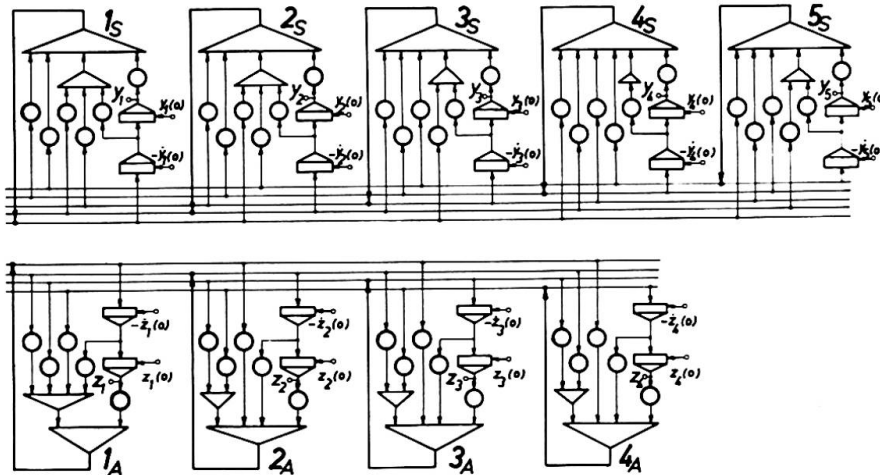


Fig. 10.

The numerical determination of the natural frequencies leads to the roots of a polynomial with a degree - in our case of the illustrative problem -  $n = 9$  with relative differences in the coefficients of 437 dB /namely from the numerical order of the extreme  $10^{22}$ !/. The relative order of the roots however are only yet about 64 dB, /namely  $10^3$ /. But the technically interesting natural frequencies varies only in the range of  $f_n = 1 - 15$  c/s /See Fig. 11/.

With respect to the eigenvalues in analogue computation, only the approximative shape  $\underline{x}'_n$  of the exact natural mode  $\underline{x}_n$  is needed, but not the value of the natural frequency. Giving to the

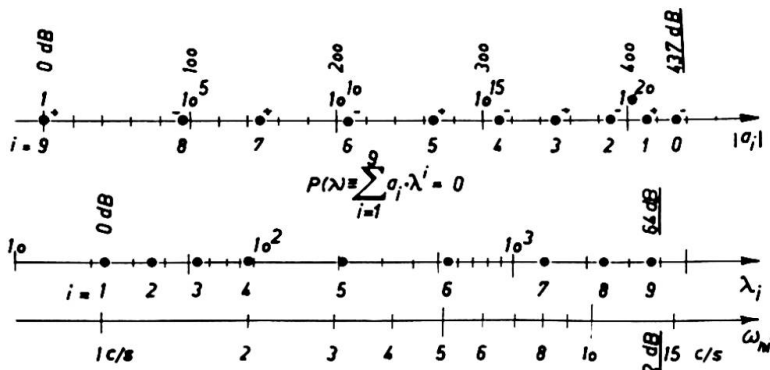


Fig. 11.

simulating dynamic model the initial conditions  $\underline{x}(0) = \underline{x}'_n$  and  $\dot{\underline{x}}(0) = \underline{0}$ , then dominant oscillations with the natural frequency  $f_n$  occurs, superposed by decaying transients of the other harmonics. For the exact, accurate natural mode  $\underline{x}_n$  /ini-

tial conditions:  $\underline{x}(0) = \underline{x}_n$  and  $\dot{\underline{x}}(0) = \underline{0}$  / the system-oscillations are

$$\underline{x}(t) = \underline{x}_{no} e^{-\lambda t} \cos(2\pi f_n t)$$

without any other harmonic transients. This procedure can be used for the very easy and very rapid iterative determination of the unknown eigenvalues: both modes and shapes.

The dynamic behaviour of a structure can be characterized by the answer given to ideal Dirac-pulses  $\delta_i(t)$

$$J = \int_0^T F dt = m_i \int_0^T \ddot{x}_i(t) dt = m_i [\dot{x}_i(T) - \dot{x}_i(0)] = m_i \Delta \dot{x}_i$$

The application of ideal pulses of the same intensity  $J$  at the mass  $m_i$  can be expressed mathematically by the initial conditions:

$\underline{x}(0) = \underline{0}$  ;  $\dot{\underline{x}}(0) = [0, 0, \dots, \dot{x}_i(0) = J/m_i, \dots, 0]^T$ . The answer functions  $\underline{x}(t)$ , the weight-functions contain all the eigenvalues and thus several natural frequencies can be determined from the diagrams of  $\underline{x}(t)$  too.

Applying a constant force  $F_{i0}$  suddenly to the mass  $m_i$  is equivalent with the excitation caused by the step-function  $F_{i0} \cdot 1(t)$ ;

initial conditions:  $\underline{x}(0) = \underline{0}$  ;  $\dot{\underline{x}}(0) = \underline{0}$ , and generating vector:

$G(t) = [0, 0, \dots, G_i(t) = F_{i0} \cdot 1(t), \dots, 0]^T$ ; oscillations with decaying transients occur, having the asymptotes

$$\underline{x}(\infty) = F_{oi} \underline{\eta}_i = F_{oi} [\eta_{1i}, \eta_{2i}, \dots, \eta_{ii}, \dots, \eta_{ni}]^T$$

This procedure offers simultaneously an easy and very suitable testprogram, controlling totally the entire modeling, both the developed mathematical equations Eq./5/, both their electrical realization.

The possibility of solving Eq./1/ for general optional excitations necessitates an analogue computer with suitable capacity, but offers the great advantage of analysing the dynamical behaviour by laboratory measurements; cumbersome site measurements are

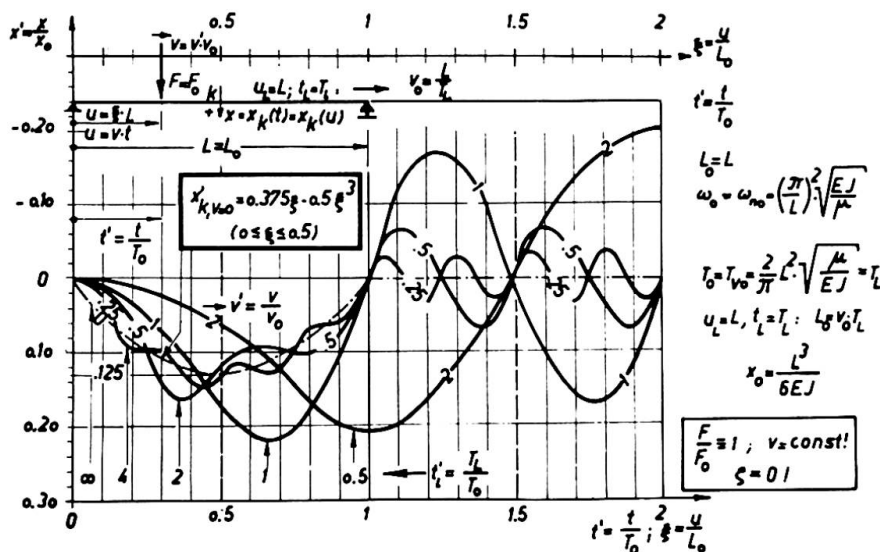


Fig.12.

to be made only to verify the theoretical results gained by dynamic modeling.

Such a pure theoretical result is the dynamical effect of a unique - massless force rolling with different, but constant velocity over a simply supported beam [4], [5]. The results of elaborate digital calculations are to be verified easily by analogue modeling. The several functions of oscillations can be gained instantly in graphical manner /Fig.12/. Such methods give sharp inside views into an important, but nevertheless very complex problem, covered in statics by the concept of the so called "impact factor" [6].

If diagrams of the stochastic variations of wind- or earthquake effects are available, then these can be considered as the input force and displacement generation of the structure under discussion. Dynamic modeling conversely gives the response by simple recording of the output.

#### A c k n o w l e d g e m e n t s .

The necessity of the method outlined grew out and was based on researches and nondestructive site investigations, made on the initiative of the Council of the Hungarian Capital Budapest and the Hungarian Ministry of Post and Communication. Considerable help in assistance was given in analogue computation of the developed equations by Mr. G á b o r L a d á n y i, E.E. /Department for Process Control, Technical University, Budapest/ and in digital computing by Mr. G y ö r g y P o p p e r /Computing Centre of the Ministry of Heavy Industries/. Site investigations were made on the authority of the Ministry of Post and Communications and the Council of Budapest in assistance of the Institute for Quality Control of Building Materials and Constructions.

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## SUMMARY

The electric simulation of mechanical systems by means of analogue computers makes laboratory researches of the actual dynamic behaviour of structures possible. The influence of space- and timevariable coupling of load and structure, changes and steps in vertical trace before or on the bridge, wind and earthquake effects respectively can be analysed separately or simultaneously.

## RÉSUMÉ

La simulation électrique de systèmes mécaniques à l'aide de calculateurs analogiques permet l'étude au laboratoire du comportement dynamique réel des structures. Ainsi l'on peut analyser séparément ou simultanément l'influence d'un accouplement (variable dans l'espace et dans le temps) des vibrations de la charge et de celles de la structure, les changements ou les gradins dans le tracé vertical avant ou sur le pont, les effets du vent et de tremblements de terre.

## ZUSAMMENFASSUNG

Durch die elektrische Simulierung mechanischer Systeme mit Hilfe von Analogrechner kann das wirkliche dynamische Verhalten von Bauwerken im Labor untersucht werden. Dabei können die Einflüsse der im Raum und Zeit veränderlichen Kopplung der Belastung und der des Bauwerkes, Gradientenänderungen, Sprünge in der Fahrbahn, vor und auf der Brücke, sowie Wind- und Erdbebenwirkungen gesondert, oder simultan untersucht werden.

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