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Safety Analysis of Suspension Bridges

Analyse de la sécurité de ponts suspendus

Sicherheitsbetrachtungen an Hängebrücken

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1. Introduction

Within the context of Theme I of the 8th congress, this paper establishes a method of structural safety analysis for the lateral vibration of aerodynamically stable suspension bridges under stormy winds.

The recent use of the so-called gust response factor in the dynamic analysis of structures subjected to gusty winds indicates an achievement of a higher level of sophistication in the structural safety analysis compared with the use of conventional safety factor, since the introduction of the gust response factor is based on the recognition that the wind velocity and hence the structural response have to be treated realistically as random processes.

The present paper demonstrates that a further effort will make it possible to estimate, in approximation, the probability of survival or failure of the suspension bridge (in the lateral mode of vibration) which is a more direct measure of safety in accordance with the probabilistic concept of structural safety[1] [(*)].

Since the type of failure considered in this paper is either buckling or yielding of a chord member of the stiffening truss due to its lateral bending under the wind pressure (this defines a critical bending moment at each cross-section), the linear equations of motion can be employed in the response analysis. Such failure modes are also assumed implicitly or explicitly in the previous papers[2,3] dealing with the same problem.

(*) Numerals indicate references at the end.

2. Structural Analysis

In the present paper, as in References 2 and 3, the wind velocity $U_z(t)$ at the height z above ground is the sum of the mean wind velocity $\bar{U}_z(t)$ and the fluctuating part $u_z(t,x)$.

The pressure due to the wind velocity $U_z(t)$ is, as usual, assumed to consist of two parts: the pressure due to the mean wind velocity

$$\bar{P}(t) = \frac{1}{2}\rho c A \bar{U}_z^2(t) \tag{1}$$

and the pressure due to the fluctuating part

$$p(t,x) = \rho c_d A \bar{U}_z(t)\, u_z(t,x) \tag{2}$$

where ρ is the density of air, c and c_d the static and the dynamic drag coefficient and A the exposed area of the structure considered.

It is usually observed from wind velocity records that $u_z(t,x)$ is nonstationary with a larger variance at a larger mean wind velocity. In the present study, however, it is assumed that $u_z(t,x)$ is stationary with a (constant) variance equal to that associated with the maximum mean wind velocity $\bar{U}_z$. Furthermore, $\bar{U}_z(t)$ in Eq.(2) is replaced by for simplicity. Hence, the following stationarized and conservative expression is used for $p(t,x)$.

$$p(t,x) = \rho c_d A \bar{U}_z\, u_z(t,x). \tag{3}$$

Since the variation of $\bar{P}(t)$ in time is much slow compared with the fundamental period of lateral vibration of the system of the cables and truss, the response $\bar{y}_T(t,x)$ and $\bar{y}_c(t,x)$ to $\bar{P}(t)$ is obtained performing a quasi-static analysis, while the mean square value of $y_T^*(t,x)$ and the bending moment $M^*(t,x)$ of the truss to $p(t,x)$ is evaluated on the basis of the standard equations of motion:

$$EI\,\bar{y}_T^{IV} + k(x)(\bar{y}_T - \bar{y}_c) = \bar{P}_T(t) \tag{4}$$

$$-H\,\bar{y}_c'' - k(x)(\bar{y}_T - \bar{y}_c) = \bar{P}_c(t) \tag{5}$$

$$m_T\ddot{y}_T^* + \mu_T\dot{y}_T^* + EI\,y_T^{*\,IV} + k(x)(y_T^* - y_c^*) = p_T(t,x) \tag{6}$$

$$m_c\ddot{y}_c^* + \mu_c\dot{y}_c^* - H\,y_c^{*\,''} - k(x)(y_T^* - y_c^*) = p_c(t,x) \tag{7}$$

with

$$k(x) = m_T g / h(x) \tag{8}$$

where the primes and the dots indicate differentiation with respect to x and t respectively, $h(x)$ is the hanger length, EI the bending rigidity of the truss in the horizontal direction, H the sum of the horizontal forces in the cables, m the mass per unit length, μ the linear viscous damping with subscripts T and C indicating that the quantities with T are associated with the truss and those with C are with the cables. The lateral bending moment of the truss can be obtained from its lateral displacement in the usual fashion.

The finite sine transform technique or the sine series expansion of $\bar{y}_T$ and $\bar{y}_C$ can be used to solve Eqs.(4) and (5) for $\bar{y}_T$ and $\bar{y}_C$. To evaluate the mean square response of M_T^* , the frequency response functions $H_{TT}(\omega, x, x_o)$ of $y_T^*(t,x)$ and $H_{TC}(\omega, x, x_o)$ of $y_C^*(t,x)$ due to an input $e^{i\omega t}\,\delta(x-x_o)$ applied at $x = x_o$ on the truss are first obtained by employing the finite sine transform technique. After some manipulation, one can show that the sine transforms $\widetilde{H}_{TT}(j) = \widetilde{H}_{TT}(\omega, j, x_o)$ and $\widetilde{H}_{TC}(j) = \widetilde{H}_{TC}(\omega, j, x_o)$ (with respect to x over $x = 0 \sim \ell$) of $H_{TT}(\omega, x, x_o)$ and $H_{TC}(\omega, x, x_o)$ satisfy the following equations.

$$\sum_{j=1}^{\infty} \widetilde{H}_{TT}(j)\, d_{nj} - \sum_{j=1}^{\infty} \widetilde{H}_{TC}(j)\, c_{nj} = \sin \frac{n\pi}{\ell} x, \qquad (n = 1, 2, \cdots) \tag{9}$$

$$-\sum_{j=1}^{\infty} \widetilde{H}_{TT}(j)\, c_{nj} + \sum_{j=1}^{\infty} \widetilde{H}_{TC}(j)\, e_{nj} = 0, \qquad (n = 1, 2, \cdots) \tag{10}$$

where

$$d_{nj} = (-\omega^2 m_T + i\,\omega\,\mu_T + EI\frac{n^4\pi^4}{\ell^4})\,\delta_{nj} + c_{nj} \tag{11}$$

$$c_{nj} = \frac{1}{\ell}\sum_{r=0}^{\infty} k_r \left\{ \delta_{(j+r),n} + \frac{|j-r|}{j-r}\,\delta_{|j-r|,n} \right\} \tag{12}$$

$$e_{nj} = (-\omega^2 m_C + i\,\omega\,\mu_C + H\frac{n^2\pi^2}{\ell^2})\,\delta_{nj} + c_{nj} \tag{13}$$

where ℓ is the span length, δ_{ij} the Kronecker delta, and k_r the coefficients of cosine series expansion of $k(x)$:

$$k(x) = \frac{2}{\ell}\sum_{r=0}^{\infty} k_r \cos\frac{r\pi}{\ell}x. \tag{14}$$

Eqs.(9) and (10) represent two sets of infinite number of equations for $\widetilde{H}_{TT}(n)$ and $\widetilde{H}_{TC}(n)$. By taking only first N terms each of $\widetilde{H}_{TT}(n)$ and $\widetilde{H}_{TC}(n)$ ($n, j = 1,2,\ldots, N$, and $r = 1,2,\ldots,2N$), one can obtain a set of $2N$ equations for $2N$ unknowns $\widetilde{H}_{TT}(n)$ and $\widetilde{H}_{TC}(n)$ ($n = 1, 2,\ldots,N$). Solving these and applying the inverse sine transformation, the frequency response function $H_{TT}(\omega, x, x_o)$ can be written as

$$H_{TT}(\omega, x, x_o) = \sum_{k=1}^{N} \alpha_k(\omega, x)\, \sin\frac{k\pi}{\ell}x_o \tag{15}$$

where

$$\alpha_k(\omega, x) = \sum_{j=1}^{N} a_{jk}^{-1}\, \sin\frac{j\pi}{\ell}x \tag{16}$$

In the Eq.(16) a_{jk}^{-1} is the $j - k$ member of the inverse matrix of a symmetric $2N \times 2N$ matrix

$$A = \left[\begin{array}{c:c} D & -C \\ \hdashline -C & E \end{array}\right] \quad \text{with } D = [d_{ij}] \;, C = [c_{ij}] \quad \text{and } E = [e_{ij}] \,.$$

The frequency response functions $H_{CT}(\omega, x, x_o)$ of $y_T^*(t, x)$ and $H_{CC}(\omega, x, x_o)$ of $y_C^*(t, x)$ due to an input $e^{i\omega t}\,\delta(x - x_o)$ on the cable can also be obtained in a similar fashion.

$$H_{CT}(\omega, x, x_o) = \sum_{k=1}^{N} \beta_k(\omega, x) \sin \frac{k\pi}{\ell} x_o \qquad (17)$$

where

$$\beta_k(\omega, x) = \sum_{j=1}^{N} a_{j,N+k}^{-1} \sin \frac{j\pi}{\ell} x \,. \qquad (18)$$

The functions $H_{CC}(\omega, x, x_o)$ and $H_{TC}(\omega, x, x_o)$ are not needed in the following analysis.

Making use of $\alpha''_k(\omega, x)$ and $\beta''_k(\omega, x)$, one can show that the mean square spectral density function of $M_T^*(t, x)$ is

$$\begin{aligned} S(\omega, x) = \sum_{r=1}^{N} \sum_{s=1}^{N} \Big[& \bar{\alpha}_r''(\omega, x)\, \alpha_s''(\omega, x)\, S_{rs}^{TT}(\omega) \\ & + 2\, Re \left\{ \bar{\alpha}_r''(\omega, x)\, \beta_s''(\omega)\, S_{rs}^{TC}(\omega) \right\} \\ & + \bar{\beta}_r''(\omega, x)\, \beta_s''(\omega, x)\, S_{rs}^{CC}(\omega) \Big] \end{aligned} \qquad (19)$$

in which $Re\, z$ and $\bar{z}$ respectively indicate real part and complex conjugate of z ,

$$S_{rs}^{XY}(\omega) = \int_0^\ell \int_0^\ell S_{p_1 p_2}^{XY}(\omega) \sin \frac{r\pi}{\ell} x_1 \sin \frac{s\pi}{\ell} x_2 \, dx_1 \, dx_2 \qquad (20)$$

with X and Y standing either for T or C and $S_{p_1 p_2}^{XY}(\omega)$ being the cross-spectral density of $p_X(t, x_1)$ and $p_Y(t, x_2)$.

The variances σ_M^2 and $\sigma_{\dot{M}}^2$ of $M_T^*(t, x)$ and $\dot{M}_T^*(t, x)$ are then obtained as

$$\sigma_M^2 = \int_{-\infty}^{\infty} S(\omega, x)\, d\omega \,, \qquad \sigma_{\dot{M}}^2 = \int_{-\infty}^{\infty} \omega^2 S(\omega, x)\, d\omega \qquad (21)$$

In the following discussion, however, the second and third terms within the square brackets of Eq.(19) are neglected because of their small contributions (as also done in Refs.2 and 3), and $S_{p_1 p_2}^{TT}$ is approximated by

$$S_{p_1 p_2}^{TT}(\omega) = \exp\left(-\frac{k\,\omega}{2\pi \bar{U}_z}\,|x_1 - x_2|\right)\Phi(\omega) \tag{22}$$

where the exponential term is the square root of the coherence, $2\pi\bar{U}_z/(k\,\omega)$ the scale of turbulence at the wave length $2\pi\bar{U}_z/\omega$, and $\Phi(\omega)$ is the mean square spectral density of $p_T(t, x)$ and is given by[2]

$$\Phi(\omega) = 4\,(\rho\, c_{d_T} A_T \bar{U}_z)^2 K \frac{\bar{U}_{33}}{\omega} \frac{\left(\dfrac{\phi\,\omega}{\pi \bar{U}_{33}}\right)}{\left[1 + \left(\dfrac{\phi\,\omega}{\pi \bar{U}_{33}}\right)^2\right]^{4/3}} \tag{23}$$

in which K is the surface drag coefficient, $\bar{U}_{33}$, the mean wind velocity at the reference height of 33 ft above ground, is related to $\bar{U}_z$ by

$$\bar{U}_1 = \bar{U}_z\left(\frac{33}{z}\right)^{\alpha} \tag{24}$$

with α being a constant.

3. Safety Analysis

In previous papers[4,5], one of the present authors developed a method of estimating upper and lower bounds of the probability that a Gaussian random process $z(t)$ will not be confined in a domain defined by $-a(t) \le z(t) \le a(t)$ in a specified time interval, where $a(t)$ (≥ 0) is a deterministic function of time.

Consider the standard design procedure for wind loads where the stiffening truss is designed so that it can withstand, with a safety factor n, the bending moment $M_d(x)$ produced by a specified (uniform) design wind pressure p_d. This implies that the critical bending moment at cross-section x is $nM_d(x)$. Suppose that the suspension bridge is subjected to a storm with mean wind velocity $\bar{U}(t)$ or mean wind pressure $\bar{p}(t)$ producing the bending moment $\bar{M}(t,x)$. Then, $a(t,x) = M^*(x) - M(t,x) = n M_d(x) - \bar{M}(t,x)$ is the maximum value of the bending moment $M^*(t,x)$ that the fluctuating part of wind pressure $p(t,x)$ can produce without failure. Since the variances of $M^*(t,x)$ and $\dot{M}^*(t,x)$ are evaluated in the preceding section, the method developed in References 4 and 5 can be applied to estimate upper and lower bounds of the probability of failure p_f or the probability that

$M^*(t,x)$ will not be confined in the domain defined by $-a(t,x) \leq M^*(t,x) \leq a(t,x)$.

Evidently, for a storm with a different mean value velocity function $\bar{U}_z(t)$, a different value of p_f is obtained. In fact, $\bar{U}_z(t)$ itself is usually a random function of time containing a number of random parameters, say $\bar{U}_z$ and T_0; $\bar{U}_z(t) = \bar{U}_z(t; \bar{U}_z, T_0)$. For example, the following forms of $\bar{U}_z(t, \bar{U}_z, T_0)$ are mathematically expedient and at the same time agree with observations reasonably well.

$$\bar{U}_z(t; \bar{U}_z, T_0) = \bar{U}_z \cdot e^{-(t/T_0)^2} \qquad -\infty < t < \infty \tag{25}$$

and

$$\bar{U}_z^2(t; \bar{U}_z, T_0) = \bar{U}_z^2 (1 - |t|/T) \qquad -T \leq t \leq T_0$$
$$= 0 \qquad \text{otherwise} \tag{26}$$

where T_0 is a measure of the duration of a storm in Eq. (1) while it is the duration in Eq.(2). Eq.(1) is used in Reference 3.

The probability of failure p_f is then computed for a storm with a particular set of $\bar{U}_z$ and T_0; $p_f = p_f(\bar{U}_z, T_0)$. Therefore, the probability of failure p_f^* due to a single application of a tatistical storm with $\bar{U}_z$ and T_0 being random is the expected value of $p(\bar{U}_z, T_0)$ with respect to $\bar{U}_z$ and T_0:

$$p_f^* = \iint p_f(\bar{U}_z, T_0) f(\bar{U}_z, T_0) \, d\bar{U}_z \, dT_0 \tag{27}$$

where $f(\bar{U}_z, T_0)$ is the joint density function of $\bar{U}_z$ and T_0. Hence, one can obtain the upper and lower bounds of p_f^* from those of $p_f(\bar{U}_z, T_0)$ using Eq.(27).

4. Numerical Example

As an example, a suspension bridge of the same dimension as the Forth Bridge is considered with $EI = 1.842 \times 10^{13}$ lb-ft^2, $h(x) = 309 - 1200(x/\ell)(1 - x/\ell)$ ft, $m_c g = 2.52 \times 10^3$ lb/ft, $m_T g = 8.38 \times 10^{13}$ lb/ft, $\ell = 3300$ ft, $H = 4.934 \times 10^7$ lb (Eqs.(4) - (8)), and such values of the linear viscous damping coefficients μ_T and μ_c (Eqs.(6) and (7)) that the logarithmic damping decrements of the first mode of independent lateral vibration of the truss and of the cables are both equal to 0.05. In Eqs.(15) - (19), $N = 5$ and in Eqs.(22) - (24), $k = 7$, $\phi = 2000$ ft, $K = 0.01$, $\alpha = 0.2$ and $z = 200$ ft (height of the truss above ground as in Refs.2 and 3).

With these parameter values, the variances of $M^*(t,x)$

and $\dot{M}^*(t,x)$ can now be evaluated numerically (IBM 7090 is used) following the method described in Section2. Because of the same assumption on the structure and the wind, the variance of $M^*(t,x)$ computed here is found to be close to those in Refs.2 and 3. Once these variances are computed, the bounding technique in Refs. 4 and 5 can be applied for the probability of failure $p_f(\bar{U}_z)$ with the time dependent barrier $a(t,x)$. Since in the present study, Eq.(26) is assumed for simplicity, $a(t,x)$ becomes

$$a(t,x) = \xi(x)\left(\frac{1}{2}\rho c A\right)\left\{ n U_d^2 - \bar{U}_z^2 \left(1 - \frac{|t|}{T}\right)\right\} \qquad (28)$$

where $\xi(x)$ is the bending moment of the truss at point x due to $\bar{p}_T$ = 1 lb/ft and $\bar{p}_c$ = 1/8.9 lb/ft (this value 8.9 is taken from Ref.3 and it is the ratio between the corresponding values of cA for the truss and the cables) and U_d is the design wind velocity which is taken as 110 mph in this study.

If the maximum mean wind velocity $\bar{U}_z$ is assumed to have the second asymptotic distribution of largest values[6] under a further assumption that $\bar{U}_z \geq 110$ mph has a return period of 3450 years[3], then the density function $\bar{U}_z$ is given by

$$f(\bar{U}_z) = \frac{\gamma}{\bar{U}_c}\left(\frac{\bar{U}_z}{\bar{U}_c}\right)^{-\gamma-1} \exp\left[-\left(\frac{\bar{U}_z}{\bar{U}_c}\right)^{-\gamma}\right] \qquad (29)$$

where γ is assumed to be 9.0 and $\bar{U}_c = 110\left[-\ln\left(1-\frac{1}{3450}\right)\right]^{1/\gamma}$ mph.

An additional assumption is made at this point that $\bar{U}_z^2$ and T_0 are proportional (or the intensity of storm and its duration are proportional) which appears to reflect the reality at least in approximation. In fact, a value $\bar{U}_z^2 / T_0 = 5$ ft/sec[3] observed from some Japanese records[3] is used here. Because of this assumption, Eq. (27) becomes a single integration hence considerably reducing the computational work:

$$p_f^* = \int_0^\infty p_f(U_z) f(\bar{U}_z)\, d\bar{U}_z \qquad (30)$$

It is evident from Eq.(26) that $p_f(\bar{U}_z) = 1$ when $\bar{U}_z \geq \sqrt{n}\, U_d$. A further assumption $c_T = c_{dT}$ (see Eqs.(1) and (2)) is made here so that the following analysis becomes independent of the value of $\rho c_T A_T$.

The upper and lower bounds of p_f^* are computed as a function of the safety factor n (Fig.1). To be precise, the probability of failure $p_f(\bar{U}_z)$ and therefore p_f^* vary along x. However, the variation is negligible because the quantities $\xi(x) / \sigma_M(x)$ and $\xi(x) / \sigma_{\dot{M}}(x)$ on which the variation depends, are almost constant according to the

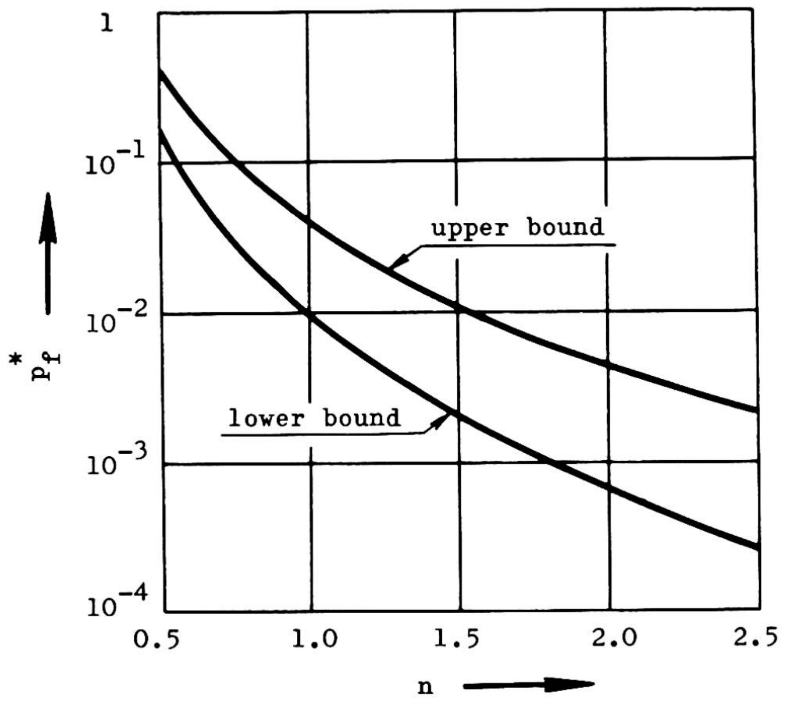

Fig. 1 **Probability of Failure p_f^* as a function of safety factor n**

numerical computation. In spite of the rather wide differences between the upper and lower bounds, the result shown in Fig.1 is quite useful in many respects. For example, using Fig.1 one can examine the effect of increasing the safety factor n . In fact, Fig. 1 indicates that the probability of failure decreases by one order of magnitude from the order of 10^{-2} to that of 10^{-3} by increasing n from 1.0 to 2.0. This implies the increase of the mean life by one order of magnitude from the order of 100 years to that of 1000 years, if it is assumed that significant storms occur on the average once a year. It is pointed out that from the view point of structural reliability analysis, the probability of failure estimated even only within the order of magnitude is a significant information.

5. Conclusion and Acknowledgement

A method of safety analysis by which the probability of failure of a suspension bridge due to lateral wind pressure caused by a (statistical) storm can be evaluated, is presented with a numerical example. The numerical example indicated that the probability can at least be estimated within the order of magnitude. This seems significant and satisfactory enough in view of the various assumptions one has to make as to structural response properties as well as statistical characteristics of the wind.

This study identified the information that is needed to make such an analysis more reliable. Other than those already identified elsewhere (for example, Refs. 2 and 3), the following quantities have to be known with reasonable accuracy; the cross-spectral density $S_{p_1 p_2}^{TC}$ (Eq.(20)) and more importantly, the mean wind velocity $\bar{U}_z(t)$ as a function of time t (Eqs.(25) and (26)) and its statistical nature, and the frequency of occurrence of significant storms.

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SUMMARY

This study presents a method of safety analysis of aerodynamically stable suspension bridges subjected to lateral wind pressure. The pressure is treated as a random process in space as well as in time. A numerical example is given under certain assumptions of statistical characteristics of the wind velocity. Importance of such a study lies not only in the development of a method of probabilistic safety analysis but also in the fact that it indicates what further information, statistical or otherwise, is needed to make the safety prediction more reliable.

RÉSUMÉ

Cette étude présente une méthode d'analyse de sécurité pour ponts suspendus aérodynamiquement stables soumis à une pression de vent latérale. La pression est supposée arbitraire dans l'espace et dans le temps. Un exemple numérique a été calculé à partir de certaines hypothèses des caractéristiques statistiques de la vitesse du vent. L'étude ne développe pas seulement une méthode d'analyse de sécurité probabiliste, elle indique avant tout quelles informations supplémentaires, statistiques ou autres, sont requises pour rendre les estimations de sécurité plus précises.

ZUSAMMENFASSUNG

Dieser Beitrag zeigt ein Verfahren für die Sicherheitsbetrachtung aerodynamisch stabiler Hängebrücken, die seitlichem Winddruck ausgesetzt sind. Der Druck wird als zufälliges Ereignis in Raum und Zeit behandelt. Ein numerisches Beispiel für bestimmte Annahmen der statistischen Charakteristiken der Windgeschwindigkeit wird angegeben. Die Wichtigkeit solcher Untersuchungen liegt nicht allein in der Entwicklung der wahrscheinlichen Sicherheit, sondern auch darin, daß erkannt wird, welche statistischen oder sonstigen Auskünfte künftig für die Sicherheitsvoraussage zuverlässig sein werden.