Stability of thin-steel hyperbolic paraboloid roofs

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Stability of Thin-Steel Hyperbolic Paraboloid Roofs

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Stabilität dünner hyperbolischer Paraboloid-Stahldächer

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Several types of roofs for a variety of structures may be built of thin-steel hyperbolic paraboloid (hypar) units. The basic unit of such roofs is composed of structural steel or thin-steel edge members and of a warped surface made of thin (light gage) steel panels [1]. Attractive roofs can be constructed by various combinations of these units. The resulting structure has high strength to weight ratio. Some aspects of metal hypar structures have been described by the theme reporter, Monsieur P. A. Lorin.

A remarkable example is shown in Fig. 1. This hangar,

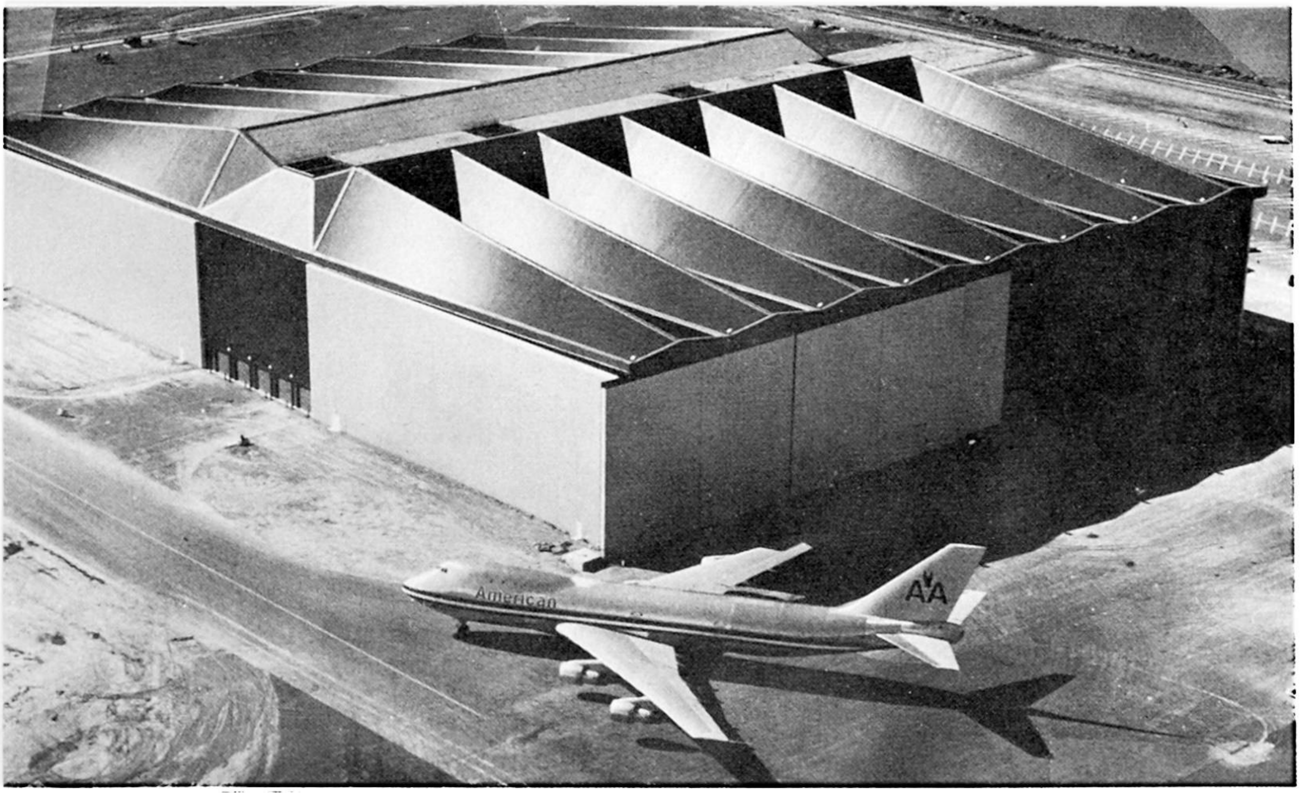

Fig. 1

designed by Lev Zetlin Associates, New York, N.Y., is covered by 230 ft (70 m) long and 56 ft (17 m) wide thin-steel hypar modules. The depth of these free cantilevers varies from 40 ft (12 m) to 4 ft (1,2 m) to accommodate the largest aircraft.

The key to the success of this type of structures lies in the well demonstrated fact [2, 3] that thin-steel diaphragms can resist in-plane shear forces quite well and that the forces in hypar shells are dominated by membrane shear forces.

Design Problems

The calculation of stresses in thin-steel hypars can usually be estimated using the simple membrane theory. This was demonstrated during an extensive analytical and experimental study at Cornell University. However, design is ordinarily controlled by stiffness (deflection and buckling) considerations.

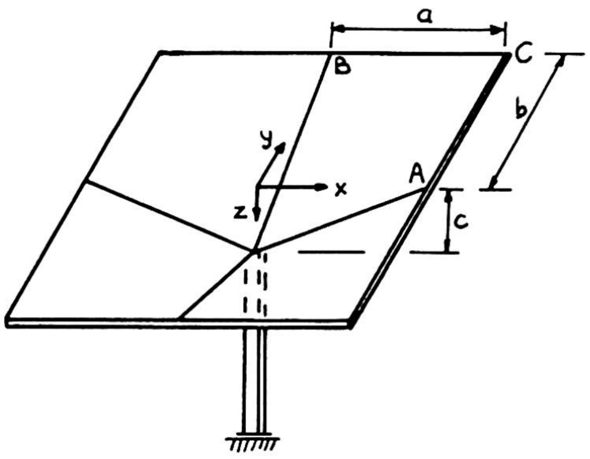

Fig. 2

Deflections may be a problem at flat corners of several types of hypar roofs (such as at the outside corners of the inverted umbrella roof of Fig. 2), or at the center of shallow deck units.

Instability may develop in three ways: a) the deck may buckle under the membrane shear forces, b) the compression edge members may buckle due to the nonconservative force system transmitted by the membrane shear [1], and c) local buckling of the deck may occur. The first and most important type of instability is the subject of this contribution. Edge member buckling was found to be unimportant in well designed structures [3], and local buckling is discussed in detail in papers on diaphragm design.

Deflections and instability depend strongly on the effective shear rigidity of the deck. This quantity is influenced by several factors [2], primarily by the connections between the deck and the edge members and between the panels of the deck, in addition to the geometry of the deck deformations. Frequently two layers of decking, with the corrugations or deformations running in orthogonal directions, are used to increase stiffness and, in some cases, to decrease bending stresses. The second important factor is the curvature (loosely speaking the rise-span ratio) of the structure.

Shell Buckling

The buckling of thin-steel decks is caused by diagonal compressive stresses associated with membrane shear stresses. The situation is very similar to the behavior of thin webs or shear diaphragms. Whereas the stability of isotropic hypar shells with fully supported edges was evaluated explicitly in a classical paper by Reissner [4], the buckling of orthotropic shells is a much more complex matter because of the presence of several rigidities D_x, D_y, D_{xy}, and D_1.

Considering the importance of shell buckling in the design of thin-steel hypar structures, the question was approached from several directions. An experimental study consisted of small and medium-scale model tests. Two analytical methods were developed: a rigorous analysis based on the finite element technique, and an approximate analysis using energy methods.

Experimental Investigation

Several tests were performed on small scale, 12 in. by 12 in. (30 cm by 30 cm) umbrella type hypars with sinusoidal corrugated decking [1]. However, it was difficult to assess the basic properties of the deck, connections, and edge members, and therefore these tests resulted in little quantitative data.

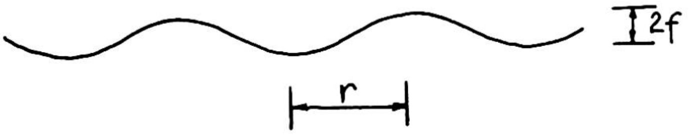

Fig. 3

Four medium-scale, 12 ft by 12 ft (3,65 m by 3,65 m) umbrella type hypars were tested. The decking consisted of one or two layers of standard corrugated sheets (Fig. 3), for which f = 0.25 in. (6,4 mm) and r = 1.33 in. (3,4 cm). The edge members were structural tubing. The decks were attached with #8x5/8 in. self-tapping screws at 2-2/3 in. (6,8 cm) spacing. The models were placed upside down, and the extension of the columns were secured to the floor. The loading was applied with air pressure in rubberized canvas bags. The buckled shell in one of the tests is shown in Fig. 4.

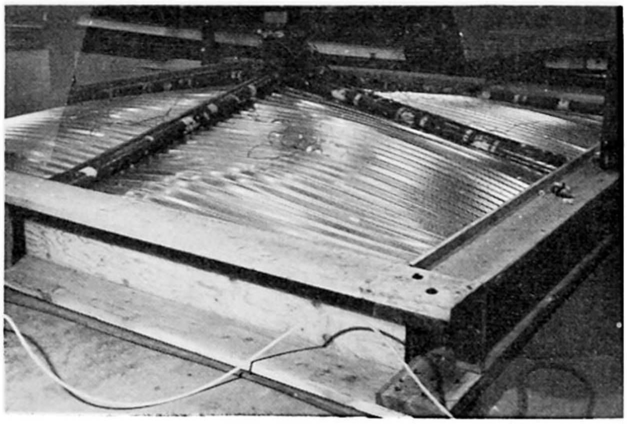

Fig. 4

Finite Element Analysis

The analysis of hypars is a highly complex subject. A number of approaches were based on shallow shell theories to evaluate bending effects and buckling. Often finite-difference solutions have been used. The difficulty with these approaches is the proper consideration of complex boundary conditions, especially in the case of thin-steel structures.

The finite element technique offers a method that can include a number of variables, such as orthotropy, edge member bending in two directions, the eccentricity of connection between the deck and the edge members, local loads, and various support conditions, in a systematic fashion. Two finite element solutions were developed by Banavalkar [3] as part of the present investigation: one used flat rectangular elements, the other employed curved shell elements rectangular in plan. The former is a special case of the latter.

The displacement fields for the curved elements were assumed to be linear for u and v, and the cross product of Hermitian polynomials for w. Linear strain-displacement relationships were utilized. The third strain component is

$$\gamma_{xy} = u_y + v_x - \frac{2c}{ab} w \qquad (1)$$

since the equation of the surface is

$$z = \frac{c}{ab} xy \qquad (2)$$

The total strain energy is caused by membrane stresses, shell bending, and edge member bending, twisting, axial straining, and warping. The displacement fields for the edge members were linear for axial displacements, and nonlinear cubic for transverse and twist displacements. The possibility of relative displacements at the connection between the deck and the edge members was considered in the analysis.

In the instability analysis, the change of potential energy caused by membrane forces was accounted for. This resulted in the following form of the stiffness equation

$$\{P\} = \Big[[K] + [N]\Big]\{D\} = [K_{eff}]\{D\} \qquad (3)$$

where [K] is the master stiffness matrix, [N] is the incremental stiffness matrix that is equal to the second derivative of the potential energy of the membrane forces, and {P} and {D} are the load and displacement vectors, respectively. The matrix [N] depends on the in-plane forces, and therefore also on the normal displacements.

In the present investigation a load incrementation method was used instead of a direct eigenvalue evaluation. The load-deflection curve exhibits a rapid change in slope at instability. In each load step the in-plane forces N_x, N_y, and N_{xy} were evaluated from {D} using a linear analysis. The new stiffness matrix $[K_{eff}]$ in Eq. (3) was used in subsequent iterations. This procedure was repeated at each load level. The analysis was verified by comparisons with Reissner's approach for the isotropic, fully supported case, and with other finite element studies.

Example

A hypar roof is composed of four units and is supported by four 10 in. (25,4 cm) square tubular columns (Fig. 5). It covers an area of 60 ft by 60 ft (18,3 m by 18,3 m). The low points A, B, C, D are connected with 3 in. (7,6 cm) diameter rods along the perimeter. The top horizontal edge members are W30X99 and the sloping outside members are W18X64. The deck is connected eccentrically to the horizontal edge members. The deck is made of two corrugated layers (Fig. 3) with thicknesses of h = 0.0747 in. (1,9 mm), and f = 0.375 in. (9,5

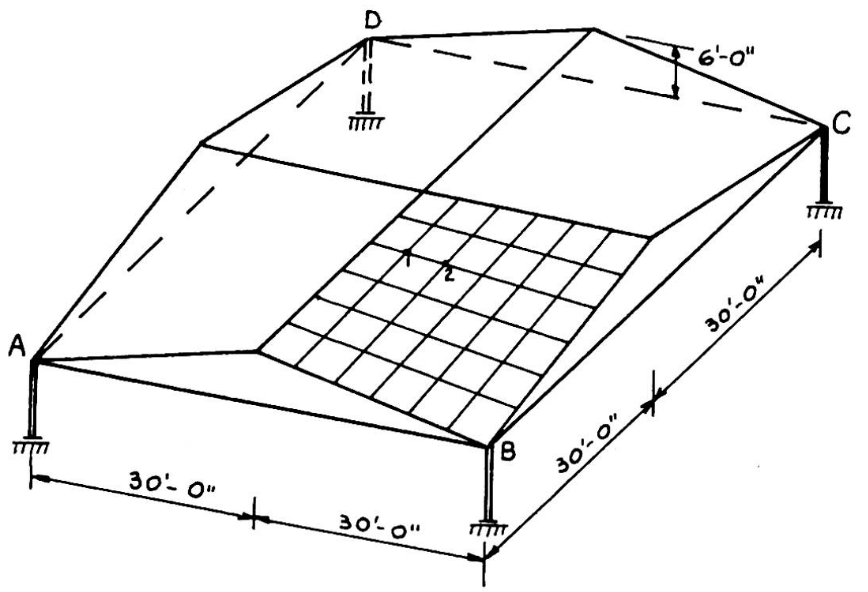

Fig. 5

mm), r = 1.5 in. (3,8 cm). The effective shear stiffness of the units was estimated from flat shear tests to be 0.10Gh.

Deck buckling analysis was carried out by specifying several load steps and three iterations at each load level. The deflections at two selected points (Fig. 5) are plotted in Fig. 6. It is seen that buckling occurs at a load of about 65 psf (320 kg/m^2).

The buckling analysis predicted about 60 psf (295 kg/m^2) critical load for the experiment of Fig. 4; the measured value was about 75 psf (370 kg/m^2). It is suspected that the actual shear rigidity was greater than the value assumed in the analysis.

Analyses indicate that prebuckling deflections and the bending stiffness of edge members have little effect on instability, but the axial stiffness of edge members does influence it somewhat. The deck buckling load of hypars with two layers is ordinarily at least three times greater than that of similar structures with single layers of decking.

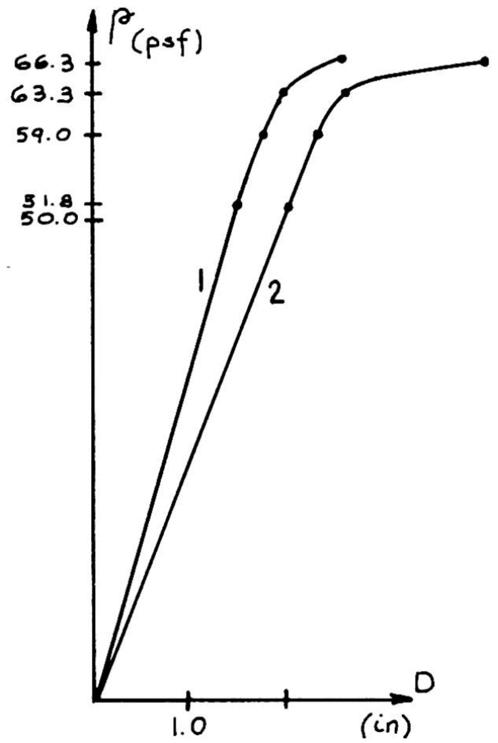

Fig. 6

Energy Analysis

The finite element instability analysis involves considerable computer expense. In order to aid in preliminary designs, an approximate deck buckling analysis was also developed.

The potential energy V of an orthotropic shell is [3, 5]

$$V = \frac{1}{2} \int_0^B \int_0^A [D_x w_{xx}^2 + 2D_1 w_{xx} w_{yy} + D_y w_{yy}^2 + 4D_{xy} w_{xy}^2 + 4G_{eff} h(c/ab)^2 w^2 + 2N_{xy} w_x w_y] dxdy \qquad (4)$$

where the subscripts denote partial derivatives and N_{xy} = pab/2c is the membrane shear force in the shell under uniform loading p. The assumed deflected shape was

$$w = \sin \frac{\pi y}{b} \sin[\frac{n\pi}{a} (x - sy)] \qquad (5)$$

where s is the tangent of the angle of the buckles measured from the y axis, and n is the number of buckled waves. This displacement function gave good results in the buckling analysis of flat shear diaphragms.

The substitution of w into Eq. (4) yields

$$p_{cr} = \frac{c}{abs} \left(\frac{\pi}{b}\right)^2 [D_x \beta^2 + 2D_1(1+\beta^2 s^2) + D_y(1/\beta^2 + \beta^2 s^4 + 6s^2) + 4D_{xy}(1+\beta^2 s^2) + \bar{G}/\beta^2] \qquad (6)$$

where

$$\beta = \frac{nb}{a} \quad \text{and} \quad \bar{G} = \frac{4G_{eff}h}{\pi^4}\left(\frac{bc}{a}\right)^2$$

This expression has to be minimized with respect to s and n. The buckled shape for a hypar unit with single layer of corrugated sheet decking is shown in Fig. 7.

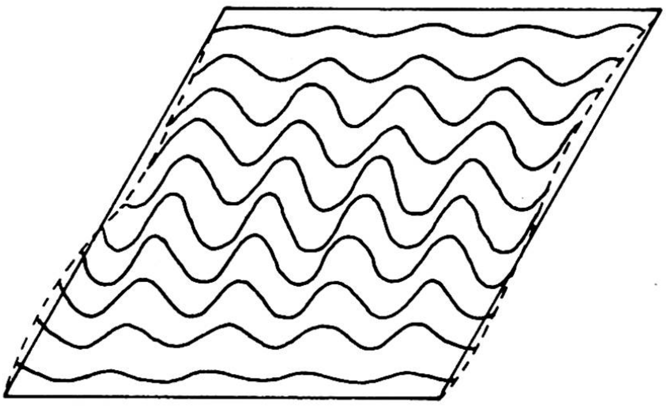

Fig. 7

Computer-aided analyses gave results that agreed well with finite element calculations and with tests. For the aforementioned example the buckling load from the energy analysis is 51 psf (250 kg/m²) that occurs at 44.4 degrees and with three half sine waves.

The computer time involved in the search of the minimum of p_{cr} is very small and therefore numerous analyses can be performed with little expense. Comparative studies to determine the relative importance of variables indicate that the buckling load of single corrugated shells is roughly proportional to

$$\frac{\sqrt[3]{\alpha}}{b^2}\left(\frac{hc}{a}\right)^{\frac{3}{2}} \qquad (7)$$

where α is a nondimensional shear rigidity factor $\alpha = G_{eff}/G$. The accuracy of the factor (7) is indicated in Fig. 8 for standard (f = 0.25 in. and r = 1.33 in.) and deep (f = 0.375 in. and r = 1.50 in.) corrugated decks. In the case of double layers, the factor (7) is approximately linear with h.

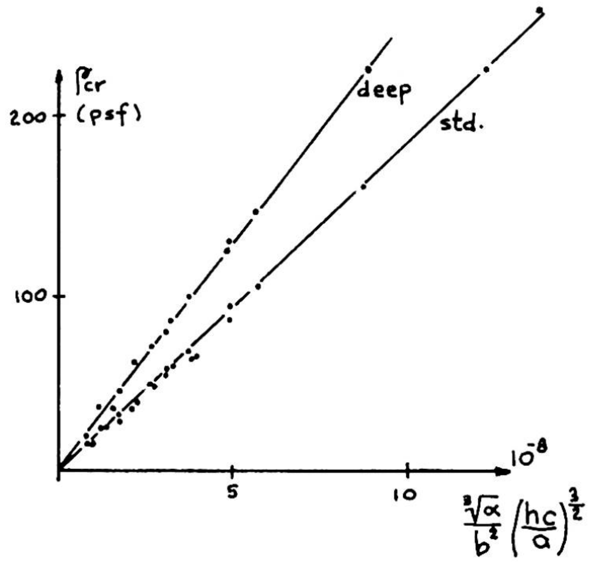

Fig. 8

For single layer shells the bending rigidity in the strong direction (D_y) is, by far, the most important rigidity factor, and p_{cr} varies approximately as $\sqrt{D_y}$. For corrugated decks D_y is approximately equal to

$$D_y \approx \frac{Ef^2h}{3}\left(1.6 + \frac{f}{2r}\right) \qquad (8)$$

The effect of D_x is much less, and the influence of D_{xy} and D_1 is negligible for corrugated shells, in fact, D_1 can be taken as zero. But, as mentioned previously, the most important variables affecting the behavior of thin-steel hypars are the curvature c/ab and the effective shear rigidity.

The approximate buckling analysis and the simple membrane theory may be sufficient in the design of minor structures, such as roofs for service stations. However, in the final design of major structures more complex analyses, for example using the finite element method, must be used to calculate buckling loads, deflections, and stresses.

Acknowledgments

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Summary

The design of hypar shell roofs with thin-steel decking is often controlled by deflections and buckling. Finite element and approximate energy instability analyses are described that may be used by designers. Experiments confirmed the analytical approaches.

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