Hull-core structures subjected to bending and torsion

Hull-Core Structures subjected to Bending and Torsion

Eléments hull-core soumis à la flexion et à la torsion

Hull-core Elemente beansprucht durch Biegung und Torsion

A. COULL   N.K. SUBEDI
Department of Civil Engineering
University of Strathclyde
Glasgow, Scotland

1. Introduction

During the past few years a number of new structural systems have been developed for high-rise buildings. A natural evolution of the familiar rigidly-jointed frame which has been used increasingly is the rigid box or tube type of system [1]. This form consists essentially of four orthogonal frame panels forming in plan a framed tube (cf. Fig. 1). Each panel consists of a number of exterior columns connected by stiff spandrel beams at each floor level. Although the exterior 'perforated tube' may be used solely to resist all wind forces, it is commonly designed to act in conjunction with an inner core to form a hull-core or tube-in-tube system. This inner core, which may be used to contain all building services, will consist of some combination of beams, columns, shear walls and box elements surrounding lift shafts and stair wells. The open area between the outer frame and inner core allows flexibility in layout planning. The system has been employed for both steel and concrete construction.

The behaviour of a perforated tube is more complex than that of a simple closed-tube, and the stiffness is less. In addition to the cantilever bending action, which produces tensile and compressive stresses on opposite faces of the tube, the side frames undergo the usual plane-frame shearing action in each storey. The primary action is complicated by the fact that the flexibility of the spandrel beams produces a shear lag which has the effect of increasing the stresses in the corner columns, and reducing them in the inner columns of the normal panels. The latter effects will produce warping of the floor slabs and consequent deformations of interior partitions and secondary structure.

The inclusion of an inner core which is connected to the outer hull by moment-resistant members will have the effect of inducing axial forces in the columns of the normal panels, which will tend to offset the reduction of stresses caused by shear lag. However, it appears to be more economic to use simply supported floor systems which do not resist lateral forces; for this reason, the floor system can be relatively shallow, and longer spans can be used between the outer and inner structures.

It is important to be able to assess both the warping effect and the amount of sway produced by lateral forces, since either may control the design of the structure. The object of the present paper is to demonstrate how the analysis of the complex three-dimensional structure may be simplified through a recognition of the dominant structural actions, particular attention being paid to the torsional behaviour. In addition, experimental results from tests on model structures are presented to show the different modes of behaviour of framed-tube, hull-core, and unperforated tube structures.

2. Analysis

Consider the hull-core structural system of Fig. 1 subjected to some lateral loading. It is assumed that the floor system is so stiff in its own plane that all cross-sections of the building undergo only rigid body movements in plan. Any applied load may then be considered equivalent to the superposition of a bending force and a torsional moment referred to the centre of rotation. If the behaviour of the structure is elastic, the two may be treated independently.

Since the overall action is complex, it is advantageous to consider the action of the outer tube alone before examining the composite hull-core interaction.

2.1 Outer Tube

(i) Bending action.

In a framed-tube subjected to bending by lateral forces, (cf. Fig. 1) the loads are resisted by two primary actions – the axial deformations of the normal panels (AB and CD) and the racking actions in the side panels (AD and BC). The primary interactive forces between the normal and side panels will be vertical (shear) forces transmitted through the corner columns. Secondary out-of-plane deformations will occur, but it is assumed that these will be restricted sufficiently by the high in-plane stiffness of the floor system to be neglible in comparison with the primary effects. All individual elements will then deflect equally at each floor level.

By assuming this dominant mode of deformation, the three-dimensional frame may be replaced by the equivalent plane frame shown in Fig. 2. It is assumed that, as is generally the case, the structure is symmetrical about both centre-lines so that only one-quarter (FDE) need be considered in the analysis, the appropriate joint conditions on the centre-lines being as indicated. The high in-plane stiffness of the floor system allows the horizontal forces at each floor level to be applied in the plane of the side frames (Fig. 2).

It is desired to transmit only vertical forces between side and normal panels, whilst maintaining conditions of compatibility at the corner. This may be achieved very simply in a normal plane frame analysis by the use of fictitious 'vertical shea transfer' beams DD′ (Fig. 2) at each floor level. In the stiffness matrix for the fictitious beams, the shear elements must be made a very large quantity, (relative to the elements in the stiffness matrices for the real members) with all other elements zero [2]. In practice, it has been found that a value of around 10^4 times the largest element gives the desired transfer. The corner columns can be considered to contribute half their cross-sectional areas to each panel, together with their appropriate second moment of area in each orthogonal direction.

From the frame analysis, the flexibility matrix F_H may be determined to give the relationship between applied loads and the resulting horizontal deflections,

$$Y_H = F_H \, P_H \tag{1}$$

where Y_H and P_H are column vectors of the horizontal deflections and applied horizontal forces at each storey level.

The accuracy of the technique was checked by comparing the results obtained for a number of plane and space frames with those from standard computer programmes. In one typical test, the six-storey structure shown in plan in Fig. 3 was chosen, this being the largest configuration which could be solved by the available program within the given constraints. The section properties of the columns and spandrel beams were as follows: columns – 8 in x 8 in x 58 lb; $I_{xx} = 227.3$ in^4; $I_{yy} = 74.9$ in^4;   A = 17.06 in^2;   J = 3.37 in^4;   beams – 5 in x 12 in x 31.8 lb; $I_{xx} = 215.8$ in^4; $I_{yy} = 9.5$ in^4,   A = 9.26 in^2;   J = 0.92 in^4.   The storey height was maintained constant at 12 ft.   The results obtained from the two analyses are

compared in Tables 1 and 2, the latter giving deflections throughout the height, and the former axial forces and bending moments at the most heavily loaded first storey level.   The results refer to a unit load (1 ton) at each floor level.

Column	Space Frame Analysis			Simplified Method		
	Axial Force	Bending Moment (ton in)		Axial Force	Bending Moment (ton in)	
	(ton)	Lower End	Upper End	(ton)	Lower End	Upper End
1	0	-93.9	-84.1	0	-97.5	-87.4
2	+0.16	-188.6	-169.8	+0.19	-195.9	-176.6
3	-21.51	-167.2	-127.0	-10.90	-173.9	-132.5
3'		-2.68	-5.13	-10.90	-2.65	-5.14
4	-3.63	-2.42	-4.65	-3.66	-2.39	-4.66
5	-0.97	-0.63	-1.15	-0.97	-0.60	-1.12
6	-0.30	-0.15	-0.26	-0.30	-0.14	-0.25

Table 1.   Axial Forces and Bending Moments in First Storey Columns.

Storey	1	2	3	4	5	6
Deflection (Space Frame) in.	0.234	0.484	0.695	0.863	0.984	1.058
Deflection (Simplified Method) in.	0.242	0.497	0.711	0.879	0.999	1.072

Table 2.   Deflections

The agreement was even better at the upper levels, the axial forces and moments corresponding to within 1% at the third storey level.   Small discrepancies will always occur since the three-dimensional analysis does not include the in-plane restraint of the floor slabs.

The accuracy of the simplified techniques has also been demonstrated experimentally[2] .

(ii) Torsional action.

The application of a twisting moment to a framed-tube structure produces two forms of deformation, a pure rotation and an out-of-plane warping displacement of the cross-section.   The combined action may be considered as a combination of the plane frame actions of the four panels and the effects of the interactions between the panels.   It is again assumed that the floor slabs act as rigid diaphragms so that all structural elements at any cross-section rotate equally under torque.   Twisting moments will be resisted primarily by shearing actions in the orthogonal panels, and the torsional moments resisted by individual beam and column elements are assumed negligible.   If the rotations are small, the frame panels may be assumed to deform in their own plane.

In Fig. 4, let θ be the rotation of any cross-section of the structure under

the action of an applied torque T.   If Δ_1 and Δ_2 are the deflections of panels DC and DA in their own planes, then

$$\theta = \frac{\Delta_1}{c} = \frac{\Delta_2}{b} \qquad (2)$$

For torsional equilibrium,

$$2q_1c + 2q_2b = T \qquad (3)$$

where q_1 and q_2 are the horizontal shear forces resisted by panels DC and DA respectively.

Equations corresponding to (2) and (3) may be written for all storey levels and, expressing them in matrix form, the compatibility and equilibrium conditions for the entire structure become,

$$\underset{\sim}{\theta} = \frac{1}{c} \underset{\sim}{\Delta}_1 = \frac{1}{b} \underset{\sim}{\Delta}_2 \qquad (4)$$

$$2c \underset{\sim}{q}_1 + 2b \underset{\sim}{q}_2 = \underset{\sim}{T} \qquad (5)$$

where $\underset{\sim}{\theta}$, $\underset{\sim}{\Delta}_1$ and $\underset{\sim}{\Delta}_2$ are column vectors of rotations and horizontal deflections, $\underset{\sim}{T}$ is a vector of total applied twisting moments, and $\underset{\sim}{q}_1$ and $\underset{\sim}{q}_2$ are vectors of the total horizontal forces in the panels, at each floor level.

The panel horizontal displacements may be related to the horizontal shear forces by the relationship

$$\underset{\sim}{\Delta}_1 = \underset{\sim}{F}_1 \, \underset{\sim}{q}_1 \qquad \text{and} \qquad \underset{\sim}{\Delta}_2 = \underset{\sim}{F}_2 \, \underset{\sim}{q}_2 \qquad (6)$$

where $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$ are square matrices of horizontal deflection flexibilities for panels DC and DA, which may be derived from a standard plane frame analysis. When evaluating the flexibility matrices the interactions between orthogonal panels must be included;   Fig. 5 indicates the unit forces required for the evaluation of $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$ respectively.   By this means, both in-plane rotations and out-of-plane warping effects may be included.   Again only one-quarter of a symmetrical structure need be included in the analysis, using the appropriate skew-symmetrical boundary conditions at the axes of symmetry.

The matrix of rotations θ may be obtained from equations (4), (5) and (6) to be,

$$\underset{\sim}{\theta} = \left[2c^2 \, \underset{\sim}{F}_1^{-1} + 2b^2 \, \underset{\sim}{F}_2^{-1} \right]^{-1} \underset{\sim}{T} = \underset{\sim}{F}_3^{-1} \, \underset{\sim}{T} \qquad (7)$$

where $\underset{\sim}{F}_3$ is the torsional flexibility matrix for the outer hull.

The displacements and horizontal forces are obtained from equations (4) and (6) respectively, and all internal stress-resultants follow from the frame analysis.

2.2 Hull-Core Structure

In view of the high in-plane stiffness of the floor slabs, it is assumed that the hull and core are constrained to deflect together in the composite structure.   Consequently, under pure bending action, all elements have the same horizontal deflection, whilst under a pure twisting action, the rotations of both hull and core will be equal at each floor level.   The two actions may again be considered independently.

    (i) Bending action.

If, as is generally the case, the floor structure is designed to be

effectively pin-connected to both hull and core, no moments are transmitted
between the two.   Under the action of wind forces, the floor system then acts
as a rigid pin-ended link transmitting horizontal forces only.

If the applied loads and resulting deflections of the core are related in
an analogous manner to equation (1) by a flexibility matrix $\underset{\sim}{F}_c$, then

$$\underset{\sim}{Y}_c = \underset{\sim}{F}_c \underset{\sim}{P}_c \qquad (8)$$

The flexibility matrix $\underset{\sim}{F}_c$ may be determined from ordinary beam theory if the
inner core can be assumed to act as a pure cantilever.   If it consists of an
inner framed tube, the method described previously may be used.   The continuous
connection technique may be utilised to give an assessment of the flexibility of
a coupled shear wall core [3] .

The conditions of horizontal compatibility and equilibrium at each level are,

$$\underset{\sim}{Y}_c = \underset{\sim}{Y}_H \qquad \text{and} \qquad \underset{\sim}{P} = \underset{\sim}{P}_H + \underset{\sim}{P}_c \qquad (9)$$

where $\underset{\sim}{P}$ is the column vector of resultant wind forces at each floor level.

The solutions of equations (1), (8) and (9) yields the distribution of forces
between the hull and core,

$$\underset{\sim}{P}_H = \left[\underset{\sim}{I} + \underset{\sim}{F}_c^{-1} \underset{\sim}{F}_H \right]^{-1} \underset{\sim}{P}$$

$$\underset{\sim}{P}_c = \left[\underset{\sim}{I} + \underset{\sim}{F}_H^{-1} \underset{\sim}{F}_c \right]^{-1} \underset{\sim}{P} \qquad (10)$$

Once the distribution of horizontal forces is known, the stress-resultants
in the frame panels follow from the frame analysis.

(ii) Torsional action.

The twisting moments and rotations of the inner core will be related by

$$\underset{\sim}{\theta}_c = \underset{\sim}{F}_4 \underset{\sim}{T}_c \qquad (11)$$

where $\underset{\sim}{F}_4$ is the matrix of torsional flexibility coefficients.   The matrix $\underset{\sim}{F}_4$
may be determined by the technique described earlier if the core consists of an
inner framed tube;   for thin walled elements, Vlasov's theory of thin walled
beams may be used [3] , whilst if the core consists of coupled shear-wall elements,
the continuous connection method may be utilised to give the flexibility
coefficients [3] .

The conditions of rotational compatibility and equilibrium then become,

$$\underset{\sim}{\theta} = \underset{\sim}{\theta}_c \qquad \text{and} \qquad \underset{\sim}{T} = 2c \underset{\sim}{q}_1 + 2b \underset{\sim}{q}_2 + \underset{\sim}{T}_c \qquad (12)$$

The solution of equations (2), (4), (6), (11) and (12) yields the matrix of
rotations,

$$\underset{\sim}{\theta} = \left[2c^2 \underset{\sim}{F}_1^{-1} + 2b^2 \underset{\sim}{F}_2^{-1} + \underset{\sim}{F}_4^{-1} \right]^{-1} \underset{\sim}{T} \qquad (13)$$

The horizontal shear forces in the outer frame panels and twisting moments
in the core follow from equations (4) and (6), and (11) respectively.

The analysis assumes that no torsion coupling of the hull and core occurs
through the floor slabs.   That is, the floor system is assumed to offer no
restraint against warping of either hull or core.

3. Experimental Investigations

In parallel with the theoretical studies, a series of tests was carried out on model framed-tube and hull-core structures, with a view to assessing the accuracy of theoretical predictions and to examining the influence of an inner core on the deformations and stresses in the composite structure.

Experiments were performed on two fifteen-storey perspex models, one with a closed box (torsionally stiff) core and the other with an open channel (torsionally weak) core. The columns for the outer framed tube and the plates for the inner cores were cut from 3/16 in. thick perspex sheet, and the floor slabs from 1/8 in. thick sheet. The hull had eight columns along one side and five along the other, each column being 1/2 in. wide , the corner ones being glued together to form an angle section. The plan dimensions are shown in Fig. 1. The storey height was 2-1/8 in., with a 2 in. clear height between floor slabs. Because of the small size of the models, it was not possible to form the desired hinged connections between vertical and horizontal members, and all joints were glued. The perimeter columns were then connected solely by floor slabs at each floor level, the columns being glued into edge slots in the slabs.

The models were tested initially as framed-tube structures, and subsequently, after inserting and glueing the inner cores into central holes left in the floor slabs, as composite hull-core systems. The inner cores were tested individually to check their calculated bending and torsional stiffnesses. A rigid-base condition was achieved by glueing all columns and cores into slots passing through a one-inch thick perspex base plate. The models were cantilevered horizontally in a test frame, the base being further stiffened by passing steel sections across the base plate as near as possible to the model.

Lateral loads were applied by hanging dead weights from nylon cords at each floor level, deflections and strains being measured by dial gauges and electrical resistance strain gauges, respectively. A concentrated twisting moment was applied at the free end by means of a twelve-inch diameter perspex disc containing a rectangular hole to fit over the end of the model (cf. Fig. 6). Equal and opposite tangential loads were applied at opposite ends of a diameter by dead weights which hung from thin nylon threads passing over pulleys. Similar devices were used to apply twisting moments to the inner cores before they were inserted into the framed-tube models to form hull-core structures. Loads were applied in increments, at fixed time intervals to minimise creep effects, and unit values obtained from the best linear curve of strain or deflection against load.

Separate tests were performed on a representative section of floor slab and columns in order to assess the effective stiffness of the floor slab in connecting the perimeter columns.

In order to compare the mode of behaviour of a perforated tube with that of a pure tube, similar tests were carried out on models of the same overall dimensions as Fig. 1, but with the outer columns replaced by continuous plates. These were cut from 3/32 in. thick sheet in order to make the cross-sectional area of the pure tube as nearly equal to that of the columns as possible (i.e. within 9 per cent

Some of the results are shown in Figs. 6, 7 and 8, Fig. 6 shows the measured and calculated rotations due to a unit torque at the free end. Figs. 7 and 8 show the bending-stress distributions and the deflection profiles for a uniformly distributed load.

4. Discussion

It has been shown that the complex three-dimensional behaviour of a framed-tube and a hull-core structure can be approximated by an equivalent plane frame technique. The sizes of the matrices involved in an analysis are thereby much

reduced, and a much larger structure may be analysed by a given size of computer.

Some experimental results have been presented from tests on small-scale model structures. It is seen in Fig. 6 that the torsional stiffness of a framed-tube can be very much less than a pure tube of the same cross-sectional area as the perimeter columns. Reasonable agreement is achieved between theory and experiment, which probably indicates that the warping of the floor slabs is of less significance than the bending coupling action. It was not possible to measure the axial stresses in the columns due to torsion, and deformations only were measured. It was found that the results obtained using the standard sectional torsional stiffness of St. Venant were much less accurate than those obtained by Vlasov's theory, which takes account of the warping of the cross-section. The latter was thus used exclusively for the inner core sections. Fig. 7 indicates how the stress-distribution in the framed-tube can be altered significantly by the addition of a core if the floor slabs are able to transmit bending actions between the two. The stresses in the columns opposite the core are subjected to additional axial stresses, which has the effect of reducing the shear lag. Similar results were obtained in the case of a channel core, although little coupling was developed on the open side of the channel [3]. The stresses in the unperforated tube are not shown, since they followed closely engineer's beam theory, with only a very small degree of shear lag being measured. At the moment it is not possible to estimate accurately the effective bending stiffness of a floor slab connecting a core and perimeter column, and the coupling effect cannot as yet be treated by the simplified technique. However, this is of little significance if in practice the floor system is not designed to transmit bending moments. Fig. 8 demonstrates how the bending stiffness of the perforated tube can be considerably less than a plain tube of the same cross-sectional area.

As well as being of interest in their own right, it is hoped that the experimental data may be useful in the assessment of the accuracy of more sophisticated methods of analysis for this type of structure.

References

1. KHAN, F.R.   "Recent Structural Systems in Steel for High-Rise Buildings"
                       B.C.S.A. Conference on Steel in Architecture, London, 1969.

2. COULL, A. and   "Framed-tube Structures for High-rise Buildings"
   SUBEDI, N.K.   Jnl. Struct. Div., A.S.C.E., Vol. 97, No. S.T.8, 1971, p.2097.

3. SUBEDI, N.K.   "Hull-core Structures for High-rise Buildings",
                       Ph.D. Thesis, University of Strathclyde, 1971.

Summary

A method is presented for reducing the analysis of a three-dimensional hull-core structure to that of an equivalent plane frame. Some experimental data are presented from tests on model structures subjected to bending and torsion.
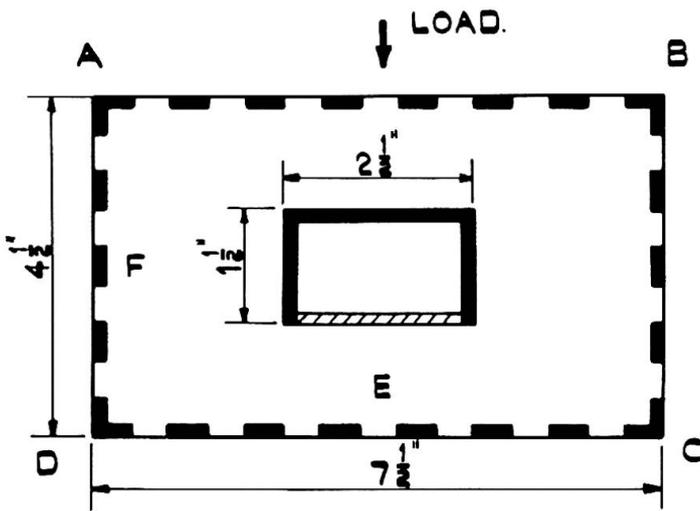
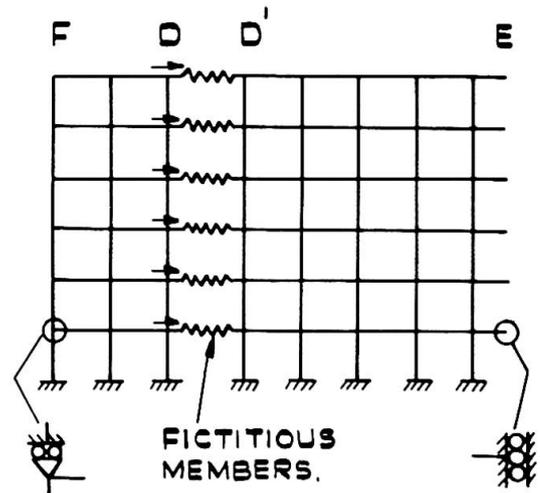FIG. 1. HULL - CORE STRUCTURE WITH BOX OR CHANNEL CORE.
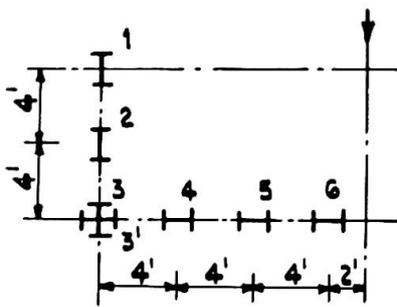

FIG. 2. EQUIVALENT PLANE FRAME.

FIG. 3.
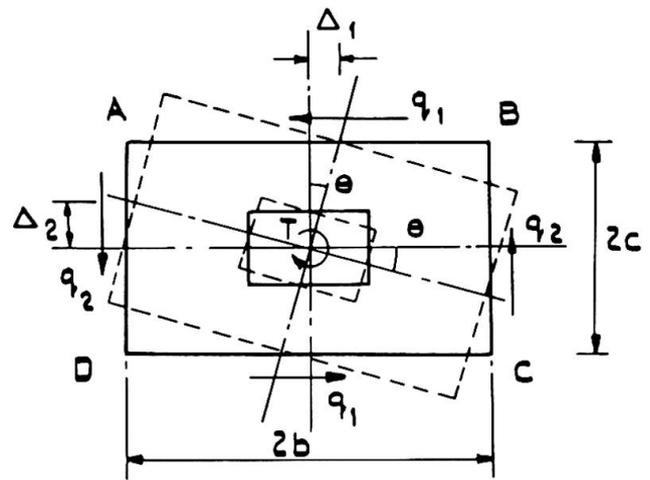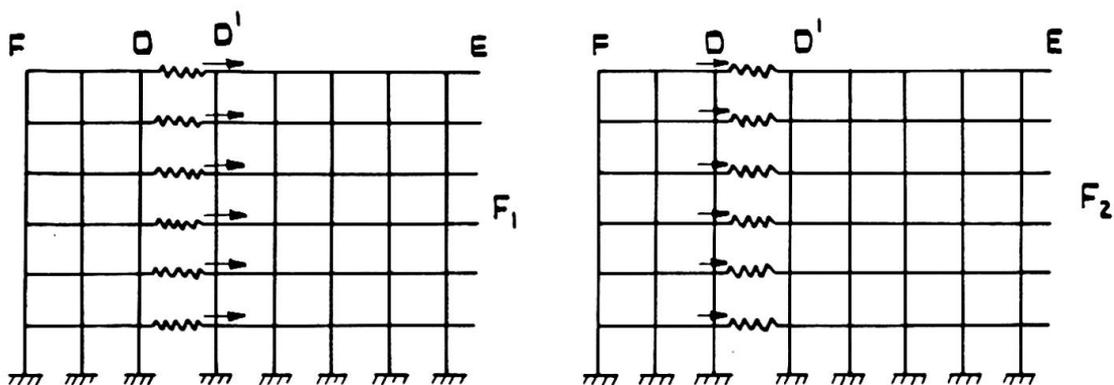

FIG. 4. ROTATION OF STRUCTURE.

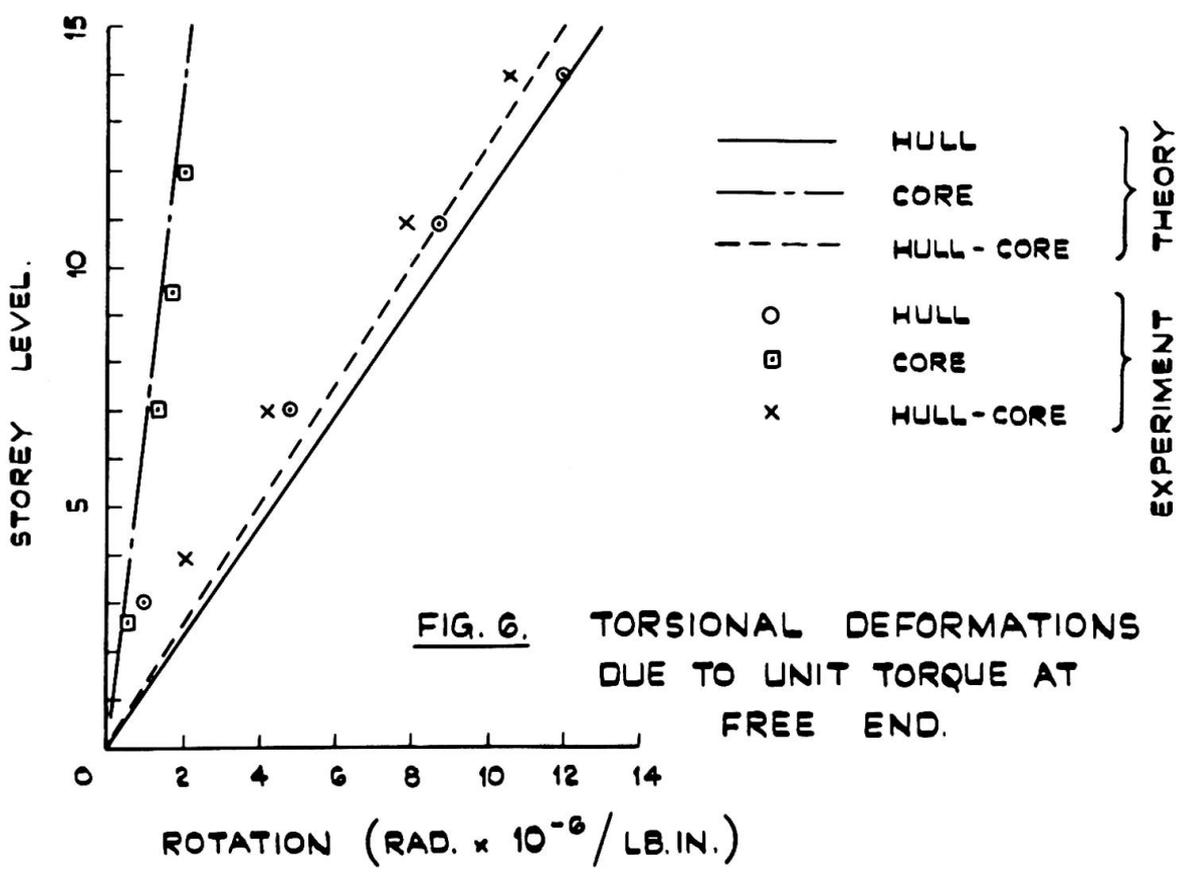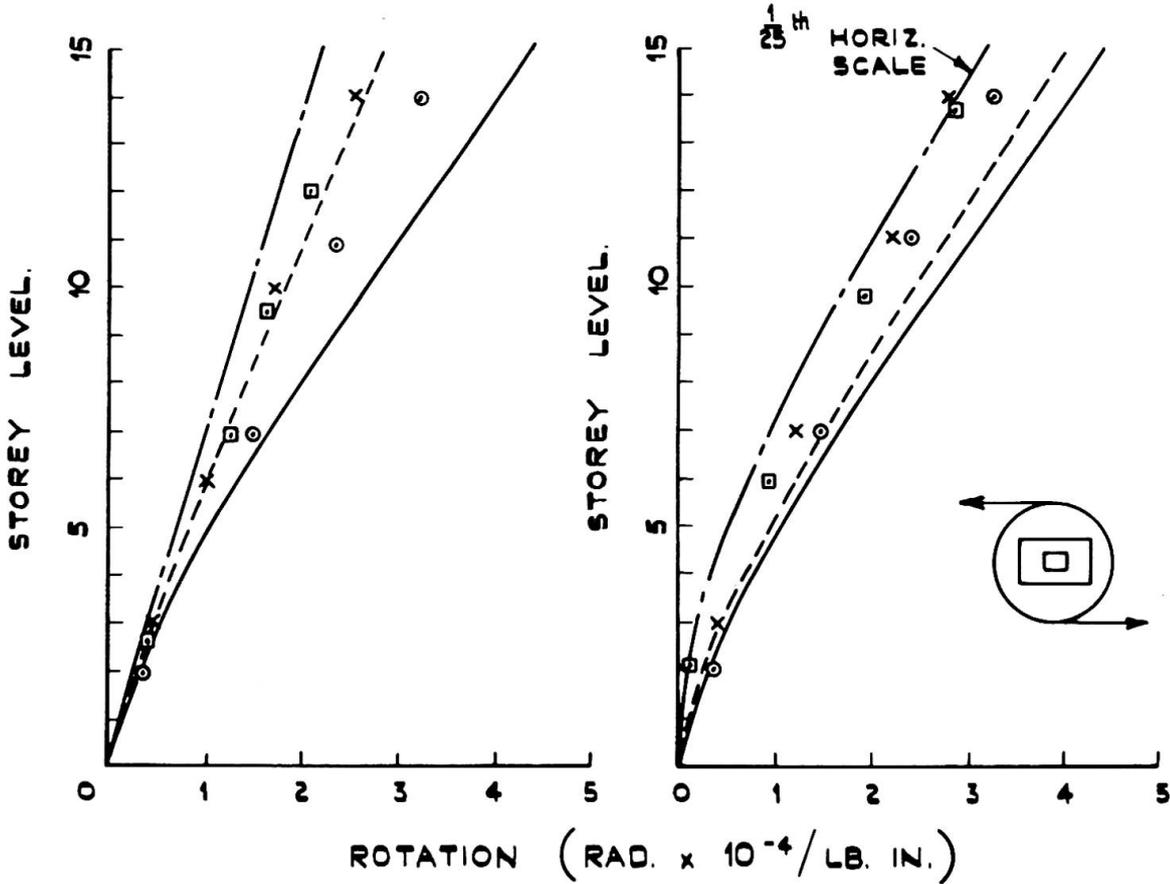FIG. 5. DETERMINATION OF FLEXIBILITIES F_1 & F_2
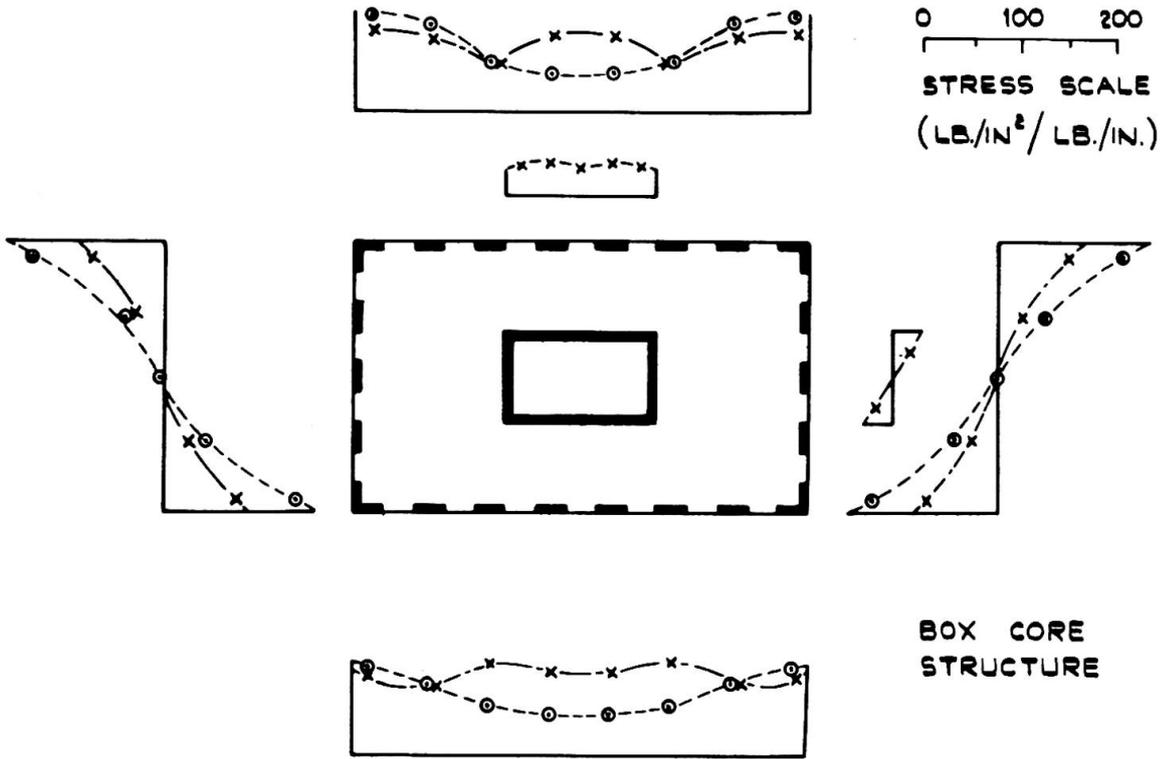
FIG. 6.   TORSIONAL DEFORMATIONS
DUE TO UNIT TORQUE AT
FREE END.

FIG. 7.   STRESS DISTRIBUTION AT THIRD FLOOR LEVEL.

FIG. 8.

DEFLECTION PROFILES
(UNIFORMLY DISTRIBUTED
LOAD.)

⊙ HULL
▢ HULL - CORE (BOX CORE)
● HULL - CORE (CHANNEL - CORE)
◇ UNPERFORATED TUBE.
✕ UNPERFORATED HULL - CORE.