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Minimum Weight Design of Frameworks

Projet de minimalisation de poids de charpentes

Entwurf einer Gewichtsminimalisierung bei Fachwerken

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1. Introduction

Full stress design is commonly adopted for design of frameworks. For a given set of joint locations j, and a system of self equillibrating loads, a framework of most general topology can have a total number of members $m = j(j-1)/2$. This, in general will be statically indeterminate. A feasible full stress design may or may not exist for a statically indeterminate structure [1,2]. However, full stress design of an indeterminate framework is feasible through geometrically controlled prestress [3,4]. Any such fully stressed structure may have its weight less than some determinate forms but not necessarily lower than all the feasible determinate forms of the indeterminate framework [5,6].

Least weight design for a fixed load condition is known to be statically determinate [7,8,9,10,11]. Hence the problem of Minimum weight design of a framework reduces to the identification of the least weight statically determinate form, out of the several feasible determinate forms. Present methods [8,10,11] of identifying the least weight statically determinate form use,either special mathematical tools or certain theorms of plastic design. In the present work, a direct method using wellknown principles of structural analysis is proposed.

2. Full Stress Design With Prestress

An initial lack of fit in a statically indeterminate system will induce prestress in the system. The forces in a statically indeterminate framework are given by

$$\{F\} = \{F^{\circ}\} + [f] \{X\} \qquad \ldots . (1)$$

in which {F} = element force vector; {F°} = element force vector of a determinate structure; [f] = influence coefficient matrix associated with redundant forces; and {X} = vector of redundant member forces.

If the members are to develop preassigned stresses at working load, the compatibility of deformations requires

$$\{f\}^T \, [L] \, \{\sigma\} + E \, \{\lambda\} = 0 \qquad \ldots . (2)$$

in which [L] = diagonal matrix of member lengths; $\{\sigma\}$ = vector of member stresses; E = youngs modulus; and $\{\lambda\}$ = vector of initial lack of fit in redundant members. Preassigned stresses at working load of a statically admissible force system, automatically fixes the sizes of the members of the framework. It is seen from Eq. (2) that for a given set of member stresses, $\{\lambda\}$ is unique. Thus the member forces in no load condition (only prestress condition) are fixed and can be determined. The prestress in any member is given by

$$\sigma_{Pi} = \frac{F_{Pi}}{a_i} \qquad \ldots . \quad (3)$$

in which σ_{Pi} = prestress in the ith member due to initial lack of fit; F_{Pi} = force in the ith member in prestress condition; and a_i = cross sectional area of the ith member.

The design will be an acceptable one, if the stresses under no load condition are also within the allowable limits i.e.

$$\sigma_{Pi} \leqslant \sigma_{ai} \qquad (i = 1, 2, \ldots, m) \qquad \ldots (4)$$

in which σ_{ai} = permissible stress in ith member.

3. Optimal Statically Determinate Form

Prestress changes the datum level of a member capacity and in the presence of prestress the effective capacity of the member is

$$F_{ei} = F_i - F_{Pi} \qquad \ldots . . \quad (5)$$

in which F_{ei} = effective capacity of the ith member at working load; and F_i = capacity of the ith member at full stress. The effective capacity of a member is increased by the presence of a compensatory type of initial prestress. Therefore, the efficiency of a member in transferring the external loads to the supports may be represented by a nondimensional factor ρ_i given by

$$\rho_i = \frac{F_{ei}}{F_i} = 1 - \frac{F_{Pi}}{F_i} \qquad \ldots . . (6)$$

in which ρ_i is defined as efficiency factor for the ith member. The efficiency factor can be as high as 2 for equal permissible stresses in tension and compression. The position of the member in the framework and the loading system are reflected in the efficiency factor. Therefore, the efficiency factor ρ is a direct indication of the effective utility of a member for the particular load condition. The optimal statically determinate form can be obtained by eliminating the required members of least efficiency. Application of this simple logic is illustrated through examples.

4. Illustrative Examples

Example 1: It is required to find the optimal statically determinate form corresponding to an indeterminate framework shown in Fig. 1(a). The detailed calculations for member efficiency factors are given in Table 1. Fig. 1(b) shows the variation of the volume (or weight) of truss for full stress design, with respect to force in redundant member at working load. The choice of the redundant member has no effect on the nature of this curve. The dotted lines in Fig. 1(b) indicate that the prestress in some members exceed the allowable values. The Kink points correspond to statically determinate forms of the system. The optimal topology will not get affected by adopting different allowable stresses in tension and compression. The optimal statically determinate form (point b) is obtained by removing the least efficient member 4.

Table 1. - Computations of Example 1

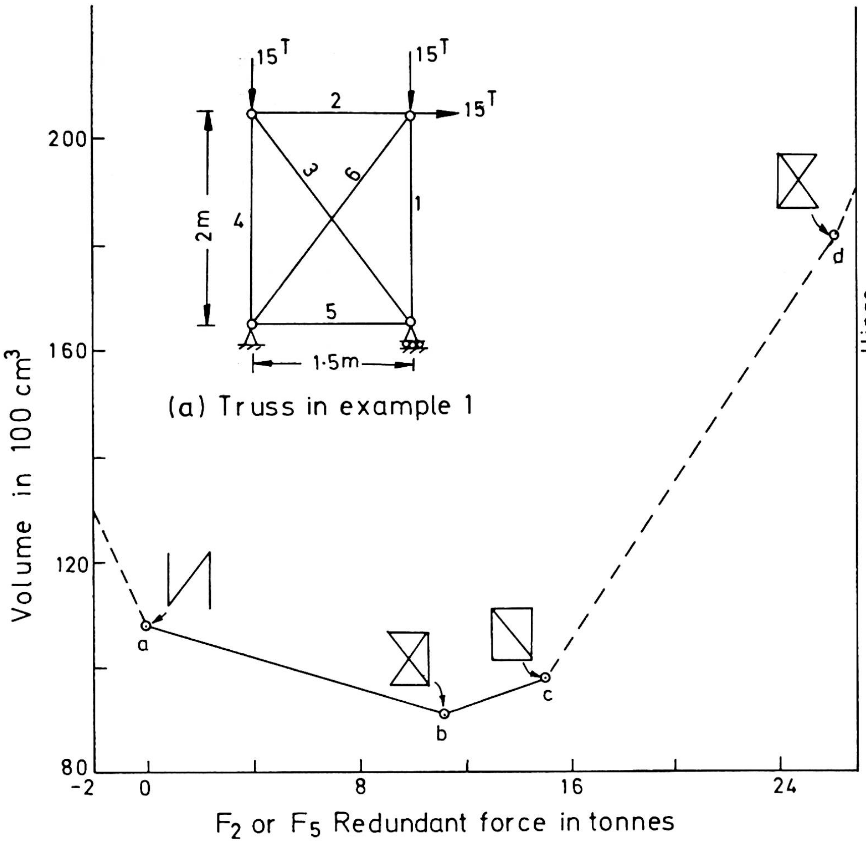
Member	F_i°	f_{i1}	F_i	a_i	F_{Pi}	σ_{Pi}	σ_{ai}	ρ_i
1	2	3	4	5	6	7	8	9
1	-15.0	-0.80	-27.00	18.00	-1.354	-0.075	-1.50	0.950
2	15.0	-0.60	6.00	4.00	-1.016	-0.254	-1.50	1.169
3	-25.0	1.00	-10.00	6.67	1.693	0.254	1.50	1.169
4	5.0	-0.80	-7.00	4.67	-1.354	-0.290	-1.50	0.806*
5	15.0	-0.60	6.00	4.00	-1.016	-0.254	-1.50	1.169
6		1.00	15.00	10.00	1.693	0.169	1.50	0.887

F_i° = force in ith member due to external loading when the redundant members are removed; f_{ij} = force in ith member due to unit tensile force in the jth redundant member when the other redundant members and external loads are removed.

Units: Force in Tonnes; area in sq. cm.; and stress in Tonnes per sq. cm.

* Least efficient member.

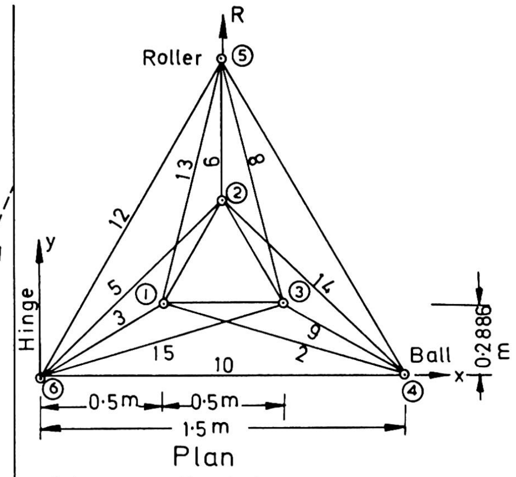
Example 2: It is required to find the optimum design of the space framework loaded as shown in Fig. 2. The calculations for the efficiency factors are listed in Table 2.



(a) Truss in example 1

(b) Weight of truss Vs redundant force

FIG. 1 ILLUSTRATIVE EXAMPLE 1



Joint i	Loading in tonnes P_x	P_y	P_z
1	0	0	-10
2	2	0	-10
3	0	0	-10

Nodes 1,2,3 are at a height of 2·5 m above the plane 4,5,6

FIG. 2 SPACE TRUSS-EXAMPLE-2

Table 2. - Computations of Example 2

Member	F_i^o	F_i	a_i	F_{Pi}	σ_{Pi}	σ_{ai}	ρ_i
1	2	3	4	5	6	7	8
1	-1.33	-0.22	0.150	0.063	0.419	1.50	1.279
2	0.00	-2.00	1.333	-0.114	-0.085	-1.50	0.943
3	-10.26	-4.58	3.052	0.222	0.072	1.50	1.048
4	-1.33	0.88	0.588	0.066	0.112	1.50	0.926*
5	3.61	-0.39	0.260	-0.118	-0.455	-1.50	0.696*
6	-13.68	-6.10	4.069	0.166	0.041	1.50	1.027
7	-3.33	-1.12	0.745	0.031	0.042	1.50	1.028
8	3.61	-0.39	0.259	-0.056	-0.217	-1.50	0.855*
9	-13.68	-8.00	5.333	0.161	0.030	1.50	1.020
10	1.78	2.15	1.432	0.021	0.015	1.50	0.990
11	1.78	2.52	1.677	0.010	0.006	1.50	0.996
12	2.44	3.18	2.122	0.022	0.010	1.50	0.993
13	..	-4.0	2.667	-0.118	-0.044	-1.50	0.970
14	..	-4.0	2.667	-0.056	-0.021	-1.50	0.986
15	..	-2.0	1.333	-0.114	-0.085	-1.50	0.943

Units: Force in Tonnes; area in sq. cm.; and stress in Tonnes per sq. cm.

* Least efficient members for elimination.

Removal of the least efficient members 5, 8 and 4 will result in a statically unstable situation. So only members 5 and 8 are omitted in the first instance. The method is repeated with the reduced framework having one order indeterminacy. The calculations for member efficiency factors are not listed here. The optimal statically determinate form obtained by removing the members 2 and 15 having the same lowest efficiency factor is shown in Fig. 3(a).

5. Conclusions

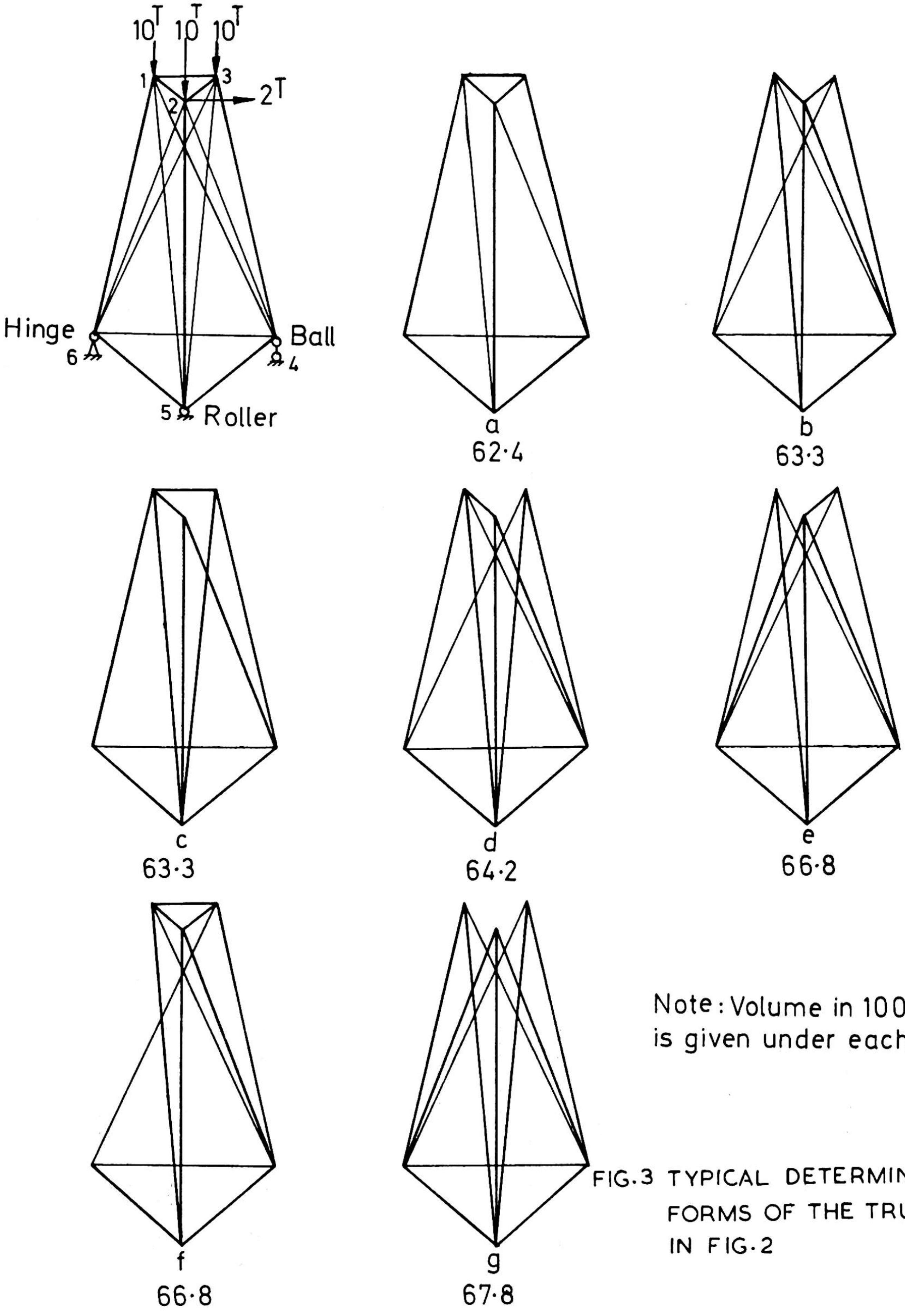
1. The optimal statically determinate form can be obtained using the following sequence of operations:

   (a) If the order of indeterminacy of the framework is n, then n members having the lowest efficiency must be eliminated

   (b) If two or more members have the same efficiency factor, they must be treated as a unit in the member elimination process. This may lead to the elimination of a joint.

   (c) If the operation (a) leads to a statically unstable system, remove less than n members so that the resulting system is stable.

   (d) As a consequence of (b) and (c) some times the reduced framework obtained will be indeterminate of reduced order;



Note: Volume in 100 cm³ is given under each fig

FIG. 3 TYPICAL DETERMINATE FORMS OF THE TRUSS IN FIG. 2

in which case the method is to be repeated starting with the reduced framework.

2. The process of eliminating members of least efficiency to get the optimal statically determinate form corresponding to an indeterminate framework was applied to several other examples successfully.

References

1. Dayaratnam, P., and Suryanarayan, P., "Feasibility of Full-stress Design", AIAA Journal, Vol. 7, No. 4, April 1969, pp. 773-774.

2. Suryanarayan, P., and Dayaratnam, P., "Behaviour and Design of Pin-connected Structures", International Journal for Numerical Methods in Engineering, Vol. 2, 1970, pp. 579-595.

3. Francis, A.J., "Direct Design of Elastic Statically Indeterminate Triangulated Frameworks for Single Systems of Loads", Australian Journal of Applied Science, Vol. 4, No. 2, June 1953, pp. 175-185.

4. Owen, J.B.B., The Analysis and Design of Light Structures, American Elsevier Publishing Co., Inc., N.Y., N.Y., 1965, pp. 32.

5. Venkateswara Rao, S., and Dayaratnam, P., "Full Stress Design of Prestressed Statically Indeterminate Frameworks", Presented at the Feb. 26-28, 1971, 23rd Annual General Meeting, Aeronautical Society of India, held at Indian Institute of Technology, Kanpur.

6. Venkateswara Rao, S., and Dayaratnam, P., Discussion on "Optimization of Redundant Prestressed Structures", Journal of Structural Division, ASCE, Vol. 97, No. ST9, Sept. 1971, pp. 2431-2434.

7. Cilley, F.H., "The Exact Design of Statically Indeterminate Frameworks: An Exposition of its possibility, But Futility", Trans. ASCE, Vol. 43, 1900, pp. 353-409.

8. Sved, G., "Minimum Weight of Certain Redundant Structures", Australian Journal of Applied Science, Vol. 5, No. 1, 1954, pp. 1-9.

9. Barta, J., "On the Minimum Weight of Certain Redundant Structures", Acta Technica, Vol. 18, Budapest, Hungary, 1957, pp. 67-76.

10. Dorn, W.S., Gomory, R.E., and Greenberg, H.J., "Automatic Design of Optimal Structures", Journal de Mechanique, Vol. 3, No. 1, March 1964.

11. Save, M.A., "Some Aspects of Minimum-Weight Design", Engineering Plasticity, Papers for Conference held in Cambridge,

March 1968, The Cambridge University Press, pp. 611-626.

Summary

The problem of finding the minimum weight topological form for a given loading from a general configuration which includes all possible member locations with respect to the given joint locations, is solved by using the concept of efficiency of a member. Relative efficiency factors of members are generated by introducing virtual prestress into the system. The optimal topology which is statically determinate is obtained by eliminating members with least efficiency.