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An Approximate Procedure for Probabilistic Limit Analysis

Une méthode approchée pour l'analyse limite probabiliste

Eine Näherungsmethode für wahrscheinliche Grenzwerte

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In the final paragraph of their Introductory Report (Ref.1), Massonnet and Save underline the necessity of "making a considerable effort towards a better knowledge of the laws of material behaviour and of the ways of load application, in order not to lose in the uncertainty of the data all the advances achieved in the methods of calculation".

However, the data of any structural problem (loads, material characteristics, etc.) are, and indeed always will be by their very nature, uncertain: as a matter of fact, they are essentially random, like most physical quantities.

Therefore, it is my strong opinion that further "advances in the methods of calculation" are in danger of remaining pure academic exercises, unless this concept of unavoidable randomness is accepted and rationally taken into account. In other words, the engineering profession will have to be persuaded that certain answers to many problems can be given only by appropriate descriptions (in probabilistic terms) of the uncertainty.

Following this approach, as noted by Massonnet and Save themselves (Ref.1, p.3), the solution of a structural problem could be formulated as a problem of stochastic programming. Unfortunately, this technique appears to be too complicated to be at present acceptable as the basis for practical design procedures, notwithstanding some very recent promising results (Ref.2).

Because of these considerations, in collaboration with Dr. Baratta of the University of Naples, I have been engaged in the development of simpler techniques, based on a fully rigorous and exact theory, but at the same time able to give, by appropriate simplification of the numerical procedures, acceptable approximations of the complete (exact) probabilistic solution with a reasonable computational effort.

So far (Refs.3-8) our attention has been confined to structures that satisfy the basic hypotheses of limit analysis, as summarized in Ref.1, the only difference being that the limit moment Mp (or in general, the local yield strength) is, in each point of the structure, a random quantity with a given probability distribution law. Consequently, if the load magnitude depends on one scalar parameter P(*), the overall strength of the structure is described by the probability distribution function of the limit (collapse) value of the load parameter, P_L; i.e., by the function

$$F(P) = \text{Prob}\ (P_L \leqslant P)\ ;\quad 0 \leqslant P < \infty \tag{1}$$

The procedure that we have proposed consists in the determination of functions that bound F(P) from above and below, thus allowing an evaluation of the degree of approximation achieved. To this aim, in Refs.3 and 4 two theorems have been demonstrated, which are the probabilistic analogues of the two bounding theorems of limit analysis (static theorem and kinematic theorem: cf. Ref.1, ineqs. 2 and 5 respectively).

The bounding theorems, whose practical relevance in classical limit analysis had been rather obscured in the last few years by the development of optimization techniques, regain thus their usefulness in probabilistic limit analysis, where the computational problems are still overwhelming.

In the formulation of the probabilistic theorems, auxiliary distribution functions are defined as follows:

a) let $F_\psi(P)$ be the probability that, investigating a number of stress fields in equilibrium with the loads measured by P, none of these fields is statically admissible (i.e. satisfies the yield condition throughout the structure);

b) let $F_\gamma(P)$ be the probability that, investigating a number of possible collapse mechanisms, at least in one case the loads measured by P are found kinematically sufficient (i.e., such that the power of the loads exceeds the dissipated plastic power).

It has then been proved that

$$F_\psi(P) \geqslant F(P) \geqslant F_\gamma(P) \tag{2}$$

for any function $F_\psi(P)$ and any function $F_\gamma(P)$.

It has also been shown that the average and the variance (dispersion) associated with the function F(P) can be conveniently bounded when a function $F_\psi(P)$ and a function $F_\gamma(P)$ are known.

The ease of calculation of the bounding functions $F_\psi(P)$ and $F_\gamma(P)$ depends on the number and complexity of the investigated mechanisms and stress fields. Numerical examples have shown that even the crudest assumptions may yield technically relevant results (Refs.3-4); however, if the initial choices did not lead to acceptably close bounds, the process could be repeated with different assumptions. Some appropriate artifices to increase convergence have been proposed (Ref.5); in general it is worth underlining that an increase in the closeness of the bounding functions requires a great increase in the amount of computational work, often unwarranted by the present scarce knowledge of the input statistical properties.

Further current studies deal with the extension of our procedures to multi-parameter loading (Ref.6), in particular to variable repeated loads that may cause incremental collapse (Ref.7), and with the formulation of the static approach in very general terms (Ref.8).

(*) The same symbols as in Ref.1 (rather than those of Refs.3-8) are used, as far as possible, in this contribution.

For a comparison with works by other authors, it is interesting to remark that the presentation of ineqs.(2) as the basis of our treatment shows that the results obtained via the kinematic approach are inherently unsafe: in fact, unless all possible collapse mechanisms are taken into account, for any value of the load parameter P a probability of collapse $F_\gamma(P)$, smaller than the actual one F(P), is calculated. However, all other works with similar aims that are known to the writer, have made exclusive use of the kinematic approach (cf. e.g. Refs.9-11): these works, explicitly or implicitly, rely on the fact that, in most if not all of the few examples of probabilistic limit analysis so far published, $F_\gamma(P)$ is a very close evaluation of the true distribution, even if not many mechanisms are investigated. Therefore, when by an appropriate truncation in the calculation of $F_\gamma(P)$, an upper bound to this distribution

$$F_+(P) > F_\gamma(P) \qquad (3)$$

is obtained (Ref.9), it is quite likely that the true distribution has been bounded on both sides, i.e.

$$F_+(P) \geqslant F(P) \geqslant F_\gamma(P) \qquad (4)$$

The present writer, however, thinks that the validity of ineq.(4), which is based on the closeness of $F_\gamma(P)$ and F(P), should still be tested in many more examples before being taken for granted in general: it is worth underlining that the number of mechanisms which may contribute to the probability of collapse, is very large even for the simplest structure. For example, the portal frame investigated in Refs.3-4, where the possibility of plastic deformations is concentrated in 11 sections, has more than 200 significant mechanisms (Ref.8).

The only rigorously safe (upper) bound $F_\psi(P)$ to the true probability of collapse F(P) is obtained via the static (equilibrium) approach, which has been proposed for the first time in Refs.3 and 4. It must, however, be admitted that in simple (perhaps unrealistic) examples the latter approach yields, with comparable amount of computations, a much worse approximation (albeit on the safe side) than the kinematic approach: on the other hand, the static approach appears to yield better to generalizations and extensions (Ref.8).

As a matter of fact, the determination of an $F_\psi(P)$ is immediate if the joint probability distribution of the local strengths is known. Therefore, if for instance the individual distributions of the local strengths are not the normal ones usually assumed in the demonstrative examples (but are statistically independent), each point of $F_\psi(P)$ is still given by a product of n terms (n being the number of elements into which the structure has been divided for the computations); on the contrary, each point of $F_\gamma(P)$ involves the calculation of an n-fold integral.

Lifting of the unrealistic assumption of statistical independence of the local strengths is a very complicated (and as yet, little investigated) problem, whose solution the writer thinks essential to increase the practical relevance of probabilistic structural analysis. However, once this problem is solved, the static approach guarantees that no new computational problems will be raised.

Finally, note that, differently from other authors (e.g. Refs.2, 12), we have separated the variables concerning strength (among which geometrical and other parameters could be included) from the variables concerning loads: each group of variables can be deterministic or stochastic. For example, if the strength variables are deterministic, all procedures reduce to the classical limit analysis, with all its properties, as explicitly pointed out in Ref.6.

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