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Alternative Analysis of Stress and Frequency of a Structure of the Type of West Coast Transmission Building in Vancouver, B.C. Canada

Analyse alternative des tensions et des fréquences d'un immeuble du type "West Coast Transmission Building" à Vancouver, B.C. Canada

Alternativberechnung der Spannungen und Schwingungen eines Gebäudes vom Typ "West Coast Transmission Building" in Vancouver, B.C. Kanada

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General

The recently constructed West Coast Transmission Office building in Vancouver is very original in design (Fig.1). It was designed and analyzed by B. Babicki and Associates, Consulting Structural Engineers. The authors were not connected in any way with the design and analysis of that building, and this paper presents an alternative method of analysis to the one actually used, of which the authors have no particulars. Its thirteen main floors, 108' x 108' in plan hang by cables on the central reinforced concrete core 36' x 36' x 265' high. This type of support improves greatly the behaviour of the structure in an earthquake, and the prestressing effect of the weight of the building makes the core much stronger in resistance to lateral loads. Furthermore, the absence of columns provides maximum usable space in the floor areas.

The floors are made of reinforced concrete slab extending over steel beams, whose inner ends rest in the core wall recesses, and the outer ends are attached to the cables, hanging vertically on the outside of the building. Above the upper floor the cables are sloped and draped over the core. With this arrangement part of the weight of the floors is carried by the beams directly to the core and the rest is applied by cables at the top of the core. The floor areas inside the core are used for elevators, stairs and services, requiring numerous large openings in the walls.

Although the distribution of load between the core and the cables is statically determinate, the stress analysis of the core is somewhat uncertain. The conventional beam formula is inapplicable in view of the presence of openings, and its approximate adaptations are unsuitable because the openings are irregular. The authors feel that their treatment of the problem by the method of finite element, although not fully rigorous, is most appropriate.

The Finite Element Model of the Core and the Loads

In a structure of high complexity the use of simplifying assumptions is unavoidable. The core is definitely unsymmetrical about the central plane XZ (or North-South), and is treated as such, but its minor asymmetry about the plane YZ (or E-W) is disregarded. Since the core walls are comparatively thin (14" and 10"), their flexural rigidity is ignored, and they are treated as in a state of plane stress.

The finite element model of the half of the structure on one side of the plane of symmetry (Fig.2) consists of as many as 726 rectangular two-dimensional finite elements or no-bar cells of 172 different sizes filling the exterior and most of the interior (not shown in Fig.2) walls. That so many cells are required in the model is determined by the need to match the wall openings. Two tiers of elements are as a rule required for the height of each storey. Some of the interior walls, judged non-contributory to the stiffness of structure are omitted from the model, and so are the interior stairs and the floors within the core. The elastic properties of the model are taken: the modulus of elasticity $E = 3{,}000$ kips/in^2 and the Poisson's ratio $\mu = 0.2$. The contribution of reinforcement to the stiffness of the structure is not considered.

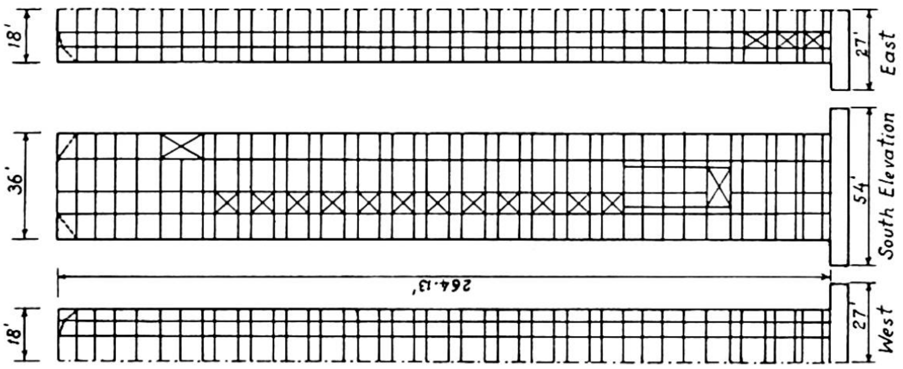

Fig. 2

The loads used in the analysis are: dead load, live load on all floors of one half of the maximum intensity, or in lieu of it, of full intensity on one half of the whole building, and the horizontal static loads, purported to imitate the actions of wind and earthquake. The intensities of all these were established by the designer in accordance with specifications.

The additional effects on stresses of the lateral deflection of the structure and of tilting of its foundation are also provided for. Although of minor importance in this particular structure, they may be very significant in more slender buildings.

By way of simplification all loads at their respective levels are applied at the core corners. The error resulting from this partial misplacement of loads is purely local, and it vanishes quickly on the way down. All nodes not common to two intersecting walls are restrained from moving out of the plane of the wall to which they belong. In the actual structure this restraining action is provided by the floors and the flexural rigidity of the walls, both of which are left out in the model.

Outline of Analysis.

The computer combines the matrices of individual cells into the stiffness matrix of the model and solves the numerous simultaneous equations for the displacements of the nodes. Products of the cell stiffness matrices and their displacement vectors give the load vectors of cells, from which are found the normal and tangential force concentrations at all nodes in several more highly stressed horizontal sections. By spreading these over the tributary areas of the adjacent cells stresses in the prototype are determined,- this is known as the method of nodal force concentrations. The other method for finding stresses, making use of nodal displacements, is mostly suitable for models composed of regularly spaced cells of one kind, uninterrupted by openings and irregularities. It was not used in this case.

Stress analysis outlined here is based on undeformed structure. Actually however, there is some lateral deflection bringing with it additional flexural effects, and a special procedure is devised to provide for them.

From the study of soil conditions an estimate is made of the amount of tilting of foundation by the action of horizontal forces and the one-sided live load, both assumed to act in easterly direction, more unfavorable than northerly. Tilted base makes the axis of the core inclined, which results in creation of bending moments by the dead and live loads all along the core. These flexural effects are augmented by the core's horizontal deflection.The amount of this deflection is unknown at the start, but the shape of the deflected axis may be closely approximated by any reasonable curve, such as a quadratic parabola. The amount of deflection at the top is assumed, and the deviation of the axis, and with it the resulting bending moments all along the axis are determined. The flexural deflection produced by them is now found by computer and compared with the assumed. If they disagree, the procedure is repeated with a new estimate of deflection.The assumed tilting of foundation may also be revised in response to the changed base moment.

Stresses by Nodal Force Concentrations.

The horizontal sections chosen for stress determination are the ones whose stresses are likely to be high, as on the lines of openings,especially where the wall thickness changes from 14" to 10". The procedure for converting the nodal force concentrations in the model into the stresses of the structure itself is more complicated here than in plates under plane stress,because of the walls meeting at an angle and the presence of openings. The method is illustrated on the example of junction K of three cells 1, 2 and 3, whose arrangement and dimensions are shown in Fig.3. Three

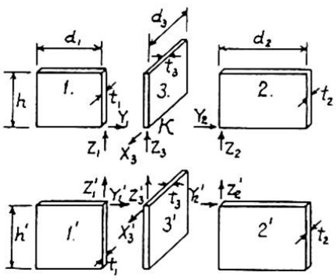

Fig. 3

similarly arranged cells 1', 2' and 3' are present underneath. For simplicity they are assumed of the same thicknesses as the ones above. The equality of vertical strains in the walls at the node K, combined with the low stress level in the horizontal direction and the smallness of Poisson's ratio, allow to assume that the vertical normal stresses in all walls are equal. With further assumption, that the contributions of the nodal concentrations Z to the normal stress decrease linearly to nothing from K to the adjacent nodes, the normal stress on the horizontal plane at K is

$$\sigma_z = \frac{2(Z_1 + Z_2 + Z_3)}{t_1 d_1 + t_2 d_2 + t_3 d_3} \qquad (1)$$

The part of the component Z_1 contributing to this normal stress is $\frac{t_1 d_1 (Z_1 + Z_2 + Z_3)}{t_1 d_1 + t_2 d_2 + t_3 d_3}$. Therefore, the balance of it

$$V_{Z1} = Z_1 - \frac{t_1 d_1 (Z_1 + Z_2 + Z_3)}{t_1 d_1 + t_2 d_2 + t_3 d_3} \tag{2}$$

contributes to the vertical shearing stress. Similar nodal force V'_{Z1} (upward) is found in the cell 1' below the cell 1, and so the the vertical shearing stress in the structure to the left of the point K is

$$\tau_{yz} = \frac{2(V_{Z1} + V'_{Z1})}{t_1 (h + h')} \tag{3}$$

With no Y concentrations in the cells 3 and 3' the normal stress on the plane Y to the right of the point K is

$$\sigma_y = \frac{2(Y_2 + Y'_2)}{t_2 (h + h')} \tag{4}$$

The shearing concentration between the cells 2 and 2' is

$$V_{zy} = \left[Y'_2 - \frac{h'(Y_2 + Y'_2)}{(h + h')} \right] \tag{5}$$

and the shear stress $\tau_{zy} = \frac{2V_{zY}}{t_2 d_2}$ (6)

The shear stress τ_{zy} on the left of the node K is numerically different from the one in Eqn.(6) in view of the presence of shear flow, coming from the interior wall of the cell 3, but it can be found by an equation similar to Eqn.(6). For the same reason the shear stresses τ_{yz} in Eqn.(3) are also different on the right and left of K.

Stresses Near Openings.

The presence of openings leads to some difficulties. Consider an element A between any two of the lower openings in the core wall. In general each of the two perpendicular nodal components in a cell represents the sums of the effects of the normal stresses on one of its adjacent sides and of the shear stresses on the other. Since however the cell A has no adjacent neighbors above and below it, its vertical nodal components designated S in Fig.5 must be viewed as representing only the shear contributions from the vertical sides, and, for the same reason, the horizontal components N - the normal effects on the vertical sides.

Determination of stresses in the prototype structure by distributing the nodal forces in the cell A in a manner described earlier leads to the presence of shear stresses on the vertical sides of the area A, as in Fig.6, and their absence along the horizontal sides contrary to the basic principle of statics.

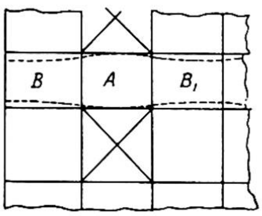

Fig. 4

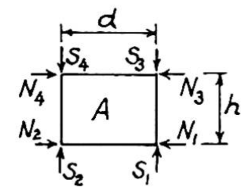

Fig. 5

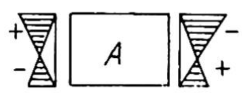

Fig. 6

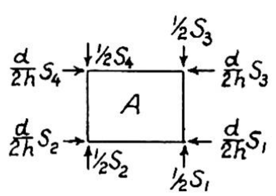

Fig. 7

It appears that the procedures devised for determination of stresses in the opening-free areas are not consistent with the stress - deformation conditions present around the openings. A qualitative examination of these conditions, illustrated on the example of the cell A and its neighbors in Fig.4 is useful. The part of the core on both sides of the opening becomes compressed and shortened, while the area A remains largely uncompressed, except on the sides, as indicated by the dotted lines. The inclinations of these lines near the sides of the area A point to the presence of vertical shear stress in substantial agreement with Fig.5, and also to some edge compression in the area A, in contradiction to the earlier advanced significance of the nodal forces in the figure. It appears then that the nodal forces S should stand partly for the normal stress on the horizontal sides of the area, unprovided in the distribution procedure formulated earlier. Similarly, the nodal forces N should partly stand also for the shearing stresses on the horizontal edges of A. With no rigorous resolution for this dilemma, the authors' suggestion based on judgment is to attribute to shear actions the parts of the nodal forces shown in Fig.7, relegating their balances to the normal stresses.

It is necessary to point out that the stress irregularity caused by openings is not confined to their immediate vicinity, but is extended to the nodes one or even more steps away from them, where the computed shearing stresses on the horizontal and vertical planes are likely to come out unequal, and for this reason should be averaged up.

A considerable improvement of stress results may be effected by subdividing the cells one in four. A fine cell model of this kind of a complete structure may increase unreasonably the computer time, but it is quite appropriate to restrict this subdivision to a part of the structure under immediate investigation, attributing to its boundary nodes the displacements found in the coarse cell model.

Analysis and Design.

It is desirable to say a few words on the design of the structure, as distinct from the subject of this work, the stress analysis. The full extent of the high shearing stresses in the vicinity of openings created by the dead weight should be considered primarily as a warning, rather than the actual design condition. On the one hand it may be improved by reinforcement and prestressing and on the other be relieved automatically by creep in concrete as the stress builds up in the course of construction, while the material is comparatively fresh. The high shear stresses are also largely participation stresses, and not the load carrying stresses.

Vibration Frequency.and Finite Element Model.

In the earthquake prone region like Vancouver the behavior of the structure under an earthquake is of primary importance. This is largely characterized by the magnitude of the lowest frequency of its vibration, which in case of the West Coast Transmission building corresponds to the flexural deflection of its core as a cantilever beam.

This vibration frequency is also determined by the method of finite element with employment of some additional assumptions. The structure is treated as symmetrical about both XZ and YZ planes, thus making the vibratory oscillation from the position of equilibrium fully antisymmetrical with reference to XZ plane.

The core is subdivided into one storey box-like elements consisting of floors and core walls (Fig.8). The massive foundation slab, resting on elastic underground forms the element #1 surmounted by the core elements 2 to 22. The last of these, trapezoidal in shape, is replaced by a rectangular one for the sake of a substantial simplification of analysis. Vertical cable lengths within the storey heights and the sloping parts of cables at the top form additional units.

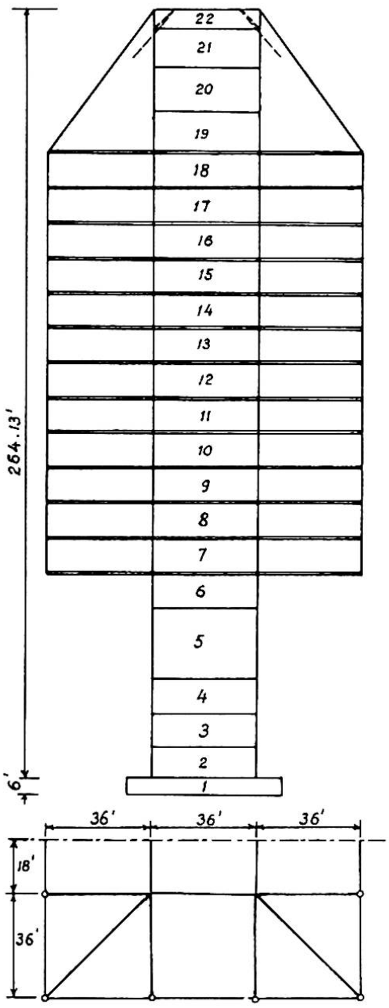

Fig. 8

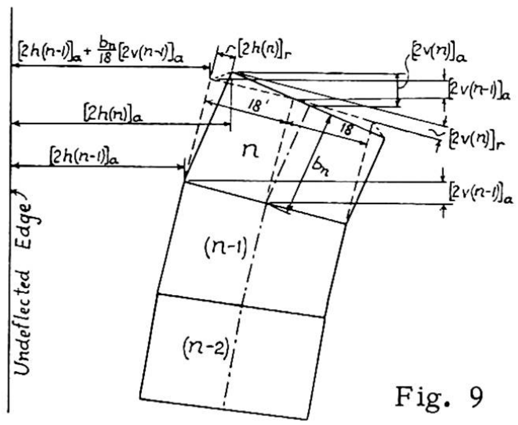

Fig. 9

In view of dual symmetry of the structure and antisymmetry of the vibratory motion, its displacements are fully described by the movements of nodes of a single quadrant. The rigidity of suspended floors in their planes makes their horizontal displacements equal with those of the core corners. On the other hand the concrete floor slabs are thin and flexible, while the floor beams carrying their loads to suspenders are very rigid. This permits to assume the vertical deflections of the cable nodes independent of similar deflections of the other cable nodes and the core corners in the same floor.

Motion of cell 1 is described by one node moving only vertically, while motions of the other core cells are described by two nodes moving in Y and Z directions. The top node of a cell below is at the same time the bottom node of the cell above. The same applies to the top and bottom nodes of the three cable lengths in one storey within a quadrant of the structure. This makes the number of independent displacements of the structure

$$1+2(21)+3(13) = 82.$$

Eigenvalue Equation. Stiffness and Mass Matrices.

Vibration of the structure is subject to the eigenvalue equation:

$$\left([K] - \omega^2[M]\right)\{\delta\} = 0 \qquad (7)$$

in which $\{\delta\}$ is the 82 term vector of absolute displacements of the model, ω is its angular frequency and $[K]$ and $[M]$ are the stiffness and the mass matrices of the model.

The terms of the matrix $[M]$ are the masses of the parts of the core and the suspended floors tributary to each δ and they are determined directly by apportioning between the nodes the masses of all parts, i.e. of the interior and exterior walls of the core and the floors both inside and outside of it.

The terms of the stiffness matrices of core cells are determined by the relative and not the absolute displacements of nodes. In each element for these are used the horizontal and vertical displacements of the upper corners in relation to the lower corners, thus forming 4 x 2 stiffness matrices of core cells except the cell 2, having 3 x 2 stiffness matrix, because its bottom node, sitting on the foundation cell, does not move horizontally. The matrix of the foundation cell possesses only one term. The contributions of the cable lengths to the stiffness matrix are determined by their cross-section areas and the relative movements of the ends.

To solve the eigenvalue equation the stiffness matrices of cells must be related to the absolute displacements, similarly to the mass matrices. Call the vertical and horizontal relative displacements of the upper node 2 relatively to the lower node 1 in the core cell n respectively $[2v(n)]_r$ and $[2h(n)]_r$, and the absolute displacements of the same node $[2v(n)]_a$ and $[2h(n)]_a$. Consider these quantities as infinitesimals of the first order and ignore the infinitesimals of the second order. This means that the vertical projection of the deflected axis of the core remains equal to its undeflected length, and the horizontal sides of the cells are no different in length from their horizontal projections after deflection . The following relations are then obtained by geometry (Fig.8):

$$\left.\begin{aligned} [2v(n)]_r &= [2v(n)]_a - [2v(n-1)]_a \\ \text{and}\quad [2h(n)]_r &= [2h(n)]_a - [2h(n-1)]_a - \frac{b_n}{18}\,[2v(n-1)]_a \end{aligned}\right\} \quad (8)$$

In these equations (n-1) is the number of the cell below n, b_n is the height of the cell n and 18 feet is its half-width.

The terms of the stiffness matrix [K] of the whole model in Eqn.(7) are found by combining the terms in the two adjacent elements with replacement of the relative nodal movements by their absolute equivalents in Eqns.(8).The eigenvalue equation is solved for its first frequency and mode vector by one of the standard procedures.

Check of the Method.

As a numerical check on the finite element method of frequency analysis employed here it was applied to a vertical fixed-ended cantilever beam of constant box section with dimensions and length comparable to the core of the structure under consideration, subdivided into a similar number of equal box-like cells without tops and bottoms. The finite element frequency was found close to the one determined by the standard formula of elasticity.

Conclusion

The authors consider it unnecessary to include here the numerical results of their calculations both for the stresses and the frequency of vibration, since present work represents an alternative method and not a part of the design and original analysis for which they were not responsible and were not in possession of all the necessary data. Thus the actual rigidity of the underground and its elastic response to vertical deformation were unknown to them and were simply assumed. The same applies to the elastic properties of the cables, known only by their diameters and not by the composition and sizes of strands and wires.

The authors admit some arbitrariness in the recommended procedur for determination of stresses near the openings, but they believe the finite element method proposed here is most suitable for analysis of structures of the type of West Coast Transmission building.

Notation

a	subscript signifying "absolute"
b	height of cell number n.
d	with subscripts, - widths of cells.
h,v	parts of symbols, signifying horizontal and vertical.
h,h'	heights of cells.
t	with subscripts, - thicknesses of cells.
r	subscript signifying "relative"
E	modulus of elasticity.
[K]	stiffness matrix.
[M]	mass matrix.
N	with subscripts, - nodal components caused by normal effects.
S	with subscripts, - nodal components caused by shear effects.
V,V'	shearing parts of nodal force concentrations.
Y,Z	with subscripts, - nodal force concentrations.
XYZ	coordinate axes.
$\{\delta\}$	vibratory deflection vector.
μ	Poisson's ratio.
σ	with subscripts, - normal stresses.
τ	with subscripts, - shearing stresses.
ω	angular frequency.

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SUMMARY

The thirteen main floors of this office building 108' x 108' in plan hang by cables on the central reinforced concrete core 36' x 36' x 265' high. The small area of the foundation and the prestressing effect of loads make this structure highly resistant to earthquakes and flexure. For the purpose of our theoretical analysis, the structure is replaced by a model consisting of numerous two-dimensional rectangular units and it is analysed for stresses and vibration frequency by the method of finite element.

RESUME

Les treize maitresses-planchers de cet immeuble de bureaux d'un plan de 108 x 108 pieds sont suspendus par câbles ancrés au noyau central en béton armé qui mesure 36 x 36 pieds en plan et 265 pieds en hauteur. La petite surface de fondation et l'effet de la précontrainte des charges rendent cet immeuble très résistant aux tremblements de terre et aux fléchissements. Pour l'analyse théorique la structure est remplacée par un modèle composé de rectangles bidimensionnelles. L'analyse des tensions et des vibrations est opérée par la méthode des éléments finis.

ZUSAMMENFASSUNG

Die dreizehn Hauptdecken dieses Bürohauses mit einem Grundriss von 108 x 108 Fuss hängen an Kabeln, die im Stahlbetonkern verankert sind, der im Grundriss 36 x 36 Fuss und in der Höhe 265 Fuss misst. Die kleine Gründungsfläche sowie die Vorspannwirkung der Lasten machen dieses Gebäude sehr widerstandsfähig gegenüber Erdbeben und Biegung. Für die theoretische Untersuchung wird das Gebäude durch ein Modell ersetzt, welches aus zweidimensionalen Rechtecken zusammengesetzt ist. Die Spannungs- und Schwingungsberechnung erfolgt nach der Methode der endlichen Elemente.

Fig. 1

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