Shear-wall bracing criteria for tall buildings

IIb

Shear-Wall Bracing Criteria for Tall Buildings

Critères d'interaction entre cadre et noyau dans des maisons tours

Interaktionskriterien für Scheiben-Rahmen-Kombinationen in Hochhäusern

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INTRODUCTION

In current design practice rigid jointed building frames are classified as braced and unbraced [1], [2]. The recommended design results in widely different member proportions for the two classes of frames. However, no clear guidance on the basis for this classification is offered in the literature.

Some studies of the bracing problem are concerned with the shear truss type of bracing [3], [4]. The results of these studies are not applicable to shear wall bracing because of the basic difference in behaviour of the shear wall and the shear truss types of bracings.

The object of this paper is twofold, (1) to identify the roles that lateral bracing plays in limiting sway movements in building frames, and, (2) to present related criteria in order that sway effects can be eliminated from the design of frames with shear wall bracing.

ROLE OF LATERAL BRACING

The current practice of designing braced frames suggests that:

1. Lateral displacements of frames under their horizontal loads should be small and no larger than a specified allowable limit. The criterion controlling the sway movements is referred to as the *lateral displacement criterion*.
2. Lateral stiffness of a braced system should be such that sway instability effects, before the ultimate stage, are small. The criterion developed for this purpose is referred to as the *stability criterion*.
3. Relative stiffnesses of the frame and its bracing should be such that a major portion of the lateral loads is assigned to the bracing elements, resulting in a design for which the frame proportions are controlled by gravity loads alone. The criterion that ensures such a behaviour is referred to as the *primary loading criterion*.
4. Moments and joint rotations of the frame members due to unsymmetrical gravity loading and/or geometry should be similar to the behaviour under infinitely stiff lateral restraints. The shear wall criterion intended to produce such a response is referred to as the *symmetric loading criterion*.

The object of this paper is to study the satisfaction of the above criteria for shear wall braced frames.

THE ANALYTICAL MODEL

The main features of the analytical model adopted for developing criteria 1 to 3 are (Fig. 1):

1. The columns of a multibay frame are lumped in a single continuous column which is restrained at every floor level by beams with pinned far ends.
2. The shear walls are represented by a single cantilever wall.
3. The interconnection between the equivalent frame and wall components is made by axially rigid pinned linkages.
4. The effect of axial loading on member stiffness is neglected, but the P-Δ effects are taken into account.

Similar models have been extensively used in past [5], [6], [7] and the approximations resulting from the adopted approach do not significantly affect the results of this study.

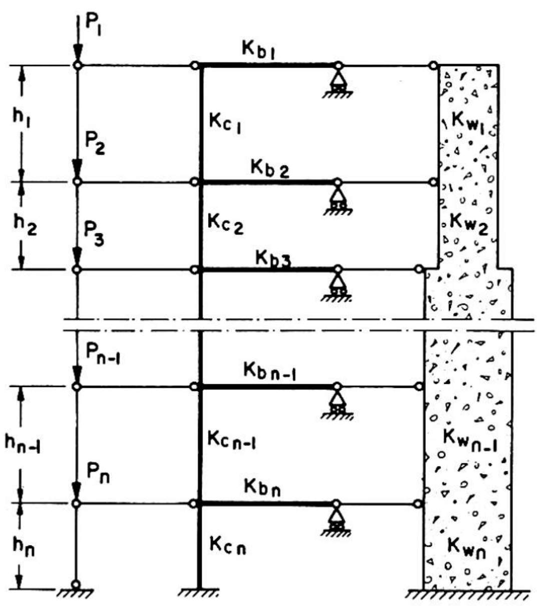

Fig. 1 *Analytical Model*

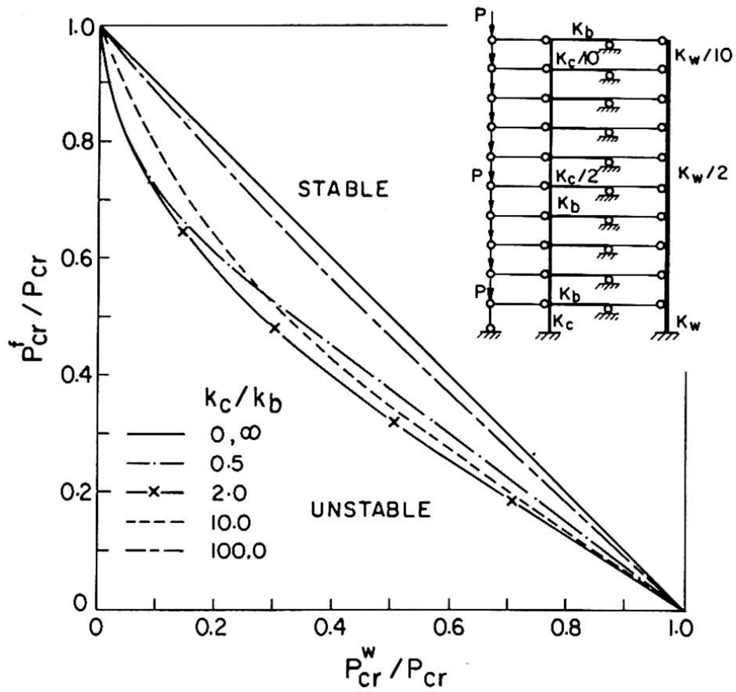

Fig. 2 *Interaction of Critical Loads*

DEVELOPMENT OF CRITERIA

The study shows that three parameters P_{cr}, P^w_{cr} and P^f_{cr} i.e., the critical loads associated with the lateral buckling of the structure, the free-standing wall and the unbraced frame respectively, control the development of the criteria. P^w_{cr} and P^f_{cr} are independent parameters, whereas P_{cr} depends upon the buckling loads and shapes of the frame and wall components. The interaction of these parameters for a ten story structure is shown in Fig. 2.

1. *Lateral Displacement Criterion*: In a single story model, a lateral force H causes a story rotation $\rho = H/P_{cr}$. The axial load P increases this primary rotation by a multiplier $\beta = 1/(1 - P/P_{cr})$. Thus, the relationship $\rho = \beta H/P_{cr}$ accounts for both the primary and secondary displacements of a single story model.

The validity of a similar relationship for multi-story structures is studied. An investigation on a number of frames suggests that the maximum story rotation ρ_m in a multi-story structure, in which the ratio of lateral to gravity loads, H/P, at all floors is constant, is approximately given by:

$$\rho_m = c\beta \, H/P_{cr} \qquad (1)$$

where the constant c depends primarily on the relative stiffness of the beams in the structure and β, as defined above, is a multiplier accounting for the P–Δ effect. The value of c(which is 1.0 for infinitely stiff beams) may be taken as 1.2 for all practical unbraced frames and 1.5 for cantilever walls. For frames braced according to the criteria in this paper, a value of c = 1.4 seems appropriate. The correlation of eq. (1) with the exact values of ρ_m is good.

In real structures, H and P generally are not in a constant ratio. Assuming the total gravity load distributed in the same manner as the lateral loads, eq. (1) yields only slightly conservative values of ρ_m.

Then eq. (1) expresses the lateral displacements in terms of the critical loads. If the serviceability conditions limit ρ_m to an allowable value ρ_a, eq. (1) with c = 1.4 yields:

$$P_{cr}/P \geqslant (1.4/\rho_a)(H/P) + 1 \qquad (2)$$

Current building codes [1],[2] leave the matter of the allowable lateral displacements open. Various technical committees [8],[9] limit the maximum horizontal sway under wind forces to 1/500 of the overall height of the structure. Reference [9] allows an alternative interpretation of this limitation as "the ratio of the relative lateral story displacement to the story height, assuming a more or less uniform story height". Accordingly, accepting $\rho_a = 1/500$, the lateral displacement criterion for design wind load becomes:

$$P_{cr}/P \geqslant 700 \, H/P + 1 \qquad (3)$$

2. *Stability Criterion*: The sway instability phenomenon affects the behaviour of structures mainly through the amplification of their primary lateral displacements, which may result due to horizontal loads or lack of symmetry in geometry or loading. In braced buildings the effects of such an amplification are completely neglected [1], [2]. In order for this practice to be justified, the value of the maximum sway multiplier must be negligible. Any value which may be considered negligible for this purpose is subjective. In this paper, an amplification of 5% at working loads and of 10–15% at the ultimate stage is considered acceptable.

Lateral displacement studies have shown that the multiplier of ρ_m is $1/(1-P/P_{cr})$. This also happens to be the maximum amplification for any story rotation. If the assumption introduced with the displacement criterion is accepted, a limitation on the maximum amplification of 5% at working loads yields:

$$P_{cr}/P \geqslant 20 \tag{4}$$

If a structure is designed to remain elastic until the ultimate stage, with a load factor of 1.25 (for combined lateral and gravity loading) this amplification becomes about 7% and with a load factor of 1.7 (for gravity loads alone) its magnitude becomes about 9%.

If the beams of a structure develop plastic hinges at their right ends before the ultimate stage and the frame is braced according to the criteria in this paper, the maximum amplification of primary story rotations increases to about 12% for the combined loading and to about 16% for gravity loading alone.

3. *Primary Loading Criterion*: The extent to which a frame contributes in resisting the applied lateral forces depends upon the relative stiffnesses of the frame and the shear wall. If critical loads are taken as a measure of stiffness, the lateral loads carried by the shear wall should bear some relationship to P^w_{cr}/P^f_{cr}. In fact, in a single story model, the distribution of lateral shears between the wall and the frame follows strictly this ratio.

In multi-story structures, such proportinality in individual stories does not exist, but the shear distribution is determined by the relative stiffnesses of the wall and the frame [7].

Studies on multi-story structures indicate that the average percentage of shear carried by the shear wall in different stories is approximately proportional to the parameter P^w_{cr}/P_{cr} with the critical loads evaluated with a distribution of P similar to H. One such study is presented in Fig. 3. In this figure, i refers to the number of stories from the top and the dotted curve also represents the average wall shear. The deviation from the average of wall shear percentages in individual stories varies with the type of frame and wall and the loading, but is mainly controlled by the ratio P^w_{cr}/P_{cr}: the larger is this ratio, the smaller is the deviation. From Fig. 3, it is clear that values of P^w_{cr}/P_{cr} above about 0.5 help the frame to a lesser degree in the lower part of the structure. A smaller value of this parameter causes a disproportionately large increase of the shears carried in the lower part of the frame. Thus, it is recommended that for braced frames:

$$P^w_{cr} \geqslant 0.5\, P_{cr} \tag{5}$$

This is a necessary but not sufficient provision for ensuring that the frame can be designed for gravity loads only.

4. *Symmetric Loading Criterion*: The case of a single column frame is considered. The column is loaded by joint moments of different ratios. Comparisons are made between column end moments and rotations for two cases: (a) column laterally restrained by a finite shear restraint, and (b) column with infinite shear restraint. Such studies indicate that a column bent in double curvature without sway exhibits the largest departure from the no sway behaviour. This case is more critical for joint rotations than for end moments. Denoting the average rotations for the sway cases by $\bar{\theta}$ and for the no sway case by θ, the behaviour of this model is described by:

$$\bar{\theta}/\theta = 1 + [12k_c/h - P^f_{cr}]/P_{cr} \tag{6}$$

With P_{cr} corresponding to a shear restraint $\alpha_\ell = 12k_c/h$, this ratio ranges from 2 for no rotational to 1 for infinite rotational restraint. This value of the shear restraint α_ℓ has been found adequate for reinforced concrete columns [10] and is used as a basis for developing this criterion.

The behaviour of double curvature frame columns, loaded primarily by moments at their joints, is approximated from eq. (6) as:

$$\bar{\theta}/\theta = 1 + [(1/i)(12k_c/h) - (k_c/\Sigma k_c)P^f_{cr}](\bar{H}^f/\bar{H}^c)/P_{cr} \tag{7}$$

where $\bar{H}^f/\bar{H}^c$ is the ratio of the no sway frame shear to the column shear under the loading being investigated, Σk_c denotes the summation of column stiffnesses EI_c/h over the story i (counting from top) containing the column under consideration, and P_{cr} corresponds to equal distribution of gravity loads between floors. The correlation of behaviour predicted by eq. (7) with the exact analyses is satisfactory. From the comparison of eqs. (6) and (7), it follows that restraint equivalent to $\alpha_\ell = 12k_c/h$ for the single column model is provided if:

$$P_{cr} = \text{Max.}[(1/i)(12k_c/h) + (k_c/\Sigma k_c)P^f_{cr}]\,\bar{H}^f/\bar{H}^c \tag{8}$$

This criterion need only be applied to a column with the maximum H^c in a story under a loading causing a maximum unbalanced shear $\overline{H}^f$. The critical story will usually be one of the few top or bottom stories of the structure.

Whereas the previous criteria apply to the service conditions, eq. (8) refers to the ultimate stage. This criterion may not be critical for elastic frames since $\overline{H}^f$ is usually small, but it may prove to be the most critical if applied to a pseudoelastic frame similar to the one described earlier for the stability criterion. In limit design, with materials having limited ductility, the use of this pseudoelastic frame for applying eq. (8) is recommended. For such frames, the contribution of P^f_{cr} may be dropped from either side of eq. (8) which thus simplifes to:

$$P^w_{cr} = \text{Max.}[(1/i)(12k_c/h)(\overline{H}^f/\overline{H}^c)] \tag{9}$$

DESIGN EXAMPLE

A twelve story four bay reinforced concrete frame, Fig. 4, with the preliminary member sizes and moments of inertia in Table 1, is to be designed for collapse at the specified ultimate loads. The frames are 20 ft. apart and carry a service live load of 100 lb/sq.ft., a wind load of 20 lb/sq.ft. and a dead load of 50 lb/sq.ft. The load is transferred to the frame through cross beams 7 ft. apart. The materials have f'_c = 4 ksi and f_y = 60 ksi. The preliminary design is based on American practice [1]. Gross moments of inertia are used for columns while some adjustments for cracking are made in computing the moments of inertia for the beams, which include a 3.5 in. slab on the top.

With these data, the gravity load per story is P = 305 kips or $1.05\ EI_c/h^2$ with $E = 60{,}000\sqrt{f'_c}$. The wind load per story is H = 5 kips, giving a ratio H/P = 1/61. From eq. (3), a value of $P_{cr}/P = 12.45$ is required for the displacement criterion. This factor is less than the minimum $P_{cr}/P = 20$ required for the stability criterion, which yields a minimum value $P_{cr} = 21\ EI_c/h^2$.

A stability analysis of the continuous column modelling the frame by Grinter's approach [5], yields $P^f_{cr} = 17.14\ EI_c/h^2$. Assuming that the shear wall stiffness varies along its height as the sum of column stiffnesses $P^w_{cr} = 0.107\ EI_w/h^2$. Studies of the interaction between P_{cr}, P^f_{cr} and P^w_{cr} show that with the type of frame and shear wall in this example, $P^w_{cr}/P_{cr} = 0.5$ requires $P^f_{cr}/P_{cr} = 0.33$, so that $P^w_{cr}/P^f_{cr} = 1.5$. Thus, in order to satisfy the primary loading criterion, a $P^w_{cr} \geqslant 1.5\ P^f_{cr}$ is required. This results in a minimum $I_w/I_c = 1.5 \times 17.14/0.107 = 240$ yielding a shear wall critical load $P^w_{cr} = 25.71\ EI_c/h^2$ which is greater than $P_{cr} = 21\ EI_c/h^2$ required for the stability and the displacement criteria.

The frame is analysed (no sway) for full loading on all beams with their right ends assumed hinged. Columns on line D (Fig. 4) exhibit higher values of $k_c\overline{H}^f/i\overline{H}^c$ in all stories, indicating that a larger amount of bracing is required for this column line. This results from a smaller end span carrying a lighter loading. Determining the amount of bracing for this column line, however, is considered unreasonable because of the smaller shears associated with this column line. Columns on lines B and C carry maximum shears in all stories and are made the basis for satisfactions of the symmetric loading criterion. For either of these column lines the maximum $k_c\overline{H}^f/i\overline{H}^c = 2.88\ EI_c/h$ occurs in the top story and, from eq. (9), a minimum $P^w_{cr} = 34.56\ EI_c/h^2$, and corresponding ratio $I_w/I_c = 34.56/0.107 = 322$ are found.

Thus, the symmetric loading criterion controls the stiffness of the shear wall required to brace the frame in this example. However, if the frame is to be designed for an elastic ultimate stage, the primary loading criterion will control, since under a loading maximizing $\overline{H}^f$ the value of $\overline{H}^f/\overline{H}^c$ in eq. (8) is relatively small.

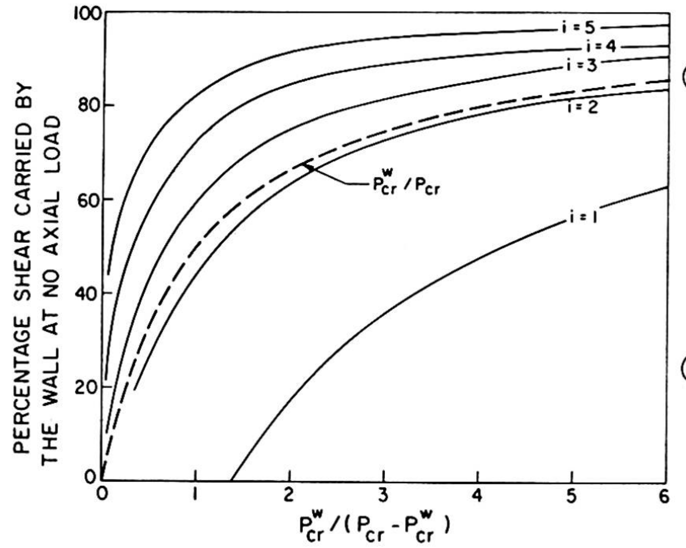

Fig. 3 Typical Lateral Load Distribution

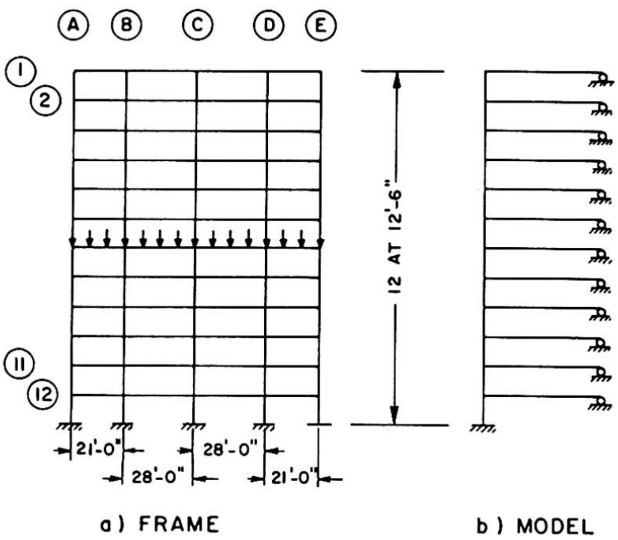

Fig. 4 Example Frame

TABLE 1

PROPERTIES OF EXAMPLE FRAME

STORIES	FRAME						MODEL	
	EXT. COLUMNS		INT. COLUMNS		BEAMS		COLUMNS	BEAMS
	Section (in. x in.)	I	Section (in. x in.)	I	Section (in. x in.)	I	k	k
1	12 x 12	I_c	12 x 12	I_c	12 x 24	$12I_c$	$5k_c$	$100\ k_c$
2	12 x 12	I_c	12 x 12	I_c			$5k_c$	
3	12 x 12	I_c	12 x 14	$1.6\ I_c$			$6.8\ k_c$	
4	12 x 12	I_c	12 x 14	$1.6\ I_c$			$6.8\ k_c$	
5	12 x 12	I_c	14 x 18	$3.95I_c$			$13.85k_c$	
6	12 x 12	I_c	14 x 18	$3.95I_c$			$13.85k_c$	
7	12 x 14	$1.6\ I_c$	16 x 21	$7.15I_c$			$24.65k_c$	
8	12 x 14	$1.6\ I_c$	16 x 21	$7.15I_c$			$24.65k_c$	
9	14 x 14	$1.85I_c$	18 x 21	$8.05I_c$			$27.85k_c$	
10	14 x 14	$1.85I_c$	18 x 21	$8.05I_c$			$27.85k_c$	
11	16 x 16	$3.16I_c$	21 x 21	$9.40I_c$			$34.52k_c$	
12	16 x 16	$3.16I_c$	21 x 21	$9.40I_c$			$34.52k_c$	

CONCLUSIONS

The purpose of the study is to develop criteria for braced frames when shear walls are used as lateral supports in tall buildings. Four basic criteria are used to develop the equations. Only the shear walls extending over the full height of the frame are considered. No allowance is made for the stiffening effects of partitions and cladding or any torsional effects arising out of an unsymmetry in the plan of the building. The study demonstrates that all the proposed criteria can be satisfied by placing suitable limits on the critical loads of the structural system and its frame and shear wall components.

The parameters used in the study may be too time consuming for exact calculations in design offices. However, if the analytical model suggested in this paper is accepted, such calculations may futher be simplified by developing standard design aids.

ACKNOWLEDGEMENT

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NOTATION

E	= Modulus of elasticity.
f_c'	= Concrete cylinder strength.
f_y	= Steel yield strength.
h	= Story height.
H	= Lateral forces at floor level.
$\bar{H}^c, \bar{H}^f, \bar{H}^w$	= Lateral shears on a column, frame and wall, respectively.
I	= Moment of inertia.
I_c, I_w	= Moments of inertia of a column and a wall, respectively.
k	= EI/L for a member.
k_b, k_c	= EI/L for a beam and a column, respectively.
n	= Number of stories in a structure.
P_{cr}	= Critical load of a structure against lateral buckling.
P^f_{cr}, P^w_{cr}	= Same as P_{cr} but for the unbraced frame and the free standing shear wall respectively.
$\alpha_\ell = \bar{H}/\rho$	= Lateral restraint magnitude.
β	= $1/(1 - P/P_{cr})$.
$\theta, \bar{\theta}$	= Average end rotations of a column, sway prevented and allowed, respectively.
Δ	= Relative lateral movement between consecutive floors.
ρ	= Δ/h.
ρ_a	= Allowable value of ρ for serviceability.
ρ_m	= Maximum value of ρ in a structure.

SUMMARY

The object of the paper is to derive rational criteria for shear wall bracing of tall building concrete frames with particular reference to limit design. Four basic conditions referred to as 1) *lateral displacement*, 2) *stability* 3) *primary loading* and 4) *symmetrical loading criteria* are developed. Approximate relations for practical design based on these criteria are suggested and their application to a typical building is illustrated on a numerical example.