Computer analysis and model experiment of cable structures

IIIa

Computer Analysis and Model Experiment of Cable Structures

Analyse par ordinateur et expérience sur modèle d'une structure de câbles

Computer-Analyse und Modellversuch von Kabelstrukturen

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1. Introduction

In this paper a computational method of two-dimensional cable structure is proposed, in which emphasis is laid on the problem of determination of structural member lengths. In construction of cable structures full knowledge about structural member forces under given loading conditions and especially, about the determination of correct length of each member is indispensable so that the completed structure forms strictly a shape of required geometry. When cable structures are constructed by connecting and tensioning many members with certain lengths (unstrained length), then their final shape should agree with those prescribed beforehand. Unsuitable choice of unstrained lengths of members makes it impossible to set up the desired structure, that there may be found many members left unstrained even in completed state.

From another point of view, we may say that the very problem is to know the completed shape and stress state of the cable structures when the structural members with certain lengths are assembled with some boundary members anchored with initial tension.

The authors report here on the nonlinear analysis of two-dimensional cable structure covering the above-mentioned problems, and on the experimental work which was done so as to certify the pertinency of the theory.

The theory is not limited to stress and deformation analysis of structures under given conditions (initial member forces and geometry), but makes it possible not only to clarify the stress and deformation states of cable structures but also to determine the correct unstrained lengths of members which are needed to set up the structure with desired geometry.

Computation starts from the estimate of pretension in each member utilizing the method of least squares, and then equilibrium state is determined by energy method. Computation is repeated, changing the values of pretention step by step, until the final shape of the structure is sufficiently conformed to prescribed one.

Laboratory experiment was made on a large-sized cable truss model of 23.6m length. In this kind of experiment the influence of errors upon displacement measurement should be strictly restricted to minimum, for deformation itself is the dominant factor to determine an equilibrium state. As the accuracy in setting-up and measurement of the model, however, is evidently restricted to a certain limit, relative errors should be made as small as possible by employing a large-sized model. Experimental results are shown and compared with theoretical values.

2. Statical Analysis

2-1. Basic Assumptions

Following assumptions are made in the analysis: (i) Stress-strain relationship of the material is linear. (ii) Bending stiffness of the member is neglected. (iii) Every loads act only at joints. The members are straight between the joints. (iv) Joints are considered to be frictionless hinges.

2-2. Estimate of Initial Tension by the Method of Least Square

Fig. 1 shows a joint j where N members are assembled. N member forces P_{jn} (n=1...N) and two external forces F_{jx} , F_{jy} act at this joint. Equilibrium conditions at joint j are written in the form

$$\sum P_{jn}(X_j - X_n)/L_{jn} = F_{jx}$$

$$\sum P_{jn}(Y_j - Y_n)/L_{jn} = F_{jy} \qquad (1)$$

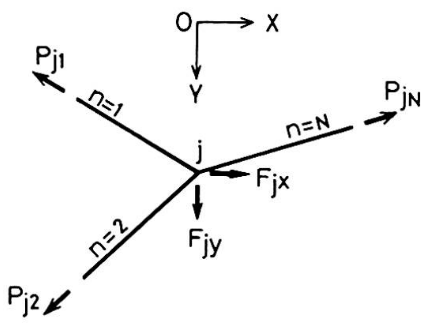


Fig. 1. Force system at a joint

When the structure is in equilibrium, Eq. (1) holds at all joints, i. e.

$$\mathbf{T} \cdot \mathbf{P} = \mathbf{F} \qquad (2)$$

where $\mathbb{T}$ is an equilibrium matrix of order (f x m) consisting of direction cosines of every members, $\mathbb{P}$ is a (m x 1) vector of every member forces and $\mathbb{F}$ is a (f x 1) vector of external forces. m and f mean the numbers of members and degrees of freedom respectively. We suppose here f > m, that is, the system to be treated is a structural mechanism, which is often the case in cable truss structures. In such cases Eq. (2) cannot be solved uniquely and the consideration of finite deformation is needed.

Now, Eq. (2) can be written in the form

$$\mathbb{T}_0 \cdot \mathbb{P}_0 = \mathbb{F} + \mathbb{r} \qquad (3)$$

where $\mathbb{T}_0$ is an equilibrium matrix which satisfies the prescribed configuration condition, $\mathbb{P}_0$ is an internal force vector which satisfies Eq. (2) approximately and $\mathbb{r}$ is the vector of unbalanced forces at every joints. We now estimate the most probable values of $\mathbb{P}_0$ making unbalanced force vector $\mathbb{r}$ minimum. The Euclidian norm of $\mathbb{r}$ is

$$\|\mathbb{r}\|^2 = (\mathbb{T}_0\mathbb{P}_0 - \mathbb{F})^T(\mathbb{T}_0\mathbb{P}_0 - \mathbb{F}) = \mathbb{P}_0^T\mathbb{T}_0^T\mathbb{T}_0\mathbb{P}_0 - 2\mathbb{P}_0^T\mathbb{T}_0\mathbb{F} + \mathbb{F}^T\mathbb{F} \qquad (4)$$

The necessary and sufficient condition to reduce $\|\mathbb{r}\|^2$ to minimum is obviously $\partial\|\mathbb{r}\|^2/\partial P_{oj} = 0$, which gives the normal equations as follows,

$$\mathbb{T}_0^T\mathbb{T}_0\mathbb{P}_0 = \mathbb{T}_0^T\mathbb{F} \qquad (5)$$

Eq. (5) can be solved uniquely and gives the most probable values of member forces at the required state $\mathbb{T}_0$, which are utilized as the initial values for finite deformation analysis which follows. It is not always easy to solve Eq. (5) directly with sufficient accuracy, since the calculation of inverse matrix $(\mathbb{T}_0^T\mathbb{T}_0)^{-1}$ is contained in its procedure. We adopted Golub's method (1) with successful results.

2-3. Finite Deformation Analysis by Energy Method

The approximate values of $\mathbb{P}_0$ have thus been obtained, but the unbalanced forces $\mathbb{r}$ still exist at the joints. In order to make these unbalanced forces vanish finite deformation analysis is carried out utilizing the theory based upon the principle of minimum potential energy. Buchholdt's works ((2), (3)) with regard to this problem furnish us much information.

The total potential energy of the cable structure is shown as

$$W = \sum U_S - \mathbf{F}^T\boldsymbol{x} \qquad (6)$$

where $\boldsymbol{x}$ is the displacement vector of joints, and U_S is the strain energy of each member and is shown in the form

$$U_S = U_{jn} = \int_0^e P_{jn}\, de = (P_o e + EAe^2/2L)_{jn} \tag{7}$$

where P_{ojn}, e_{jn}, $(EA)_{jn}$ and L_{jn} are initial tension, total elongation, extensional rigidity and final length of member $\overline{jn}$ respectively, further,

$$e_{jn} = \frac{1}{L_{jn}}\left\{\Delta X \cdot \Delta x + \Delta Y \cdot \Delta y + \frac{(\Delta x)^2 + (\Delta y)^2}{2}\right\}_{jn} \tag{8}$$

where $\Delta X = X_n - X_j$, $\Delta x = x_n - x_j$ etc. (cf. Fig. 2)

The principle of minimum potential energy leads to the equilibrium conditions at every joints, i. e.

$$\nabla W = \left\{\frac{\partial W}{\partial x_1}, \text{-------}, \frac{\partial W}{\partial y_1}, \text{-------}\right\}_f = 0 \tag{9}$$

where

$$\frac{\partial W}{\partial x_j} = \sum_n \frac{\partial U_{jn}}{\partial e_{jn}} \cdot \frac{\partial e_{jn}}{\partial x_j} - F_{jx}$$
$$= -\sum_n \frac{P_{jn}}{L_{jn}}(\Delta X + \Delta x) - F_{jx} \tag{10}$$

In order to find the displacement vector $\boldsymbol{x}$ which satisfies Eq. (9), the conjugate gradient method is used. Letting $\boldsymbol{x}^r$ be the displacement vector at the r-th step of repeated calculation, $\boldsymbol{x}^{r+1}$ at the (r+1)-th step is obtained by the relation

$$\boldsymbol{x}^{r+1} = \boldsymbol{x}^r + S^r \mathbf{v}^r \tag{11}$$

where S^r is a line element along the descent vector $\mathbf{v}^r$ to minimize W, that is, S^r minimizes

$$q(S^r) = W(\boldsymbol{x}^r + S^r \mathbf{v}^r) \tag{12}$$

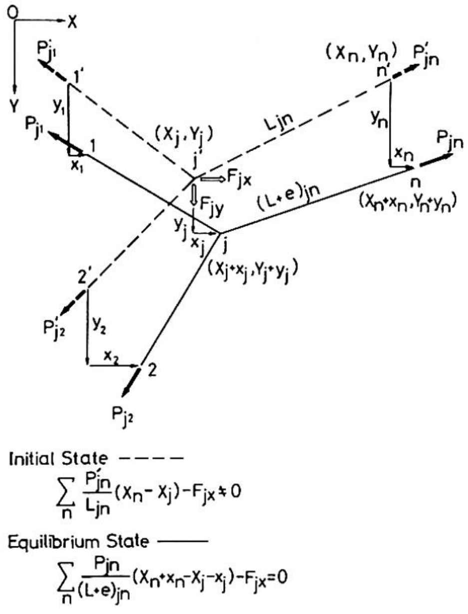


Fig. 2. Equilibrium at a joint

Fletcher-Reeves method (1) is effective to find the value of descent vector $\mathbf{v}^r$. Its sequence of calculation is as follows:

(i) for r=1, put

$$\mathbf{v}^1 = \{\text{-----}(v_x^1)_j\text{-----}\}_f = \{\text{-----} -\partial W/\partial x_j \text{-----}\}_f^{\boldsymbol{x}=\boldsymbol{x}^0} \tag{13}$$

($\boldsymbol{x}^0$ may be assumed to be zero vector.)

(ii) for r=2 ~ (f+1) calculate

$$\mathbf{v}^r = -\mathbf{g}^r + \left[(\mathbf{g}^r)^T(\mathbf{g}^r)/(\mathbf{g}^{r-1})^T(\mathbf{g}^{r-1})\right] \cdot \mathbf{v}^{r-1} \tag{14}$$

where

$$\mathbf{g} = \{\text{-----}(g_x)_j\text{-----}\}_f = \{\text{-----} -\partial W/\partial x_j \text{-----}\}_f^{\boldsymbol{x}=\boldsymbol{x}^r}$$

(iii) for r=f+2 turn to (i).

2-4. Evaluation of Member Lengths

The main purpose of our analysis is to find the correct member lengths at unstrained state. Combining the method of analysis mentioned above, we can find the required unstrained lengths of every members. Assembling such members the structure having desired shape can be obtained. When the completed state of the structure is thus obtained, it is not difficult to analyse it under any additional loading condition. The flow diagram of analysis is shown in Fig. 3.

3. Model Experiment

Fig. 4 shows the cable truss model which is to be thought of as a model of catwalk for long-spanned suspension bridge (about 1,000m long) with scale 1:40. Pieces of piano wire cut in calculated unstrained lengths were assembled to form the cable truss, which was subjected to concentrated loads at every joints (corresponding to dead loads of prototype structure) and finally tensioned by pulling and anchoring the both ends of the lower chord member.

Table 1 shows the prescribed coordinates and concentrated vertical loads (dead loads) of every joints. A part of least square solution for member forces is shown in Table 2, which is used as input data for subsequent finite deformation analysis. By this analysis the joint coordinates are obtained as shown in Table 3 (X-coordinates omitted).

The theoretical values in Table 3 (Th.) seem to agree fairly well with the prescribed one in Table 1. From this result the unstrained lengths of members are determined, which makes it possible to set up the model in required geometry. The experimental values (Ex.) at the completed state of the model are also shown in Table 3. Differences between theoretical and experimental values are very small for the size of the model.

Table 4 shows the result with regard to member forces, and Fig. 5 shows deflection curves of cable truss due to additional concentrated load (live load) applied at the mid-span. In Figs. 6 and 7 the load vs. deflection curves and load vs. member force curves are shown respectively. Every experimental results seem to agree very well with theoretical values.

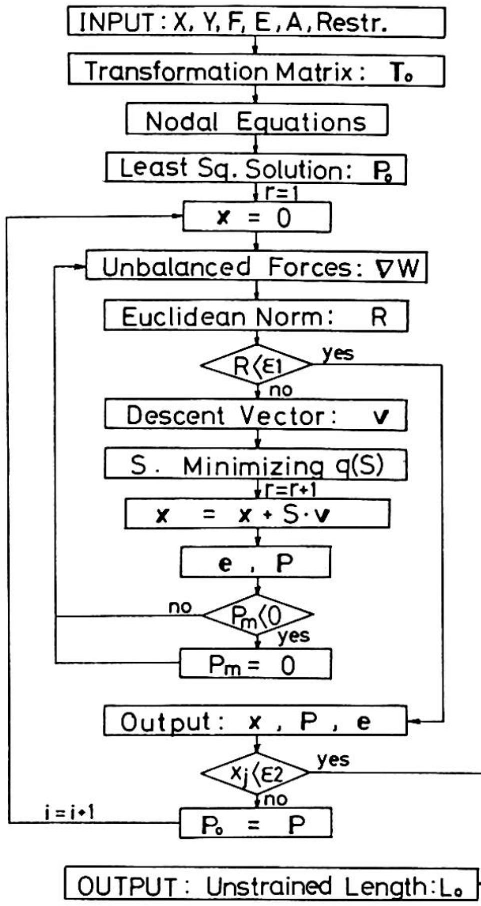


Fig. 3. Flow diagram of computation

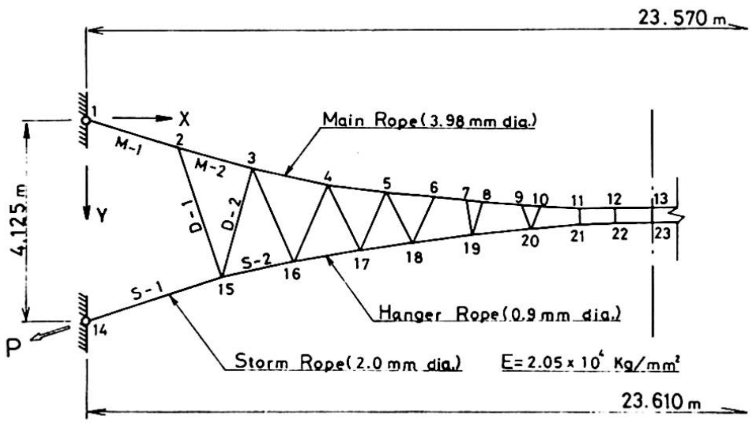


Fig. 4. Cable truss model

Main Rope				Storm Rope			
No. of Joint	Coordinates X (mm)	Coordinates Y (mm)	Load (kg)	No. of Joint	Coordinates X (mm)	Coordinates Y (mm)	Load (kg)
1	0	0	0	14	−20	4125	0
2	1930	605	26.9	15	2713	3374	4.4
3	3437	1001	23.7	16	4220	3046	3.0
4	5001	1341	21.0	17	5640	2792	2.6
5	6183	1550	16.9	18	6787	2625	2.4
6	7223	1702	12.8	19	8021	2485	2.5
7	7868	1780	7.1	20	9234	2385	2.2
8	8168	1812	9.0	21	10206	2333	1.7
9	9056	1893	9.3	22	10956	2309	1.6
10	9406	1918	8.6	23	11785	2300	1.6
11	10206	1963	11.6				
12	10956	1989	11.8				
13	11785	1999	12.4				

Table 1. Prescribed shape and loads for cable truss model

Member	P_0(kg)
M − 1	799.4
3	775.4
5	764.4
7	762.5
10	760.1
12	759.0
S − 1	207.4
3	206.9
6	204.6
9	204.1
D − 1	13.0
4	3.8
7	4.1
10	4.3
13	5.7
15	2.9

Table 2. Least square solution

Main Rope				Storm Rope			
No. of Joint	Y-Coordinates Ex.	Y-Coordinates Th.	Diff.	No. of Joint	Y-Coordinates Ex.	Y-Coordinates Th.	Diff.
1	0	0	0	14	4125	4125	0
2	603	601	2	15	3371	3371	0
3	1000	998	2	16	3044	3044	0
4	1344	1340	4	17	2795	2791	4
5	1555	1550	5	18	2633	2625	8
6	1709	1702	7	19	2495	2486	9
7	1787	1782	5	20	2395	2388	7
8	1823	1814	9	21	2345	2337	6
9	1904	1895	9	22	2320	2314	6
10	1931	1921	10	23	2310	2305	5
11	1975	1968	7				
12	2000	1994	6				
13	2010	2004	6				

Ex.=Experimental Value
Th.=Theoretical Value
Diff.=Ex.− Th.
(Unit : mm)

Table 3. Joint coordinates of cable truss model

Main Rope				Storm Rope				Hanger Rope		
Member	Member Force Ex.	Member Force Th.	Ratio %	Member	Member Force Ex.	Member Force Th.	Ratio %	Member	Member Force Ex.	Member Force Th.
M − 1	799	800	99.9	S − 1	200	196	102	D − 1	15	11
2	779	787	99.0	2	205	195	105	2	5	5
3	776	778	99.7	3	210	194	108	3	12	7
4	772	771	100.1	4	198	195	102	4	5	4
5	773	766	101.0	5	205	194	106	5	10	6
6	768	766	100.3	6	204	194	105	6	5	3
7	759	763	99.0	7	203	195	104	7	10	5
8	—	763	—	8	206	194	106	8	10	5
9	707	760	93.0	9	204	194	105	9	10	5
10	713	760	94.0					10	10	3
11	753	759	99.0					11	10	5
12	738	759	97.0					12	5	3

(Unit : kg)

Table 4. Member forces of cable truss model

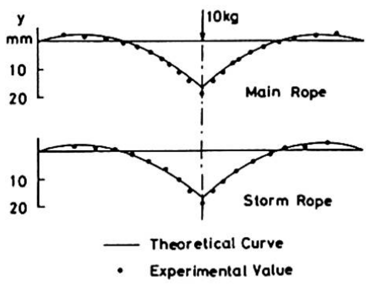


Fig. 5. Deflection curves due to load at mid-span

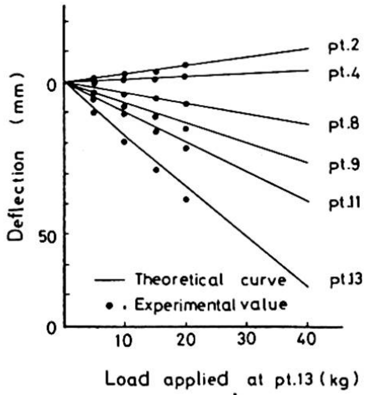


Fig. 6. Load vs. deflection curves

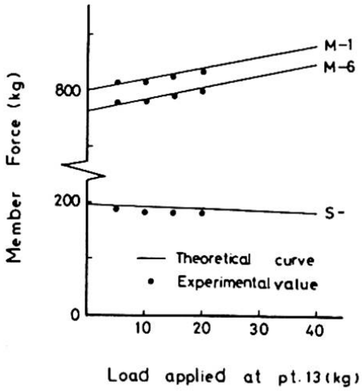


Fig. 7. Load vs. member force curves

4. Vibrational Analysis and Experiment

4-1. Method of Analysis

Equation of motion of the cable structure can be written in the form

$$\mathbb{M}\ddot{x} + \mathbb{K}x = \mathbb{F} \tag{15}$$

where $\mathbb{M}$ and $\mathbb{K}$ mean mass and stiffness matrices respectively. Stiffness matrix $\mathbb{K}$ is the superposition of every member stiffness matrices $\mathbb{K}_{jn} = [k_{ik}]_{jn}$ (i,k=1....4). k_{ik} are obtained by Castigliano's theorem, i. e.

$$k_{ik} = \partial^2 U_{jn} / \partial x_i \, \partial x_k \tag{16}$$

where U_{jn} is the strain energy of member $\overline{jn}$ as given by Eqs. (7) and (8).

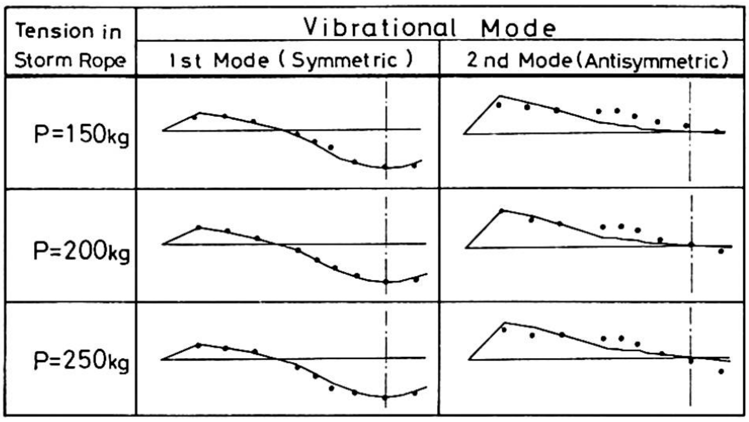
Frequency equation is

$$\det \left| \mathbb{M} - \lambda \mathbb{K} \right| = 0 \tag{17}$$

of which roots give natural frequencies of the structure. Householder's method was successfully used to give the roots of Eq. (17).

4-2. Experiment

Vibrational tests were carried out on the model structure. Natural frequencies and vibrational modes are obtained by giving harmonic excitation to the model. Results are shown in Fig. 8 with sufficient agreement between theory and experiment.



Natural Frequency

P (kg)	1st Ex.	1st Th.	2nd Ex.	2nd Th.
150	1.42	1.43	1.51	1.65
200	1.47	1.48	1.61	1.70
250	1.51	1.53	1.73	1.74

(Unit : sec^{-1})

Fig. 8. Vibrational mode and natural frequency

Acknowledgement

The authors are greatly indebted to Professor Dr. A. Nishimura (Kobe University) for his important advice and encouragement. Thanks are also due to Mr. Y. Nakajima, manager, department of construction, Kobe Steel, Ltd. for his cooperation.

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Summary

A computational method of two-dimensional cable assembly is proposed, where emphasis is laid on the problem of determination of member lengths, so that the final shape of the structure satisfies the configuration condition prescribed beforehand. Experimental study was made on a large-sized model of a cable truss. The results of both statical and dynamical experiments showed good agreement with theoretical values.