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Autor: Klingmüller, O.

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Results of the Application of Stochastic Programming for the Computation of Safety of Structures

Résultats de l'application de la programmation stochastique pour le calcul de la sécurité des structures

Ergebnisse aus der Anwendung der stochastischen Programmierung für die Berechnung der Sicherheit von Konstruktionen

O. KLINGMÜLLER
Dipl. Ing.
Gesamthochschule Essen
Essen, BRD

To the proposed application of stochastic programming for the computation of safety I want to make some additional remarks. My first point deals with the comparison of the well-known method of computing the probability of failure of structures on the basis of Failure Modes /1/ with the proposed concept. The second remark concerns the application of the proposed concept to the limit load analysis by nonlinear programming.

1. On the left hand side of figure 1 we have a formulation of the first limit load theorem as a linear programming problem. The solution of this primal problem gives us the maximum of the load factor which holds the equilibrium conditions as well as the linearized yield conditions, both combined in the matrix $\underline{B}$. In this special formulation the vector of primal variables $\underline{y}$ consists of the load factor y_1 and the redundant forces $y_2 \ldots y_{p+1}$; p is the redundancy of the structure.

On the right hand side of figure 1 we have a formulation of

Primal Problem - First Limit Load Theorem

maximize y_1

subject to

$$\underline{B} \cdot \underline{y} \leqq \underline{F}_o$$
$$\underline{y}_1 \geqq 0$$

Dual Problem - Second Limit Load Theorem

minimize $\underline{F}_o' \cdot \underline{z}$

subject to

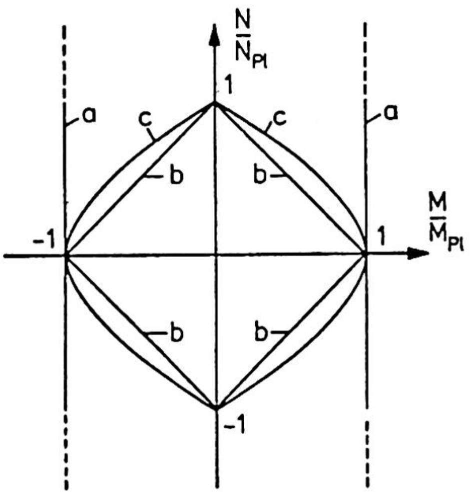
$$\underline{B}_1' \cdot \underline{z} \geqq 1$$
$$\underline{B}_i' \cdot \underline{z} = 0, \; i=2 \ldots p+1$$
$$z \geqq 0$$

$\underline{y}$: vector of primal variables
y_1: load factor , $y_2 \ldots y_{p+1}$: redundant forces
$\underline{z}$: vector of dual variables, i.e. strain velocities
$\underline{B}$: matrix, combining linearized yield conditions and equilibrium conditions
$\underline{F}_o$: vector of the right hand sides (e.g. fully plastic moments)
$\underline{F}_o$ z : work of the internal forces

Figure 1 : Linear Programming Problems for Limit Load Analysis

the second limit load theorem as a linear programming problem. The solution of this dual problem gives us the minimum of the work of the internal forces, i.e. the scalar product $\underline{F}_o'\underline{z}$. The work of the external forces , given by $B_1'\underline{z}$ must be not less unity. The equality constraints $B_i'\underline{z}$ are the conditions of kinematic compatibility. B_i are columns of $\underline{B}$. Solving this dual problem is equivalent to finding the most critical out of all failure modes.

By the theorem of duality in linear programming we know that the load factor, that is the value of the objective function must be the same for both problems. Computing the probability of failure with the proposed concept then means just to look at the probability of failure by the most critical failure mode. Thus we have a lower bound for the probability of failure /1/. This is the reason why the sensitivity analysis becomes very important.



a) Failure Mode concept (Stevenson)

$$\frac{M}{M_{Pl}} \leq 1$$

$$-\frac{M}{M_{Pl}} \leq 1$$

b) linearized yield condition of proposed concept

$$\frac{M}{M_{Pl}} + \frac{N}{N_{Pl}} \leq 1$$

$$\frac{M}{M_{Pl}} - \frac{N}{N_{Pl}} \leq 1$$

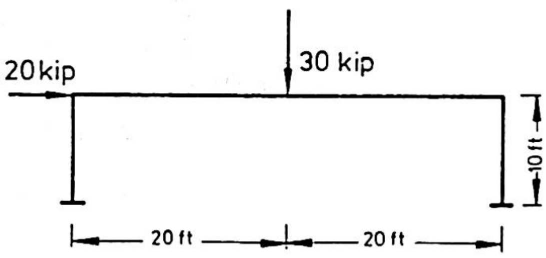
$$-\frac{M}{M_{Pl}} - \frac{N}{N_{Pl}} \leq 1$$

$$-\frac{M}{M_{Pl}} + \frac{N}{N_{Pl}} \leq 1$$

c) nonlinear yield condition

$$\frac{|M|}{M_{Pl}} + \left(\frac{N}{N_{Pl}}\right)^2 \leq 1$$

Figure 2 : Yield Conditions of Beam Elements



coefficients of variation: 0.1

beam	column	Failure Mode concept		proposed concept	
mean M_{Pl}	mean M_{Pl}	load factor	p_f	load factor	p_f
200	100	1.0	0.56	0.99	0.54
320	50	1.0	0.63	0.98	0.59
450	75	1.5	$2.1 \cdot 10^{-4}$	1.46	$0.9 \cdot 10^{-4}$

Figure 3: Comparison of Different Computation Concepts

On the other hand in the computing of the probability of failure with regard of all possible failure modes the yield condition "a" in figure 2 is used. An interaction of normal forces and bending moments is either neglected, or the computation demands additional iteration. The lines "b" in figure 2 are bounding the feasible region of the proposed concept. The curves "c" belong to the quadratic function for the interaction of normal forces and bending moments.

For the comparison of the results which are given in figure 3 for Stevensons' test example /1/ we therefore must have in mind different yield conditions and different computation concepts.

2. The difference between the nonlinear yield condition and the linearized yield condition (figure 2) leads to the idea of computing the limit load by means of nonlinear programming. The nonlinear programming problem for limit load analysis is given in figure 4. An iterative method yields a solution of this problem. In the solution a number of yield conditions hold the equality sign. These nonlinear yield conditions are expanded into a Taylor series around the solution and around the stochastic variables $\underline{F}_o$. The nonlinear terms are suppressed and this results in a linear transformation of the stochastic variables $\underline{F}_o$ into the unknowns $\lambda, \underline{X}$. With such a linear transformation we can apply the described concept to the limit load analysis by nonlinear programming.

maximize λ

subject to

$$\Phi_i\ (\lambda, \underline{X}, F_{oi}) \geqq 0 \qquad i = 1 \ldots r$$
$$\lambda \geqq 0$$

λ : load factor

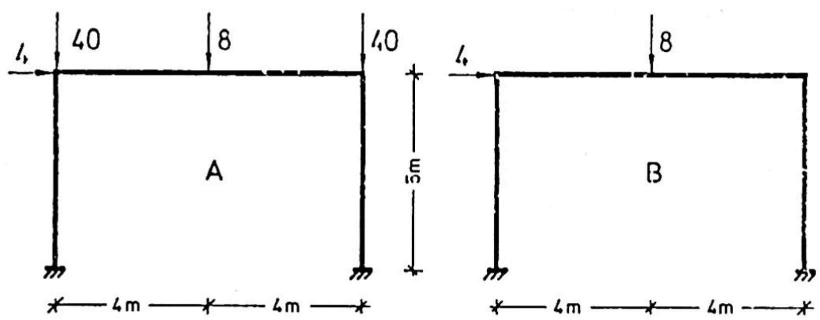
$\underline{X}$: vector of redundant forces

F_{oi}: plastic load bearing capacities (e.g. fully plastic moments)

Φ_i : nonlinear yield conditions

r : number of control points

Figure 4 : Nonlinear Programming Problem for Limit Load Analysis



M_{Pl} = 17.4 [Mpm] , N_{Pl} = 210.9 [Mp] in all sections

coefficients of variation : $c_v^P = 0.2$, $c_v^{M_{Pl}} = 0.1$

	system A		system B	
	linear	nonlinear	linear	nonlinear
load factor	1.56	1.81	1.93	2.00
p_f	$5.3 \cdot 10^{-3}$	$6.6 \cdot 10^{-5}$	$1.9 \cdot 10^{-5}$	$4.5 \cdot 10^{-6}$

Figure 5 : Comparison of Linear and Nonlinear Programming in Limit Load Analysis.

The results of linear and nonlinear programming in limit load analysis are compared in figure 5 with two different loading conditions. The differences in the probability of failure are caused mainly by the different load factors. The greater load factors of the nonlinear solution are due to the larger feasible region of the quadratic yield condition.

The design of structures for a given allowable probability of failure requires a solution of the above described problems in every step of iteration, i.e. computing the load factor and the probability of failure for given cross sections of the elements. For a single structure the computertime for a reliability-based design might cost too much. But if we look at problems like optimizing prefabricated elements for a large number of buildings or like the optimization of a building code for given restricted resources /2/ it is important to find structures with minimum cost which are safe. With the proposed concept I hope to have given a more general way of evaluating the safety of structures with regard to their mechanical properties.

References

/1/ Stevenson, J.: Reliability Analysis and Optimum Design of Redundant Structural Systems with Application to Rigid Frames. Ph.D. Thesis, Case Western Reverse University 1968

/2/ Ravindra,M.K. and Lind,N.C. : Theory of Structural Code Optimization. Journal of the Structural Division,ASCE,Vol.99 Nº ST 7 , July 1973 , p. 1541-1553

SUMMARY

Results obtained with the proposed concept for the computation of the probability of failure as it was described in the preliminary report are compared to results which are given by the well-known method of computing the probability of failure on the basis of failure modes. A second part of the paper deals with the application of the proposed concept to the limit load analysis by nonlinear programming.

RESUME

Les résultats de l'application de la programmation stochastique pour le calcul de la sécurité des structures - méthode présentée dans le Rapport Préliminaire- sont comparés avec les résultats donnés par le calcul de la probabilité de ruine usant la méthode des chaînes cinématiques . La seconde partie de l'exposé montre l'application de la méthode proposée au calcul à l'état limite, utilisant la programmation non-linéaire.

ZUSAMMENFASSUNG

Ergebnisse des im Vorbericht beschriebenen Verfahrens zur Berechnung der Versagenswahrscheinlichkeit von Konstruktionen werden mit der bekannten Methode, die Versagenswahrscheinlichkeit mit Hilfe der kinematischen Ketten zu bestimmen, verglichen. Der zweite Teil des Beitrags behandelt die Anwendung des vorgeschlagenen Verfahrens auf die Traglastberechnung mit nichlinearer Programmierung.