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Comments by the Author of the Introductory Report

Remarques de l'auteur du rapport introductif

Bemerkungen des Verfassers des Einführungsberichtes

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Optimization Concepts & Techniques in Structural Design

In preparing the Introductory Report for Theme II, 'Progress in Structural Optimization' the three reporters each took a specific aspect of the theme. My objective was to provide an introduction to Structural Optimization; to describe its philosophical goals and to outline in brief and simple terms some of the mathematical techniques which are most frequently used to solve structural optimization problems. It was intended to be a 'beginner's guide' to the topic which would be expanded in more detail by the other Reporters and by the authors of papers in the Preliminary Report.

I will not dwell on an introduction to structural optimization but will assume that you are familiar with it from the Introductory Report. Towards the end of the Report I have remarked that by 1970 most of the simple problems of structural optimization had been solved, only the difficult ones being left. I think this point is demonstrated very well by the six papers in the Preliminary Report under Theme IIa. None of them deals with simple, straightforward problems; they all are concerned with difficult aspects of the topic and they all give a very fair indication of the present-day complexities of structural optimization.

I would like to consider first the papers by Anraku and by Balasubramonian and Iyer since they represent the forefront of technologically difficult problems. Anraku is concerned with designing steel frames to withstand dynamic earthquake loadings. Balasubramonian and Iyer are concerned with random variable variable loadings. It is significant that throughout my Introductory Report I have not mentionedloadings such as these. Much of the research done in structural optimization over the last twenty years has considered only deterministic static loads. Only in the last few years have researchers begun to look at optimum design for dynamic or non-deterministic loads. The reason for this is that the complexity of the problems increases dramatically as one moves away from deterministic static loads and this is an area of work which still requires much research. It is also an essential area for future research. Structural design methods and codes of practice are now moving towards a greater recognition of the probabilistic nature of loadings. Limit state concepts in which safety factors against many different possible occurrences are assessed are also becoming widely accepted. If structural optimization is to retain any relevance for the practising engineer it is essential that it too should be able to handle dynamic loads, probabilistic loads and limit state concepts.

The papers by Anraku and by Balasubramonian and Iyer are therefore welcomed because they are pointing in the right direction for the future of practical structural optimization. However, Balasubramonian and Iyer's paper dealing with structural optimization under random loading effects is entirely theoretical. One of the main reasons why structural optimization methods are not now used more widely in practical design is that there has too often been a large gap between what is correct in theory and what works in practice. This is particularly so when using applied probabilistics. There is a world of difference between defining in theoretical terms the probability of failure of a structure and actually evaluating it accurately for a real-world structure. Nevertheless Balasubramonian and Iyer have made a start in rationalising the effects of random loadings.

Anraku's paper deals with optimum design of frames for earthquake loading - once again a technologically complex form of loading. Some codes of practice incorporate requirements for designing against earthquakes and Anraku is to be complimented on attempting to extend structural optimization into this difficult area of work. As an optimization method Anraku has used sequential linear programming. This method is often used when no other method is available or when the problem is very complicated. Unfortunately it is on these highly nonlinear problems that its performance is worst and it is evident from Anraku's paper that he has experienced difficulties with this method. He comments that an accurate analysis of the dynamic loading is essential if the method is to converge and it appears from his Figure 6 that his optimized design violates some design restrictions by as much as 20%. Both these effects are inherent in the sequential linear programming method. Any linearisation of a highly nonlinear model is bound to be both sensitive to error and inaccurate.

The paper by Brozzetti *et al* is a complete contrast to the preceding papers. It is concerned with a very practical, pragmatic approach to using a commercially available computer package program for designing steel structures. In particular they consider the minimum weight design of practical steel frames so as to satisfy a large number of limit state criteria. The paper highlights the philosophical point that structural optimization is not a mathematical discipline but is, and will always continue to be, an engineering discipline. The objective of structural optimization is to produce the best possible engineering structure. Sometimes precise mathematical methods will allow this to be done mathematically but usually the practical limitations of codes of practice, methods of construction and aesthetics.make a completely mathematical formulation of the design problem impossible. Here the expertise of the engineer is essential. Sometimes those researching new structural optimization methods ignore practical considerations or make dubious assumptions in order to force a practical problem into a mathematically amenable form. While this may be possible for research purposes it is not possible for practical design purposes. Practical structural optimization very often has to be an inexact process relying sometimes upon rigorous mathematics, sometimes upon heuristics and always relying upon engineering experience. Brozzetti *et al* do not describe their optimization technique in detail - it seems to be sequential linear programming but coupled with a lot of engineering knowledge in order to produce real-world structural designs. In their paper they demonstrate that in order to produce really economical designs it is necessary to include the nonlinear interactions of axial forces and bending moments in steel framed structures. Very often these interactions are ignored by researchers when studying these structures since they introduce awkward mathematical nonlinearities.

The remaining three papers all deal with almost classical topics in structural optimization. Structural optimization has always been concerned with two basic questions - one practical, the other more theoretical. The practical question is - 'How can I design the most efficient structure to perform a specific task?' The more theoretical question is - 'What are the funadmental laws which govern structural efficiency?' It is important to distinguish between these two questions and theoretical work which attempts to answer the second question should not be criticised because it seems irrelevant to practical design. Work in theoretical structural optimization is important and essential because it adds to our fundamental knowledge of structural behaviour. An increased awareness of why some structures are more efficient than others will eventually benefit practical design engineers but the immediate practical relevance of such work may not be apparent.

Nakamura and Nagase consider the optimum rigid-plastic design of multi-storey plane frames for multiple load cases. In my Introductory Report I mention in Section 4.4 that optimum rigid plastic design can be represented as a linear programming problem. Nakamura and Nagase have done this and have then considered some of the more advanced aspects of linear programming theory using duality theory in order to reduce the size of the problem and solve it rapidly. This area of work, optimum rigid-plastic design is much researched and it can truthfully be said that our knowledge of the mechanics of structures in the plastic régime has been greatly advanced by such work. Nakamura and Nagase have made an important contribution to this topic by considering multiple loading cases and their paper is well worth further study. They do not claim to be able to produce an optimum practical design but their method can be used for rapidly producing an efficient and economical initial design which can then be analysed and modified in minor ways to satisfy engineering criteria. I commend their treatment and uses of duality and I believe their work could be developed to form the basis of really efficient design programs for practical structural design.

The two final papers, one by myself and one by Lipp and Thierauf both deal with the same classical problem. How can one design truss-type structures for minimum weight in the presence of restrictions upon member stresses, nodal displacements, member size limits? The optimum design of trusses has always been a subject of much research for several reasons. First of all trusses are practical engineering structures and so it is a relevant area of work. Secondly, the problem is a nonlinear one of a most interesting mathematical form and thirdly the methods which can be used to design trusses can also, with minimal modifications, be used to optimally design certain classes of more complex finite element plate structures. Perhaps the major difficulty which any optimum truss design method has to face is that of problem size. For each truss member there is usually one variable (the cross-sectional area) for which an optimal value is to be found. Trusses of several hundred members are not uncommon so for these structures the optimum design problem expressed mathematically is nonlinear, has several hundred variables and even more constraints.

A straightforward numerical search for an optimum of such a large problem is not possible as it is wasteful of time and computer resources. Recently engineers have looked more deeply at this problem and have found that by examining the theory of optimality more carefully new, more rapid design methods for trusses can be developed. My own paper explores this topic further and describes how duality principles can be used to develop new design methods. The paper by Lipp and Thierauf is concerned with the same approach - indeed the mathematics of the two papers is remarkably similar. I do not have time in this summary to talk about the differences and similarities in these papers in detail but I would like to add a final comment. In my Introductory Report I mention that duality might prove to be a mathematical concept of great value to those interested in the optimum design of large structures. My own paper reflects this of course but it should be noted that the Lipp-Thierauf paper is also concerned with duality via the Lagrange multiplier technique thus strengthening my earlier opinion.

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Comments by the Author of the Introductory Report

Remarques de l'auteur du rapport introductif

Bemerkungen des Verfassers des Einführungsberichtes

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System and Geometrical Optimization for Linear and Non-Linear Structural Behaviour

Please accept by apologies for being unable to attend and report personally to this meeting but circumstances beyond my control have led to my absence. I offer my best wishes for the success of this session and I thank in advance Dr. Templeman for substituting on my behalf.

Many discussions at earlier IABSE sessions have considered optimization. Professor Courbon defined optimization as designing and constructing a structure at the lowest cost with the object of fulfilling a well-defined purpose. Cost consideration must be given to safety, service life, maintenance and future adaptability. Within this broad context, the speciality of structural optimization arose to provide specific design purposes and methods which will aid in reaching an optimum structure. Thus, in the same way that matrix methods or finite elements aid in structural analysis, techniques of structural optimization have been developed to improve design procedures. Its applicability depends as much on reducing the ultimate cost of the structure as on savings in time and cost for the design engineer.

Historically, optimization has used simple design rules to check optimum designs. Gradually, more sophisticated mathematical methods applied with computer programs arose to systematically search and locate optimum structures.

A description of formal optimization methods taken from fields of mathematical programming and Operations Research has been presented by Dr. Templeman in his survey paper published in the Introductory Report. Such methods have found widespread application in the design of structural elements which are described by a number of design variables and constraints determined by codes of practice.

Figure 1 of my Introductory Report shows examples of such element designs. There is an example of a welded box girder for which I have had occasion to design large numbers for crane structures. Another example, is the welded plate girder which we designed based on the rather complex provisions for unbraced members in the AISC specifications. Also shown is a prestressed concrete beam with eleven design variables. The element design is controlled by contraints on loading and prestress force and deformations. Other element designs reported include welded columns, stiffened ship plates, shear walls, prestressed plates and reinforced concrete beams.

Element optimization has led to a number of computer programs whose function is to efficiently design a variety of elements and perform the tedious calculations required by the designer in trying to proportion such elements. The programs have usually been based on penalty or geometric programming methods of optimization. Professors Ohkubo and Okumura in their Preliminary Report paper have derived the optimum design of elements such as bridge girders and truss members using the method of sequential linear programming. This was then adapted by them to a branch and bound procedure for solving discrete variables such as steel type and flange thickness. A different approach to the optimization of element, in this case concrete bridges, is presented by Ulizkij and Jegoruschkin. It uses influence factors for predicting the behaviour of the bridge and therefore simplifies subsequent optimisation.

A combination of elements as in a total structural framework requires a different approach to optimization. Any changes in the design on the path to the optimum may subsequently require complete reanalysis of the structure to determine new stresses and deflections. In Figure 2 of my Introductory Report, a grillage is shown in which redistribution of forces occurs following each design change. The optimum design procedure for this case was reported by Moses and Onoda. Other examples of system optimization are statically indeterminate trusses and frames.

A system optimization, to be efficient, requires techniques such as the sequential linear programming shown by Ohkubo and Okumura. It is important that the number of cycles of reanalysis does not become large leading to excessive demands for computer time.

Inclusion of gross geometrical variables of the structures represents an important improvement in the class of problems for which optimization may be applied. Figure 4 of my Introductory Report shows a transmission tower in which the tower shape and location of nodes is permitted to change leading to significant reductions in structural weight. The left figure is the original design while the right is an optimized case. The optimization takes place automatically with a program using methods of minimisation working with respect to the geometric or shape design variables.

Another example of geometric optimization is the arch dam reported by Vitiello and shown in Fig. 5 of my Introductory Report. The mesh shown is the finite element analysis while X_1 - X_4 are the geometric design variables. Such applications show that major improvements in structural efficiency can often come from variations in geometric design variables. This is investigated for arches and suspension bridges by Professor Hirai and Yoshimura in their Preliminary Report.

Form and type of structure represent a high level of optimization for which programs have only recently been attempted. Figure 6 from my Introductory Report paper shows a schematic diagram for optimizing the cost of single storey factory buildings. The variables include structural layout such as bay spacing and also the type of joists, girders, columns and foundation including material type and detailed design variables. The design methods, automatically performed by the computer, can lead to important structural savings and can be updated following changes in individual construction and material costs.

Bomhard in the Preliminary Report shows a comparison of structural form with an illustration of beams, arches and suspensions to cope with long-span structures. Suruga and Maeda have developed a very interesting concept of a decision matrix to compare structural forms for their application to floor systems of long-span bridges. Each type of floor system such as composite girder or orthotropic deck is rated according to cost, construction and performance before a final decision can be made. This leads to a multi-objective criteria for optimization which may have important applications to other examples such as comparing economy with safety. The inclusion of safety directly in the optimization methodology is covered by Tegze and Lenkei with an example of collapse analysis of statically indeterminate plane structures.

The inter-relationship of safety and economy of structure has been recognised by many authors, but more effort is required to bring these factors into both the code specifications and the programs for optimum design.

Comments by the General Reporter

Remarques du rapporteur général

Bemerkungen des Generalberichterstatters

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Examples of Computer-Aided Optimal Design of Structures — General Report

The main subject of Theme II is on "Progress in Structural Optimization", which was originally proposed by the Japanese National Group of IABSE, because it was intended to stimulate and encourage Japanese engineers to apply the concept and method of optimization to problems of structural design, since the structural optimization has very recently been introduced into Japan. Along this intention we are very thankful for the three excellent Introductory Reports.

Dr. Gellatly and Mr. Dupree presented a very excellent paper as an introductory report on applied structural optimization in terms of examples of computer-aided optimal design of structures. They covered two different approaches to the optimum design of complex structural systems, emphasizing the practical aspects of design problems intended for producing a useful tool for designers.

The first approach, "Optimality Criteria Approach" will be accepted by designers because of its simplicity and effectiveness. The approach to the weight minimization of fixed-geometry structures with constraints based on the use of optimality criteria, appears to offer considerable advantage over mathematical-programming based methods. At comparative studies, the present method seems to reach a similar or better design in considerably fewer iterations than most numerical search methods with the reduction of computational costs.

They presented five examples, and also Dr. Gellatly discussed this approach at his other paper[1)] at an example of "Twenty-five-bar Transmission Tower" in which, using the current program, convergence was obtained in seven iterations to get its minimum weight, although, using a numerical search method, over one hundred analyses might have been required.

These results are very encouraging us, because they indicate that some, if not all, of the difficulties encountered in large-scale optimization problems for the very large number of variables in finite element representation of real structures, can be eliminated through this type of approach. However, certain problems may still remain to be unsolved, particularly with regard to convergence characteristics.

The second approach is labeled Sieve-Search Procedure developed at Bell Aerospace Company, the guiding philosophy of which is that an optimum system is an optimum arrangement of pre-optimized components. The results obtained from the design studies on high-speed vessels and a design study on a complete bridge structure have indicated that, firstly, the method will permit the full variation of construction method, materials and configuration as well as component sizing, and secondly, this method is also an efficient cost-effective approach to automated optimum design.

Dr. Gellatly and Mr. Dupree suggested finally that the ideal solution for optimization problems would possibly appear to be a combination of the two approaches, in which the sieve-serach defines configuration and non-continuous variables and the optimality criteria method will be used for refinement of the design, expecting a considerable potential for overall system optimization at various design problems.

We have been expecting a number of papers to be presented at the Preliminary Report under the stimulus and for the discussions of the Introductory Reports. For the Sub-Theme IIc, the following five papers have been accepted:

1. The paper presented by Mr. Gurujee

The paper shouldhavebeen discussed at Theme IIa. He proposed a general optimization algorithm for a structure. A structural optimization problem can be generally solved as a sequence of analysis-programming cycles by the mathematical programming. In the optimization process which the author proposed in the form of a chart shown in Fig.1 at the Preliminary Report, p.179, the relation between the changes in the behavior variables due to a specified change in each of the design variables, is found and stored in the form of "Sensitivity Matrix". Then, the programming problem can be solved by using the penalty function method. In this paper, however, he did not show any specific examples to which his proposed method was applied.

2. The paper presented by Prof. Yamada and Mr. Furukawa

They treated the optimal design of a system of tower and pier of a suspension bridge, on the elastic foundation subjected to earthquake ground motion. They showed an example how to combine mathematical programming and dynamic structural analysis through response spectrum for a dynamic loading problem, referring to Figs. 1 and 2 at the Preliminary Report, p.184. To simplify very complicated real dynamic behavior of the system, two design variables were selected: longitudinal width of the pier and stiffness of the tower. A generalized cost was selected as the objective function, and requirements for stress of the tower and displacement of the pier at its top, and buckling of the tower, overturning of the pier, and physical limits, were constraints.

Since the problem is non-linear and undifferential, the Sequential Unconstrained Minimization Technique by Powell's direct search method was applied to optimization, probably because the method is more reliable in terms of guaranteed convergence if the first derivatives or no derivatives are available. At a numerical example, the authors found out that the generalized cost is greatly affected by the modulud of elasticity of the foundation. This problem is overall system optimization of a simple tower-and -pier system. Shape and geometry optimization and combination with detailed element optimization will be a future problem.

3. The paper presented by Prof. Konishi and Prof. Maeda

The paper on "Total Cost Optimum Design of I-Section Girders for Bridge Construction" treated examples of detailed design optimization of main elements of girder bridges. Generally, at the problem of bridges, cost optimization is selected as the objective function, but the cost used to be defined material cost only or material plus overall fabrication cost. At the present paper, the objective function consists of material and fabrication costs, which cover costs of full-scale drawing, machining, shop welding, shop assembly and shop painting base on actual detailed informations obtained at fabricating shops in Japan.

A computer-aided optimum design of girders by the method of "Sequential Linear Programming" was illustrated at I-shaped, deck-type, welded plate girders with five different span lengths, and sixteen design variables including material selection (See Fig.2 at the Preliminary Report, p.192). The influence of material and size selections on the total cost was discussed in detail, to help designers carry out a detailed element design efficiently from the point of optimization, taking into consideration not only material cost, but also shop fabrication cost.

For a specific or individual bridge, it would be required to study on an overall optimization design including transportation and erection costs for a system of main girders, laterals and decks.

4. The paper presented by Professor Schindler

He proposed an optimization method to combine design-oriented approach and computer-oriented approach, in which a designer can search for a range of approximation near an optimum value with a design program, within the capacity of a computer, not spending so much money and time for computer calculation.

He illustrated his method at the optimum design of a railway truss bridge shown in Abb.1 at the Preliminary Report, p.196, taking into account three kinds of deck system , two kinds of steel, two kinds of bridge class, five kinds of span length. The objective function was total steel weight, and the design variables were span length, number of panels, height of the truss, and width of chord members. For various truss heights, steel weights were calculated by a computer with parameters of span length and number of panels. By comparison of each steel weight, the minimum weight was found out for a certain value of span length and of number of panels.

This approach is not straightforward, but rather comparative or selective. Sometimes depending on a problem, this approach may save the time and money for a computer more than mathematical programming methods. This kind of approach could be examined in contrast with a study presented by Prof. Ohkubo at Theme IIb [2)] who proposed a sub-optimizing method for trusses.

5. The paper presented by Messrs. Tanaka, Kamemura and Maruyasu

They introduced the total computer-aided design system for girder bridges. which has recently been developed at Nippon Kokan Company, Japan. Automated computer techniques for design have advanced so that various types of detailed element design and selection among alternatives for minimum cost can be carried out. In this sense, the proposed computer system is a well advanced method for automated design of a girder type bridge in its element and overall system.

As the authors pointed out, such a computer program could be used for lowering cost, increasing standardization of elements and also evaluating the effects of changing constraints on weight, cost and behavior. The authors discussed conceptually the interaction between optimum design and automated design, but they did not show concretely with an illustration how to incorporate optimization into the automated design program.

The proposed computer system should be examined in contrast with the flow chart of Sieve-Search Optimization for bridge design proposed by the Introductory Reporters, Dr. Gellatly and Mr. Dupree [3].

As a concluding remark, at the Prepared Discussion nore demonstrations of structural optimization are welcome in terms of examples to encourage designers to utilize optimization techniques at their routine office practice, and also to discuss what kinds of problems have been encountered at practical designs.

References:

1) R.A. Gellatly & L. Berke, "Optimality-criterion-based Algorithms", Optimum Structural Design, ed. by Gallagher & Zienkievicz, John Wiley, 1973, p.44.
2) S. Ohkubo & T. Okumura, "Structural System Optimization Based on Sub-optimizing Method of Member Elements", Prel. Rept. 10th Congress, IABSE,1976.
3) R.A. Gellatly & D.M. Dupree,"Examples of Computer-Aided Optimal Design of Structures", Introductory Rept., 10th Congress of IABSE, 1975.

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Optimum Design of Cable-Stayed Bridges using an Optimality Parameter

Calcul de ponts haubannés à l'aide d'un paramètre d'optimisation

Die Berechnung von Schrägseilbrücken mit einem Optimierungsparameter

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1. INTRODUCTION

The optimization discussed in this paper is applied for the design of the overall super-structure of cable-stayed bridges. Then the hierarchy of this study is belonged to category 3 described in the introductory report of the 10th congress by Templeman(1). The optimization method developed here is a kind of the optimality criterion method discussed by Templeman, Gellatly and Dupree(1)(2).

Up to the present many nonlinear programming techniques have been developed and applied for the optimum design of bridge super-structures, but successful applications are very few. Because the fully stressed design for a common type of super-structure such as girder bridge is a convenient design method and gives the satisfactory economical result. Therefore from the practical point of view, the optimum design without considering the price of sub-structure may be important for only some specific type of bridges such as cable-stayed bridges, suspension bridges.

In this study, an optimality condition parameter is obtained by a mean of the numerical calculation and the parameter is used to determine the economically proportional sizes of the cable and girder.

2. PRECONDITIONS FOR THE NUMERICAL PROCEDURE

To determine an optimality parameter by numerical process, the following preconditions must be given.

(1) Utilization of the structural nature of cable-stayed bridges is very important to find out the optimality condition. Fig. 1, 2 show the static behavior of cable-stayed bridges due to dead loads and live loads. These examples show that the each rigidity value of cable and girder is not main factor of changing the section force distribution. It is obvious that the main influence to the girder section force is a rigidity ratio, $\gamma = EG \cdot IG / EC \cdot AC$. where, EG is the modulus of elasticity of girder, IG is the moment of inertia of girder, EC is the modulus of elasticity of cable, AC is the cable area.

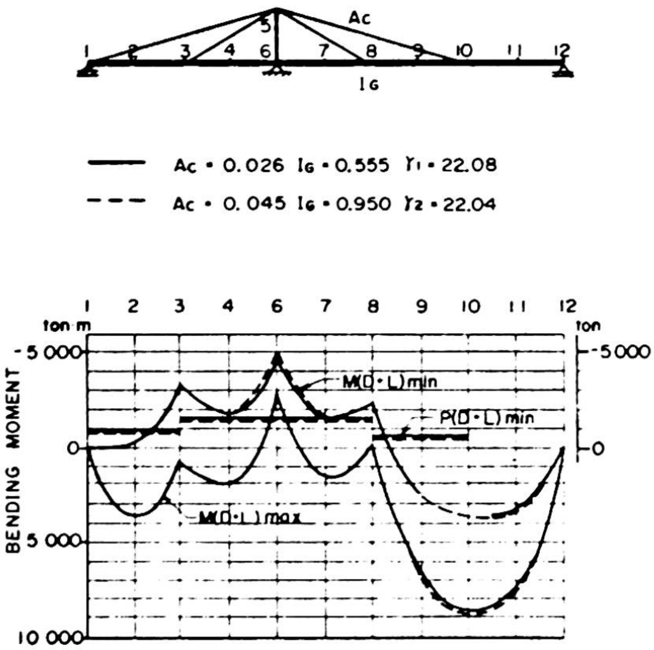

Fig. 1 2 Span Bridge

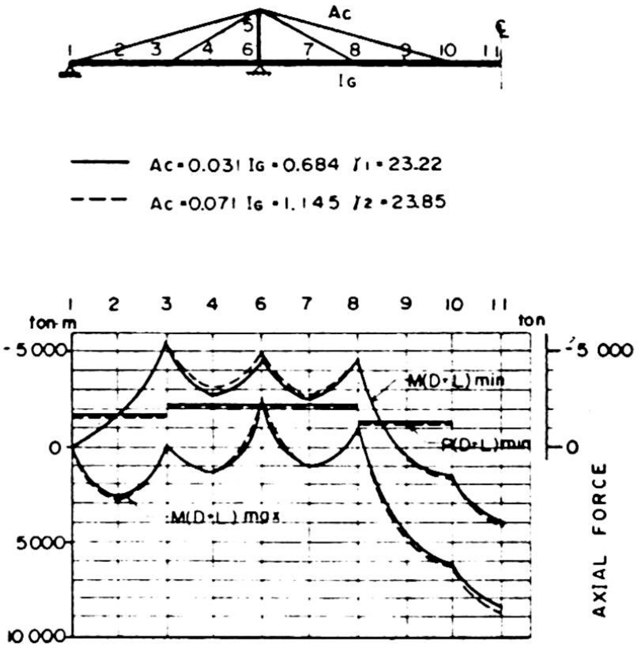

Fig. 2 3 Span Bridge

(2) The price ratio of materials including the cost of fabrication and erection is assumed as follow.

structural steel : high tensile steel : cable = 1 : 1.15 : 2.0

(3) Fig. 3 is the analyzing structural system which rigidity ratio is assumed as:

$\gamma_{try} = EGX_1/ECX_2$ X_1: moment of inertia of girder
X_2: cable area

Fig. 4 is the actual redesign structure which rigidity ratio is expressed by:

$$\gamma_{real} = \frac{EG/NG \cdot \sum_{n=1}^{NG} IGn}{EC/NC \cdot \sum_{n=1}^{NG} ACn}$$

IGn: moment of inertia of girder
ACn: cable area
NC, NG: number of cable and girder

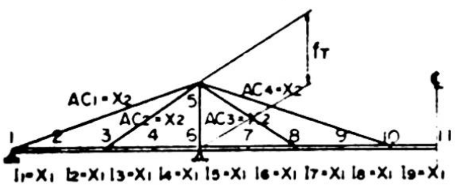

Fig. 3 Assuming Member System

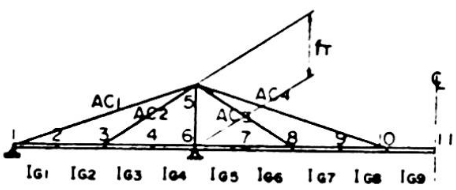

Fig. 4 Actual Member System

The approximate design process must be carried out by keeping the following ctiteria.

$$0.9 < \gamma_{try}/\gamma_{real} < 1.1$$

(4) The section and material compositions of stiffening girder are illustrated in Fig. 5, 6. The price of girder member is determined by the element design based on the fully stressed design.

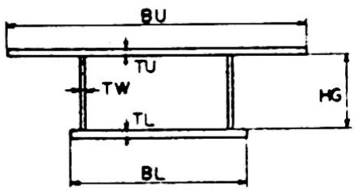

Fig. 5 Girder Section

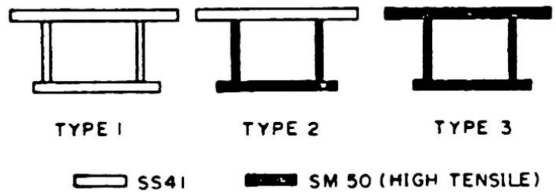

Fig. 6 Material Composition of Girder

3. DETERMINATION OF AN OPTIMALITY PARAMETER

An optimality parameter is determined after carried out the next 2 step procedure.

(1) Characteristic Parameter (Step 1)

The basic structure to be effectively prestressed is determined by the grid search procedure, because two design variables are employed for the global system optimization. The cost evaluation is made by the following equation.

$$Z(X_1,X_2) = \sum_{m=1}^{NG} \text{price } G(X_1,X_2) + \sum_{n=1}^{NC} \text{price } C(X_1,X_2)$$

X_1: moment of inertia of stiffening girder
X_2: cable area
price G: price evaluation of girder depend on X_1, X_2
price C: price evaluation of cable depend on X_1, X_2

The characteristic parameter at the grid point is expressed as:

$$KE = EG \cdot IG / EC \cdot AC \cdot HG^2$$

IG: moment of inertia of stiffening girder
AC: cable area
HG: web depth

(2) Determination of an Optimality Parameter (Step 2)

Prestressing forces (external loads) are introduced into the cable of basic structural system determined by above procedure. In this step, prestressing forces are design variables (Fig. 7). In case of two design variables, an optimality parameter minimizing the total cost is also selected among the grid points number of characteristic parameters, and it is expressed by the following nondimensional parameter.

$$KOPT = EG \cdot IGOPT / EC \cdot ACOPT \cdot HGOPT^2$$

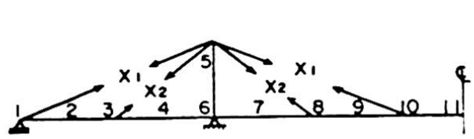

Fig. 7 Prestressing System

4. NUMERICAL MODELS AND RESULTS

Numerical calculation is carried out for nine cases of analyzing models. Structural models of 2-span and 3-span bridges are illustrated in Fig. 8, 9.

The differences of analyzing models are indicated belows.

2-span Bridge

Case	Tower Height	Web Thick	Steel Weight	Other
1	30 m	10 mm	3.3 t/m	
2	30 m	14 mm	3.3 t/m	
3	30 m	10 mm	3.3 t/m	Knie Type
4	30 m	10 mm	5.3 t/m	
5	35 m	10 mm	3.3 t/m	
6	40 m	10 mm	3.3 t/m	

3-span Bridge

Case	Tower Height	Web Thick	KE
7	30 m	10 mm	2.58
8	30 m	14 mm	2.64
9	30 m	14 mm	2.96

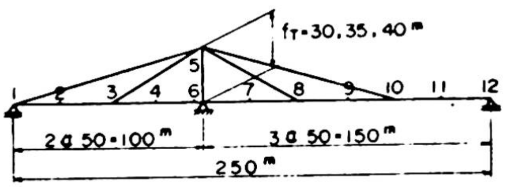

Fig. 8 2 Span Model

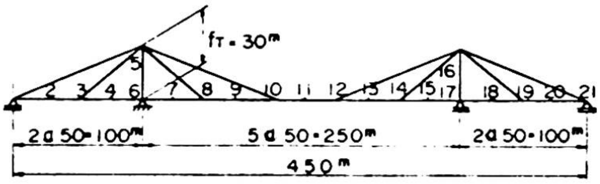

Fig. 9 3 Span Model

Table 1 Numerical Result of Parameters

	CASE	DESIGN STEP		KE PARAMETER	KOPT	PARAMETER
2 SPAN	1	A	FULLY STRESS	4.0 ~ 5.0		2.48
		B	BASIC STRUCTURE	0.92 ~ 7.56		
		C	P.S. ARRANGEMENT	2.47 ~ 2.89	2.55	
	2	B	BASIC STRUCTURE	0.96 ~ 7.88		
		C	P.S. ARRANGEMENT	2.54 ~ 2.96	2.49	
	3	B	BASIC STRUCTURE	0.92 ~ 7.73		
		C	P.S. ARRANGEMENT	2.43 ~ 2.90	2.43	
	4	B	BASIC STRUCTURE	0.92 ~ 7.03		
		C	P.S. ARRANGEMENT	2.53 ~ 2.89	2.55	
	5	B	BASIC STRUCTURE	0.92 ~ 7.05		
		C	P.S. ARRANGEMENT	2.41 ~ 2.89	2.45	
	6	B	BASIC STRUCTURE	0.92 ~ 7.50		
		C	P.S. ARRANGEMENT	2.41 ~ 2.89	2.41	
3 SPAN	7	A	FULLY STRESS	3.22 ~ 4.73		2.46
		B	BASIC STRUCTURE	0.92 ~ 5.67		
		C	P.S. ARRANGEMENT	2.14 ~ 2.58	2.58	
	8	B	BASIC STRUCTURE	0.96 ~ 5.81		
		C	P.S. ARRANGEMENT	2.25 ~ 2.65	2.27	
	9	B	BASIC STRUCTURE	0.96 ~ 5.81		
		C	P.S. ARRANGEMENT	2.53 ~ 2.96	2.53	
TOYOSATO BR.		SPAN 80.5 · 216 · 80.5 (PWS)				2.87
SUEHIRO BR.		SPAN 109 · 250 · 109 (PWS)				2.89
ONOMICHI BR.		SPAN 85 · 215 · 85 (LOCKED-COIL)				2.39

Numerical results of characteristic parameter and optimality parameter are listed in Table 1. The KE parameter of the cable-girder system determined by the fully stressed design is in the small range (3.∿5.). On the other hand the KE parameter determined by the approximate design process developed in this study is in the fairly large range (1.∿8.). Prestressing forces are introduced into the suitable basic structure which cable components are not fully stressed. From the results of case 7 ∿ case 9, it is obvious that KE value reduces about 15 percents by introducing the prestressing forces.

5. AN EXAMPLE OF THE DESIGN USING AN OPTIMALITY PARAMETER

The main difference of this method from the usual design method is the use of the parameter KOPT = 2.5 obtained by numerical calculation as shown in Fig. 10.

dead load .... 10.0 t/m
line load .... 50.0 t
uniform load .... 3.5 t
impact 0.2
optimality
  parameter...... KOPT=2.5
assuming rigidity
  girder .... IG=1.0 m^4
  cable .... AC=0.046

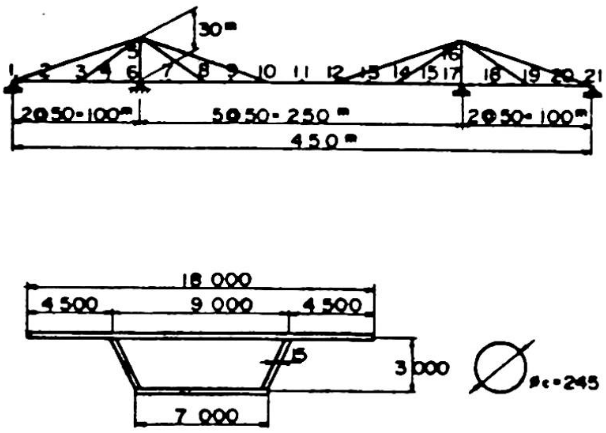

Fig. 10.
Design Conditions
and Basic Dimensions

The optimum bending moment arranged by prestressing forces and the moment inertia of girder members are also illustrated in Fig. 11.

By using the optimality parameter, structural designer can get the reasonable sections of girder and prestressing forces of cable-stayed bridges by one time trial. The assuming rigidity of analyzing system determined by using an optimality parameter is very close to the real rigidity of the final structure.

6. CONCLUSION

Unexperienced structural engineer may feels some difficulties to design the economical cable-stayed bridge. Because allowable stress guarantee the safety of structures, but it does not always guarantee the economical condition. The prices and strength of the cable and steel girder are extremely different. Furthermore the arrangement of the bending stress of the stiffening girder causes the more complicated problem. Therefore the economical criterion for cable-stayed bridges may be important as same as the factor of safety.

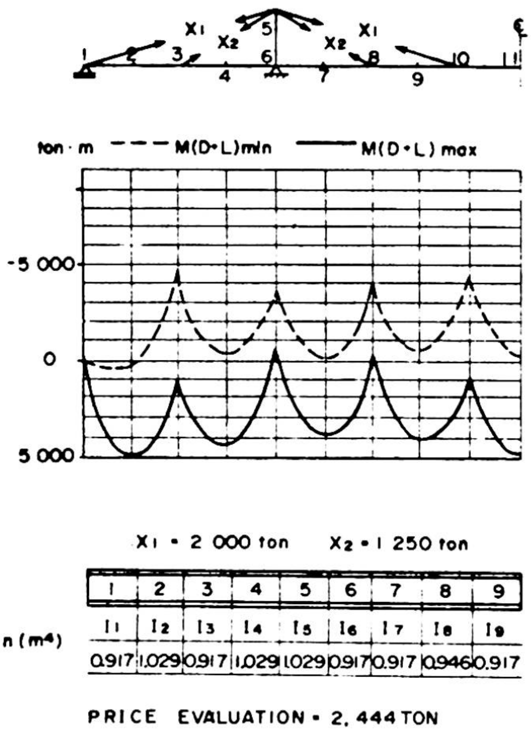

Fig. 11. Optimum Design

Finally the conclusions of the basic study for the optimality criterion method are outlined by the next statements.

(1) The optimality parameter value of the radial type of the cable-stayed bridge based on the price ratio (structural steel : cable = 1 : 2) exists in the range of 2.0 ∿ 3.0.

(2) The moment of inertia of stiffening girder can be considered as uniform along the girder axis by effectively prestressed.

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SUMMARY

A convenient optimization method using an optimality parameter has been discussed. The optimality parameter is based on the structural nature and the approximate design procedure. This parameter is used to determine the optimum rigidity ratio of the cable-girder system after introducing the prestressing forces. The optimum design using an optimality parameter will be easily accepted by structural engineers as the economical criterion.

RESUME

Une méthode pratique d'optimisation à l'aide d'un paramètre d'optimisation est présentée. Le paramètre d'optimisation est obtenu à partir du caractère structural et de la méthode approximative de calcul. Ce paramètre est employé pour déterminer la rigidité la plus favorable du câble et de la poutre après avoir introduit les forces de précontrainte. Le calcul à l'aide d'un paramètre d'optimisation sera accepté par les ingénieurs comme un critère économique.

ZUSAMMENFASSUNG

Eine anwendbare Optimierungsmethode mit einem Optimierungsparameter wird dargestellt. Der Optimierungsparameter wird aufgrund des Tragwerkssystems und der Näherungsberechnungsmethode bestimmt. Mit diesem Parameter werden die günstigsten Seil- und Trägersteifigkeiten unter Berücksichtigung der Vorspannkräfte bestimmt. Die Berechnung mit einem Optimierungsparameter wird von den Ingenieuren zur Steigerung der Wirtschaftlichkeit angenommen werden.

Preponderance of Idealization in Structural Optimization

Prépondérance de l'idéalisation dans les problèmes d'optimisation structurale

Die überragende Bedeutung der Idealisierungen bei der Optimierung von Tragwerken

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The optimal design of a structure may be divided in two steps. In the first one - the *idealization* - the structural problem is put in following mathematical formulation :

"Find $\vec{X}$ such that :

$$f_k(\vec{X}) < 0 \quad \text{for} \quad k = 1,2,\dots m ; \qquad (1)$$
$$h_j(\vec{X}) = 0 \quad \text{for} \quad j = 1,2,\dots l ;$$

and :

$$F(\vec{X}) = \text{minimum (maximum)} \text{"}$$

where $\vec{X}$ is a vector which contains the design variables,

f and h are the constraints of the problem,

and F is the objective function to optimize.

The second step - the *solution process* implies (a) the choice of the solving procedure and (b) the search of the solution of the problem formulated as in (1).

In the opinion of the authors, a good idealization is the basic condition for obtaining a good value of the solution, while a more or less refined mathematical treatment of it plays a rather secondary role |1| .

In many papers of the literature, emphasis is too often brought on the choice of the solution procedure rather than on that of a heuristic which does not modify in anyway the sense of the actual problem.

So long as the structural problem is small - about ten variables and constraints - many methods are available in the literature. However, various numerical experiments have shown that the choice of a method depends on the problem to be solved, for most of the algorithms cannot be used economically in all cases |2|. As a consequence, conclusions concerning the use range and the efficiency of an algorithm for a structural problem can rarely be extended to another one.

If emphasis is almost brought on the idealization, the designer may be sure of obtaining a realistic solution of the problem and, in addition, important simplifications in the mathematical treatment of the second step become possible. Indeed, on one way, a judicious choice of variables or an ingenious variable transformation often enable to present the complex problem in a more simple form, and, on another way, by means of a previous evaluation of the several variables, the designer can establish a hierarchy of the variables and divide the complex problem into smaller ones, which are then more easier to solve quickly.

For example, in |3|, MYLANDER demonstrates that a rather simple variable transformation changes a mathematical non-linear and non-convex problem into a linear programming system. It is worthwhile to recall the following basic non-linear problem which is considered as a very difficult one. The objective function is :

$$f(x) = b_o + a_{o1}\, x_1 + \left(\sum_{j=2}^{5} a_{oj}\, x_j\right) x_1 \rightarrow \min$$

subject to constraints :

$$0 \leqslant a_{i1}\, x_1 + \left(\sum_{j=2}^{5} a_{ij}\, x_j\right) x_1 \leqslant b_i \quad i = 1,2,3 \qquad (2)$$

$$x_1 \geqslant 0 \; ; \; 1.2 \leqslant x_2 \leqslant 2.4 \; ; \; 20.0 \leqslant x_3 \leqslant 60$$

$$9.0 \leqslant x_4 \leqslant 9.3 \; ; \; 6.5 \leqslant x_5 \leqslant 7.0.$$

where the values of the constants are :

a_{01}	= - 8,720,288.795	a_{21}	= - 155,011.1055
a_{02}	= - 150,512.524	a_{22}	= 4,360.5334
a_{03}	= - 156.695	a_{23}	= 12.9492
a_{04}	= - 476,470.319	a_{24}	= 10,236.8839
a_{05}	= - 729,482.825	a_{25}	= 13,176.7859
a_{11}	= - 145,421.4004	a_{31}	= - 326,669.5059
a_{12}	= 2,931.1506	a_{32}	= 7,390.6840
a_{13}	= - 40.4279	a_{33}	= - 27.8987
a_{14}	= 5,106.1920	a_{34}	= 16,643.0759
a_{15}	= 15,711.3600	a_{35}	= 30,988.1459
b_o	= - 24,345.0	b_2	= 294,000.0
b_1	= 294,000.0	b_3	= 277,200.0

By putting, according to MYLANDER

$$y_i = x_1 \,.\, x_i \qquad i = 2, 3, 4, 5$$

and (4)

$$y_1 = x_1$$

above non-linear problem takes following linear formulation :

$$g(y) = b_o + \sum_{j=1}^{5} a_{oj}\, y_j \rightarrow \min$$

$$0 \leqslant \sum_{j=1}^{5} a_{ij}\, y_j \leqslant b_i \quad i = 1, 2, 3 \qquad (5)$$

$$y_i \geqslant 0 \qquad i = 1, 2, \ldots, 5$$

$$y_2 - 1.2\, y_1 \geqslant 0 \;; \quad 2.4\, y_1 - y_2 \geqslant 0$$

$$y_3 - 20.0\, y_1 \geqslant 0 \;; \quad 60.0\, y_1 - y_3 \geqslant 0$$

$$y_4 - 9.0\, y_1 \geqslant 0 \;; \quad 9.3\, y_1 - y_4 \geqslant 0$$

$$y_5 - 6.5\, y_1 \geqslant 0 \;; \quad 7.0\, y_1 - y_5 \geqslant 0.$$

which may directly solved by means of the classical simplex routine.

The optimal solution, obtained after six iterations, is given by :

$$g = -\ 5{,}280{,}344.9$$

$$y_1 = 4.53743 \;; \quad y_2 = 10.88983 \;; \quad y_3 = 272.24584 \qquad (6)$$

$$y_4 = 42.19811 \;; \quad y_5 = 31.76202$$

which in terms of the original variables gives f = - 5,280,344.9

$$x_1 = 4.53743 \;; \quad x_2 = 2.40000 \;; \quad x_3 = 60.00000 \qquad (7)$$

$$x_4 = 9.30000 \;; \quad x_5 = 7.00000.$$

The solution of the original problem by means of non-linear programming methods |4, 5| lead, after a lot of iterations, to values of f which are 2 or 3 % below the true optimum but, in some cases, with value of the variable x_3 which is about 50 % erroneous.

In |6|, the authors show how a suitable choice of the behaviour model for a complex structural design - indeterminate prestressed bridges - leads to a benefit similar to that obtained by MYLANDER.

The idealization of the problem is based on an approach with sensitivity coefficients, as that proposed by GURUJEE |7|, and on a variable transformation; it is then allowed to solve this complex design problem by means of linear programming, without the actual problem be denaturated and taking account of all the technological requirements (cover thickness, anchorage dimensions, redundant effects of prestressing, friction losses, anchorage slippage,...). After the variable transformation, the problem remains partially non-linear but the authors have shown in |8| that the non-linear term, being of the order of 1 % with respect to its corresponding linear component, may be neglected in practice.

The authors would like to conclude by saying that for optimum design, as for all the other engineering activities, mathematics are a good servant but a bad master.

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SUMMARY

In structural optimization problems, it is nearly always observed that, in the search for a realistic solution, the suitability of idealization is more important than the choice of the solving algorithm.

RESUME

Dans les problèmes de dimensionnement optimal, il est généralement constaté que la recherche d'une solution réaliste dépend davantage de l'idéalisation du problème que du choix de l'algorithme de résolution.

ZUSAMMENFASSUNG

Bei der Optimierung von Tragwerken wird allgemein festgestellt, dass die Suche nach einer realistischen Lösung mehr von der Idealisierung des Problems als von der Auswahl des Lösungsalgorithmus abhängt.

Minimum Weight Plastic Design of Regular Rectangular Plane Frames

Calcul plastique pour un poids minimum de cadres plans rectangulaires

Plastische Bemessung auf Minimalgewicht für rechteckige, ebene Rahmen

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1. INTRODUCTION

Structural engineers have been concerned more with practical computational techniques for optimum structural designs than with theoretical results. On the other hand, scientific investigations on optimality criteria and optimal structures have been carried out mostly by researchers in the field of structural or applied mechanics. These two approaches are mutually compensating in order to develop more rational methods of structural designs.

The introductory report by A.B.Templeman has been primarily concerned with the hierarchy of optimum structural design problems and the corresponding computational techniques. Reference is made in his report to the linear theory of minimum weight plastic design and to the advantage of linear programming. It should also be recognized that the theoretical results on optimality criteria and optimal structures not only have the scientific significance but also lay the foundations and stimulate new ideas for developing practical computational techniques.

The purpose of this discussion is to call attention to the recent results [2-10] by the author and his colleagues on some general solutions derived analytically in closed forms to the problems of minimum weight plastic design of regular rectangular plane frames of practical interest and then to point out the theoretical and practical significances of those solutions.

2. FRAME MOMENT FOR REGULAR RECTANGULAR FRAME

Fig.1 shows a regular rectangular plane frame and one set of vertical and lateral design loads. The geometrical regularity in such a frame not only is reflected in design loads but also characterizes its structural behaviors and optimal plastic designs. In many practical design problems, the story shear resultant increases rapidly from the top floor toward lower stories as compared with the variation of vertical gravity loads. In those countries where frames must withstand against strong-motion earthquakes and strong gusts, *fairly large* lateral design loads are assigned. Under these circumstances, the first step of analytical treatment of a Foulkes-type problem is to assume an extremely deteriorated collapse mecha-

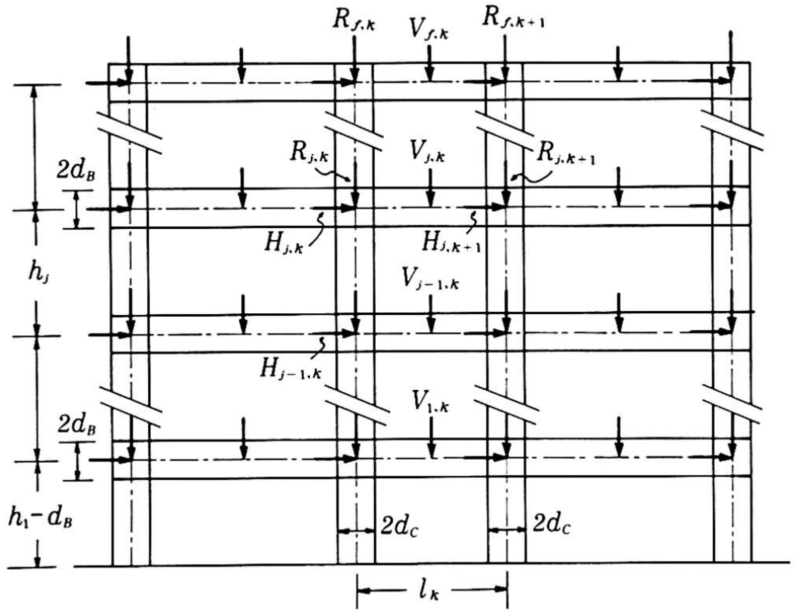


Fig.1
Regular
rectangular
frame

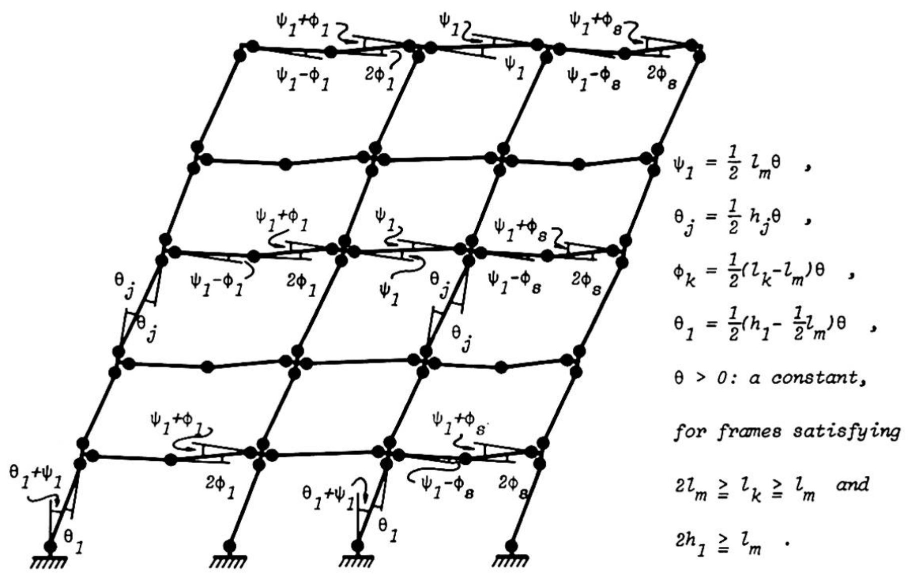


Fig.2
A Foulkes
mechanism
for
regular
rectangular
frame

nism shown in Fig.2, in which *simple* plastic hinges have formed at almost all the potentially critical sections except at the midspan sections of some particular bay(s) to be found as a part of the solution. Fig.3 shows that the corresponding bending moment diagram at plastic collapse may be conceived as the result of two-fold superpositions of decomposed diagrams. Each decomposed diagram is such that the moment equilibrium is maintained at the four corners with the same absolute value in the manner shown in Fig.3. This equal corner moment associated with this elementary moment diagram is called a "frame moment". A restricted minimization may then be carried out analytically in terms of the frame moments, and some statical conditions are derived under which the assumed bending moment diagram corresponds indeed to a general solution. It is then shown that the Foulkes mechanism condition can also be satisfied for a class of frames satisfying two simple geometrical conditions as shown in Fig.2.

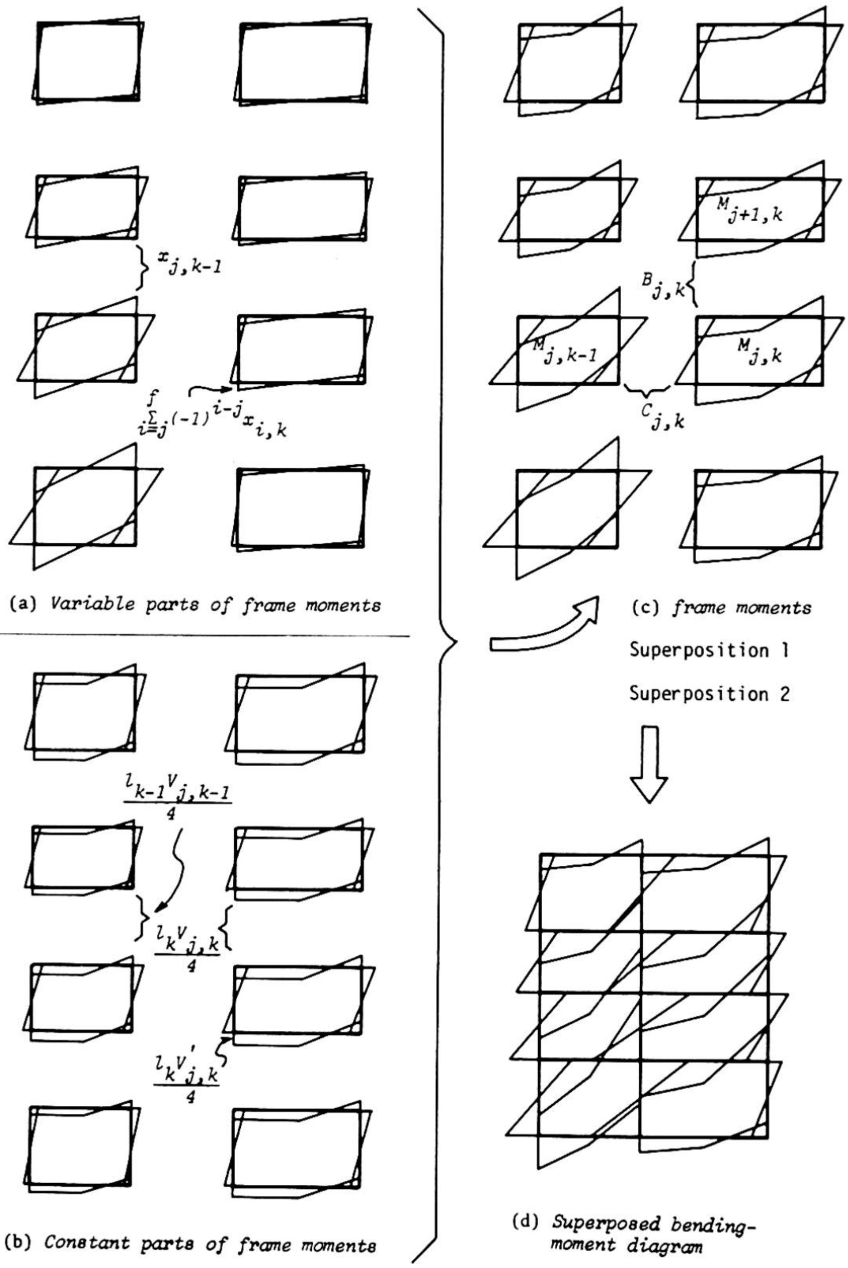


Fig.3 Frame moments and two-fold superposition procedure

3. APPLICATION OF FRAME MOMENT DECOMPOSITION TECHNIQUE

The two general solutions mutually exclusive and compensating on a design chart [2] not only clarify the general features of the classes of the minimum weight designs, but also provide a basis on which some modified general solutions can be derived to problems formulated more realistically by incorporating the axial force-bending moment interaction yield conditions for idealized beams and columns [3] shown in Fig.4. Fig.5 shows a part of modified Foulkes mechanism in a theory [4] in which only the idealized columns are required to satisfy the interaction yield condition shown in Fig.4. The regularity in the frame geometry enables one again to derive the general solutions and the statical and geometrical conditions analytically in closed forms [3, 4].

For the problem where reaction constraints have been incorporated within the framework of Foulkes' theory, a *bay shear distribution law* has been derived in [5] also on the basis of the concept of the frame moment and of the afore-mentioned two-fold superposition procedure.

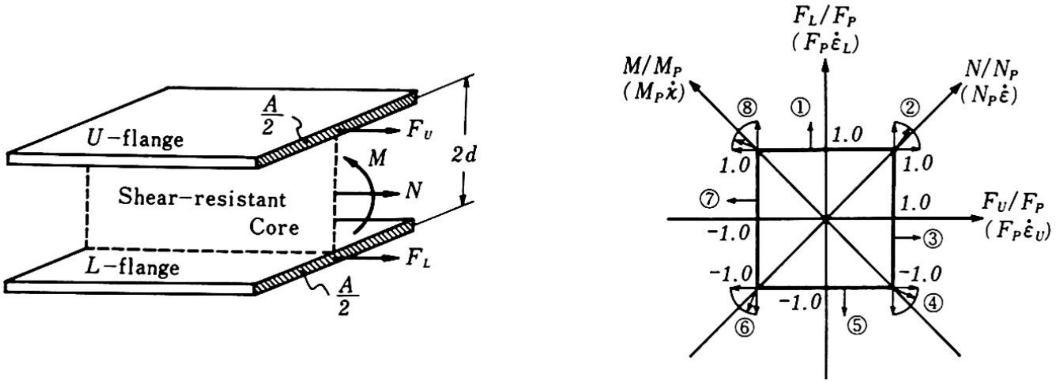


Fig.4 Axial Force-Bending Moment Interaction Yield Condition for an Idealized Sandwich Member

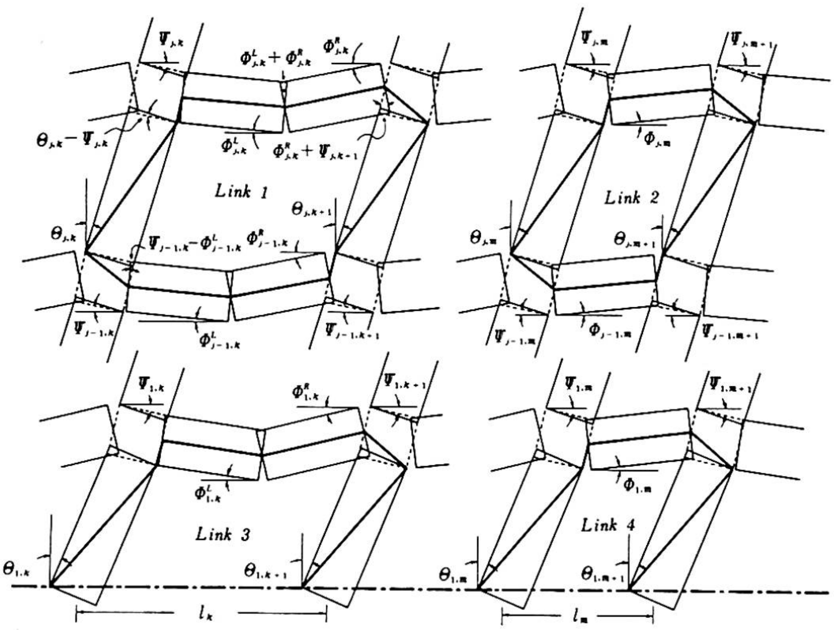


Fig.5
A modified
Foulkes
mechanism

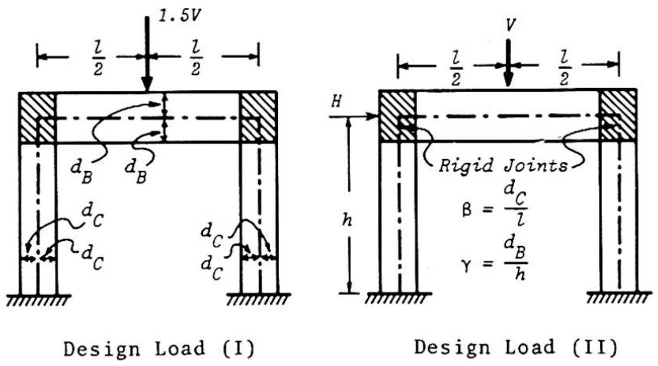
For the problem of multi-story multi-span frames to be designed for five sets of design loads, a kinematical restricted maximization procedure has been developed in [6] by combining the primal-dual method of LP with a semi-inverse approach similar to [2]. Some general solutions have thereby been derived analytically in closed forms. Fig.6 shows a portal frame obeying an idealized interaction yield condition and subjected to two sets of design loads. Fig.7 shows a fundamental design chart for this frame. This chart together with the theory in [7] constitutes the foundation for a possible analytical attempt of incorporating the result of [3] and [4] in [6].

4. SIGNIFICANCE OF THE CLOSED FORM GENERAL SOLUTIONS

The theoretical significance of these general solutions are now obvious. Each general solution provides a basis for developing practically useful general solutions to problems of more realistic formulations, though some modifications may become necessary for the topmost few stories. The afore-mentioned results may be said to provide ample grounds for the fruitfulness of this successive refining process.

For practical application, these solutions must first be modified for the effect of inelastic stability and member design requirements. The author and his colleagues have already clarified to a certain extent through numerical large-deflection analyses that minimum weight frames can indeed withstand against static al-

Fig.6
Portal frame
and
the two design
loads



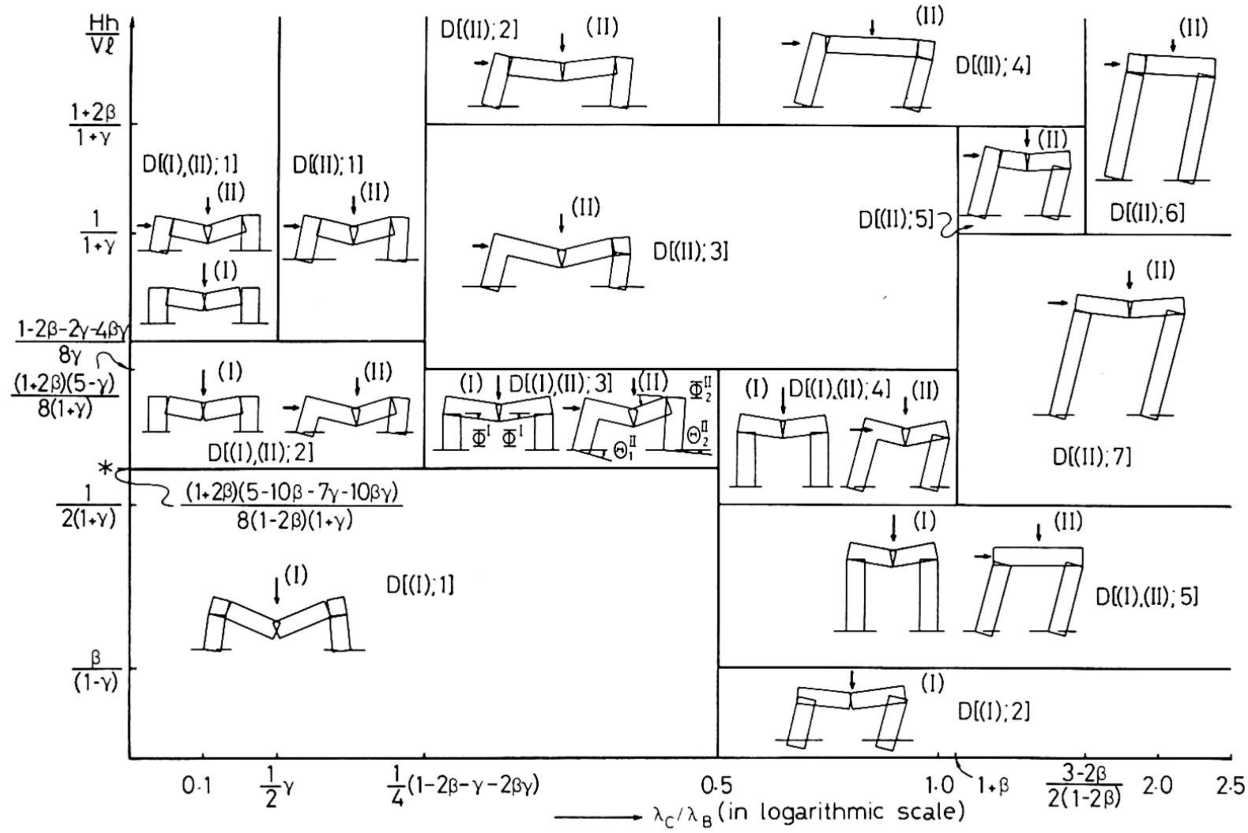


Fig.7 Design chart for a portal frame subjected to two sets of design loads

ternating lateral loads [8] and strong-motion earthquake disturbances [9] and are not particularly imperfection sensitive [10]. It should also be noted that the afore-mentioned solutions are the necessary consequences of the one-sided optimization using an approximate "failure" design criterion aside from the "serviceability" design criterion to be satisfied in practice.

Yet it can be said that the afore-mentioned solutions have the following significances: (i) they clarify the intrinsic features of the minimum weight plastic designs of regular rectangular frames at various levels considerably well; (ii) they will provide good initial solutions, if properly incorporated in a program, to start a numerical search for an optimal solution under additional constraints and may also be utilized as some standards for program verification. (iii) It may be well expected that the closed form solutions will be useful for seeking for optimum span length combinations analytically.

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SUMMARY

The frame moment decomposition technique due to the author and its applications to several more realistically formulated problems have been briefly described. The theoretical and practical significances of the analytical approach to the minimum weight plastic design problems have been explained in reference to the papers by the author.

RESUME

On décrit la méthode de décomposition des moments du cadre, proposée par l'auteur, et ses applications pratiques. La valeur théorique et pratique de cette méthode de calcul plastique, pour un poids minimum, est discutée.

ZUSAMMENFASSUNG

Die vom Autor entwickelte Methode der Momentenzerlegung sowie deren praktische Anwendungsmöglichkeiten werden beschrieben. Der theoretische und praktische Wert dieser Methode der plastischen Bemessung auf Minimalgewicht wird untersucht.

Structural Optimization via Penalty Methods: A New Type of Penalty Function

L'optimisation structurale par les méthodes de pénalisation: un nouveau type de fonction de pénalité

Optimierung von Tragwerken durch Strafmethoden: ein neuer Typ von Straffunktionen

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1. Introduction

Sound mathematical idealizations of practical design problems lead as a rule to highly nonlinear, and possibly nonconvex, programming problems.

The main effort in the field of computerized design methods should therefore be concentrated upon the implementation of versatile numerical procedures capable of solving, at least in principle, general mathematical programming problems. It is obvious that particular problems can be solved more cheaply by means of 'ad hoc' techniques exploiting their special properties, but it is the authors' opinion that the general approach should yield the major improvements to structural optimization, at the present stage of its development.

In this note, the attention is focussed on sequential unconstrained minimization techniques, which seem to be among the most interesting approaches for general automated design routines. A new kind of penalty function is introduced, and applied to a typical design problem, with the aim of assessing its capabilities.

2. Mathematical formulation

We consider the following type of problem

$$\begin{aligned} &\text{minimize} \quad f(x_i) \\ &\text{subject to} \quad g_j(x_i) \leq 0 \end{aligned} \qquad (i=1, \ldots, n;\ j=1, \ldots, m) \qquad (1)$$

From problem (1) the following parametric problem is derived

$$\text{minimize} \qquad f(x_i) + \sum_{j=1}^{m} < 1 + g_j(x_i) >^{\alpha} \tag{2}$$

where the symbol $<\cdot>$ has the meaning

$$<\cdot> = \max\,(0,\ \cdot)$$

and the parameter α ranges over the open interval $(1, +\infty)$.

Each inequality constraint $g \leq 0$ is accounted for by a penalty term

$$p(g) = <1+g>^{\alpha} \tag{3}$$

From fig. 1 it is apparent that function (3) is neither an interior nor an exterior penalty function.

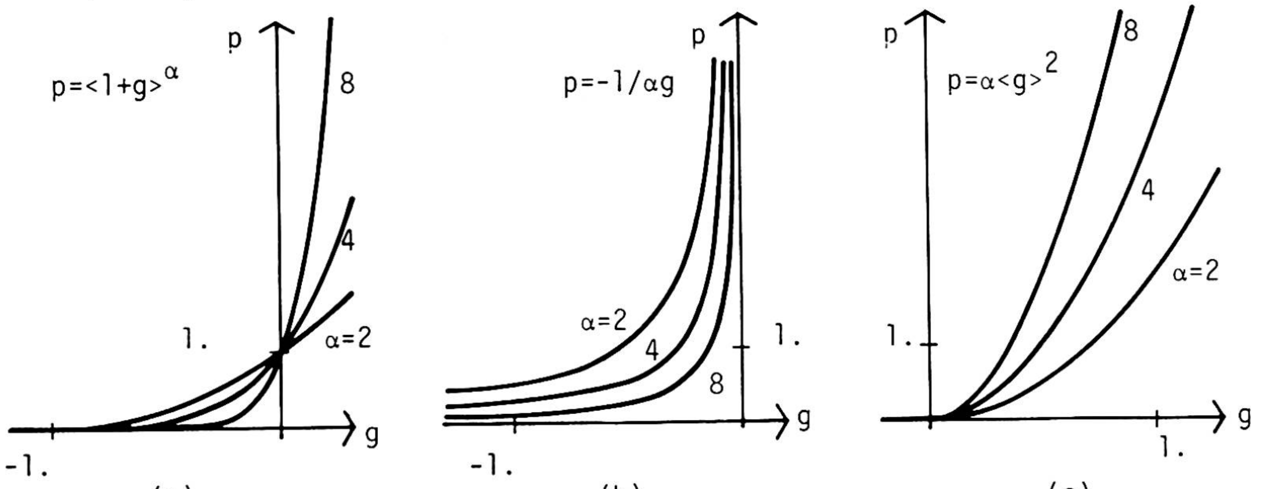

Fig. 1: Proposed penalty function (a) versus interior (b) and exterior (c) penalty functions.

The main properties of formulation (2) may be stated as follows:

i) if problem (1) has a (local) solution, a solution of problem (2) will approach it, when α approaches infinity;

ii) in contrast with interior penalty functions, penalty function (3) is defined over the range $-\infty < g < +\infty$;

iii) in contrast with exterior formulations, formulation (2) yields feasible minima for sufficiently large values of α, i.e. the solution of problem (1) is approached from the inside of its feasible region.

Properties ii) and iii) give an obvious advantage to penalty function (3) over interior and exterior penalty functions, respectively.

3. Allowable stress design of a truss with assigned topology

If a minimum weight design is sought, the objective function is easily ex pressed in terms of member cross-sectional areas and joint coordinates. For each member and each load condition, the following constraints are considered

$$g_+ = \sigma/\sigma^+ - 1 \leq 0$$
$$g_- = \sigma/\sigma^- - 1 \leq 0 \qquad (4)$$

where σ^+ (σ^-) is the allowable tension (compression) stress of the considered member. If member buckling is accounted for, the compression limit σ^- depends on the (minimum) radius of gyration of the member cross-section. For a given type of cross-section, the radius of gyration can usefully be expressed as a function of the area, thus leaving only one design variable for each member. A second set of constraints will impose a minimum admissible value to each area. Displacement constraints may be obviously included.

The major task is to compute the stress and its gradient (the displacement method of analysis is of course preferable). Special attention must be devoted to the fact that stress constraints (4) are not defined over the entire design space: in fact, there exist (unfeasible) designs for which in one or more members the stress grows to infinity. This difficulty can be cured by introducing suitable modifications of the stress constraints (4) outside the feasible region, and by adopting a careful minimization strategy.

4. Numerical results

An algorithm (AUDE) for the numerical solution of automated design problems, based on the described formulation, has been developed. The minimization (2) is performed, for a sequence of suitably increasing (integer) values of α, using the Davidon-Fletcher-Powell method. Two-point cubic fit for successive unidirectional searches is used. Size and geometry variables are treated simultaneously.

The results obtained for a sample design problem, relative to a steel planar truss, are represented in fig.2. The lower chord is assumed to be straight and made up of six bars, long 5 m each. The total span of the upper chord is 30 m also, but its shape is free. All members are tubular, and their thickness is supposed to be adequately represented by the relationship

$$t = 1.5 + 0.02\,D \qquad (t, D \text{ in mm})$$

D being the diameter. Load conditions are specified by a single 10,000 kg concentrated load, moving along the lower chord. The allowable tension stress is assigned a value of 2,400 Kg/cm^2, and the allowable compression stress is computed in terms of the member slenderness ratio according to the Italian Code requirements.

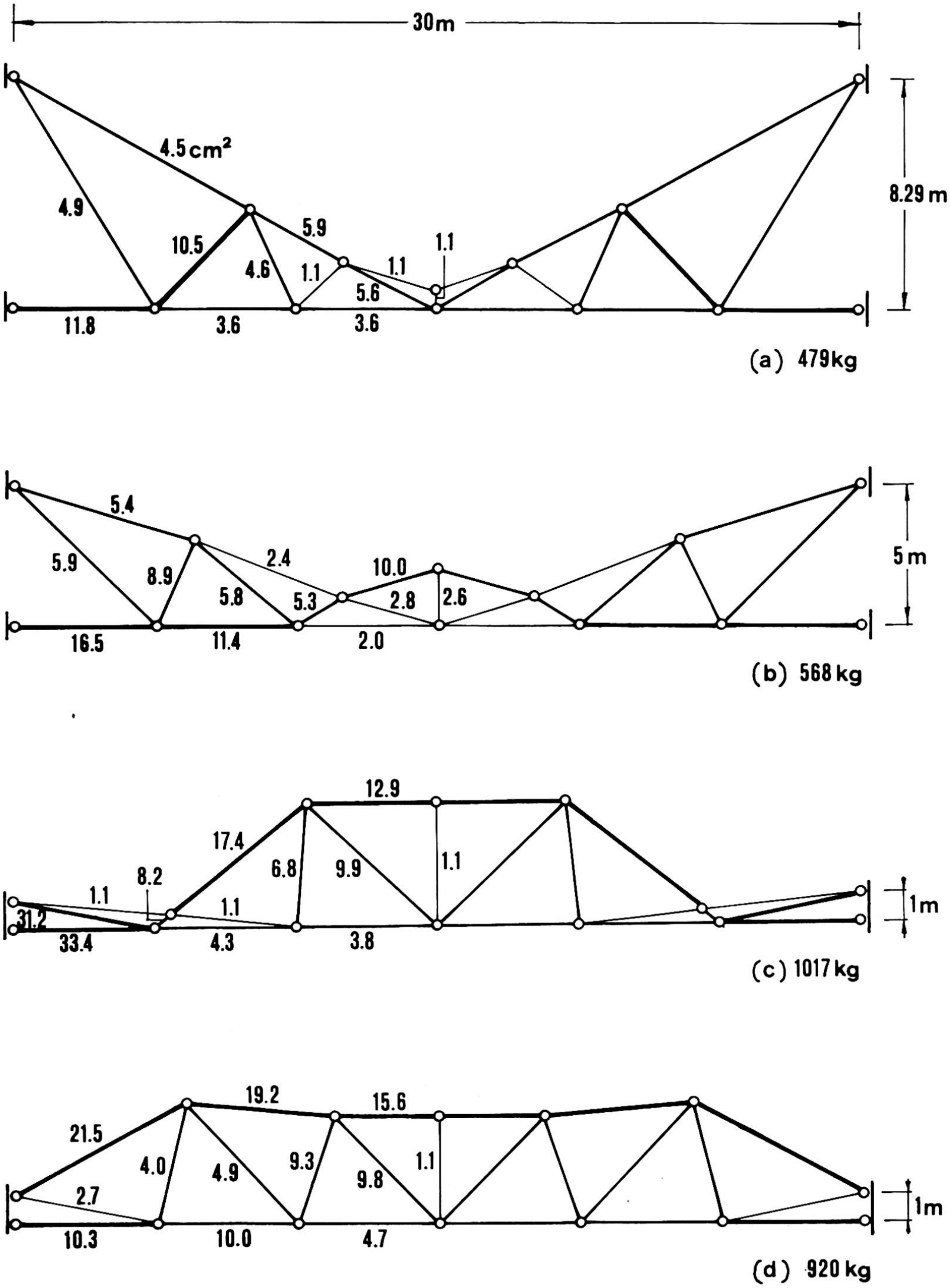

Fig. 2: Truss design

The lower chord should not undergo vertical displacements greater than 1/800 of the span. The truss should be designed for minimum weight.

Taking into account the obvious symmetry of the optimal solution(s), the above stated problem can be treated with 12 size variables, 6 geometry variables, and 3 load conditions. The optimal design obtained by AUDE is depicted in fig. 2a, where the member areas (in cm^2) are also reported. Note that 1.1 cm^2 was the minimum allowable area used in the computation. The weight of the optimum truss is 479 Kg, its height 8.29 m.

If now the distance H between the supports if given a fixed value, the geometry variables reduce to 5, and the optimum weight should obviously increase. Fig. 2b shows the solution obtained for H = 5 m. For H = 1 m two local optima have been detected (figs. 2c, d), the second one being a good candidate for the global solution.

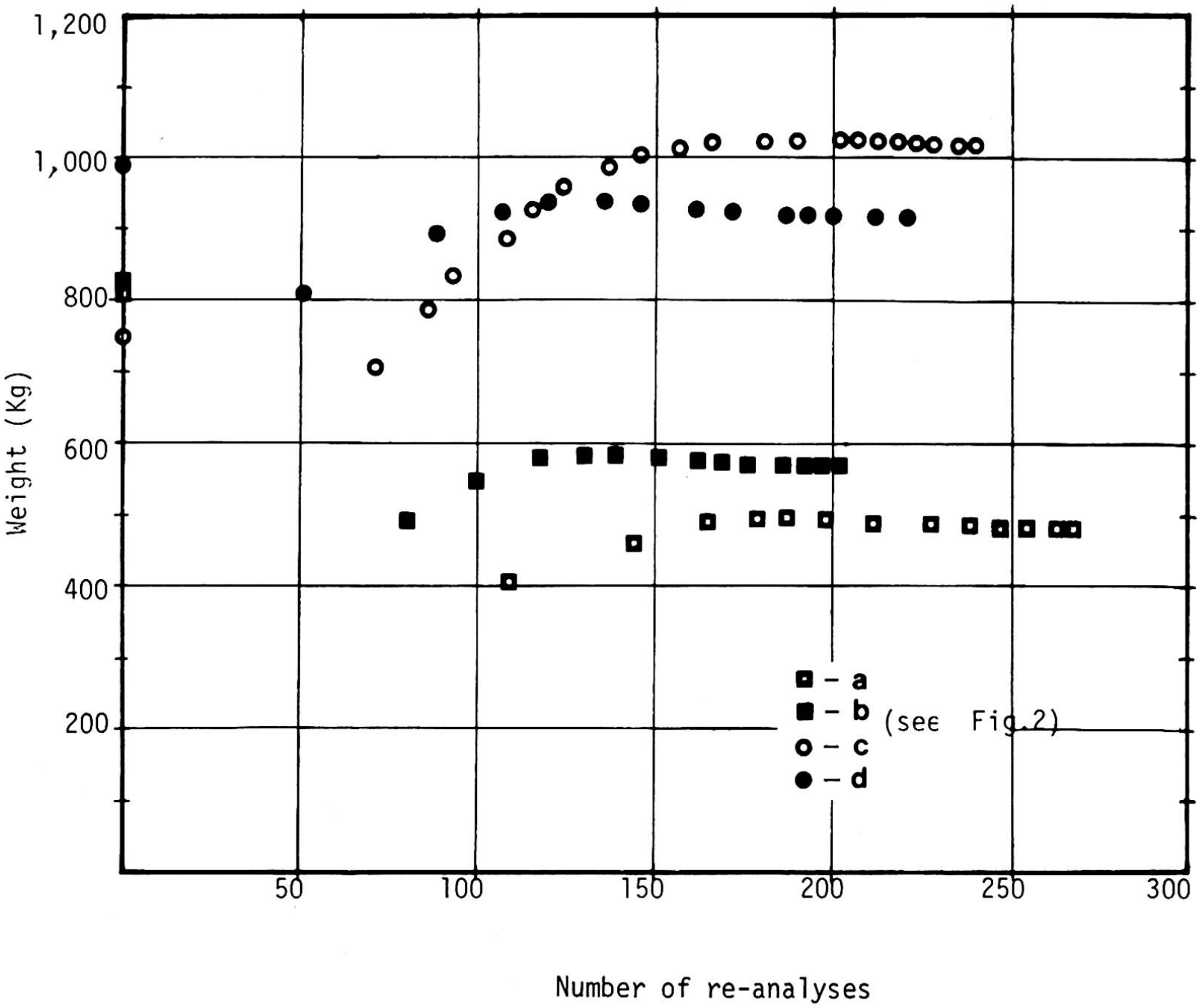

Fig. 3: Minimization trend

In each of these calculations, the parameter α was increased until a value of about 8000, and 200 ÷ 250 re-analyses were performed. Fig. 3 shows the sequences of minima relative to the four cases of fig. 2. As it is seen, after a drastic change on the first response surface, the objective function approaches rather smoothly its asymptotic value.

SUMMARY

An exponential penalty function is introduced and applied to a typical nonlinear and nonconvex design problem. Some results on geometry optimization of plane trusses are presented and discussed.

RESUME

On introduit une fonction de pénalisation exponentielle, et on l'applique à un problème typiquement non linéaire et non convexe d'optimisation structurale. On présente et on discute quelques résultats relatifs à l'optimisation géométrique de structures réticulées planes.

ZUSAMMENFASSUNG

Eine exponentielle Straffunktion wird auf ein typisch nichtlineares und nichtkonvexes Tragwerksproblem angewandt. Einige Ergebnisse über die Optimierung der Geometrie von ebenen Fachwerken werden angegeben und besprochen.

IIa

Über die Grundlagen und Methoden der Optimierung

On the Fundamentals and Methods of Optimization

Sur les principes et les méthodes d'optimisation

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1. Einleitung

Die analytischen Methoden des konstruktiven Ingenieurbaus sind in den letzten Jahren durch verstärkten Einsatz von Digitalrechnern mehr und mehr verfeinert worden. Das dabei erreichte hohe Niveau ist die Folge einer langen Tradition und einer daraus resultierenden Formalisierung der Berechnungsverfahren. Abgesehen von weiteren - sicherlich wichtigen - Verbesserungen (Berücksichtigung nichtlinearen Werkstoffverhaltens und großer Verformungen) sind jedoch keine grundlegend neuen Erkenntnisse mehr zu erwarten. Anders liegen die Verhältnisse bei der Tragwerkssynthese, bei der die Charakteristika des Bauwerkes als Unbekannte betrachtet und im Hinblick auf ein Bewertungskriterium unter Beachtung von technologisch - mechanischen Restriktionen festgelegt oder optimiert werden. Hier befindet man sich erst am Anfang einer Entwicklung, die sicherlich auch Rückwirkung auf die Tragwerksanalyse selbst haben wird.

Innerhalb der Synthese kommt den Optimierungsmethoden tragende Bedeutung zu, da erst mit ihrer Hilfe Syntheseprobleme zu lösen sind. Die Vielzahl der Optimierungen, die nur unzureichend die spezifischen Belange der Ingenieurpraxis berücksichtigen, gibt Veranlassung - in Ergänzung zum Aufsatz von TEMPLEMANN [E, S. 46 ff] einige kritische Anmerkungen zu machen.

2. Kritische Anmerkungen

Zwei Ursachen sind hauptsächlich dafür verantwortlich, daß die Optimierung zu zweifelhaften, da nur akademisch interessanten Lösungen führen kann:

- unrealistisch konzipierte Optimierungsmodelle,
- rein mathematisch orientierte Lösungsverfahren.

Im folgenden soll hierauf kurz eingegangen und einige Anregungen zur Überwindung einer Fehlentwicklung dargelegt werden.

2.1. Konzeption des Optimierungsmodells

Aus der Fülle akademischer Beispiele soll der beidseitig eingespannte Biegeträger minimalen Gewichtes als besonders typisches Beispiel dafür, daß die gefundene "Optimallösung" irrelevant ist, herausgegriffen werden (siehe Bild 1).

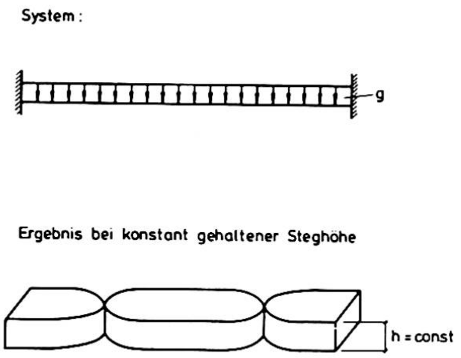

Bild 1: Optimierung des Gewichtes eines beidseitig eingespannten Trägers, ein "akademisches Beispiel" nach Hupfer (1)

Das gefundene Optimum ist die Folge eines schlecht konzipierten (ill - conditioned) Optimierungsmodells, das sich im Normalfall immer aus den drei Elementen

- Optimierungsvariablen
- Optimierungskriterium
- Optimierungsrestriktionen

zusammensetzt. Ist auch nur eine dieser Größen unzutreffend formuliert, zum Beispiel die Zielfunktion, verzerren sich die Optimierungsergebnisse. Viel weitreichendere Folgen stellen sich allerdings ein, wenn Restriktionen fehlen, fehlerbehaftet sind oder nicht in die Form gebracht werden können, wie es das später benutzte Lösungsverfahren verlangt, weil in diesem Fall der Lösungsraum ein völlig anderes Aussehen haben kann als in Wirklichkeit und somit andere Optima möglich sind. So hätte sich bei dem obengenannten Biegeträger mit Sicherheit eine andere Lösung ergeben, wenn technologische Restriktionen (Kontinuität der Kontur) berücksichtigt worden wären.

Will man praxisrelevante Optimierungsergebnisse sicherstellen, müssen alle drei Optimierungselemente ingenieurmäßig aufgebaut sein. Das ist aber im allgemeinen nur dann möglich, wenn man lediglich die Algorithmisierbarkeit der drei Elemente und nicht bestimmte mathematisch erzwungene Ausdrücke fordert.

2.2. Wahl des richtigen Lösungsverfahrens

Viele der vorgeschlagenen, heute gebräuchlichen Lösungsverfahren orientieren sich an der Denkart der Mathematiker, für jeweils eng abgegrenzte Problemklassen Lösungsmethoden zu erstellen, die nur unter bestimmten Voraussetzungen (hinsichtlich Konvexität, Stetigkeit, Differenzierbarkeit und bestimmter Formen der Nichtlinearität, etc.) anwendbar sind. Beispielsweise informiert das Bild 2 über die vielfältigen Varianten allein bei differenzierbarer Zielfunktion. Im Bereich der technischen Optimierung gibt es jedoch nur selten Fälle, die derartig mathematisch klassifizierbar sind. Infolgedessen braucht der Ingenieur eine Art "Generallöser", der universell anwendbar, flexibel und effizient ist. Die Effizienz - ein bislang nicht eindeutig definierter Begriff - soll dabei u.a. danach beurteilt werden,

- wie groß der Arbeitsaufwand zur Anpassung des Lösungsverfahrens an ein beliebig nichtlineares Optimierungsproblem ist,
- wie groß der Arbeitsaufwand zur Anpassung der Zielfunktion und Restriktionen an das Lösungsverfahren ist,
- wie groß der Rechenzeitbedarf (CPU-time) bei bestimmten Testfunktionen ist,
- wie groß der Kernspeicherbedarf ist,
- wie das Konvergenzverhalten bei pathologischen Fällen ist.

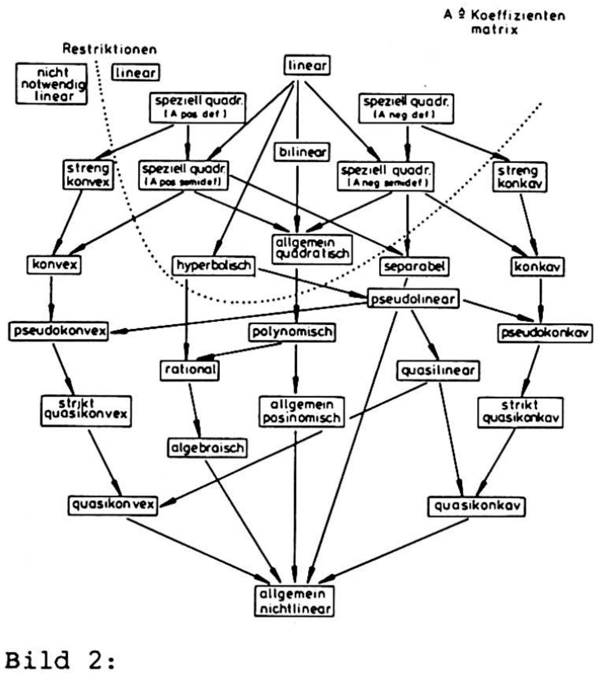

Bild 2:
Klassifikation nichtlinearer differenzierbarer Optimierungsaufgaben
nach Collatz . Wetterling (2)

Da insbesondere der Arbeitsaufwand zur Anpassung an Gegebenheiten erhebliche Kosten verursacht, muß der universellen Anwendbarkeit eines Lösungsverfahrens - unabhängig von der Klasse des Optimierungsproblems - größter Stellenwert eingeräumt werden. Die meisten der derzeitigen Lösungsverfahren genügen einer solchen Flexibilitätsforderung nicht. Die sogenannte Evolutionsstrategie dagegen [3], [4], die vom Verfasser mit Erfolg bei der Optimierung von Schalentragwerken eingesetzt wurde [4], [5], und im folgenden kurz vorgestellt werden soll, genügt dieser Forderung und kommt - bei entsprechender Weiterentwicklung - dem lang gesuchten "Generallöser" einen Schritt näher.

3. Evolutionsstrategie

Die Evolutionsstrategie ist ein sequentiell arbeitendes, iteratives stochastisches Suchverfahren mit Lernfähigkeit, bei dem die Suchschrittweite den Verhältnissen des jeweiligen Suchraumes angepaßt und selbst optimiert wird. Da die Suchschrittweite eine Zufallsvariable ist, kann man die Strategie als Monte-Carlo-Simulation höherer Stufe bezeichnen, deren Effizienz im Vergleich zu anderen Verfahren sich besonders bei vielen (ab 10 Variablen) bemerkbar macht. Sie darf aber auf keinen Fall mit der eigentlichen Monte-Carlo-Simulation verwechselt werden, weil ihr methodisches Vorgehen erheblich vom bekannten Monte-Carlo-Verfahren abweicht.

Das "Geheimnis" des Erfolgs und der Ausbaufähigkeit dieser Strategie ist darin begründet, daß die innere Logik des Verfahrens Optimierungsmechanismen der biologischen Vererbung, deren optimierender Effekt die Biologie tausendfach beweist, simuliert.

Durch eine einfache Konvergenzregel wird erreicht, daß optimale Fortschrittsgeschwindigkeit erzielt wird. Diese ist dann gegeben, wenn im Durchschnitt nach jeweils 5 zufälligen Suchvorgängen 1 Erfolg (Qualitätsverbesserung) erreicht wird. Andernfalls ist die Suchschrittweite (besser die Streuung der Suchschrittweite) zu vergrößern oder zu verkleinern. (Bild 3 zeigt die prinzipielle Arbeitsweise der Strategie an einem Beispiel).

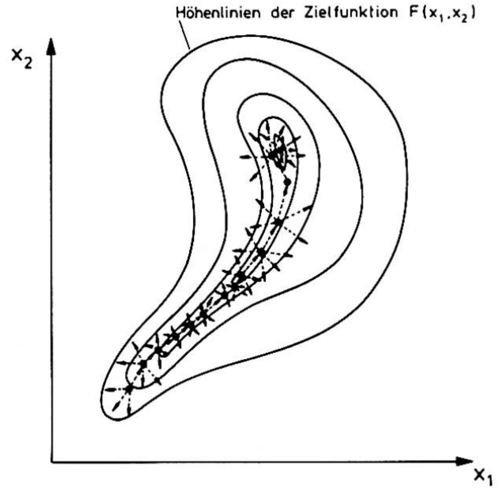

Bild 3:
Arbeitsweise der Evolutionsstrategie an einem zweidimensionalem Beispiel

4. Abschließende Bemerkung

Mit der Evolutionsstrategie besitzt der Ingenieur ein geeignetes Werkzeug, um Optimierungsprobleme im Konstruktiven Ingenieurbau zu lösen. Da die Routine in programmierter Form vorliegt und flexibel anwendbar ist, braucht sich der Benutzer nur noch um das problemabhängige Optimierungsmodell zu kümmern, eine Aufgabe, die jeder Ingenieur ohne große Mühe bewältigen kann.

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ZUSAMMENFASSUNG

Es werden kritische Anmerkungen zum Aufbau eines auf die Praxis ausgerichteten Optimierungsmodells gemacht und die universell anwendbare Evolutionsstrategie wird als Lösungsverfahren technischer Optimierungsprobleme vorgestellt.

SUMMARY

Some critical remarks are made for the establishing of practical optimization models. Furthermore, a generally applicable solution method, the "evolution strategy" is proposed.

RESUME

Des considérations critiques sont faites pour l'établissement de modèles d'optimisation répondant aux besoins de la pratique. La méthode de la "stratégie évolutive" peut être utilisée de façon universelle pour résoudre des problèmes techniques.

Cora's Lesson

La leçon de Cora

Die Lehre von Cora

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1. INTRODUCTION.

Since 25 years we have been acquainted with the development of optimization-techniques for the design of structures. The first programs were mainly based on the "mechanism" approach of limit analysis.
After a general matrix theory for structures became available about 1966, design-programs based on the equilibrium method were developed [1].
Formulating a minimum-weight design, based on the equilibrium method, with only stress-limitations as constraint conditions, leads to a problem of linear programming (L.P.).
In practice, however, a design with only stress-limitations is not acceptable. The building codes require a number of additional constraint conditions, for instance with relation to stability and rotation capacity (fig. 3). Often these special conditions are strongly non-linear and they can be linearized only in the neighbourhood of a solution, so that one has to pass on to a non-linear programming technique.

2. CORA.

Recently a design program has been developed in the Netherlands (named CORA) for the design of braced steel frames. The designs meet the requirements by the national authority. Moreover, it is possible to distinguish between welded joint connections with or without stiffeners. Given an accepted input of geometry and loading (fig. 1), the program automatically produces the design of the frames using a sequential linear programming formulation (fig. 2). The introduction of the special constraint conditions means that a very large linear programming problem has to be solved a number of times. The solution has been found in applying a sophisticated L.P. algorithm and in the approximation of the constraint conditions by one plane with the aid of the method of least squares. Instead of five sets of constraint conditions (fig. 4) only one set has to be taken into consideration (fig. 5). It proved that 3 or 4 iterations were sufficient to obtain the theoretically exact values of the solution.

Problems arose from the wish to develop an instrument which will really be used in practice. This means that it should not be too expensive in use and that it should fit realistic structures. The more difficult problems however were formed by the codes themselves. In drawing up these codes the committee has had in mind of course a more or less sensible structural engineer and a proper structure. But in applying these rules and codes in an automated design program irrevocably gaps and inconsistencies prove to be present in the codes. The computer is not a sensible structural engineer and he stumbles in the pitfalls caused by these gaps. Especially a mathematical optimization technique is a master in finding the inadmissible minima, as we noticed to our regrets several times.

3. THE LESSON.

The experience gained with this program has led us to the insight that if, in developing a design-process, one has to make allowance for requirements made by the government or a local authority, the design-program has to be separated from these prescriptions or codes. This applies to computer-aided design and the more so to computer-automated design. The reasons for this opinion - which we believe should be generally accepted - are:

a. The codes and prescriptions contain gaps and inconsistencies, which will always be recognized by the optimization-technique and unfortunately exploited.

b. The programmer who builds the design-program is not allowed to improve these inconsistencies.

c. By integrating the code into the design-program the program becomes dependent on this code. Codes have a temporary character. Adapting a design-program to code-changes will in general be very expensive or even impossible.

d. Working up codes into a form which is understandable by the computer is a lot of work which can best be left to those who draw up the codes instead of to every individual programmer.

4. CONCLUSION.

If - at least for the Building Industry - we want to leave behind us the more or less trivial examples, to proceed with our design-techniques to real life structures, we have to create the possibility to develop C.A.D.-programs that are independent from the codes. Therefore the codes have to be brought into a computer-readible form for instance in the way - indicated by Fenves [3] - by means of Decision Logic Tables. If this effort has led to success, the codes can be changed without consequences for the design-program (of course it will have consequences for the design itself).
Because the programs will be less vulnerable, software development for C.A.D. will become more popular. Two consequences seem to be of particular importance:

a. To develop C.A.D.-programs that can make codes of different countries accessible. This will make the software less dependent and less "national".

b. To study beforehand the effect of proposed code-changes, e.g. on economics or safety.

5. ACKNOWLEDGEMENT.

The CORA-program has been developed in the T.N.O. Institute for Building Materials and Structures, at the request of the Steel Structure Society and with financial aid of the C.I.A.D. (Computer Society of civil engineers). The first author was the chairman of the steering committee, the second author was responsible for the technical contents and the production. Most of the work was carried out by Ir. A.K. de Groot, Ing. G. Kusters and Ir. B. Speelpenning.

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SUMMARY

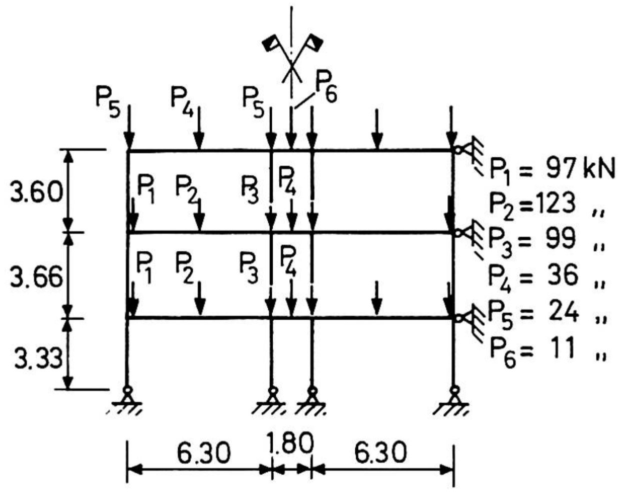
During the last three years a computerprogram (CORA) has been developed in the Netherlands for the design of braced steelframes. The frames designed with this program meet the requirements, made by the national authority. The building code requirements have been integrated into the design program, which has led to a very complex problem. The experience gained by solving this problem, has led to the insight, among other things, that design programs should be separated from rules and requirements in the building codes.

RESUME

Pendant les derniers trois ans un programme d'ordinateur (CORA) a été développé aux Pays-Bas pour le dimensionnement des ossatures en acier. Les ossatures dimensionnées avec ce programme satisfont aux conditions posées par les règles nationales pour les structures en acier. Les règles ont été insérées dans le programme de dimensionnement ce qui a conduit à un problème très compliqué. L'expérience a conduit à la conclusion que les programmes de dimensionnement doivent être séparées des conditions dans les règles ou codes.

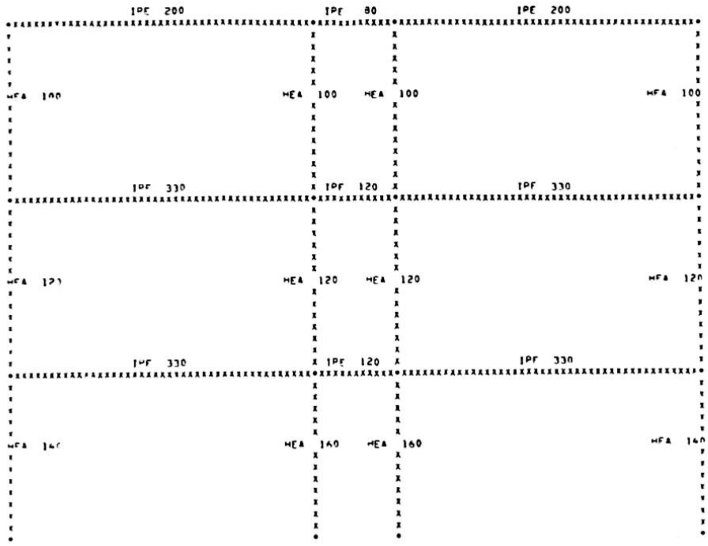
ZUSAMMENFASSUNG

Während der letzten drei Jahre ist in den Niederlanden ein Computerprogramm (CORA) für das Entwerfen von Rahmentragwerke aus Stahl entwickelt worden. Die Tragwerke die mit diesem Programm entworfen sind, erfüllen die Forderungen der nationalen Behörden. Die bautechnischen Anordnungen sind in das Entwurfsprogramm aufgenommen worden, was zu einem sehr komplizierten Problem geführt hat. Die Erfahrungen haben zu der Ansicht geführt, dass Entwurfsprogramme unabhängig und separat von Regeln und Forderungen in bautechnischen Anordnungen sein sollen.



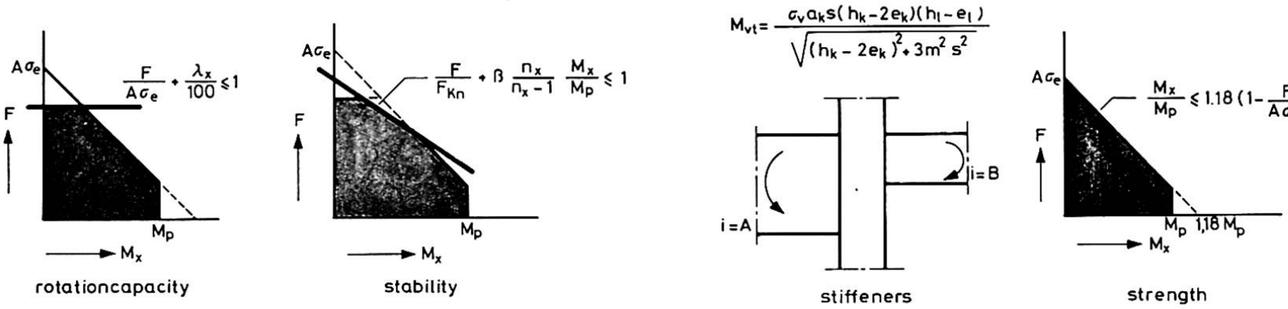
geometry and loading

fig.1



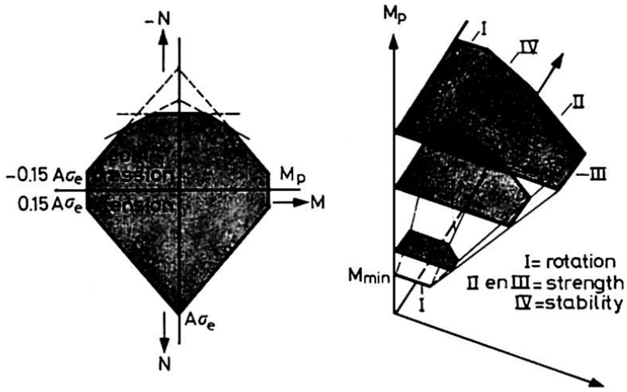
lineprinter plot

fig.2



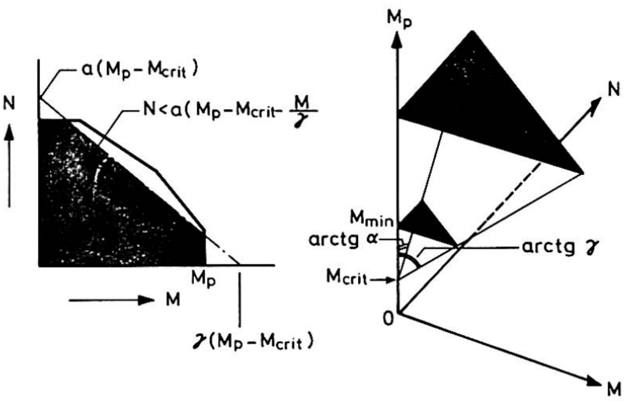
constraints

fig. 3



constraint surface

fig.4



approximate constraint surface

fig.5