Resistance of Steel Structures according to the Limit States Method

Résistance des structures en acier selon la méthode des états limites

Die rechnerische Tragfähigkeit von Stahlkonstruktionen nach der Methode der Grenzzustände

AUGUSTIN MRAZIK
Leading scientific worker
Institute of Construction and Architecture, Slovak Academy of Sciences
Bratislava, Czechoslovakia

1. Introduction

In Czechoslovakia, the method of limit states is used for the calculation of steel structures. The calculation basis corresponds with the calculation according to the International Standard of member states of the Council of Mutual Economic Aid. It is, however, adapted to Czechoslovak material and loading conditions.

The Czechoslovak standards defining the limit states of steel structures are as follows: ČSN 73 0031 Basic Regulations for the Calculation of Structures, ČSN 73 0035 Loading of Steel Structures, ČSN 73 1401 Design of Steel Struc ures. All these standards are being revised at present.

According to the standards the limit state of a structure is such a state, in which the structure loses its load-carrying capacity or it does not satisfy the demands its current use. Two groups of limit states are differentiated:

a/ limit states of the load-carrying capacity - the states in which the structure loses the load-carrying capacity, or in which further use of a structure is excluded,

b/ limit states of serviceability - the states in which the structure is no more appropriate for the common use, or in which its durability decreases.

The limit states are based on the probabilistic method of calculation. The method of mathematical statistics are used only for the determination of the initial quantities - the values of load and resistance of a material, or of a structure. As each quantity is analysed individually, the limit states method is a semi-probabilistic one.

1.1 Material resistance

The specified and factored resistance is distinguished. The specified resistance R_s is determined with respect to the method of checking the material properties and their variation. It ought to correspond to at least the probability value of 0,05. The possible unfavourable deviations of the structure resistance from the specified values are expressed by the factored resistance R. Its definition will be quoted later. The ratio of the specified resistance and the factored resistance is material factor K.

1.2 Loading of structures

Similarly, the specified, factored load and the load factor are considered for loading. Besides,a load combination factor also is applied.

These problems are not dealt with here and thus no standard definitions are stated.

2. Statistic nature of limit states

If the i-th load effect is denoted by S_i and the structure resistance by R^x, the safety margin G is defined as

$$G = R^x - \sum S_i \ . \qquad /1/$$

The limit state occurs when the safety margin is exceeded, hence if G = 0.

The load effects and structure resistance are random variable, mostly mutually independent quantities, and thus the safety margin has to be studied statistically and the limit state has to be determined for some small probability of occurrence.

These problems were studied by A.R. RZHANICYN [12], [13] and some other authors later on - i.g. [1], [2], [4], [11].

Let us pay attention to the main theoretical results and show the transition from them to the present limit states method.

The statistical characteristics [14] of difference /1/ of independent randomly variable quantity are:
mean value

$$m_G = m_R - m_S, \qquad /2/$$

standard deviation

$$s_G = \sqrt{s_R^2 + s_S^2}\ , \qquad /3/$$

skewness

$$a_G = \frac{s_R^3\, a_R - s_S^3\, a_S}{\left(\sqrt{s_R^2 + s_S^2}\right)^3} \ . \qquad /4/$$

Then, the safety margin G for a certain probability p is

$$G_p = m_G - \beta_{pa_G}\, s_G = m_R - m_S - \beta_{pa_G} \sqrt{s_R^2 + s_S^2} \geqq 0. \qquad /5/$$

Using LIND's [4] separation linearisation function α_{RS} this inequality can be written in the form

$$m_R\,(1 - \alpha_{RS}\, \beta_{pa_G}\, v_R) \geqq m_S\,(1 + \alpha_{RS}\, \beta_{pa_G}\, v_S)\,, \qquad /6/$$

where $v_R = s_R/m_R$, $v_S = s_S/m_S$.

Condition /6/ formally separates the structure resistance from the load effects. In practical cases the distribution functions of quantities R and $\sum S$ are often considered sufficiently accurately according to the normal distribution ($a_R = a_S = 0$). Besides if the separation linearisation function α_{RS} is considered as a constant value according to LIND, then the separation is complete.

The Czechoslovak limit states method considers $\alpha_{RS} \approx 1{,}0$ and studies separately the structure resistance and the load effects.

The structure resistance is expressed in terms of the material characteristics with the dimension of stress and is referred as factored resistance R.

The mathematically statistical representation of resistance R by help of the left side of the inequality /6/, is

$$R = m_R (1 - \beta_{pa_R} v_R), \qquad /7/$$

where only skewness a_R is considered.

3. Factored resistance of steel structures

The resistance of steel structures is derived from the yield point stress of steel σ_y. The specified resistance R_s is a statistical value of the yield point stress for probability $p = 0,05$.

When considering the material, the structure safety is affected not only by yield point stress, but also by random dimensions of cross-sections, by their deformation due to manufacture, by technological effects, etc. The steel behaviour in a structure is not the same as the behaviour during a tension test. The yield point stress in a cross-section is not the same as in the place from which the test specimen is taken. The test values are affected by the size and the shape of test speciment and by the testing method.

All these effects are randomly variable quantities. Only some of them can be investigated separately /e.g. size of cross-section/. The other influences are referred to as unknown imperfections. Thus all the effects are generally expressed by the conventional randomly variable cross-section area and a uniform statistical model of a tension bar and the probability of $p = 0,001$ is taken into consideration.

Let the real yield point stress be $[\sigma_y]_i$, the imperfections represented by the area $[A]_j$ and the theoretical area A, then the condition of tension is $\bar{R}A = [\sigma_y]_i [A]_j$, or

$$\bar{R} = [\sigma_y]_i \frac{[A]_j}{A} = [\sigma_y]_i [f]_j . \qquad /8/$$

It is the product of two independent randomly variable quantities.

The definition of the factored resistance is:
The factored resistance R is the boundary value $\bar{R}_{0,001}$ of the product /8/ for probability $p = 0,001$.

Using the PEARSON's density curve of type III expressed by functional relations according to VORLÍČEK [14] we can write

$$R = \bar{R}_{0,001} = m_R - \beta_{0,001\,a_R} s_R = m_y m_f (1 - \beta_{0,001\,a_R} \sqrt{v_y^2 + v_f^2 + v_y^2 v_f^2}), \qquad /9/$$

where m_R, s_R are the statistic characteristics of product /8/,
m_y, s_y - the statistic characteristics of yield point stress σ_y,
m_f, s_f - the statistic characteristics of ratio f,
$\beta_{0,001\,a_R}$ - the quantile of probability $p = 0,001$ depending upon the skewness of distribution of density

$$a_R = \frac{v_y^3 a_y + v_f^3 a_f + 6 v_y^2 v_f^2}{(v_y^2 + v_f^2 + v_y^2 . v_f^2)^{3/2}} .$$

The Author determined the numerical values of the factored resistance R according to the test results of steels in steel works in the years 1956-1959, 1966-1969 and 1971-1975. The values from the first period served for the preparation of the Czechoslovak Standard ČSN 73 1401, the values from following periods for the verification and refinement of factored resistances in this standard and for its revision.

The statistical analysis was carried out not only for the yield point stress σ_y , but also for the strength $\sigma_{T.S.}$, the ductility δ , the brittleness /results of Charpy tests/ and the ratio of areas of cross-sections. The results of the years 1956-69 are published in [5] to [10]. The recent results are prepared for publication. Some of these results are given here.

In Tab. 1 the statistical characteristics for common steels are given. Tab. 2 gives the boundary values of the yield point stress and of strength and factored resistances. The statistic values of the factored resistance R_{stat}, of the boundary yield point stress $\sigma_{y\ 0,05}$ and the strength $\sigma_{T.S.\ 0,05}$ allow a comparison with the standard values.

4. Conclusion

The paper shows that the limit states method allows the analysis of structure resistance and load effects. Even though it does not fully use the safety margins of structures, it characterizes the behaviour of structure more accurately than the method of allowable stresses based on the total safety factor.

5. References

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Table 1: Mean values m, standard deviations s, skewnesses a of yield point stress σ_y and tensile strength $\sigma_{T.S.}$

Steel Designation	Product, thickness	mm	Standard $\sigma_{T.S}/\sigma_y$ N/mm2	Yield point stress: Number of results	m_y N/mm2	s_y N/mm2	a_y	Tensile strength: Number of results	$m_{T.S.}$ N/mm2	$s_{T.S.}$ N/mm2	$a_{T.S.}$
11 373.0	Plates	>3-13	370/240	24 915	272,6	27,0	0,897	25 889	412,2	32,6	0,313
11 373.1		14-25	370/240	4 105	268,0	23,3	0,901	4 098	409,1	30,8	0,447
		26-60	370/230	1 242	266,0	26,0	0,366	1 216	414,2	31,9	0,713
11 373.0	Angles	≤ 13	370/240	8 983	273,2	21,5	0,488	8 504	406,2	29,6	0,338
	Beams	≤ 13	370/240	6 521	283,8	24,3	0,535	6 617	418,2	30,4	0,320
11 483.1	Plates	26-50	480/360	1 526	391,1	23,6	0,241	1 522	555,3	28,4	-0,103
11 523.1	Plates	>3-16	520/360	8 115	386,7	29,8	-0,209	8 142	566,0	34,1	-0,145
		17-25	520/350	2 948	387,4	26,0	-0,165	3 654	571,2	34,1	-0,067
11 523.1	Flat	≤ 16	520/360	1 976	384,9	29,1	0,312	1 994	567,2	35,1	0,212
	steel	17-25	520/350	1 804	389,4	25,9	0,656	1 997	579,5	32,9	0,534
11 523.0	Angles	≤ 13	520/360	236	379,7	28,1	0,198	243	555,0	43,8	0,561
	Beams	≤ 13	520/360	160	394,2	27,1	0,149	147	563,9	35,1	0,151

Table 2: Boundary values of yield point stress σ_y and tensile strength $\sigma_{T.S}$ for probabilities 0,001, 0,05 and 0,999
Factored resistance R according to statistic evaluation and to Czechoslovak Standard ČSN 73 1401

Steel Designation	Product, thickness	mm	Yield points stress N/mm2 σ_y 0,001	σ_y 0,05	σ_y 0,999	Tensile strength N/mm2 $\sigma_{T.S.}$ 0,001	$\sigma_{T.S.}$ 0,05	$\sigma_{T.S.}$ 0,999	Factored resistance N/mm2 R_{stat}	R_{CSN}	$\frac{R_{stat}}{R_{CSN}}$	$\frac{\sigma_y\ 0,05}{R_{stat}}$
11 373.0	Plates	> 3-13	221,2	236,2	391,1	325,8	361,7	527,3	214,4	210	1,021	1,102
11 373.1		14-25	223,7	236,3	370,3	333,0	362,6	524,3	220,8	210	1,051	1,070
		26-60	199,2	226,2	360,1	346,6	368,9	545,6	197,9	200	0,990	1,143
11 373.0	Angles	≤ 13	222,1	241,2	354,9	328,6	360,6	512,2	208,8	210	0,992	1,155
	Beams	≤ 13	226,7	247,8	377,6	337,9	371,1	526,1	214,8	210	1,023	1,154
11 483.1	Plates	26-50	326,2	353,8	472,0	463,9	501,9	638,8	324,2	290	1,118	1,091
11 523.1	Plates	>3-16	285,7	336,0	469,8	453,5	508,7	668,6	280,8	290	0,968	1,197
		17-25	301,6	343,7	421,5	471,7	514,6	673,5	298,4	290	1,029	1,152
11 523.1	Flat	≤ 16	307,8	339,8	487,8	466,9	512,1	686,2	302,6	290	1,043	1,123
	steel	17-25	332,4	352,1	493,8	501,9	530,8	706,5	326,5	290	1,126	1,078
11 523.0	Angles	≤ 13	301,0	335,0	474,4	453,4	490,6	725,8	286,7	290	0,989	1,168
	Beams	≤ 13	316,4	350,6	483,6	463,2	507,4	679,7	300,5	290	1,036	1,167

SUMMARY

The statistical basis of limit states method and their definition according to Czechoslovak standards are adduced. The limit states investigate separately load effects and structure resistance. The statistical definition of factored resistance is pointed out. Statistical results are published.

RESUME

L'auteur met en relief la nature statistique de la méthode des états limites et de leur définition d'après les normes tchécoslovaques. Les états limites étudient les effets séparés des charges et la résistance des constructions. L'auteur présente une définition statistique de la résistance. Des résultats de recherches statistiques sont publiés.

ZUSAMMENFASSUNG

Es werden die statistischen Grundlagen der Grenzzustände-Methode und deren Definition nach den tschechoslowakischen Normen angegeben. In den Grenzzuständen werden die Lastwirkung und die Tragfähigkeit getrennt. Die statistische Definition der Tragfähigkeit wird dargestellt und entsprechende Ergebnisse werden veröffentlicht.