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Computing Intrinsic Values of Flexural Vibrations of Cable-Stayed Bridges

Calcul des valeurs propres des oscillations verticales des ponts à haubans

Berechnung der Eigenwerte vertikaler Schwingungen von Schrägseilbrücken

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SUMMARY
Discrete mass systems (or mathematical models) are described rather than actual bridge cables. The mathematical modelling of a cable represents the essential features of the cable's behaviour and is at the same time sufficiently simple to enable analysis of a complex assembly of such cables in a dynamic analysis of cable-stayed bridges.

RESUME
Les modèles mathématiques représentant le comportement dynamique de câbles réels de ponts sont décrits. La valeur de ces modèles mathématiques est importante car les masses discrètes des éléments du pont et des câbles peuvent être utilisées lors de l'analyse des caractéristiques dynamiques.

ZUSAMMENFASSUNG
Es werden die mathematischen Modelle dargestellt, die das dynamische Verhalten realer Kabel einer Brücke beschreiben. Diese Modelle erlauben, für eine dynamische Analyse das Zusammenwirken der Kabel mit der restlichen Brückenkonstruktion auf einfache Weise zu erfassen.

ONE NEW PROPOSAL FOR COMPUTING EIGENVALUES OF FLEXURAL VIBRATIONS OF CABLE-STAYED BRIDGES

DISCRETE MASS SYSTEMS WHICH SUBSTITUTE THE CABLES OF THE BRIDGE

Taking into consideration that the second and all the other even oscillating cable forms do not call out the change of force in cable, the first odd cable form has predominent influence upon dynamic behaviour of the bridge. This is so because the first odd cable form, on the occasion of oscillation, causes the greatest change of static force in the cable. To this purpose such mathematical models will be made which completely substitute the behaviour and the characteristics of the cables oscillating by their first form. On this occasion in the springs of mathematical models will be formed the same force which was produced in the real cable on the occasion of its forced oscillations during the oscillation of the bridge, while masses of mathematical models oscillate by the same frequencies by which the real cables oscillate. The contribution and significance of these mathematical models reflects in the fact that the whole assembly consisting of discrete masses of the bridge structure and the discrete masses of mathematical models (instead of cables) is analysed togethar. Now it is possible to include the increased influence of the cable, which has the natural frequency close to the frequency of the oscillating assembly, upon the dynamic behaviour of the bridge.
Diferential equation of motion for cables with one and fixed and the other one connected to the oscillating structure (fig.1) has the following form [1,2] :

$$\frac{w}{g}\left[\ddot{\eta}(x,t) + \frac{x}{s}\ddot{\nu}(t)\cdot\sin\alpha\right] = T\cdot\eta''(x,t) + dT(t)\cdot y''(x) \qquad (1)$$

where w is the weight per unit length of cable, T is static force in cable before the beginning of oscillations, g is acceleration of graviti, dT is the value for which the force in cable (due to its oscillations) has changed and has the following form [1,2] :

$$dT(t) = \frac{AE}{s}\cdot\cos\alpha\left[\nu(t) + \frac{w}{T}\int_0^s \eta(x,t)\cdot dx\right] \qquad (2)$$

(A is cross sectional area of cable, E is modulus of elasticity).

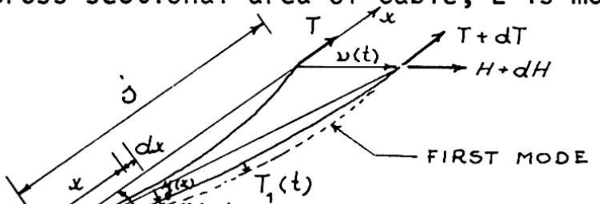


FIG.1

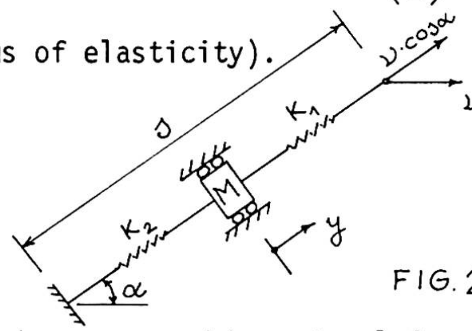


FIG.2

The form of forced vibrations of the problem can be expressed by natural forms of free vibrations of the cable X_i and can be presented by the following equation [3] :

$$\eta(x,t) = \sum_{i=1}^{\infty} X_i(x)\cdot T_i(t) \quad ; \quad X_i = \sin\frac{i\pi x}{s} \quad i=1,2,3\ldots \qquad (3)$$

where T_i is a generalized coordinate which represents the displacements in the middle of the cables span during its forced vibrations. The function T_i will be found by substitution of the eq. (3) into the eq. (1). In this case the value of the function T_1 will be required and be the resultant solution of the following differential equation:

$$\ddot{T}_1 + \omega_0^2 T_1 = -\frac{4gk}{\pi T}\cdot\nu(t) - \frac{2\sin\alpha}{\pi}\cdot\ddot{\nu}(t) \qquad (4)$$

$$k = \frac{AE}{s}\cdot\cos^2\alpha \quad ; \quad \omega_0^2 = \omega_1^2 + \frac{8sgkw}{T^2\pi^2} \qquad (5)$$

where k is a horisontal stiffness of the cable conceived as a weightless wire.

If the eq. (3) is substituted into the eq. (2)we shall obtain the value for which the static force in the cable has changed:

$$dT=\frac{AE}{s}\cdot \nu(t)\cdot\cos\alpha - \frac{2wAE\cos\alpha}{\alpha T}\cdot T_1(t) \qquad (6)$$

During the oscillation $\nu(t)$ of one end of the mathematical model its discrete mass is oscillating by the same frequency by which the real cable oscillates, and in the spring K_1 the same force (eq.6)is produced as in the real cable (while the real cable is oscillating by its first form of oscillation). For the mathematical model from fig.2 the following equations are related

$$\ddot{y}+\omega_o^2 y=\frac{K_1}{M}\nu(t)\cdot\cos\alpha \quad (7) \quad ; \quad \omega_o^2=\frac{K_1+K_2}{M} \quad (8) \quad ; \quad dT=\nu(t)\cdot K_1\cdot\cos\alpha - K_1\cdot y(t) \quad (9)$$

where M is mass and K_1 and K_2 are spring stiffnesses of the mathematical model. In order to make the eq. (7) and (9) identical to the eq. (4) and (6) the following transformations of coordinates will be introduced [3] :

$$T_1(t)=E_1\cdot y(t) - \frac{2\sin\alpha}{\alpha}\nu(t) \qquad (10)$$

where E_1 is the unknown constant of transformation. That is the method how the elements of mathematical model with single degree of freedom and constant of transformation are obtained [3,4] :

$$K_1=\frac{AE}{s} - \frac{4wAE\sin\alpha}{\alpha^2 T} \qquad (11) \qquad\qquad M=\frac{(K_1\alpha T\cos\alpha)^2}{8swgk^2} \qquad (13)$$

$$E_1=-K_1\frac{\alpha T\cos\alpha}{2wsk} \qquad (12) \qquad\qquad K_2=M\omega_o^2-K_1 \qquad (14)$$

The values ν_r and y_r are produced for each r-th form of the bridge oscillation. From the eq. (10) and (12) is obtained

$$T_{1,r}=-\frac{K_1\alpha T\cos\alpha}{2wsk}\cdot y_r - \frac{2\sin\alpha}{\alpha}\nu_r \qquad (15)$$

and the real values of the cables displacement can be found from eq. (3). Let now introduce the discrete mass system (mathematical model) substituting the cable with both ends in oscillation (fig.3). The lower end of the cable connected to the oscillating structure has vertical direction of oscillation ν_1 while the upper end of the cable has horisontal direction of oscillation ν_2 . According to the eq. (11) - (14) the mechanism may be esteblis hed, which represents two mathematical models from fig.2, whose elements can be presented by the following expressions:

$$K_{12}=\frac{AE}{s}\cos\alpha - \frac{2wAE\cos\alpha}{\alpha T}\cdot\frac{2\sin\alpha}{\alpha} \; ; \quad E_{12}=-\frac{K_{12}\alpha T\cos\alpha}{2wsk} \; ; \quad M_2=\frac{(K_{12}\alpha T)^2\cos\alpha}{8wsgk^2}$$

$$K_{11}=\frac{AE}{s}\sin\alpha - \frac{2wAE\cos\alpha}{\alpha T}\cdot\frac{2\cos\alpha}{\alpha} \; ; \quad E_{11}=-\frac{K_{11}\alpha T\cos\alpha}{2wsk} \; ; \quad M_1=\frac{(K_{11}\alpha T\cos\alpha)^2}{8wsgk^2\sin\alpha} \qquad (16)$$

$$K_{22}=M_2\cdot\omega_o^2-K_{12} \; ; \quad K_{21}=M_1\cdot\omega_o^2-K_{11} \; ;$$

Let us pass to the possibility when both ends are oscillating at the same time. In this case the principle of superposition can be applied to the previously mentioned mechanism and the next mechanism (fig.3) as a system with two degrees of freedom can be esteblished. γ is an angle under which the resultant discplacement of mass M given by coordinate y is performed. This angle is obtained by one of the following equations:

$$\gamma=\arctan\left(\frac{K_{22}}{K_{21}}\right) \quad ; \quad \gamma=\arctan\left(\frac{K_{12}}{K_{11}}\right) \qquad (17)$$

The link is made between the displacements y_1 and y_2 of the previously mentioned mechanism and the displacement y of the discrete mass of the required mathematical model

$$y(t)=y_2(t)\cdot\sin\gamma + y_1(t)\cdot\cos\gamma \qquad (18)$$

Accordingly, the rest of the unknown elements of mathematical model, which completely substitutes the oscillations of the cable with both ends connected to the oscillating structure, are :

$$K_2=\sqrt{(K_{22})^2+(K_{21})^2} \quad ; \quad M=\sqrt{(M_2)^2+(M_1)^2}$$

$$E_1=\sqrt{(E_{12})^2+(E_{11})^2} \quad ; \quad \gamma_r=\arctan\left(\frac{K_{12}\cdot\nu_{2,r}}{K_{11}\cdot\nu_{1,r}}\right)$$

$$T_{1,r}(t)=E_{1,r}\cdot y_r(t) - \frac{2\sin\alpha}{\alpha}\nu_{2,r}(t) - \frac{2\cos\alpha}{\alpha}\nu_{1,r}(t)$$

eq.(19)

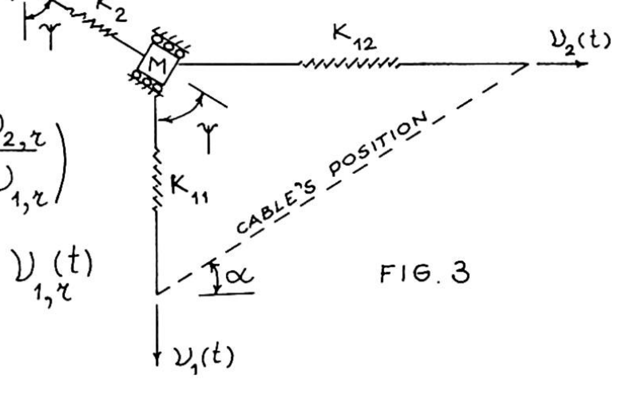


FIG. 3

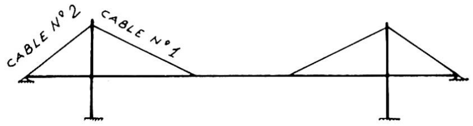

CABLE N° 2
CABLE N° 1

FIG. 4a

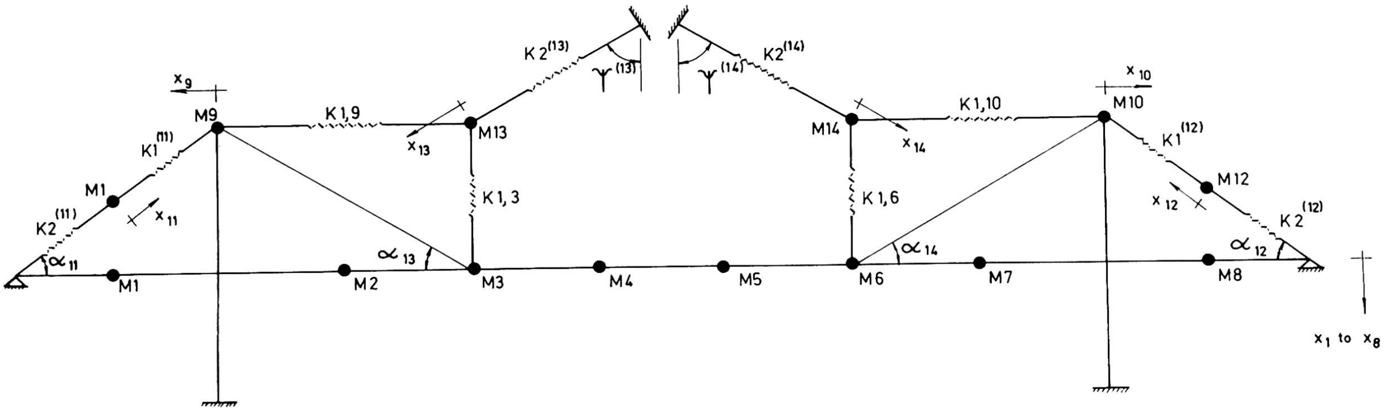
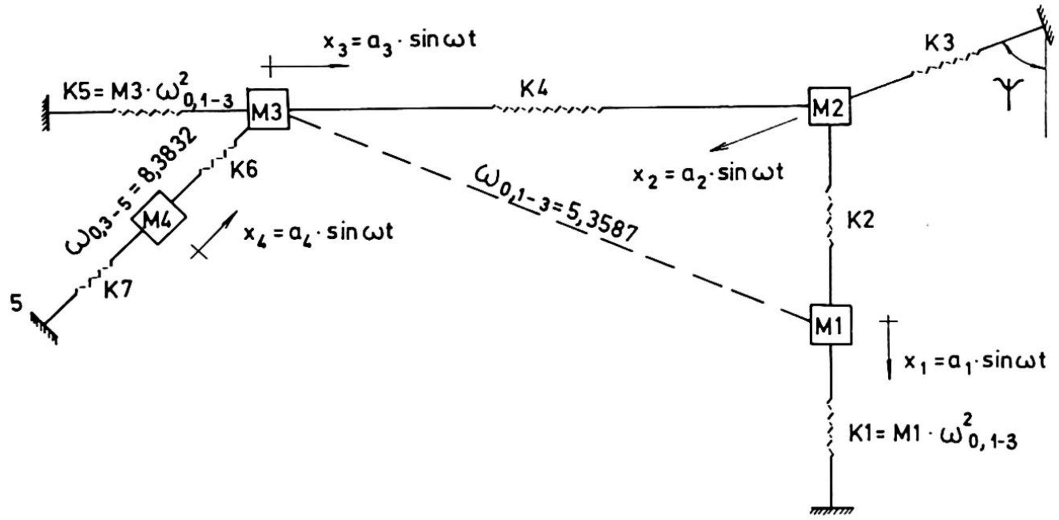
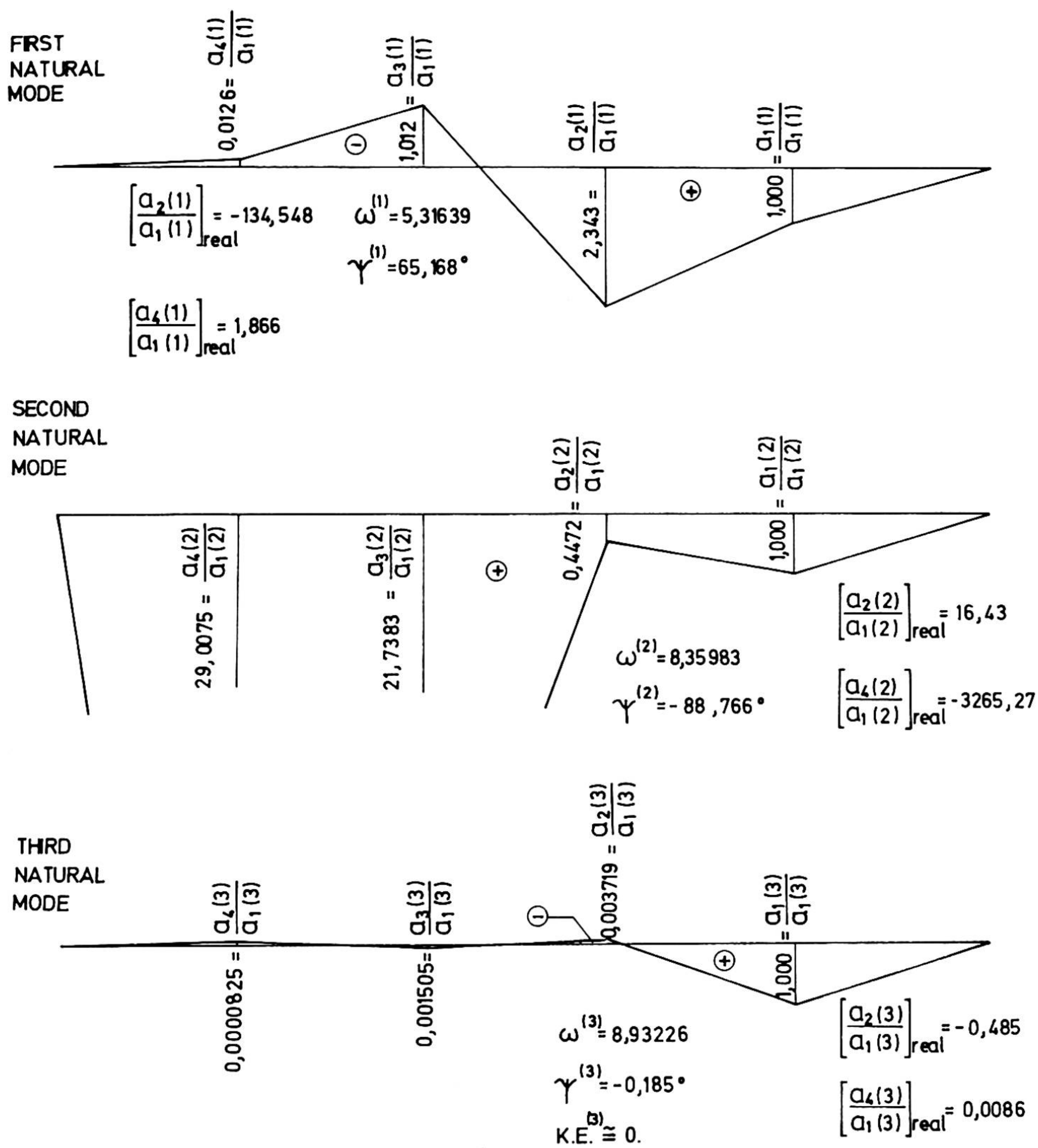

x9
M9
K1(11)
M1
K2(11)
α11
x11
M1
K1,9
x13
M13
K1,3
α13
M2
M3
K2(13)
ψ(13)
ψ(14)
K2(14)
M4
M5
M14
x14
K1,6
M6
α14
K1,10
M7
x10
M10
K1(12)
M12
x12
K2(12)
α12
M8
x1 to x8

FIG.4b



x3 = a3 · sin ωt
K3
K5 = M3 · ω²0,1-3
M3
K4
M2
Ψ
ω0,3-5 = 8,3832
K6
x2 = a2 · sin ωt
M4
x4 = a4 · sin ωt
ω0,1-3 = 5,3587
K2
5
K7
M1
x1 = a1 · sin ωt
K1 = M1 · ω²0,1-3

FIG. 5



FIRST NATURAL MODE
0,0126 = a4(1)/a1(1)
1,012 = a3(1)/a1(1)
2,343 = a2(1)/a1(1)
1,000 = a1(1)/a1(1)
[a2(1)/a1(1)]real = -134,548
ω(1) = 5,31639
Ψ(1) = 65,168°
[a4(1)/a1(1)]real = 1,866
SECOND NATURAL MODE
29,0075 = a4(2)/a1(2)
21,7383 = a3(2)/a1(2)
0,4472 = a2(2)/a1(2)
1,000 = a1(2)/a1(2)
[a2(2)/a1(2)]real = 16,43
ω(2) = 8,35983
Ψ(2) = - 88,766°
[a4(2)/a1(2)]real = -3265,27
THIRD NATURAL MODE
0,0000825 = a4(3)/a1(3)
0,001505 = a3(3)/a1(3)
0,003719 = a2(3)/a1(3)
1,000 = a1(3)/a1(3)
ω(3) = 8,93226
Ψ(3) = -0,185°
K.E.(3) ≅ 0.
[a2(3)/a1(3)]real = -0,485
[a4(3)/a1(3)]real = 0,0086

FIG. 6

THE NUMERICAL EXAMPLE AND CONCLUSIONS

The natural modes and frequencies of bridges are the result of the following matrix differential equation $[M]\{\ddot{x}\} + [k]\{x\} = \{o\}$ (20)

By introduction the mathematical models instead of cables, the matrix of masses and the matrix of stiffnesses in eq. (20) contain the elements of mathematical models as well. The results of analysis reported herein are concerned with the part of the structure in fig. 4 comprising the cable with both ends in oscillation (cable no.1), the cable with only one end in oscillation while the other one is fixed (cable no.2) and the parts of the bridge structure to which the reffering cables are connected (fig.5).

The stiffnesses of the springs K1 and K5 of the reffering discrete system will be chosen to make possible the analysis of the most interesting case when the discrete system in fig. 5 has the first natural frequency near to the natural frequency $\omega_{o,1-3}$ of the cable no.1 and the second natural frequency near to the natural frequency $\omega_{o,3-5}$ of the cable no.2.

$$K1 = M1 \cdot \omega^2_{o,1-3} \quad ; \quad K5 = M3 \cdot \omega^2_{o,1-3} \qquad (21)$$

Discrete masses M1 and M3 are the masses at the connection points of the cables and the bridge structure. The numerical values of these discrete masses as well as natural frequencies of the cables are referred to the new designed cable-stayed bridge over the river Sava in Belgrade, which is now in stage of construction [5]. By the analysis of such a system (fig.6) the following is conculuded:

1.) The results obtained in case when the analysed structure oscillates with frequency near to natural frequency of some of the cable in the system demonstrate that this cable produces kinetic energy of a great value. This energy can now be taken into account when natural modes and frequencies of the bridge structure are required.

2.) In the discrete system analysed here is easy to notice the great values of the real natural amplitudes of oscillations of the cables (1-3) and (3-5) when the analysed discrete system in its first and second form oscillates with frequencies which are near to the natural frequencies of the cables (1-3) and (3-5). When the analysed discrete system oscillates in its third and fourth natural form with frequencies different from the natural frequency of any cable in the system, the real natural amplitudes of oscillation of cables became very small and may be concerned as negligable.

3.) The designers of such kind of bridges can find out eigenvalues of flexural vibrations using here presented method. Forced or natural frequencies of the bridge should not approach the vicinity of natural frequencies of any of the cables on the bridge. The natural frequency of the cable can be found using reference [2].

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