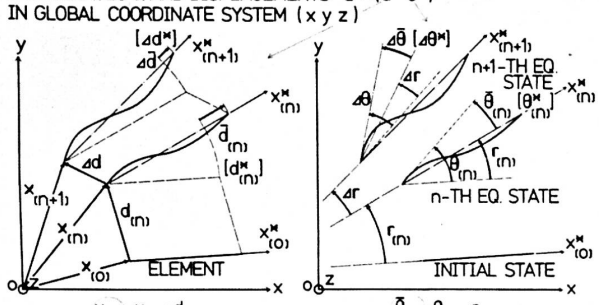
A COMPUTER ORIENTED FORMULATION FOR GEOMETRICALLY NONLINEAR PROBLEMS
EINE COMPUTER-ORIENTIERLE FORMULIERUNG FÜR GEOMETRISCHE NICHTLINEARE PROBLEME
UNE FORMATION POUR DES PROBLÈMES NONLINÉAIRE GÉOMÉTRIQUE

Yutaka YOSHIDA, TOKYO INST. TECH.
Takeo NISHIWAKI, Nobutoshi MASUDA, MUSASHI INST. TECH.

DERIVATION OF ELEMENT EQUATION

1. COORDINATES x AND DISPLACEMENTS $u^T=(d^T\ \theta^T)$ IN GLOBAL COORDINATE SYSTEM (x y z)



$x_{(n)}=x_{(0)}+d_{(n)}$ $\qquad \bar{\theta}_{(n)}=\theta_{(n)}-r_{(n)}$

2. DISPLACEMENTS $u^{*T}=(d^{*T}\ \theta^{*T})$ IN LOCAL SYSTEM $(x^*\ y^*\ z^*)$

$d^*_{(n)}=\Lambda_{(n)}\bar{d}_{(n)}=\Lambda_{(n)}x_{(n)}-\Lambda_{(0)}x_{(0)}$ $\qquad \theta^*_{(n)}=\Lambda_{(n)}\bar{\theta}_{(n)}$

$\Lambda_{(n)}$: COORDINATE TRANSFORMATION MATRIX

3. INCREMENTAL NODAL DISPLACEMENTS IN LOCAL COORDINATE SYST. (FROM N-TH TO N+1-TH EQUILIBRIUM STATE)

$$\Delta u^*=u^*_{(n+1)}-u^*_{(n)}=\Delta T\begin{pmatrix}x\\ \theta\end{pmatrix}_{(n)}+T_{(n+1)}\begin{pmatrix}\Delta d\\ \Delta\theta\end{pmatrix} \qquad ;\ T_{(n)}=\begin{bmatrix}\Lambda & \\ & \Lambda\end{bmatrix}_{(n)}$$

$$=\Delta T\begin{pmatrix}x\\ \theta-r\end{pmatrix}_{(n)}+T_{(n+1)}\begin{pmatrix}0\\ -\Delta r\end{pmatrix}+T_{(n+1)}\Delta u$$

4. FORCE-DISPLACEMENT RELATION IN LOCAL COORDINATE SYSTEM

ASSUMPTION OF SMALL STRAIN

$\Delta f^*=K^*\Delta u^*$ $\qquad$ K^*: CONVENTIONAL LINEAR STIFNESS MATRIX

5. RELATION BETWEEN LOCAL AND GLOBAL NODAL FORCES

$$\Delta f=f_{(n+1)}-f_{(n)}=\Delta T^T f^*_{(n)}+T^T_{(n+1)}\Delta f^* \qquad \because\ f_{(n)}=T^T_{(n)}f^*_{(n)}$$

6. FORCE-DISPLACEMENT RELATION IN GLOBAL SYSTEM [EQ. A]

$$\Delta f=\overbrace{\Delta T^T f^*_{(n)}+T^T_{(n+1)}K^*\left[\Delta T\begin{pmatrix}x\\ \theta-r\end{pmatrix}_{(n)}+T_{(n+1)}\begin{pmatrix}0\\ -\Delta r\end{pmatrix}\right]+T^T_{(n+1)}K^*T_{(n+1)}\Delta u}^{\text{KNOWN}}$$

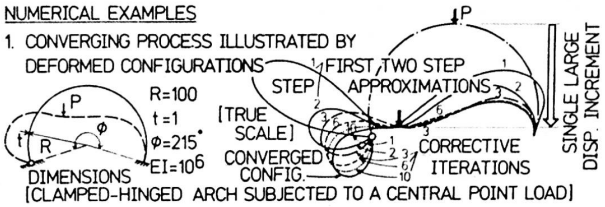
FUNCTIONS OF UNKNOWN INCREMENTAL DISPLACEMENTS Δu

DEVELOP. OF A TWO STEP APPROX. & CORRECT. ITERATION SOL. PROC.

In the first two steps, EQ.A is linearized with respect to Δu, before the assemblage of element equations. Then in iterations, only Δu_{k+1} are treated as unknowns, others are estimated by using k-th approximation of Δu.

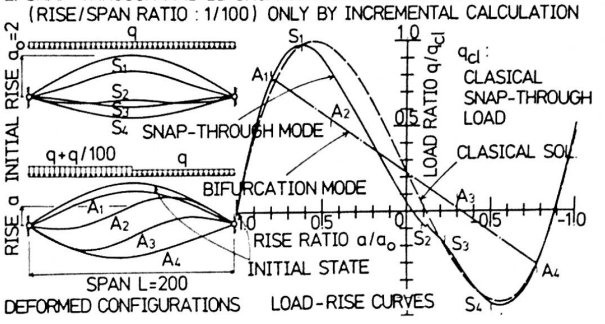
NUMERICAL EXAMPLES

1. CONVERGING PROCESS ILLUSTRATED BY DEFORMED CONFIGURATIONS



[CLAMPED-HINGED ARCH SUBJECTED TO A CENTRAL POINT LOAD]

2. SNAP-THROUGH AND BIFURCATION ANALYSIS OF A FLAT ARCH (RISE/SPAN RATIO : 1/100) ONLY BY INCREMENTAL CALCULATION



A COMPUTER ORIENTED FORMULATION FOR GEOMETRICALLY NONLINEAR PROBLEMS

Yutaka YOSHIDA
Associate Professor
Tokyo Institute of Technology

Takeo NISHIWAKI
Professor

Nobutoshi MASUDA
Lecturer

Musashi Institute of Technology
Tokyo/Japan

FORMULATION OF THE ELEMENT EQUATION

An incremental formulation is developed here, under the assumption of large displacement but small strain. The key point of the formulation is the description of nodal locations by the coordinates themselves instead of by mere displacements. A local coordinate system (x* y* z*) is introduced which is at any time on the element itself, and the element force-displacement relation is assumed to be linear in this local system. The geometrical nonlinearity is considered only through the nonlinear transforming relations of displacement components between the space-fixed global coordinate system (x y z) and the element-fixed local coordinate system. The transformations are evaluated rigorously without neglecting small terms, so that incremental relations satisfy equilibrium conditions after deformation as accurately as poissible.

SOLUTION PROCEDURE

To construct and solve structural equilibrium equations, an effective corrective iteration solution procedure is also originally developed. The process, outlined in the following, makes use of physical properties of each term in the derived element equation. At first, the changes of transformation matrix and rigid body rotation in the element equation, ΔT and Δr respectively, are linearized with respect to incremental displacement Δu at the n-th equilibrium state A. And the transformation matrix after the increment, $T^{(n+1)}$, is estimated at the same state A. Hence the element incremental equation is written in the following quasitangential form.

$$\Delta f=(K_f+K_u+K)_A\Delta u$$

Then summing up the element incremental equations thus approximated for overall structure and solving them, the first approximation of the n+1-th equilibrium state C is obtained. Secondly, ΔT and Δr are linearized at the midpoint B between A and C, meanwhile $T^{(n+1)}$ is approximated at the above obtained first approximating state C. Thus the element incremental equation is written as

$$\Delta f=(K_f+K_u+K)_{B,C}\Delta u$$

Thus the second approximating solution, denoted by D, can be obtained. Then afterwards in iterations, ΔT and Δr as well as $T^{(n+1)}$ are all evaluated by using the just preceding approximating solution, and only the incremental displacement Δu is treated as unknown variable. Namely, for the k+1-th approximation

$$\Delta f-h_k=K_k\Delta u$$

is used, where

$$K_k=T_k^T K^* T_k \quad , \quad h_k=T_k^T f^*_{(n)}+T_k^T K^*\left(\Delta T\begin{pmatrix} x \\ \theta-r \end{pmatrix}_{(n)}+T_k\begin{pmatrix} 0 \\ -\Delta r \end{pmatrix}\right)$$

and

$$\Delta T=T_k-T_{(n)} \quad , \quad \Delta r=r_k-r_{(n)}$$

The iteration is continued until satisfactory convergence is obtained.

FEATURES OF THE PROPOSED METHOD

- With mere incremental calculations and without special techniques such as eigenvalue analysis, not only snap-through and limit point phenomina but also bifurcation can be pursued.
- Even for an extraordinary large increment, numerical stability exists and sufficient accuracy is obtained.
- As the consequence of the above characteristics, calculating time can be reduced considerably.