

# Deformability of composite timber beams

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**II****Deformability of Composite Timber Beams**

Déformabilité des poutres composées en bois

Deformabilität von zusammengesetzten Holzträger

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**SUMMARY**

The report deals with the simple method for calculation of the bending and buckling behaviour of composite bars. The method gives, for the praxis, enough exact results considering any number of composite parts. The same principles can also be used for the calculation of the deformability of torsional beams and for the control of torsional stability (torsional buckling, lateral buckling).

**RESUME**

Cet article présente une méthode simple permettant de calculer la flexion et le flambement des poutres composées en bois. Les résultats obtenus sont suffisamment exacts pour une utilisation pratiques, quel que soit le nombre de composants. Les mêmes principes sont utilisables pour le calcul des phénomènes de torsion (déformabilité, flambement de torsion, déversement).

**ZUSAMMENFASSUNG**

Dargelegt wird eine einfache Methode für die Berechnung des Biege- und Knickverhaltens zusammengesetzter Holzstäbe. Die Methode bietet für die Praxis genügend genaue Resultate für eine beliebige Zahl zusammengesetzter Teile. Dieselben Prinzipien sind ebenfalls anwendbar für die Berechnung der Verformung von tordierten Trägern und für die Kontrolle der Torsionsstabilität (Drillknicken, Kippen).

1. THE BASIC PRINCIPLES FOR THE CALCULATION OF THE DEFORMABILITY OF COMPOSITE TIMBER BEAMS.

The starting point of our calculation is Fig.1, where in Fig.1/a deformations and stresses for a stiff-jointed beam are presented, in Fig.1/b an elastically jointed beam, and in Fig.1/c a composite beam without fasteners are presented. If  $r_i$  is the reduction-factor:

$$\sigma_v = \sigma_{io} + \sigma_{iv}, \sigma_{io} = r_i \cdot \sigma_{ioa}, \sigma_{iv} = \sigma_{iva} = \sigma_{ivb} = \sigma_{ivc} = \sigma_{vc}, R = \text{const} \quad \dots/1$$

Acc. to Fig.2 the stress  $\sigma = \epsilon \cdot E$  for the distance "y" from the neutral axis and thus  $\sigma_{ioa} = C \cdot y_i$  and  $\sigma_{iva} = C \cdot y_{iv}$ ,  $C = E/R$  and:

$$\sigma_v = C \cdot (y_{iv} + r_i \cdot y_i), y_{iv} = y_v - y_i, C = E/R \quad \dots/2$$

The moment, taken over by the beam, if the beam has "n" parts and  $A$  is the complete area of the cross-section, and  $A_i$  the area of the part "i"

$$M = \int_A \sigma_v \cdot y_v \cdot dA_i = C \cdot \left\{ \sum_1^n \int_{A_i} y_{iv}^2 \cdot dA_i + \sum_1^n r_i \cdot y_i^2 \cdot A_i \right\} \quad \dots/3$$

$$M = C \cdot \left( \sum_1^n I_{ti} + \sum_1^n r_i \cdot I_{Ti} \right) = C \cdot I_e = E \cdot I_e / R, I_{ti} = \int_{A_i} y_{iv}^2 \cdot dA_i, I_{Ti} = y_i^2 \cdot A_i$$

where  $I_e$  is the effective moment of inertia, and if we denote

$$I_{ae} = \sum_1^n r_i \cdot I_{Ti}, I_{am} = \sum_1^n I_{Ti} \text{ ( m...mathematical )}, I_{ti} = \frac{b_i \cdot h_i^3}{12} \quad \dots/4$$

and

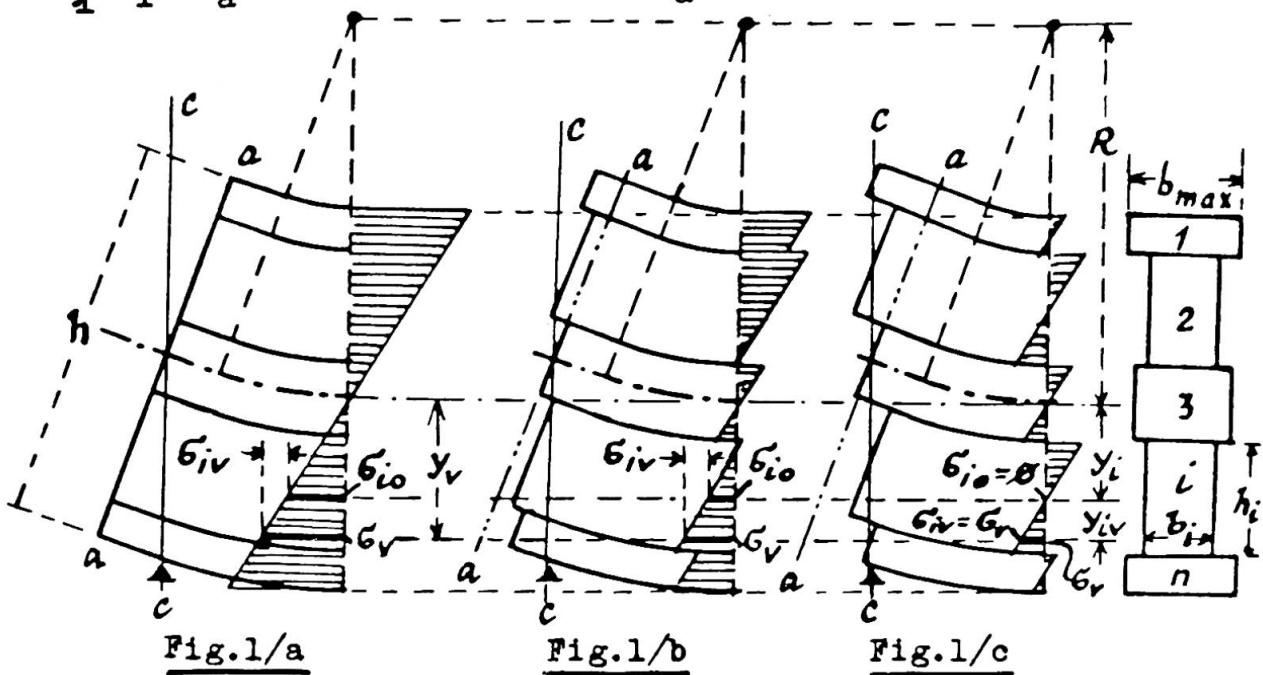
$$I_e = \sum_1^n I_{ti} + I_{ae}, I_m = \sum_1^n I_{ti} + I_{am} \text{ (=math.mom.of in.)} \quad \dots/5$$

Now  $r_i$  should be established. This can be simplified by introducing:

$$r_i = q \cdot k_i, 1,00 \geq q \geq 0,00, 1,00 \geq k_i \geq 0,00; q = \text{const.} \quad \dots/6$$

2. THE CALCULATION OF THE FACTORS OF REDUCTION AND OF THE APPROPRIATE NUMBER OF FASTENERS.

The Eq.3 can be composed so that the moment  $M$  is divided into the local moments  $M_i = C \cdot I_{ti}$  and into the associated moment  $M_a = C \cdot I_{ae}$ , i.e.  $M = \sum_1^n M_i + M_a$ . If for the moment  $M_a$  first the calculation of  $I_{ae}$  is



deformation is considered with the consideration of the effective moment of inertia  $I_{ae}$ , and then of the mathematical moment of inertia  $I_{am}$ , here including the deformations with regard to the transversal forces, it follows:

$$f = M_a \cdot L^2 / K \cdot E \cdot I_{ae} = M_a \cdot L^2 / K \cdot E \cdot I_{am} + a \cdot M_a / A_o \cdot G_d \quad \dots/7$$

Here is  $E$  the modulus of elasticity for timber,  $L$  is the span (Fig. 3/a, b),  $K$  is the factor dependent on the shape of the moment line, for the praxis,  $K \approx 10$  is sufficient; "a" is for a rectangular cross section  $a=1, 2$ , and  $A_o$  is the area of the cross-section outlined rectangle;  $G_d$  is the reduced shear modulus (deformability of fasteners!). From Eq. 7 the value for  $I_{ae}$ :

$$I_{ae} = I_{am} / (1+m), \quad m = a \cdot I_{am} \cdot K \cdot E / L^2 \cdot A_o \cdot G_d \quad \dots/8$$

Supposing that all  $k_i$  are equal to 1,00, i.e.  $r_i = q = \text{const.}$ , acc. to Eq. 4:  $I_{ae} = q \cdot I_{am}$  and finally (acc. to Eq. 8):

$$q = 1 / (1+m), \quad \text{for } k_i = \text{const.} = 1,00, \quad r_i = q \quad \dots/9$$

The reduced shear modulus  $G_d$  is shown in Fig. 4. In Fig. 4/a we have the real example with  $G=G$  and the displacement  $x_f$ , in Fig. 4/b we have the fictitious example  $G \rightarrow G_d$ , and the same displacement  $x_n$ , but now without the dislocation of the elements "i" and "j", which is acc. to Fig. 4/a equal to  $z_{pij}$  ( $\tau_{ji} = \tau_{ij} \cdot s_{ij} / s_{ji}$ !):

$$x_f = \tau_{ij} \cdot d_{ij} / G + \tau_{ji} \cdot d_{ji} / G + z_{pij} = x_n = \tau_{ij} \cdot d_{ij} / G_d + \tau_{ji} \cdot d_{ji} / G_d \quad \dots/10$$

and:

$$G_d = G / (1 + z_{pij} \cdot G / H_{ij} \cdot \tau_{ij} \cdot s_{ij}), \quad H_{ij} = d_{ij} / s_{ij} + d_{ji} / s_{ji} \quad \dots/11$$

The dislocation  $z_{pij}$  at the shear force  $P_{ij}$  in one fastener, which have the allowable loading  $N_{sij}$  and the dislocation with regard to this loading  $z_{nij}$ , is then:  $z_{pij} = z_{nij} \cdot P_{ij} / N_{sij}$ . If there are  $n_{sij}$  fasteners on the length of  $100 \text{ cm}$ , we get  $z_{pij} = z_{nij} \cdot \tau_{ij} \cdot s_{ij} \cdot 100 / N_{sij} \cdot n_{sij}$ . By introducing this in Eq. 11, and regarding  $Q_{ij} = N_{sij} \cdot H_{ij} / 100 \cdot z_{nij}$  we get:

$$G_d = G / (1 + G / Q_{ij} \cdot n_{sij}) = G_{do} / (1 + G_{do} / G), \quad G_{do} = Q_{ij} \cdot n_{sij} \quad \dots/12a$$

To have the same  $G_d$  along the whole depth of the beam,  $G_{do} = \text{const.}$ :

$$Q_{ij} \cdot n_{sij} = Q_{ts} \cdot n_{sts}, \quad \text{i.e. } n_{sij} = n_{sts} \cdot Q_{ts} / Q_{ij} \quad \dots/12b$$

Here is  $t$ - $s$  any jointed plane, where we can take any number of fasteners ( $n_{sts}$  on  $100 \text{ cm}$ ).

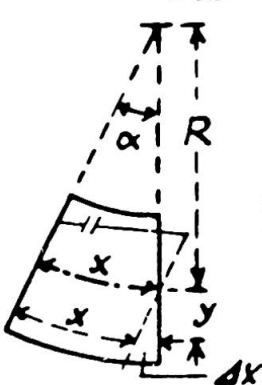


Fig. 2

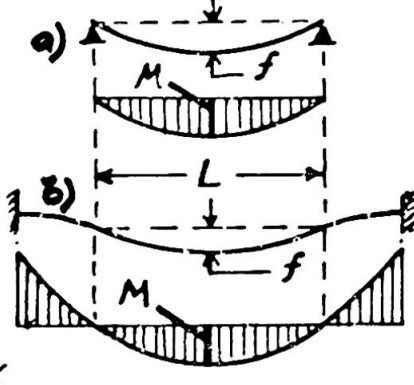


Fig. 3/a, b

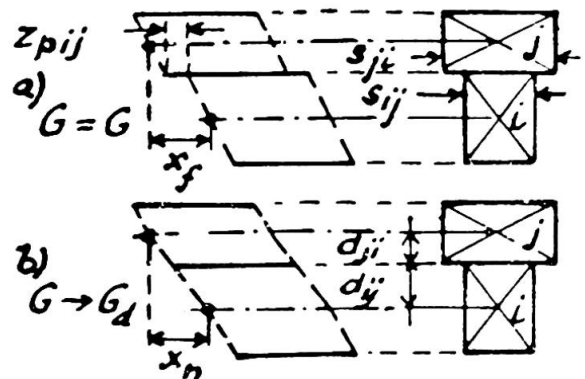


Fig. 4/a, b

If we want less fasteners  $n'$  on boundary areas, we take into account  $r \rightarrow r' = q \cdot k$ ,  $k < 1,00$  any values, for  $i=t, i=s$  recommendable  $k_t = k_s = 1,00$ . In this case the shear force in the joint  $i-j$  diminishes from  $P = \tau_{ij} \cdot s_{ij} \cdot \Delta L$  to  $P' = \tau'_{ij} \cdot s_{ij} \cdot \Delta L$ , i.e. the factor of reduction of fasteners  $t_{ij} = \tau'_{ij} / \tau_{ij}$ , so (r...boundary [edge] element):

$$t_{ij} = \frac{r}{\sum_j k_p} \cdot \frac{A_p \cdot y_p}{\sum_j A_p \cdot y_p}, \quad p = j, j+1, j+2, \dots, r \quad \dots/13$$

The second correction arises because of the increase of deformations of fasteners and is acc. to Fig.5 equal  $c_{ij}$ :

$$c_{ij} = (y_j - y_i) \cdot (1 - q) / \{y_j \cdot (1 - k_j \cdot q) - y_i \cdot (1 - k_i \cdot q)\} \quad \dots/14$$

$$n'_{sij} = n_{sij} \cdot t_{ij} \cdot c_{ij} \quad (n_{sij} \text{ for } k_i = 1,00, \text{ Eq.12/b}) \quad \dots/15$$

In Eq.7, the values "a" should be presented also for other cross-sections. To avoid the known complicated equation, we take (Fig.6):

$$a \doteq 1,2 \cdot b_{\max} \cdot \sum H_{ij} / \sum (d_{ij} + d_{ji}) \quad \dots/16$$

3. APPLICATIONS OF THE GIVEN METHOD IN BENDING, BUCKLING AND TORSION.

In bending, the effective moment of inertia  $I_e$  (from Eq.5) is taken for the calculation of deformations. To calculate the stresses we put from Eq.3 the value  $C = M/I_e$  in the Eq.2 and we get:

$$\sigma_v = (M/I_e) \cdot (y_{iv} + r_i \cdot y_i) \quad \dots/17$$

A special example is a lattice beam, flanged additionally. Here the influence of diagonals and verticals is exchanged by the web of the width  $b''$  (which is not considered in the sums acc. to Eqs.3,4,5). In buckling, the total slenderness is calculated acc. to equation:

$$\lambda_{\text{tot}} = \sqrt{\lambda_e^2 + \lambda_1^2}, \quad \lambda_e = L_1 / i_e, \quad i_e = \sqrt{I_e / A} \quad \dots/18$$

where  $L_1$  is for slenderness competent length, and  $\lambda_1$  the local slenderness of the most inconvenient part of the beam.

In torsion, the method can be used in the calculation of the composite box-cross-sections, acc. to Fig.7. Supposing the equal rotation-angle "u" of every single part and of the whole, we get:

$$u = T \cdot L / C_d, \quad C_d = \sum C_{di} + C_{da}, \quad C_{di} = G \cdot Y_i, \quad C_{da} = G_a \cdot Y_a \quad \dots/19$$

where T is the moment of torsion, L is the length, where the angle "u" is measured,  $C_d$  is the torsional stiffness, G is the timber shear modulus,  $G_a$  is the reduced shear modulus,  $Y_i$  is the local tors. moment of inertia,  $Y_a$  the associated tors. mom. of inertia:

$$G_a = G / (1 + 100 \cdot z_{nij} \cdot G \cdot D / N_{sij} \cdot n_{sij} \cdot U_{ij}), \quad Y_a \doteq 4(H-D)^2 \cdot (B-D)^2 \cdot D / \sum U_{ij} \quad \dots/20$$

