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# Alternating Plasticity Analysis of an Industrial Frame

Analyse plastique d'un cadre dans la construction lourde

Traglastanalyse eines Stahlrahmens im Industriebau

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## SUMMARY
The basic postulates of the limit analysis of structures are valid when the applied loads are acting simultaneously and are increased proportionally in their magnitude without reversing the direction until limit load is reached. If this is not the case, the carrying resistance of the structure may be reduced to shake-down load due to influence of the variable repeated loading. An industrial steel frame has been analyzed under variation of the specification snow and wind loads as given in specifications and the obtained results are discussed.

## RESUME
Les postulats de base de l'analyse limite sont valables lorsque les charges appliquées le sont simultanément et croissent constamment jusq'à la rupture. Si ce n'est pas le cas, la résistance de la structure peut être réduite à la charge "Shake-down", à cause de l'influence d'une charge répétée variable sur la structure. Le cadre d'une structure industrielle a été analysé sous l'effet de charges variables de neige et de vent — définies dans les normes — et les résultats sont discutés.

## ZUSAMMENFASSUNG
Die Grundanforderungen der Grenzanalyse gelten für die Belastungen, die ohne Richtungsänderung gleichzeitig wirken und stetig bis zur Grenzlast wachsen. Andernfalls kann die Tragfähigkeit der Konstruktion durch sogenannte "shake-down"-Lasten, d.h. durch wechselnde Lastwirkungen, vermindert werden. Im gegebenen Fall wurde ein Industrie-Stahlrahmen aufgrund der wechselnden Norm-Belastungen aus Schnee und Wind untersucht, die Ergebnisse verglichen und erläutert.

## 1. INTRODUCTION

The actual loads may vary considerably and change indenpendently of each other during the lifetime of a structure. Such loads as termed as variable repeated loads. Under such load variation, as first recognized by M.Grüning (1926) and G.Kazinczy (1931), the structure may fail under considerably lower loads than computed applying the limit analysis basic postulates. W.Prager (1948) first used the term shakedown loads, as the largest set of loads under which a structure is safe against failure under variable repeated loads.

As in the case of the limit load, the concept shake down load has a precise mathematical meaning only for idealized materials. The contributions of strain hardening are ignored. The Bauschinger effect is also disregarded.

Two types of the problem may be encountered when variable repeated loading is acting on a structure and they are differentiated as:

(a) alternating plasticity collapse, at load factor $\lambda_s = \lambda_a$, and
(b) incremental collapse, at load factor $\lambda_s = \lambda_i$.

In the alternating plasticity problem, failure occurs at a relatively small number of load cycles when the repeated loading is such that yielding of the material at the structure critical sections occurs alternately in tension and compression. When plastic flow occurs in reverse direction, material at the given section accumulates a certain amount of plastic work encircled by the $M_p/\Theta$ curve (Fig. 1). The structure can tolerate only a limited amount of plastic work. After this limit is reached, the material is weakened or becomes more brittle. This effect is similar to high-cycle elastic fatique, but the number of cycles of load application involved is only of order of tens or hundreds. During each cycle of loading the plastic flow will increase and eventually lead to structure member fracture at the load factor $\lambda_a$.

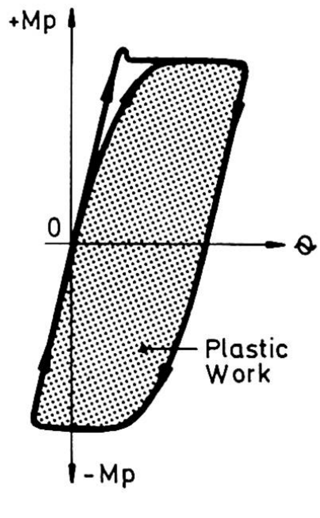



Fig. 1 Plastic work limit

In the cases just described, when the history of loading corresponds to plastic reversals in some parts of the structure, it is neccesary to evaluate the condition of reliability of alternating plasticity low cycle fatique collapse. Reduction of the structure carrying capacity should be expressed by the magnitude and number of plastic reversals. A critical number of reversals can be accumulated in a relatively short time depending on the nature of loading.

According to present specifications the plastic theory is applied to the design of structures under predominantly static loads. In these specifications there are no specific requirements related to the shake-down problem.

## 2. FUNDAMENTALS

Failure of a structure due to alternating plasticity will not occur when there exist such ranges of bending moment values for which a section behaves elastically. This condition may be stated as:

$$(M_i^e)_{max} - (M_i^e)_{min} \leq 2M_p/\alpha \tag{1}$$

Where,

$(M_i^e)_{max}$ = Maximum elastic bending moment at section i,

$(M_i^e)_{min}$ = Minimum elastic bending moment at section i,

$M_p$ = Full plastic moment at section i,

$\alpha$ = Cross section shape factor.

In the incremental collapse problem an increase in deflection occurs during each cycle of loading. It is necessary to find a load factor $\lambda_i$ for which increments of deflection stop after several load cycles and overall deflection ceases to increase.. Failure of a structure due to incremental collapse will not occur when the sum of residual moment and elastic moment of applied loading on the structure do not exceed the full plastic moment value at any given section, i.e.

$$(M_i^e)_{max} + m_i^r \leq M_p \tag{2}$$

Where,

$(m_i^e)_{max}$ = Maximum elastic bending moment at section i assuming that yielding does not occur,

$m_i^r$ = Residual bending moment at section i,

$M_p$ = Full plastic moment at section i.

The loads that exceed the incremental shakedown load are found to gradually cause structure failure due to excessive deflections. The above conditions for shakedown load should be checked at every section i where:

- concentrated load is applied,
- at the joint of two or more structure elements, and
- where element changes section properties.

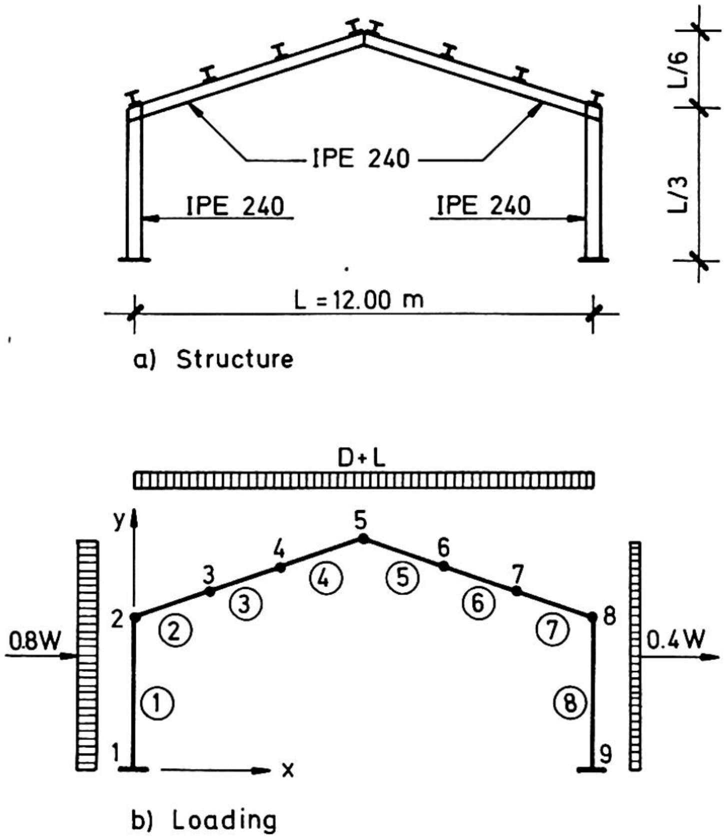
For the ordinary low frame structure the structure failure is more likely to occur due to alternating plasticity than to incremental collapse [9] . In the application of the plastic method to the design of the structure, strength is the major criterion. It is necessary, however, also to check other safety criteria such as local stability, overall stability, deflections, and if main load is variably repeated, the shakedown collapse. The most up to date research of structures under variable repeated loads has been performed on ideal structures. A 12 m industrial frame is analyzed under variable repeated loads given by load specification [11] , as follows:

- snow load: S = from 0.00 to +0.75 kN/m², and
- wind load: W = from -1.10 to +1.10 kN/m².

The whole structure is made of IPE 240 from mild steel (A36) with yield stress of 236 N/mm².

## 3. AN INDUSTRIAL FRAME ANALYSIS

Fig. 2 shows an outline of the frame, its section properties, gravity and wind loading on the frame.




Structure Properties:

IPE 240
$A = 39.1\ cm^2$
$I_x = 3\,890\ cm^4$
$M_p = 8\,638\ kNcm$

Gravity Loads:

$D = 0.417\ kN/m^2$
$L = 0.750\ kN/m^2$
$D+L = 1.167\ kN/m^2$

Wind Load:

$W = 1.10\ kN/m^2$

Load Combinations [kN]

| Joint | DL+LL | | DL+LL+W | |
|---|---|---|---|---|
| | $P_x$ | $P_y$ | $P_x$ | $P_y$ |
| 2 | 0 | -7.0 | 11.44 | -9.5 |
| 3 | 0 | -14.0 | 1.68 | -19.0 |
| 4 | 0 | -14.0 | 1.68 | -19.0 |
| 5 | 0 | -14.0 | 1.68 | -14.0 |
| 6 | 0 | -14.0 | 1.68 | -9.0 |
| 7 | 0 | -14.0 | 1.68 | -9.0 |
| 8 | 0 | -7.0 | 6.12 | -4.5 |

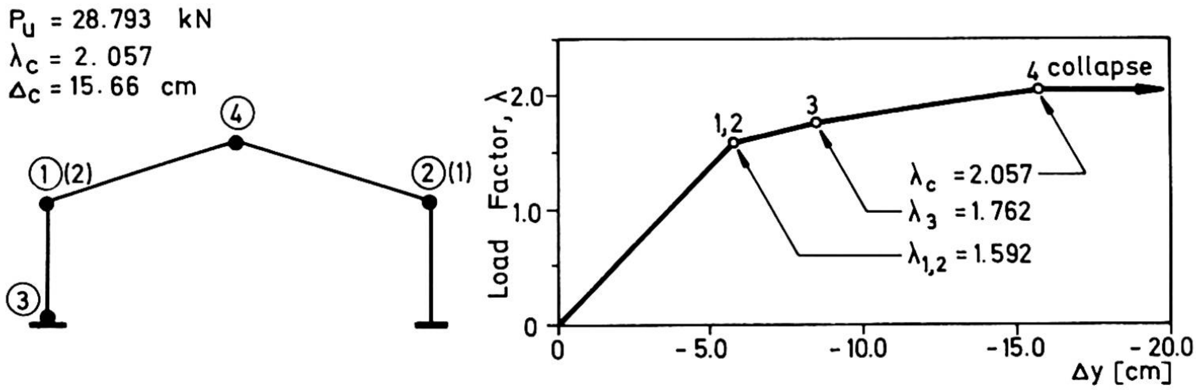
Fig. 2 Frame properties and loadings

### 3.1 Limit load calculation

(a) Case I - Dead load + Live load

From the principle of virtual displacements, the interior and exterior work on the structure collapse mechanism may be equated, i.e.

$$6M_p\Theta = 9P_uL\Theta/6 \quad \text{or}$$
$$P_u = 4M_p/L = 4\cdot 8638/1200$$
$$P_u = 28.793\ \text{kN}.$$




(a) Formation of hinges (b) Vertical deflection at joint 5

Fig. 3 Formation of plastic hinges and vertical deflection at joint 5

Collapse load factor:

$$\lambda_c = P_u/P = 28.793/14.0$$
$$\lambda_c = 2.057 > \lambda_{req} = 1.70 \text{ (see Refs. 1,4,6,12 )}$$

Maximum vertical deflection of the frame occurs at the ridge line (joint 5). Deflection of joint 5 and hinge formation history is shown in Fig. 3. First and second plastic hinge are to be formed at eave lines at load P=22.37 kN. The last hinge is to be formed at ridge line at load P=28.79 kN and vertical deflection of -15.66 cm. For this case of loading the structure is overdesigned 21% (=0.357/1.7).

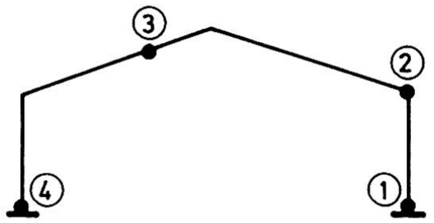
(b) Case II - Dead load + Live load + Wind

Similarly, as it was done in Case I, equating the internal and external work on the collapse mechanism, the collapse load is obtained:

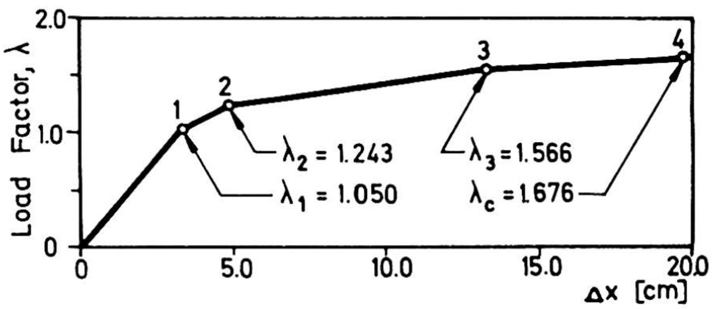
$P_u$ = 23.466 kN, at the collapse load factor

$\lambda_c$ = 1.676 > $\lambda_{req}$ = 1.50 (see Refs. 4,13 ),
= 1.36 (see Ref. 6 ), and
= 1.30 (see Ref. 1 ).

The structure has an additional carrying capacity of 11.7% (=0.176/1.50). The maximum deflection occurs at lee-ward wind eave (joint 8). The plastic hinge history formation and deflection of joint 8 are shown in Fig. 4.




(a) Formation of hinges




(b) Horizontal deflection at joint 8

Fig. 4 Formation of plastic hinges and horizontal deflection at joint 8

### 3.2 Alternating plasticity collapse analysis

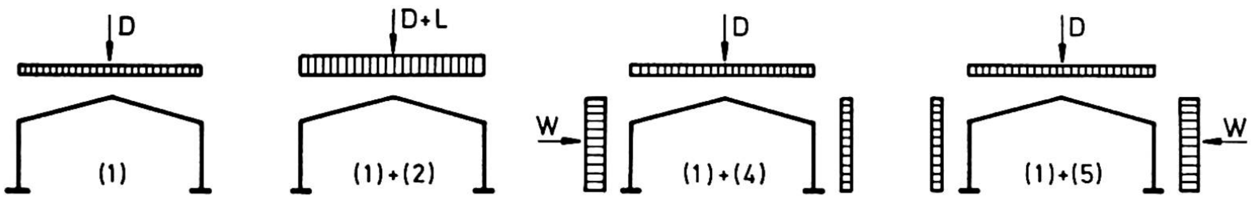
For the given specification loads the elastic bending moments at critical sections obtained by the second-order analysis are shown in Table 1.(in kNcm).

| | Loading | | Section | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Hor. | Vert. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | O | D | 1615 | -1935 | - 20 | 895 | 805 | 895 | - 20 | -1935 | 1615 |
| 2 | O | L | 2910 | -3495 | - 40 | 1615 | 1460 | 1615 | - 40 | -3495 | 2910 |
| 3 | O | D+L | 4540 | -5460 | - 65 | 2530 | 2290 | 2530 | - 65 | -5460 | 4540 |
| 4 | W | O | -4050 | 1495 | 2050 | 1490 | -180 | -1615 | -1935 | -1135 | 3700 |
| 5 | -W | O | 3700 | -1135 | -1935 | -1615 | -180 | 1490 | 2050 | 1495 | -4050 |

Table 1. Elastic bending moments for specified loading

Using the value for elastic bending moments from Table 1 , the shakedown factors for alternating plasticity are calculated using Eq. (1). The loading cycles a, b, and c are combined as follows:

Cycle a - load combination (2),(4) and (5)




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $M_i^+$ | 6610 | 1495 | 2050 | 3105 | 1460 | 3105 | 2050 | 1495 | 6610 |
| $M_i^-$ | -4050 | -4630 | -1975 | -1615 | - 360 | -1615 | -1975 | -4630 | -4050 |
| $M_i^+ - M_i^-$ | 10660 | 6125 | 4025 | 4720 | 1820 | 4720 | 4025 | 6125 | 10660 |

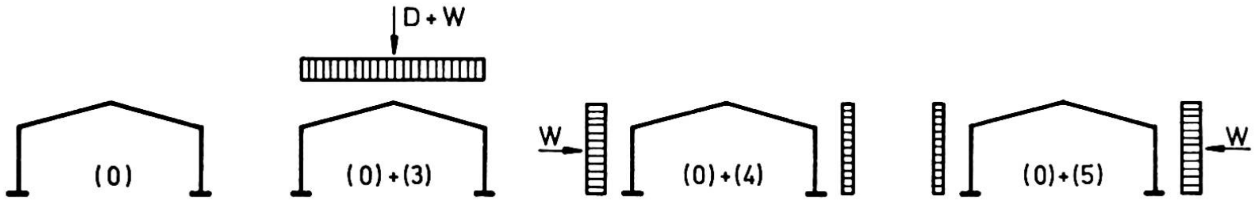
Table 2 Cycle a - Extreme bending moments for load combination (2), (4) and (5)

Applying Eq. (1) and using moment value from Table 2 the load factor is:

$$\lambda_{s,a} = \frac{2M_p}{(M_i^e)_{max} - (M_i^e)_{min}} \cdot \frac{1}{\alpha} = \frac{2 \cdot 8638}{10660} \cdot \frac{1}{1.13}$$

$\lambda_{s,a}$ = 1.434 (Resistance of the structure is reduced to 1.434/1.676 = 0.856)

Cycle b - load combination (3),(4) and (5)




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $M_i^+$ | 8240 | 1495 | 2050 | 4020 | 2290 | 4020 | 2050 | 1495 | 8240 |
| $M_i^-$ | -4050 | -6595 | -2000 | -1615 | - 360 | -1615 | -2000 | -6595 | -4050 |
| $M_i^+ - M_i^-$ | 12290 | 8090 | 4050 | 5635 | 2650 | 5635 | 4050 | 8090 | 12290 |

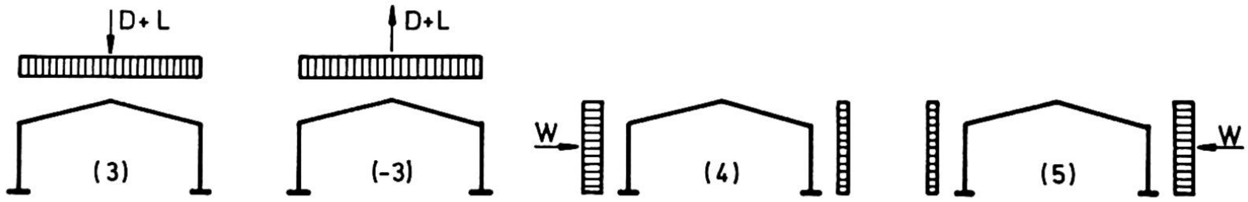
Table 3 Cycle b - Extreme bending moments for load combination (3),(4) and (5)

The load factor for cycle b:

$$\lambda_{s,b} = \frac{2 \cdot 8638}{12290} \cdot \frac{1}{1.13}$$

$\lambda_{s,b}$ = 1.244 (Resistance of the structure is reduced to 1.244/1.676 = 0.742).

Cycle c - load combination (3),(-3) and (5)




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $M_i^+$ | 8240 | 6955 | 2115 | 4020 | 2290 | 4020 | 2115 | 6955 | 8240 |
| $M_i^-$ | -8590 | -6595 | -2000 | -4145 | -2650 | -4145 | -2000 | -6595 | -8590 |
| $M_i^+ - M_i^-$ | 16830 | 13550 | 4115 | 8165 | 4940 | 8165 | 4115 | 3550 | 16830 |

Table 4 Cycle c - Extreme bending moments for load combination (3),(-3),(4) and (5)

The load factor for cycle c:

$$\lambda_{s,c} = \frac{2 \cdot 8638}{16830} \cdot \frac{1}{1.13}$$

$$\lambda_{s,c} = 0.908$$

A coefficient of the load variation $k_s$ equals to a ratio of alternating plus 50 per cent of loading to the overall load acting on the structure. For the considered loading cycles the following values are obtained:

$$k_{s,a} = (25.96+0.5 \cdot 54.0)/109.96 = 0.482$$
$$k_{s,b} = (26.96+0.5 \cdot 84.0)/109.96 = 0.618, \text{ and}$$
$$k_{s,c} = 109.96/109.96 = 1.000$$

The load cycle c corresponds to alternating of the total load acting on the structure in practical sense, but not in a true mathematical meaning because of the wind suction effect at the leeward side of the building.

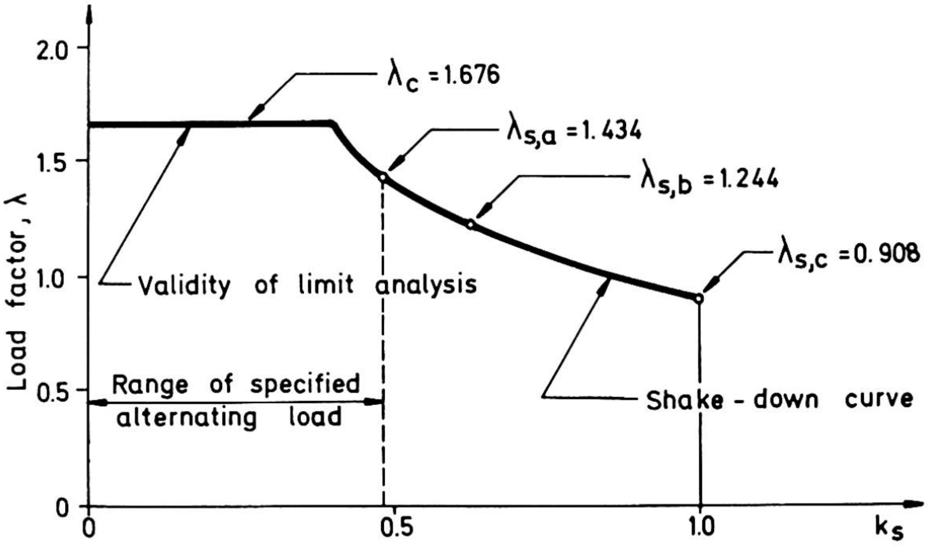



Fig. 5 Range of limit analysis and shake-down validity

In Fig. 5 the limit analysis validity range is represented by a horizontal line at $\lambda_c = 1.676$. The curved line shows the reduction of the structure resistance due to variation of repeated loads.

## 4. CONCLUSIONS

The effect of specification repeated loads on an industrial steel frame was investigated and the following conclusions can be drawn:

- The reduction of the structure carrying resistance under specification variable loading was 14.4 per cent (load cycle a);
- If the total vertical load was variably repeated then the carrying resistance reduction of the structure would be 25.8 per cent (load cycle b);
- Up to the load ratio k=0.40 there is no theoretical reduction of the structure carrying resistance due to variable repeated loading given in the specification.

Some countries are practising plastic design of steel structures [1,2,4,6,8], and others are presently in the process of either adapting or considering adaptation of their editions of steel design specification based on limit analysis and plastic design. The reduction of the carrying resistance in the considered example seems extensive and cannot be compensated by strain hardening. In view of this the limit of "predominantly static load" in these specifications should be more precisely defined for which the analysis due to variation of repeated loads is not required.

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