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SECONDARY STRESSES IN TRUSSES.

CONTRAINTES SECONDAIRES DANS LES TREILLIS.

NEBENSPANNUNGEN IN FACHWERKEN.

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The study of secondary stresses in trusses owing to rigidity in the joints can be traced back to 1878 (Manderla) and has since then often been reconsidered under very different points of view (by ENGESSER, W. RITTER, MOHR, GEHLER, BLEICH [1]) and many others).

Further WITMER [2]) has called attention to a method, the principle of which is to cut all the members in the loaded and distorted truss and at the same time retain the joints in an unrotated position (compared with the unloaded construction) [3]). Afterwards the elastic continuity of the construction is restored by forcing the sections of the various members together in their proper relative positions. By this performance moments will be introduced at the fixed joints, and those moments can readily be computed. This offers the opportunity of establishing the elastic equations, which has been done by A. OSTENFELD [4]) or, as WITMER has preferred, to apply the Cross method [5]), which after all is another way of solving these equations.

The writer, who has had the opportunity of dealing with similar problems, and in particular with the question of restoring continuity in distorted constructions [6]), calls attention to the fact, that in WITMER's case it is not necessary to construct more than the tentative WILLIOT's diagram, as this will give a sufficient characteristic of the distorsion. As a matter of fact, the relative displacement D of the ends of a member with respect to a direction perpendicular to this member, scaled on the tentative WILLIOT's diagram, can be used directly in the expression for the fixed-end moments without changing the final result, and in addition the small advantage will be gained that $D = 0$ for the member not rotated (in WITMER's example: centerpost 6-7).

Whenever using tentative or final WILLIOT's diagrams the fixed-end moments from this procedure are rather large compared with the final ones [7]), which again means considerable "moment distribution" and "carrying over" before the quantities involved become negligible. The writer therefore recommends solving the problem in a modified way, which substantially does away with the above mentioned objection.

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1) BLEICH: Die Berechnung statisch unbestimmter Tragwerke nach der Methode des Viermomentensatzes. Springer, Berlin 1918.

2) WITMER: Secondary Stresses in Trusses. Eng. News-Record 1932, page 132.

3) The same procedure has been proposed by SUTER: Die Methode der Festpunkte. Springer, Berlin 1923. S. THOMPSON and R. W. CUTLER: Proc. Am. C. E. 1931, page 1362.

4) A. OSTENFELD: Die Deformationsmethode. Springer, Berlin 1926.

5) CROSS: Analysis of continuous frames by distributing fixed-end moments. Proc. Am. C. E. 1930, page 919.

6) EFSEN: Die Methode der primären Momente. C. E. Gad, Copenhagen 1931.

7) To emphasis this, in WITMER's case we have, for joint 2:

Fixed-end moments:	3168	5056	801	135
Final moments:	156	352	136	60.

The starting point for this new procedure is the study of the distortion of the angles between the members uniting at a joint when the joints are absolutely hinged. This change in angle can be expressed by:

$$\alpha = D/l,$$

l being the length of the member, and D to be taken from any (tentative or final) WILLIOT's diagram. Now, to restore the elastic continuity in a joint, the individual members should be forced back through an angle α. This will for the member 1—2 in fig. 1 induce the following moment in section 1—2:

$$M_{1-2} = c_{2-1} \cdot s_{1-2} \cdot \alpha_{1-2}, \tag{1}$$

where

$$s = \frac{EI}{l}.$$

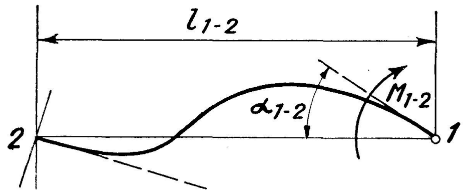

Fig. 1.

The constant c depends on the rigidity assumed in joint 2 and is for the Cross method = 4. Applying the method of the writer (denoted in the following as the P. M.-method, which stands for the Preliminary Moments Method), c will have values between 3 and 4, these values representing the solution of the basic equations for the system (3 for hinged ends, 4 for absolutely fixed ends). The restoring of the original angle between all the members in a joint means inducing the following moment:

$$\Sigma M = \Sigma c \cdot s \cdot \alpha, \tag{2}$$

where the summation is to incorporate all the members uniting at that special joint. The moment ΣM will rotate the joint through an angle:

$$\frac{\Sigma M}{\Sigma c \cdot s},$$

after which the moment in the individual member is:

$$Y_{1-2} = M_{1-2} - c_{2-1} \cdot s_{1-2} \cdot \frac{\Sigma M}{\Sigma c \cdot s} \tag{3}$$

and for the Cross method specially:

$$Y_{1-2} = M_{1-2} - s_{1-2} \cdot \frac{\Sigma M}{\Sigma s}. \tag{4}$$

Y in (3) is the preliminary moment in the sense of the P. M.-method, whilst Y in (4) is the Cross fixed-end moment.

Obviously, all the angles α in formula (2) can be changed by the same amount without altering the value of Y.

Coming to the following part of the solution, the choice between the Cross and the P. M.-methods can be considered more or less a matter of taste. Against the simple way of deriving at the various constants by the first method stands the better convergence of the other, and also the fact that the moment distribution occurs without "return" moments as is the case with the Cross method where the carried-over moment is affected by the subsequent moment distribution in the joint. This last circumstance, in the opinion of the writer, makes the P. M.-method preferable.

We shall now apply the foregoing to joint 2 in the example studied by WITMER. We start by tabulating the following data:

	Member	l	$10^6 \cdot \alpha$	$10^6 \cdot \alpha - 3{,}55$	$s/10^6$
Joint 2	2—1	478″	$+1545/478 = 3{,}232$	$-0{,}318$	165
	2—4	300″	$+1065/300 = 3{,}550$	0	237
	2—5	478″	$+1266/478 = 2{,}649$	$-0{,}901$	51
	2—3	372″	$+\ 920/372 = 2{,}473$	$-1{,}077$	9

By the Cross method:

$$\begin{aligned} M_{2-1} &= 4 \cdot 165 \cdot (-0{,}318) = -210 \\ M_{2-4} &= 4 \cdot 237 \cdot \quad 0 \quad = \quad 0 \\ M_{2-5} &= 4 \cdot \ 51 \cdot (-0{,}901) = -184 \\ M_{2-3} &= 4 \cdot \quad 9 \cdot (-1{,}077) = -\ 39 \end{aligned}$$

$$\begin{array}{llll} Y_{2-1} = -210 & + \left\{ \begin{array}{r} 165 \\ 237 \\ 51 \\ 9 \end{array} \right\} \cdot \dfrac{433}{462} = & \left\{ \begin{array}{r} -\ 55 \\ +222 \\ -136 \\ -\ 31 \end{array} \right. & \begin{array}{l} \text{In-Lb.} \\ \text{"} \\ \text{"} \\ \text{"} \end{array} \\ Y_{2-4} = \quad 0 & & & \\ Y_{2-5} = -184 & & & \\ Y_{2-3} = -\ 39 & & & \\ \Sigma M = -433, & \Sigma s = 462 & \Sigma Y = 0 & \end{array}$$

When using the P. M.-method the first step is to calculate the basic constants for the system [8]). The result of this calculation may be taken from

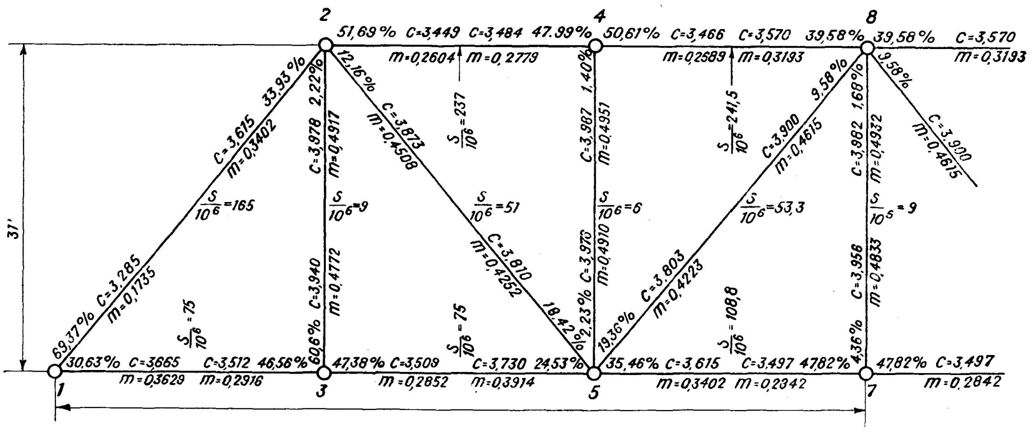

Fig. 2.
The basis constants — Die Grundkonstanten — Les constantes fondamentales

fig. 2. For joint 2,

$$c_{1-2} = 3{,}285, \quad c_{4-2} = 3{,}484, \quad c_{5-2} = 3{,}810, \quad c_{3-2} = 3{,}940.$$

[8]) See: Die Methode der primären Momente, § 13.

Applying these values to the formula above, we have:

$$
\begin{aligned}
M_{2-1} &= 3{,}285 \cdot 165 \cdot \alpha_{2-1} = 542 \cdot (-0{,}318) = -172 \\
M_{2-4} &= 3{,}484 \cdot 237 \cdot \alpha_{2-4} = 825 \cdot \quad 0 = \quad 0 \\
M_{2-5} &= 3{,}810 \cdot \ \ 51 \cdot \alpha_{2-5} = 194 \cdot (-0{,}901) = -175 \\
M_{2-3} &= 3{,}940 \cdot \ \ \ 9 \cdot \alpha_{2-3} = \ 35 \cdot (-1{,}077) = -\ \ 38
\end{aligned}
$$

$$
\begin{aligned}
Y_{2-1} &= -172 \\
Y_{2-4} &= \quad 0 \\
Y_{2-5} &= -175 \\
Y_{2-3} &= -\ 38
\end{aligned}
\quad
\begin{matrix} + \\ + \\ + \\ + \end{matrix}
\left\{\begin{matrix} 542 \\ 825 \\ 194 \\ 35 \end{matrix}\right\} \cdot \frac{385}{1596} =
\left\{\begin{matrix} -\ \ 41 \\ +199 \\ -128 \\ -\ \ 30 \end{matrix}\right.
\begin{matrix} \text{In-Lb.} \\ \text{"} \\ \text{"} \\ \text{"} \end{matrix}
$$

$$
\Sigma M = -385, \ \Sigma c \cdot s = 1596 \qquad \Sigma Y = 0
$$

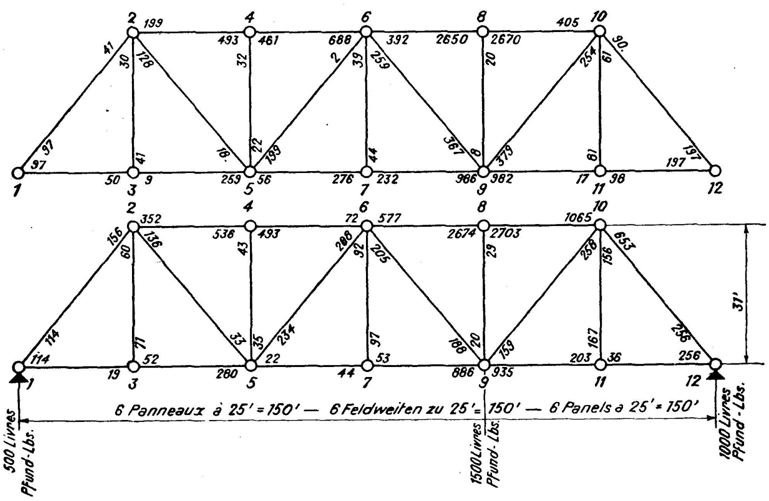

Fig. 3.
Preliminary Moments.
Vorläufige Momente.
Moments préliminaires.

Fig. 4.
Final Moments.
Endgültige Momente.
Moments finals.

Treating the other joints in the same manner gives preliminary moments as indicated in fig. 3 and this will again, through a moment distribution, result in the final end-moments seen in fig. 4. Note the excellent correspondance between these and the moments calculated by Witmer. In the figures the moments are written at the tension side of the members.

Summary.

As is well known, the main stresses in a truss are found by assuming that the members are frictionless hinged at the joints. From the main stresses the elongation of the members can be calculated, and thereafter the distortion of the truss can be found by means of a Williot's diagram.

Now, to find the secondary stresses owing to rigidity in the joints, the joints are retained in the position determined by the main stresses, thus giving a system with an immovable frame diagram. This, in fact, means adopting the assumption that the displacements of the joints due to secondary stresses are negligible as compared with the displacements caused by the main stresses.

The distortions of the angles between the members uniting in a joint when the joints are absolutely hinged can be taken from Williot's diagram. By forcing the ends of the members back through angles equal to the above mentioned distortions the elastic continuity of the construction will be restored. By this performance some external moments are introduced at the joints and these can readily be computed. The following part of the problem consists in eliminating these moments without disturbing the elastic continuity

of the system. As the system is one with an immovable frame diagram, this can be done by adopting either the Cross method or the P. M.-method given by the author.

Zusammenfassung.

In einem Gittersystem findet man bekanntlich die Grundspannungen unter der Voraussetzung, daß die Stäbe reibungslos mit einander verbunden sind. Durch die Grundspannungen berechnen sich die Änderungen der Stablängen, und die Knotenpunktsverschiebungen können darnach mittels einem WILLIOT'schen Verschiebungsplan gefunden werden.

Um jetzt die Nebenspannungen zu finden, die von der biegungsfesten Verbindung der Gitterstäbe in den Knotenpunkten herrühren, denkt man sich die Knotenpunkte in der von den Grundspannungen bestimmten Stellung festgehalten, wobei ein System mit unbeweglicher Knotenpunktsfigur entsteht. Man geht somit von der angenäherten Voraussetzung aus, daß die von den Nebenspannungen herrührenden Knotenpunktsverschiebungen im Vergleich mit den Beiträgen der Grundspannungen unbedeutend sind.

Der WILLIOT'sche Verschiebungsplan gibt ohne weiteres über die Winkeländerungen Bescheid, welche — reibungslose Verbindungen in den Knotenpunkten vorausgesetzt — zwischen den in einem Knotenpunkt zusammenstoßenden Stäben vorgegangen sind. Läßt man hiernach die Stabenden solche Winkelverdrehungen erleiden, daß die oben erwähnten Winkeländerungen verschwinden, so ist der elastische Zusammenhang des Systems wieder hergestellt; zugleich sind aber einige äußere Knotenpunktsmomente eingeführt, und diese lassen sich leicht berechnen. Die Aufgabe besteht nun darin, diese Momente zu eliminieren ohne den elastischen Zusammenhang des Systems zu stören. Dies kann, da es sich um ein System mit unbeweglicher Knotenpunktsfigur handelt, durch Anwendung der Cross-Methode oder der vom Verfasser angegebenen P. M.-Methode gemacht werden.

Résumé.

On sait que dans un système en treillis, on détermine les contraintes principales en partant de cette hypothèse que les barres sont assemblées entre elles sans frottement. En partant des contraintes principales, on calcule les modifications de longueur des barres et les déformations des noeuds d'assemblages peuvent être trouvées ensuite au moyen du diagramme de cheminement de WILLIOT.

Pour déterminer les contraintes secondaires qui résultent de la rigidité des assemblages des barres dans les noeuds, on suppose les noeuds maintenus dans la position déterminée par les contraintes principales, ce qui correspond à l'existence d'un système de noeuds d'assemblage immobile. On part ainsi de l'hypothèse approchée suivant laquelle les déformations provoquées dans les noeuds d'assemblage par les contraintes secondaires sont faibles en comparaison des valeurs des contraintes principales.

Le diagramme de translation de WILLIOT donne directement des renseignements sur les déformations angulaires qui se produisent entre les différentes barres réunies dans un même noeud d'assemblage, en admettant que les assemblages sont exempts de frottement dans les noeuds. Si l'on

laisse les extrémités des barres supporter effectivement les rotations angulaires correspondantes, de telle sorte que les déformations angulaires disparaissent, on arrive à nouveau, pour le système, à un état élastique, tout en introduisant toutefois quelques moments extérieurs par rapport aux noeuds d'assemblage, moments qu'il est difficile de calculer. On en arrive alors à la nécessité d'éliminer ces moments sans provoquer de perturbation dans l'état élastique du système. Ce résultat peut être obtenu en employant la méthode de Professor CROSS ou la méthode P. M., indiquée par l'auteur, étant donné qu'il s'agit d'un système rigide de noeuds d'assemblage.