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Autor: Wilson, W.M.

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TESTS OF REINFORCED CONCRETE ARCH BRIDGES.

VERSUCHE AN EISENBETON-BOGENBRÜCKEN.

ESSAIS SUR PONTS EN ARC DE BÉTON ARMÉ.

W. M. WILSON, Research Professor of Structural Engineering, University of Illinois, Urbana, Illinois, U. S. A.

Synopsis.

It was the privilege of the author to serve for a number of years on the Special Committee on Concrete and Reinforced Concrete Arches of the American Society of Civil Engineers [1]), and this paper ist based upon the results of tests made for the Committee.

The investigations, which included laboratory tests of specially constructed experimental structures and field observations on actual bridges, pertained to such phases of design as: Verification of the elastic theory as applied to single-span arch ribs and multiple-span bridges consisting of three arch ribs on slender piers; influence of various types of deck upon the action of the rib; influence of intermediate expansion joints; movement of piers during the construction of multiple-span arch bridges; effect of climatic changes upon a multiple-span bridge; influence of time yield in concrete upon the stresses in a single-span arch; and the strength developed by concrete in an arch rib relative to the strength of similar concrete developed in 6-in. by 12-in. control cylinders.

There were a large number of elaborate tests made and it is only possible in this paper to outline the tests and give an abstract of the results with a summary of the conclusions. References to detailed reports of the individual tests are given in the bibliography at the end of the paper.

1. Verification of Elastic Theory as Applied to a Single-Span Arch Rib: A reinforced concrete arch rib is usually constructed without hinges and the ends are assumed to be fixed by the abutments which support them. A structure of this type is statically indeterminate and the abutment reactions and the stresses in the rib can only be determined algebraically by taking into consideration the relation between stress and strain for the material of which the rib is composed. Many methods have been described for analyzing structures of this type but all methods are fundamentally alike in that they supplement the laws of statics with the effect of the relation between stress and strain upon the geometry of the structure. And, if the same relation of stress to strain is used, all methods will give the same results except for minor variations due to errors resulting from the substitution of a summation of small finite quantities for an integration of differential quantities. The relation of stress to strain being known, the solution is a problem in algebra and geometry, and the solution

[1]) See Acknowledgments, Section 14 of this paper.

needs no experimental verification. But the relation between stress and strain is not known for concrete and an assumed relation is used in the algebraic methods of analysis used by designers.

The algebraic methods usually used in the analysis of concrete arch ribs are based on three assumptions: 1. Plane sections remain plane. 2. Stress is proportional to strain. 3. The relation between stress and strain is the same at all sections and at all stresses, tension and compression. The author knows of no experimental verification of the first. There have been a large number of tests indicating that the second and third are seriously in error. Considered from a purely academic viewpoint, an error in the assumptions upon which the method is based produces an error in the results. But the engineer, granting the possibility of academic inaccuracy, is interested primarily in

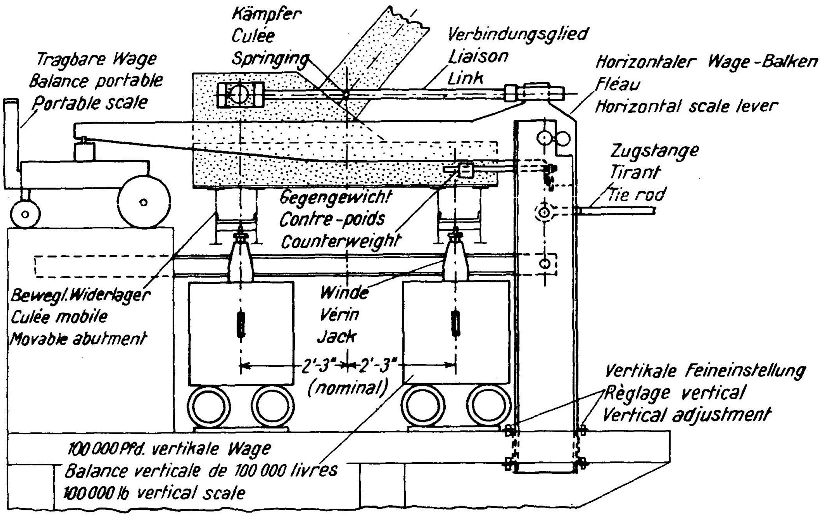


Fig. 1.
Wägeapparat und Widerlagerunterstützungen.
Appareil de pesage et appuis des culées.
Weighing Apparatus and Supports for Abutments.

the question. Is the magnitude of the error large enough to affect the usefulness of the method as a means of design? Experimental studies were made to answer this question.

A number of tests were made in which the abutment reaction components of an arch rib were weighed in order that the weighed values might be compared with values computed by an algebraic analysis. Nine similar arch ribs were used in the first series[2]) (1) of tests, each having a span of 17 ft. 6 in. A single specimen having a span of 27 ft. 0 in. was used in the second series (2). For both series, the abutments were carried on specially designed scales which measured, independently, the three reaction components, H, V and M. Moreover, provision was made to control the three components (x, y and Θ) of movement of the abutments and, if movement occurred, return the abutments to their original position relative to each other.

The weighing apparatus used in the second series is shown in Fig. 1, and the general dimensions of the specimen are shown in Fig. 2. Because the arch was so short, the stresses due to its own weight were much smaller

[2]) Numerals indicate references to be found in the bibliography at the end of this paper

than the dead load stresses in a long arch. Additional load was obtained by hanging concrete blocks of known weight at the load points, which was of such magnitude and distributed in such a manner as to produce stresses commensurate with the dead-load stresses in an arch bridge having a span of 135 ft. This is known as the design dead load and is shown in Fig. 3. The design live load is also shown in the figure and is commensurate with the live load on a 135-ft. highway bridge. The structure was designed to carry the design dead load and one live load. In the test to determine the load-carrying capacity of the structure, the load was increased in multiples of One Live Load. The position of the thrust line was determined by the elastic theory and by two experimental methods. By one experimental method, the thrust line was located from the weighed reaction components, by the other, by the measured strain. For the latter method, the strain was measured on

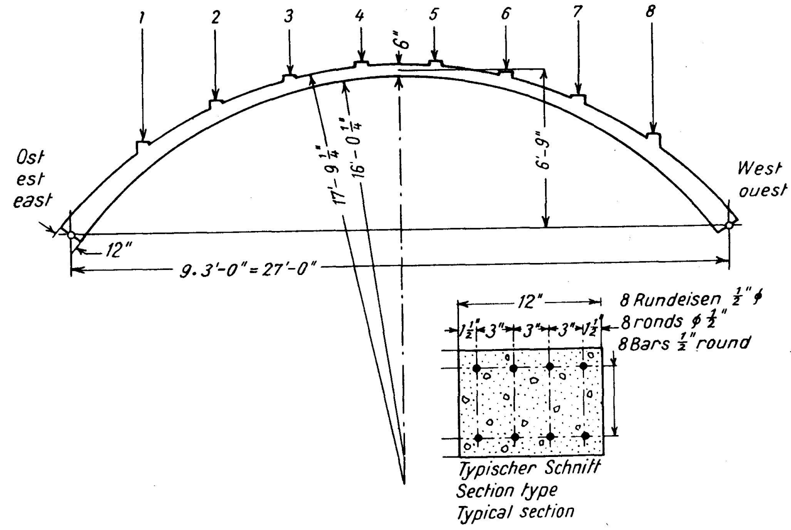


Fig. 2.

Abmessungen der Bogenrippe — Dimensions de l'arc — Dimensions of Arch Rib.

the intrados and extrados at sections midway between each pair of adjacent load points, and the center of pressure, which is a point on the line of thrust, was determined from the resulting strain diagrams. The position of the thrust line as determined by the three methods and at various loads up to the dead load plus five times the live load, is shown in Fig. 4.

A series of tests was also made on the 27-ft. arch to determine the influence ordinates for the reaction components in the following manner. With the dead load on the arch the changes in the reaction components at the abutments were weighed, first when a unit load of 2000 lb. was applied and then when it was removed, successively, for each load point. The abutments were adjusted for position before and after each load change so that none of the changes in the reaction components would be due to a movement of the abutments. The influence diagrams for the reaction components determined experimentally in this manner and as determined by the elastic theory, are compared in Fig. 5.

Longitudinal measurements on the ribs indicated that the modulus of elasticity of the concrete in the arches was much greater near the ends than at the crown, the maximum value being twice as great as the minimum value for some specimens. But this variation does not appear to greatly affect the reaction components due to load, a conclusion that was anticipated by Professor HARDY CROSS[3]) as a result of algebraic analyses (3).

Tests were also made in which, with no load change and with one abutment fixed, one abutment was given, successively, each of the three components of movement (x, y and Θ). The magnitude of each movement and the resulting changes in the reaction components were weighed, thereby determining the relation between unit movements of an abutment and the changes in the abutment reaction components (2).

As a result of the tests outlined in this section, the author has drawn the conclusions relative to the action of a single-span arch rib:

In analyzing an arch rib to determine the reaction components due to load, the modulus of elasticity of the concrete may be assumed to have the same value at all stresses and at all sections; and the moment of inertia of

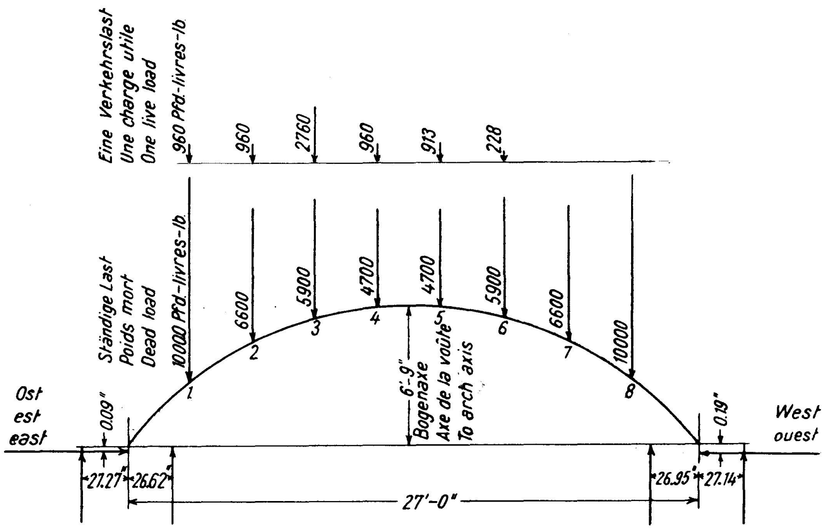


Fig. 3.
Belastungsschema zur Berechnung der Bogenrippe.
Diagramme de charge de l'arc.
Design Load for Arch Rib.

a section may be based upon the assumption that the concrete resists tension. The errors in these assumptions will not materially affect the results.

The measured and computed values of the reaction components due to abutment displacements are in close agreement for the arches tested if the computed values are based upon a value of the modulus of elasticity of the concrete determined from strain readings on the rib itself.

2. Verification of Elastic Theory as Applied to a Three-Span Structure Consisting of Ribs Without Deck Supported on Slender Piers: Tests were made on a three-span structure consisting of ribs without deck supported on slender piers (2). The object of the tests was to compare the experimental and theoretical values of the reaction components at the springings of the arches. The general dimensions of the structure and the location of the loading points are shown in Fig. 6, and the details of the ribs and piers are shown in Figs. 2 and 7. The ribs and piers are continuous with each other. The abutment supports are the same as those used in the test of the 27-ft. single span, and are

[3]) Professor of Civil Engineering, Yale University.

shown in Fig. 1. The supports for the piers are shown in Fig. 8. These apparatus provide means for weighing, separately, the three components of the reactions of the pier base (H, V and M) and for giving the pier base, separately, the three components of movement (x, y and Θ). The scales and the apparatus for moving the pier bases are attached to the auxiliary base provided for each pier, but the apparatus for determining the position of the base can be applied at any point on the pier and the weighed forces can

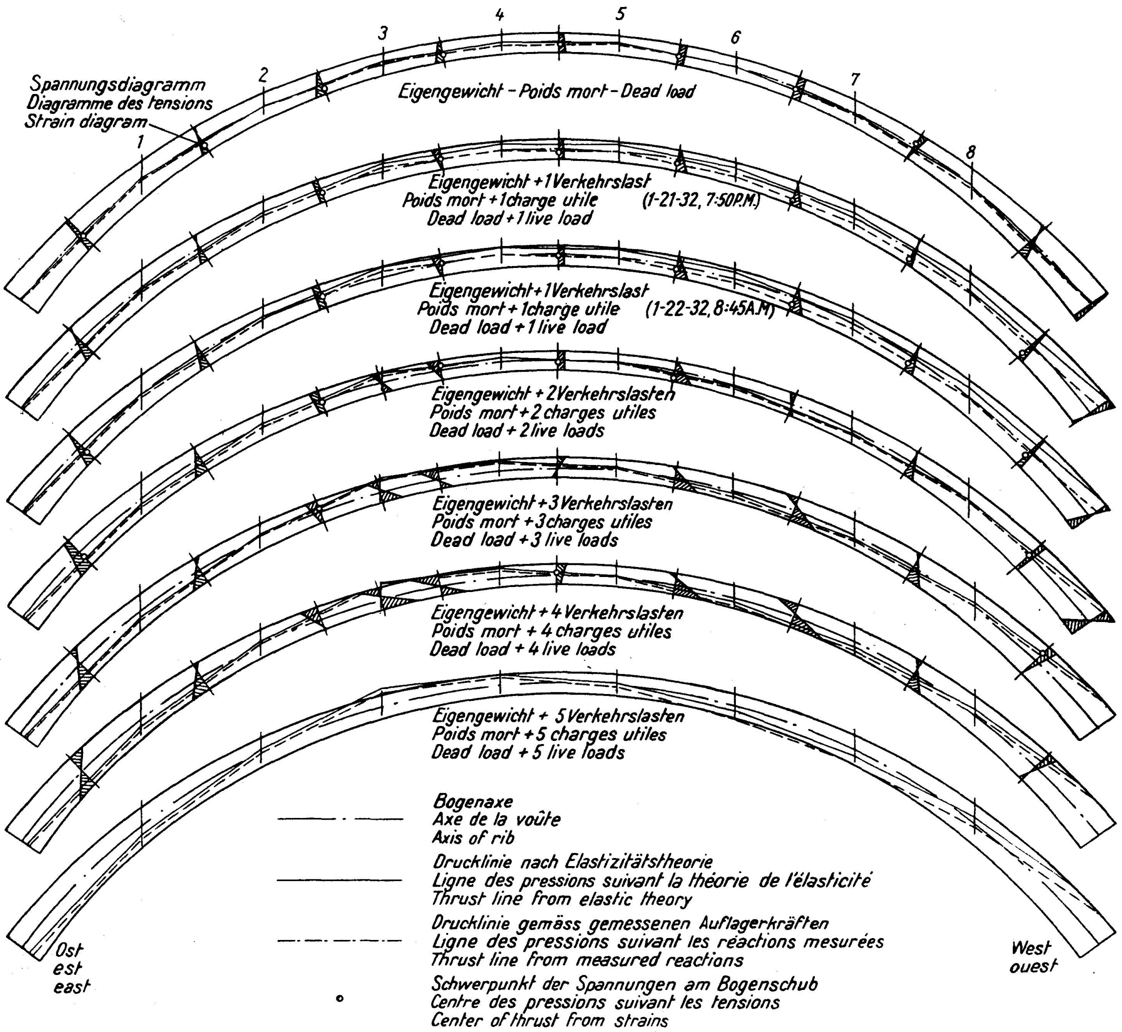


Fig. 4.
Lage der Drucklinie. Bruchversuch.
Position de la ligne des pressions. Essai de rupture.
Position of Thrust Line. Test to Failure.

be transferred to any point on the pier which it is desired to consider as the pier base. In making a load test for which the bases are to be fixed, if the apparatus for controlling the movement of a point on the pier is attached 20 ft. below the springings of the arch, the effective height of the pier is 20 ft.; if the apparatus is attached 15 ft. below the springing, the effective height of the pier is 15 ft. In this manner, the one structure was tested with 20-ft. piers, with 15-ft. piers and with 10-ft. piers.

Because the span of the arches is so small relative to real structures, the stresses due to the weight of the structure are much less than the dead-load stresses in an arch bridge. Additional load was therefore added, the same as in the test of the single-span arch, which was designated as the design dead load. This is shown in Fig. 6. It is of such magnitude and distributed in such a manner that it produces stresses in the ribs commensurate with the dead-load stresses in a 135-ft. arch span.

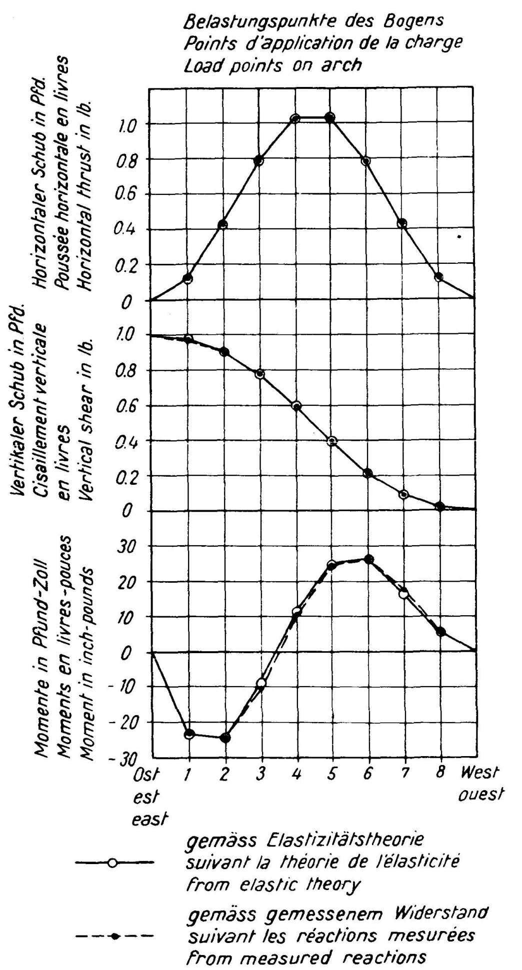


Fig. 5.

Einflußlinien für die Reaktionen am östlichen Widerlager — Lignes d'influence des réactions à la culée est — Influence Lines for East Abutment Reactions.

Two series of tests were made in the verification of the elastic theory. For one series, the design live load shown in Fig. 6 was added to the arch and the position of the thrust line was determined by two methods: By weighing the reactions at the bases of the piers; and by locating the center of pressure, which is a point on the thrust line, of the various sections from the measured strain at the intrados and extrados of the sections. For the other series, the changes in the reaction components were determined when a unit load of 2000 lb. was first added and then removed, successively, from the various load points. The reaction components (*H*, *V* and *M*) were determined for each end of all spans. The values determined from the tests were then compared with the values obtained by the elastic theory.

The positions of the thrust line, as determined by the elastic theory and by the two experimental methods, are shown in Fig. 9 for structures having pier heights of 20 ft., 15 ft. and 10 ft. The results obtained by the three methods are in almost perfect agreement.

The influence lines for the reaction components at the end *B* of the center span *BC* are shown in Fig. 10. The light full line represents values determined by the elastic theory, the heavy full line represents values at *B* determined by unit loads and the broken line represents values at C determined by unit loads with the diagram turned end for end so that it could be superimposed on the heavy full-line diagram. The dot-and-dash line, shown for the center span only, is the influence line, by the elastic theory, for a single span with fixed ends. It is of interest to note that, for the structure on high piers, the influence lines for the reaction components at the ends of the center span determined by the various methods agree closely, and that the influence lines for a single span with fixed ends differ materially from the corresponding influence lines for a three-span structure on high piers.

The complete report of these tests (2) contains diagrams similar to those contained in Fig. 10 for the reaction components at both ends of all spans for structures having pier heights of 20 ft., 15 ft. and 10 ft.

The influence ordinates for stress in the rib at load points $C\,3$ and $C\,4$, Fig. 6, were computed from the influence ordinates for the reaction components at B of BC and it was found that, for the system of live loads used, the maximum stress for the structure on 20-ft. piers occurs at $C\,4$. The influence lines for the stress at this point are given in Fig. 11.

As a result of the tests outlined above and presented in full in the detailed report (2), the author has drawn the following conclusions relative to the behavior of a three-span series of arch ribs on high piers:

The elastic theory based upon the usual assumptions and taking into account the elastic deflection of the piers, gives values for the moment, thrust

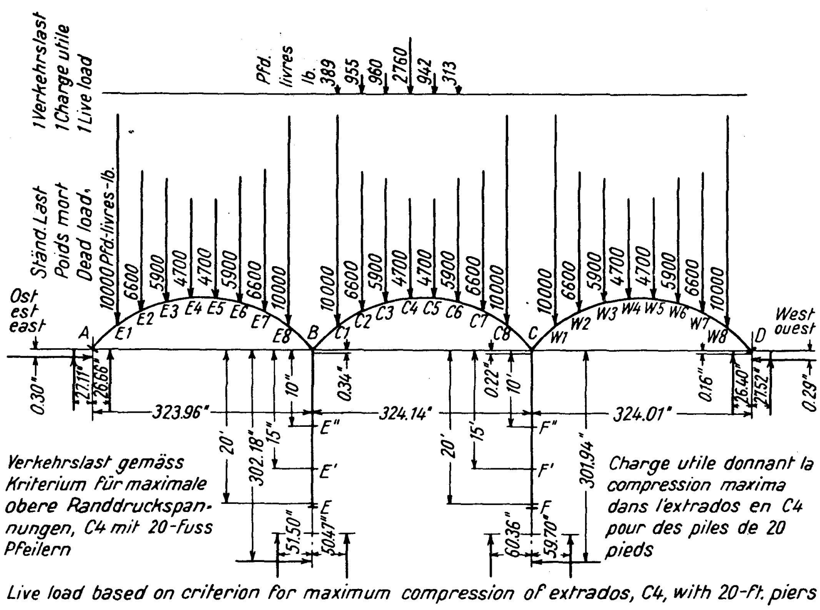


Fig. 6.
Maximal-Spannungen in $C\,4$ aus Verkehrslast.
Charge utile engendrant la tension maxima en $C\,4$.
Live Load Producing Maximum Stress at $C\,4$.

and shear at various sections that agree with the measured values within the tolerance of the tests.

In analyzing a multiple-span arch series on elastic piers it may be assumed that E has the same value at all sections and at all stresses, tension or compression, and the moment of inertia of a section may be computed on the basis that concrete takes tension. Moreover, the value of E used in the analysis may differ considerably from the true value without seriously affecting the reactions due to loads.

For the structure tested, the flexure of the piers increased the maximum live-load stress in the rib near the crown of the center span and near the abutment of the end span, but decreased the corresponding function for the springing of the center span.

The maximum unit compression due to design load (dead load plus live load) was 13 per cent greater for the three-span series on 20-foot piers than for a similar single span with fixed ends.

Considerable cracking of the arch rib did not greatly alter the position or magnitude of the thrust.

3. Effect of Climatic Changes Upon a Multiple-Span Reinforced Concrete Arch Bridge: Field observations (4) were made on the Gilbert Street highway bridge at Danville, Illinois, to determine the influence of climatic changes upon the action of the structure. The bridge, shown in Fig. 12, consists of six open-spandrel concrete arch spans having a total length of approximately 1000 ft. The deck is so low as to be integral with the rib at the crown, thus forming a saddle at the middle of the rib about one-fourth as long as the span of the arch. The piers extend upward to the surface of the deck and there is an expansion joint on each side of each pier. There ist also an expansion joint at each end of the saddle for all except the two end spans. The largest span is in the middle, and the ribs are of such shape and are so located vertically relative to each other that there is an unbalanced thrust due to both the dead load and to a rising temperature which tends to tip the piers outward.

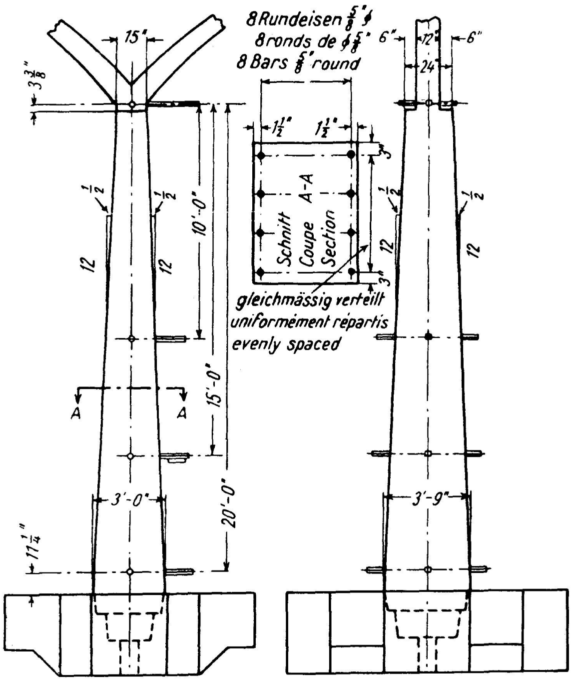


Fig. 7.
Pfeilerabmessungen.
Détails de pile.
Details of Pier.

Observations were made to determine the rotation of six piers, the opening and closing of the expansion joints in the deck for the entire bridge, and, for Span No. 2, the temperature of the concrete in the rib, the rise and fall of points on the rib, and the longitudinal deformation of the concrete and steel on several gage lines at each of five sections of the rib.

Seventeen sets of readings were taken at intervals of approximately one month, the first being taken November 20, 1923 and the last June 4, 1925. There were a number of phenomena that were of interest that are discussed in the following paragraphs.

Contrary Tipping of the Piers.

The piers tipped inward as the temperature rose and outward as the temperature fell. An analysis of the structure indicated that the reverse should have occurred and some auxiliary influence apparently entered. After reviewing the several observations that had been made it was finally noted that, whereas the bridge had been constructed with expansion joints of equal width on the two sides of the pier extension, the expansion joints outside of the piers (on the side away from the large central span) are closed and those on the inside are open and the openings have approximately twice their original width. Moreover, the opening of the outside joints did not vary in width whereas the change in width of the inside joints was equal, approximately, to the expected movement for the two joints. These facts, together with the unbalanced dead-load thrust on the piers, account for the contrary rotation of the piers that accompanies temperature changes.

The dead-load pressure of the soil on the bases of the piers was greater on the side away from the central span than on the side toward that span. The soil, shale, is one that is subject to time yield, and the time yield increases with the intensity of pressure. As a result of the unequal settlement the piers rotated until the extension of the pier at the top came into contact with the deck slab, closing the outside and opening the inside joint in the deck. This action having once taken place soon after the structure had been built, further action of the bridge was as follows:

The deck expanded with a rise in temperature and, since it butted against the city pavement at the end of the bridge, moved away from the

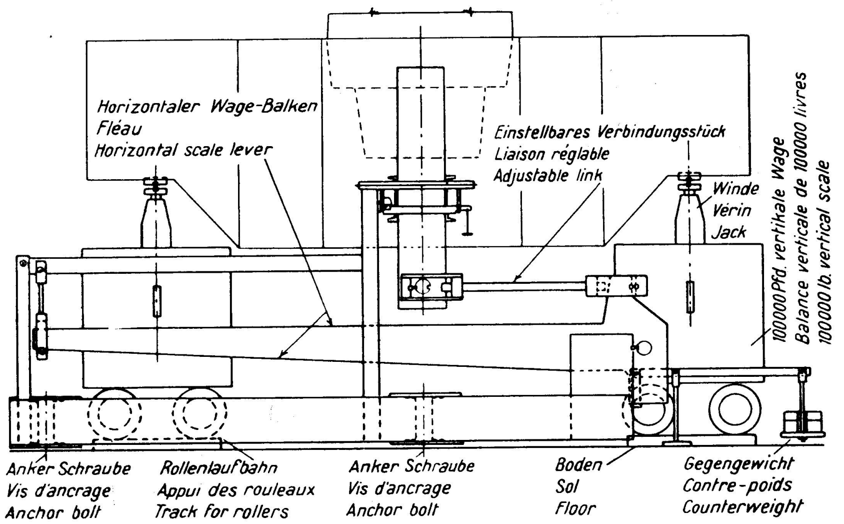


Fig. 8.

Apparat zur Messung von Auflagerkräften an Pfeilern.
Appareil permettant de mesurer les réactions à la base des piles.
Apparatus for Measuring Reactions at Pier Bases.

shore. The expansion joint on the shore side of the pier was closed, causing the deck to exert an inward thrust at the top of the first pier. The expansion joint on the channel side of the pier was open, thus preventing the pier from transmitting the thrust to the adjacent deck, and producing on the pier an inward overturning thrust at the level of the deck. As a result of this unbalanced thrust, the pier tipped toward the channel, moving the rib inward relative to the deck, thus producing a shear in the spandrel columns. This shear forced the deck in the second span against the next pier, and, since the joint on the shore side was closed and the one on the channel side open, this pier, likewise, tipped toward the channel. The action continued toward the channel, spanned by the longest arch, from each end of the bridge. Since, for the long span, both piers had open joints on the side toward that span, both had unbalanced thrusts at the elevation of the deck that made the pier tip inward as the temperature rose, thus actually shortening the span. The author believes that this is the true explanation of the peculiar behavior of the piers of this bridge. There is no doubt that the piers did tip toward the long span as the temperature rose, and that the expansion joints adjacent to the piers were, in general, open on the side of the pier toward the long span and closed on the side away from that span.

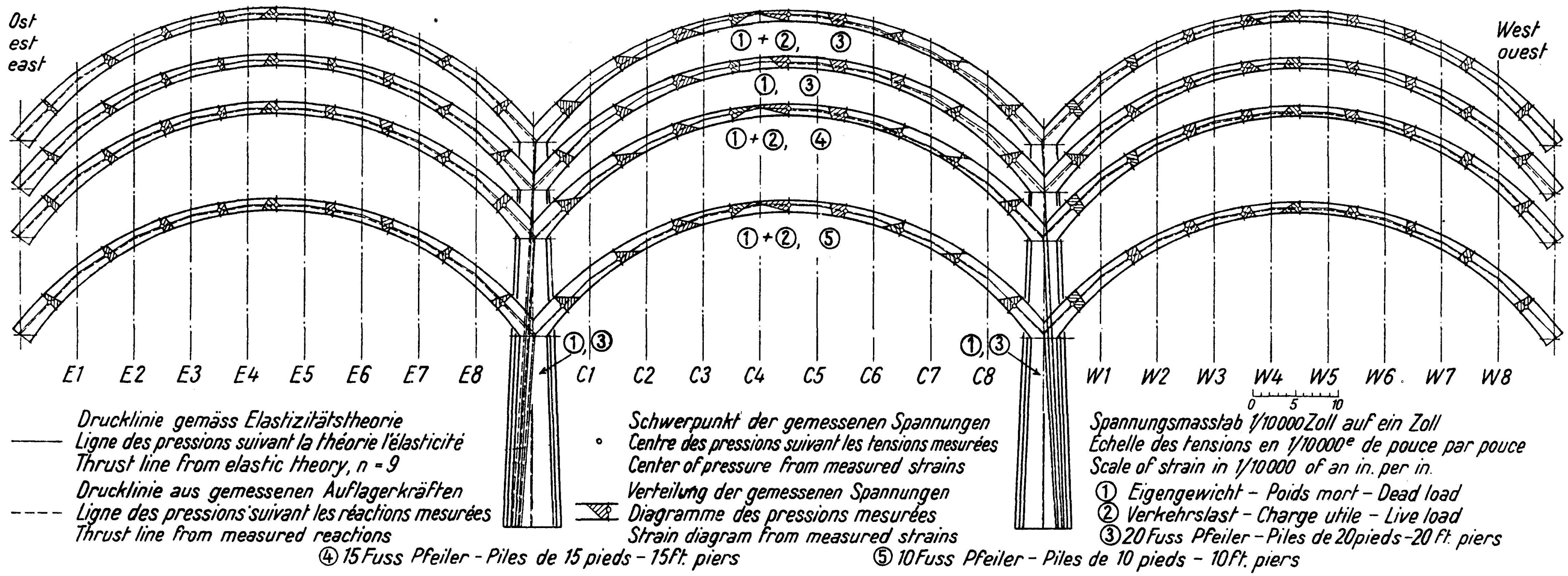


Fig. 9.

Drucklinien für Lastschema der Berechnung.
Lignes des pressions pour le diagramme de charges choisi.
Thrust Lines for Design Load.

Movement at Expansion Joints.

There are expansion joints in the deck on both sides of all piers and at the ends of the saddle for all except the end spans. There was practically no movement at the expansion joints at the ends of the saddle or at the joints on the side of the piers away from the large center span. The joints on the inside of the piers closed with a rise in temperature and opened with a fall in temperature. The sum of the movements of all the expansion joints was equal to the computed thermal expansion of a concrete slab extending from the pavement in the city streets at one end of the bridge to that at the other. The sum of the movements at all of the joints had a maximum value of 5.37 inches.

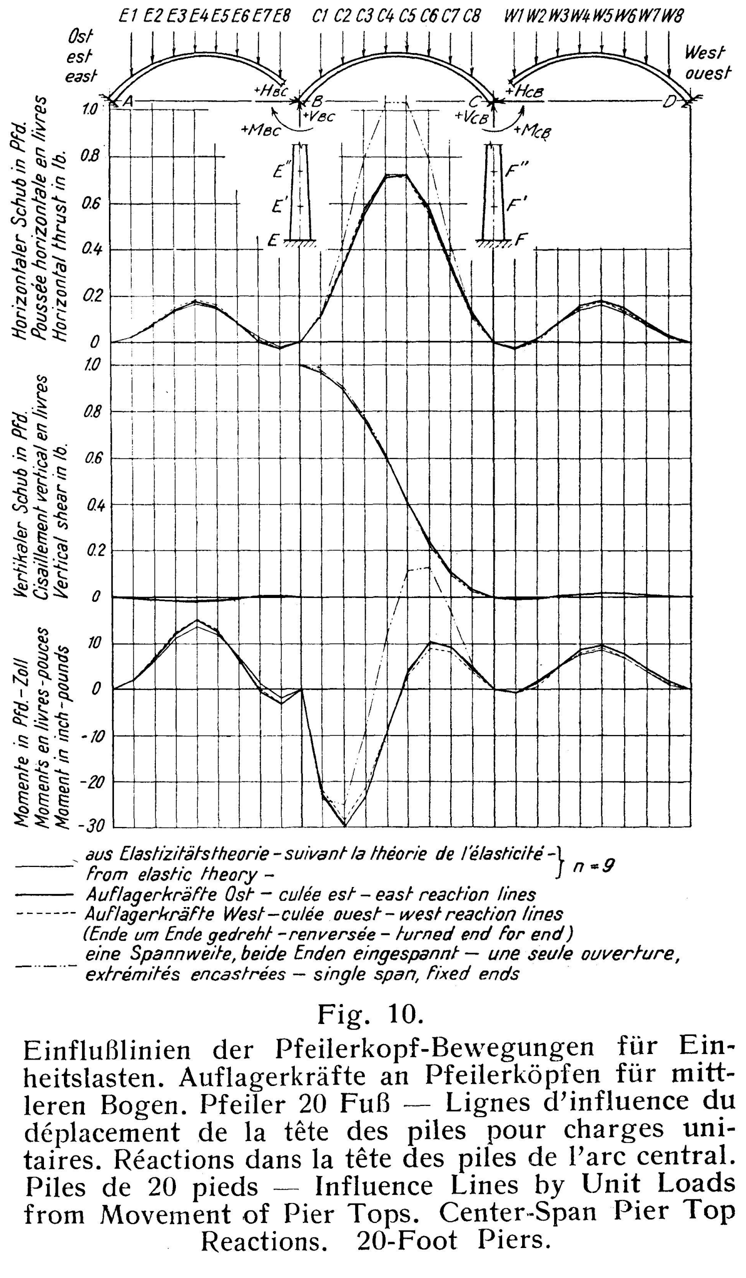


Fig. 10.

Einflußlinien der Pfeilerkopf-Bewegungen für Einheitslasten. Auflagerkräfte an Pfeilerköpfen für mittleren Bogen. Pfeiler 20 Fuß — Lignes d'influence du déplacement de la tête des piles pour charges unitaires. Réactions dans la tête des piles de l'arc central. Piles de 20 pieds — Influence Lines by Unit Loads from Movement of Pier Tops. Center-Span Pier Top Reactions. 20-Foot Piers.

Temperature Stresses in the Concrete and Steel.

Deformations were measured at five sections of the rib of Span No. 2 to determine the longitudinal stress due to temperature changes. These sections were located one adjacent to each springing, one at each end of the saddle and one at the crown. The deformation of the concrete was measured in both the longitudinal and transverse directions and, since there is no transverse stress, the difference between the longitudinal and transverse deformation was taken as the deformation due to stress. The temperature stress in the steel was obtained from the measured deformation, temperature corrections being made for both the steel and the standard bar.

The measured value of the temperature stress, in both the steel and concrete, was greater than the computed value (for the rib alone) at all sections, but the difference was not great except at the ends of the saddle; at the latter sections the measured value was very much greater than the computed value.

4. Influence of Deck Upon the Stresses in Arch Rib Due to Loads: The stresses in an open spandrel arch rib are usually determined on the basis that the deck has no restraining action upon the rib even though the deck, spandrel columns and rib are made continuous.

Tests (5) were made on arch ribs without deck and similar arch ribs poured monolithic with decks to determine the effect of the deck upon the stresses in the rib. Six types of structures having similar ribs, all shown in Fig. 13, were tested. Arches 27—3 and 27—4 were ribs without deck; 27—5 and 27—6 had a high deck without intermediate expansion joints and the moment of inertia of a transverse section of the deck about a horizontal gravity axis was relatively small. The decks of the other arches were identical in section but differed in other respects. Arches 28—1 and 28—2 had decks that were a considerable distance above the rib at the crown, but 28—3 and 28—4 had decks so low that they were integral with the rib at the crown. Arches 28—1 and 28—3 had no intermediate expansion joints in the deck, but 28—2 and 28—4 had expansion joints near the one-third points. The arches were loaded at points over the spandrel columns, the load being 50 per cent greater on the right-hand than on the left-hand half of the structure. The distribution of a 1000-lb. load is indicated in Fig. 13. The

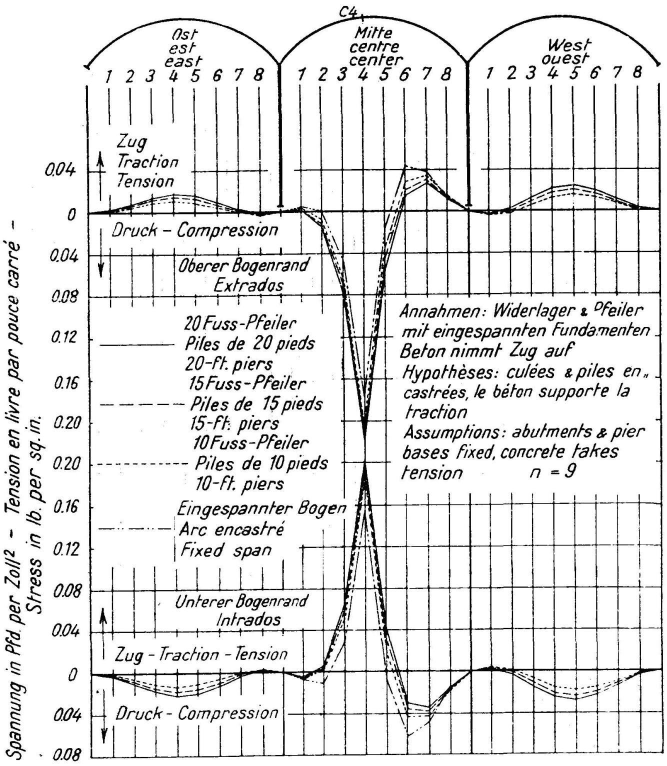


Fig. 11.
Einflußlinien der Spannungen in Lastpunkt C 4.
Lignes d'influence des tensions au point C 4.
Influence Lines for Stress at Load Point C 4.

longitudinal strain in the rib was measured at sections intermediate between the spandrel columns as indicated in Fig. 14. The abutments were returned to their original position relative to each other after each load change. The abutment reaction components were weighed, so that the magnitude of the forces (shear, moment and thrust) were determined independently of the strain, although the latter was used as a means of determining the position of the line of thrust relative to the axis of the arch. The stress distribution at various sections and at various loads is shown in Figs. 15, 16 and 17 for arches 27—3, 28—3 and 28—4. These diagrams indicate that the measured and computed values of the stress agree closely for a rib without a deck but that for a rib with a deck without intermediate expansion joints the measured value of the stress in the rib differs materially from the computed value (for a rib without deck) at sections of large theoretical moment. For the structure with a deck with intermediate expansion joints, Arch 28—4, the measured and computed values (the latter for a rib without deck) agree closely. Corresponding diagrams for the structures with a high deck, arches 28—2

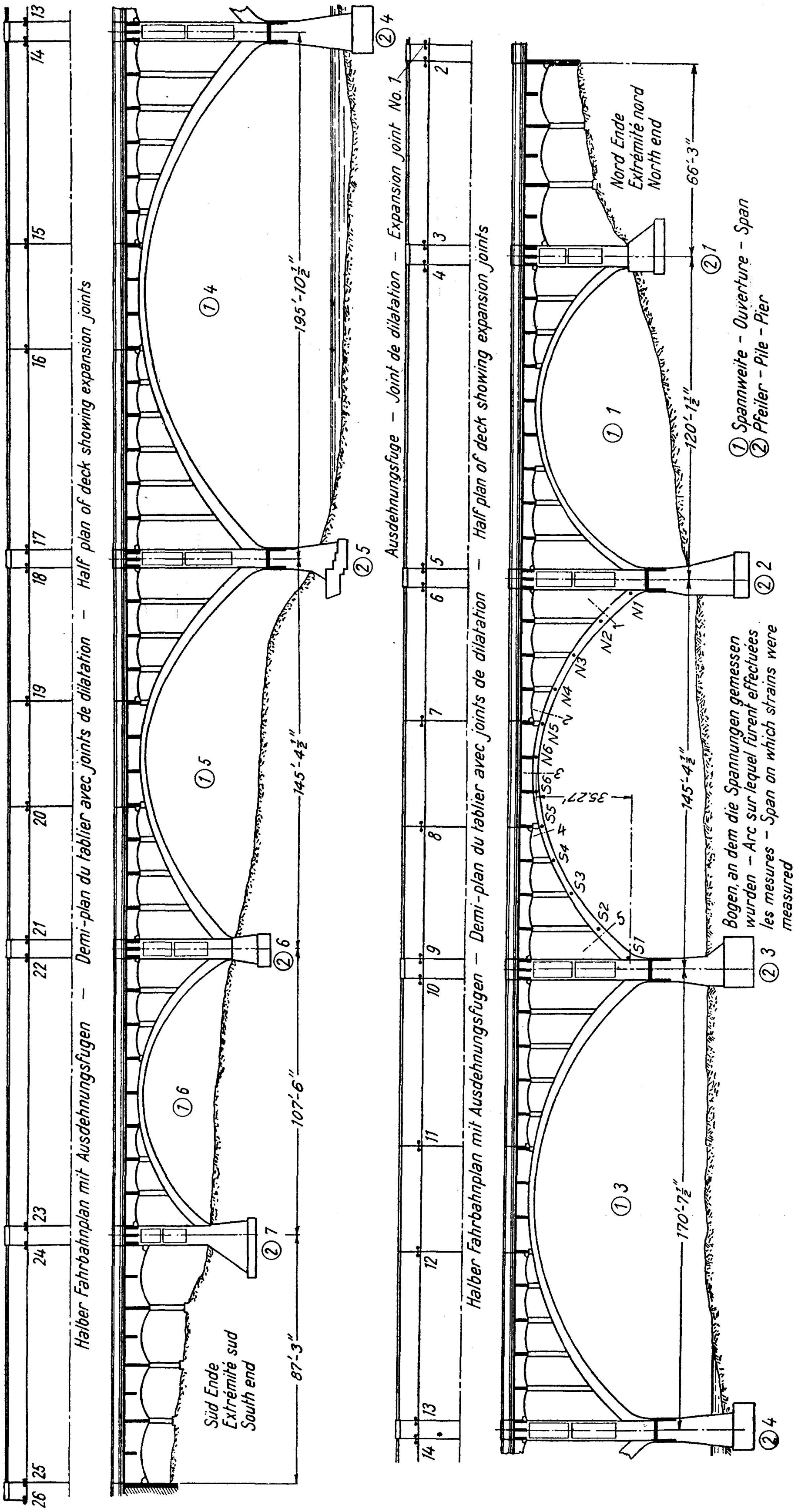


Fig. 12.

Längsschnitt der Brücke — Coupe en long du pont — Longitudinal Section of Bridge.

and 28—3, given in the detailed report of the tests (5), show that for these structures also, the measured and computed stresses differ materially for the structure without intermediate expansion joints and agree closely for the structure with intermediate expansion joints. For the structures tested, the deck without intermediate expansion joints reduces the moment in the rib

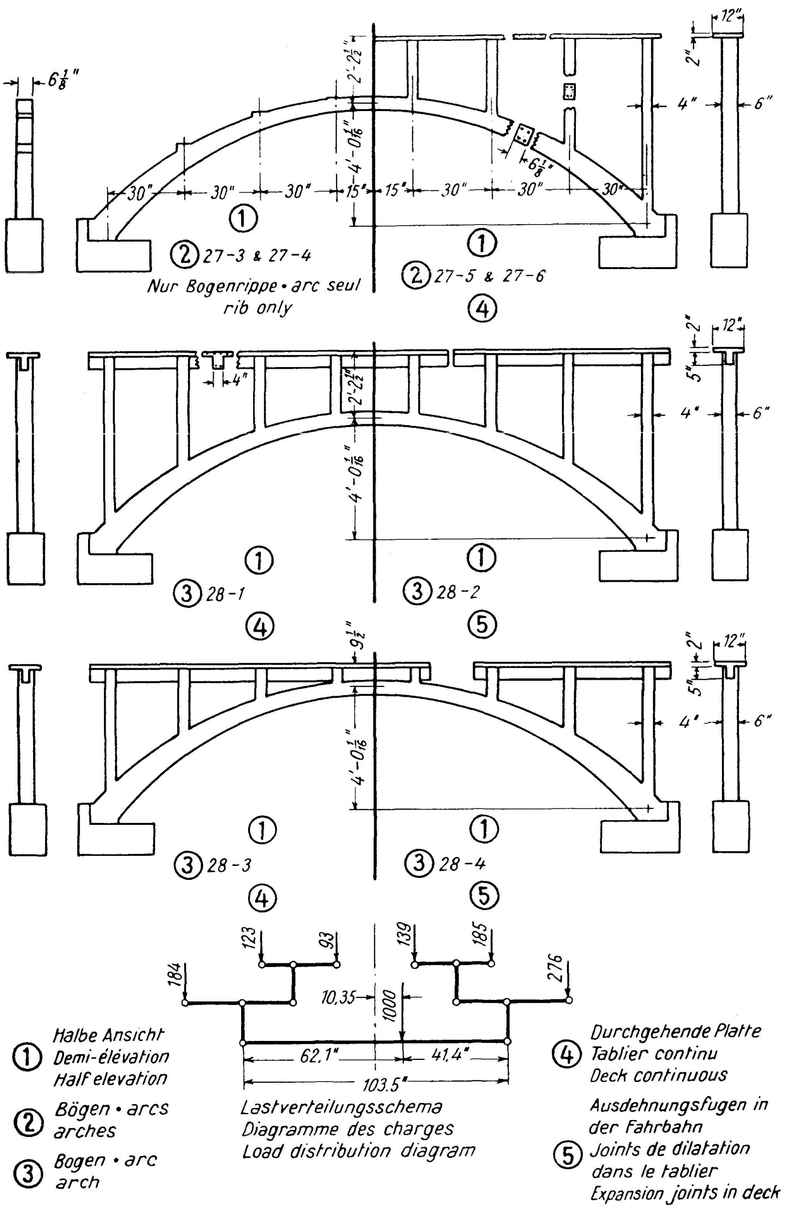


Fig. 13.
Fahrbahneinzelheiten — Détails du tablier — Details of Decks.

over the central portion of the span but the deck with intermediate expansion joints does not materially reduce this moment. The fact should not be overlooked, however, that, although the moment in the rib due to a vertical load is reduced, moment is produced in the columns and deck, a fact that must be considered in the design. It is true, however, that the development of a crack in the deck or columns reduces the moment in these parts and does not endanger the stability of the structure as a whole.

The extent to which the deck will reduce the moment in an arch rib depends upon the relative stiffness of the deck, the spandrel columns and the ribs (11). For the structures tested the deck is rather stiff relative to the rib.

There was no indication that there was any difference in their influence upon the load-carrying capacity of the arch between the high deck and the low deck. The intermediate expansion joints apparently decreased the load-carrying capacity of both types of structures.

5. Influence of Intermediate Expansion Joints in a Deck Upon the Stresses in an Arch Rib: Open-spandrel arch bridges usually have expansion joints in the deck over the piers and abut-

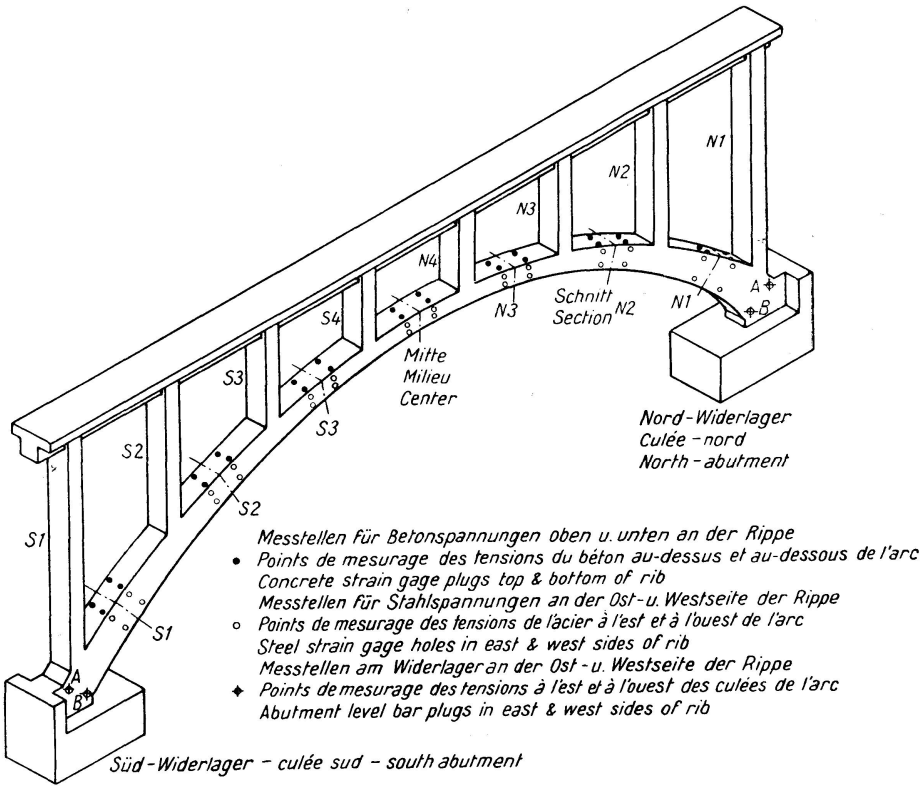


Fig. 14.
Schnitte, an denen Spannungen gemessen wurden.
Sections où furent mesurées les tensions.
Sections at Which Strains Were Measured.

ments; some bridges also have intermediate expansion joints at the center or near the one-third points of the span. Observations on the Gilbert Street Bridge at Danville, Illinois (4), presented in Section 3, indicate that, for the temperature stresses in the rib at the ends of the saddle and directly beneath the intermediate expansion joints, the measured values were much greater than the computed values for the rib alone. Because of these results, a special study was made to determine the influence of various types of decks upon the temperature stresses in an arch rib (5). Celluloid models of the structures were used to determine the position of the thrust line and the magnitude of the thrust resulting from a change in span equivalent to a one-degree change in temperature. Knowing the magnitude and position of the thrust, the temperature moment on the section of the rib at the springing and at points directly below the expansion joints can be determined. The spans studied included Span No. 2 of the Gilbert Street Bridge at Danville, Illinois,

Fig. 12; arches 27—3, 28—1, 28—3 and 28—4 of Fig. 13; and the arches of Fig. 18. The tests of the celluloid model of the Gilbert Street Bridge gave a temperature moment at the end of the saddle 2.38 times as great as the theoretical moment at the same section of a rib without deck, thus confirming, qualitatively, the field observations on the same bridge. The tests

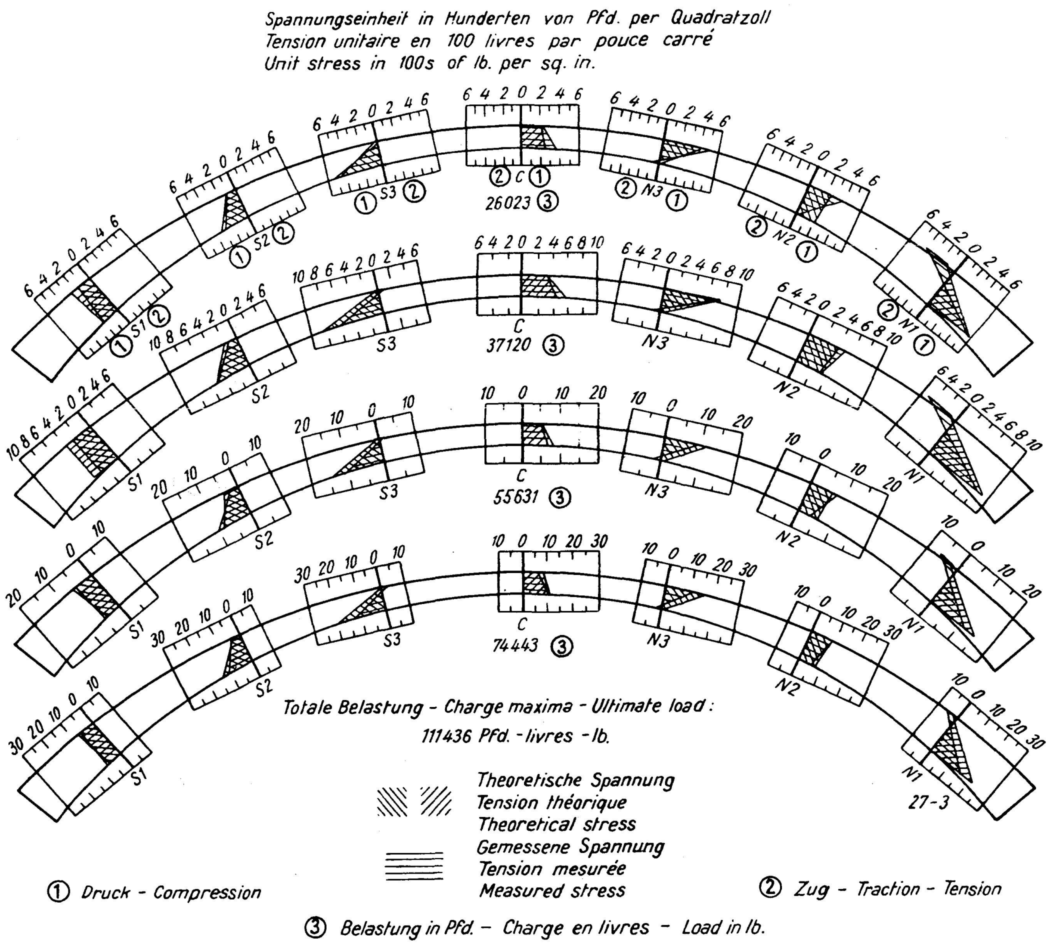


Fig. 15.

Beziehungen zwischen theoretischen und gemessenen Spannungen, Bogen 27—3. Die angegebene Belastung ist die totale Belastung des Bogens. Spannungsunterschied ergibt sich aus totaler Belastung weniger Anfangslast, 8850 Pfd. aus Eigengewicht des Bogens und der Apparatur.

Relation entre la tension théorique et la tension mesurée, arc 27—3. La charge indiquée est la charge totale de la voûte. La différence de tension est due à la charge totale moins la charge initiale, 8850 lb, représentée par le poids propre de l'arc et les appareils de mesurage.

Relation between Theoretical and Measured Stress, Arch 27—3. Load given is total load on arch. Stress increment is due to total load less initial load, 8850 lb., due to weight of arch and apparatus.

of the model of arch 28—4, very similar to the Gilbert Street Bridge except that the saddle was shorter, gave a measured value of the temperature moment 1.31 times as great as the theoretical value at the same section of a similar rib without deck. Tests of the models of the other bridges referred to appear to justify the following statements: For a structure with a deck so low as to be integral with the rib at the crown, thus forming a saddle, if expansion joints are provided at the ends of the saddle the temperature moment will be greater at the ends of the saddle than it would be at the same sections of a rib without deck; and increasing the length of the saddle increases the temperature moment at its end. Intermediate expansion joints do not increase the moment at sections of the rib directly beneath them as much for a structure with a high deck as for a similar structure with a low deck. The temperature moment at the springing is greater for a structure with a deck than it is

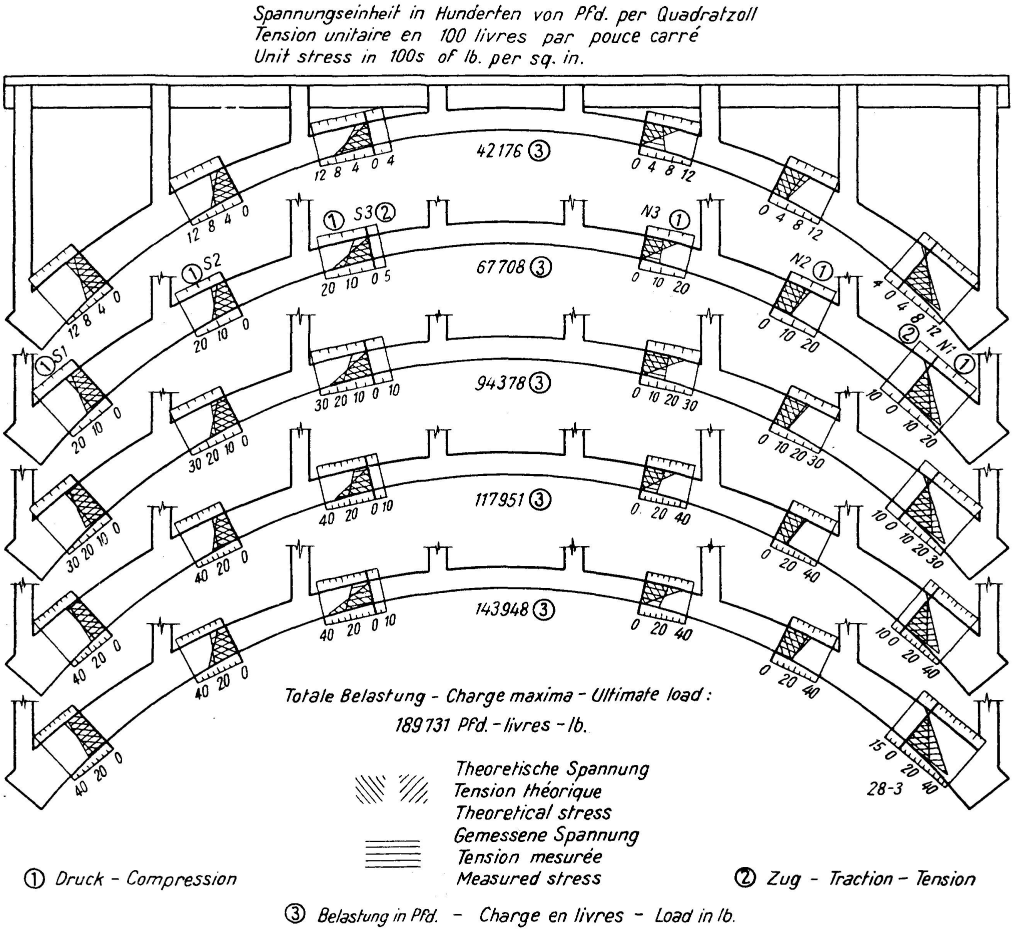


Fig. 16.

Beziehungen zwischen theoretischen und gemessenen Spannungen, Bogen 28—3. Die angegebene Belastung ist die totale Belastung des Bogens, Spannungsunterschied ergibt sich aus totaler Belastung weniger Anfangslast, 9993 Pfd. aus Eigengewicht des Bogens und der Apparatur.

Relation entre la tension théorique et la tension mesurée, arc 28—3. La charge indiquée est la charge totale de la voûte. La différence de tension est due à la charge totale moins la charge initiale, 9993 lb, représentée par le poids propre de l'arc et les appareils de mesurage.

Relation Between Theoretical and Measured Stress, Arch 28—3. Load given is total load on arch. Stress increment is due to total load less initial load, 9993 lb., due to weight of arch and apparatus.

for a similar rib without deck, and it is greater for a structure without than it is for one with intermediate expansion joints.

The purpose of expansion joints is to permit longitudinal expansion and contraction of the deck without detrimental flexure of the spandrel columns. If an expansion joint is needed at a particular section of a deck, movement will occur if a joint is provided. And conversely, if an expansion joint is provided and only a very small movement occurs, the expansion joint is not needed. Tests were made on models of the arches shown in Fig. 18 in which the length of the span was changed and the opening or closing of the expansion joints was measured, to determine whether or not the expansion joints are needed. The results of the tests, shown in Fig. 19, indicate that the movement at intermediate expansion joints is from 0.09 to 0.14 of the change in span for arches having decks so low as to be integral with the rib at the

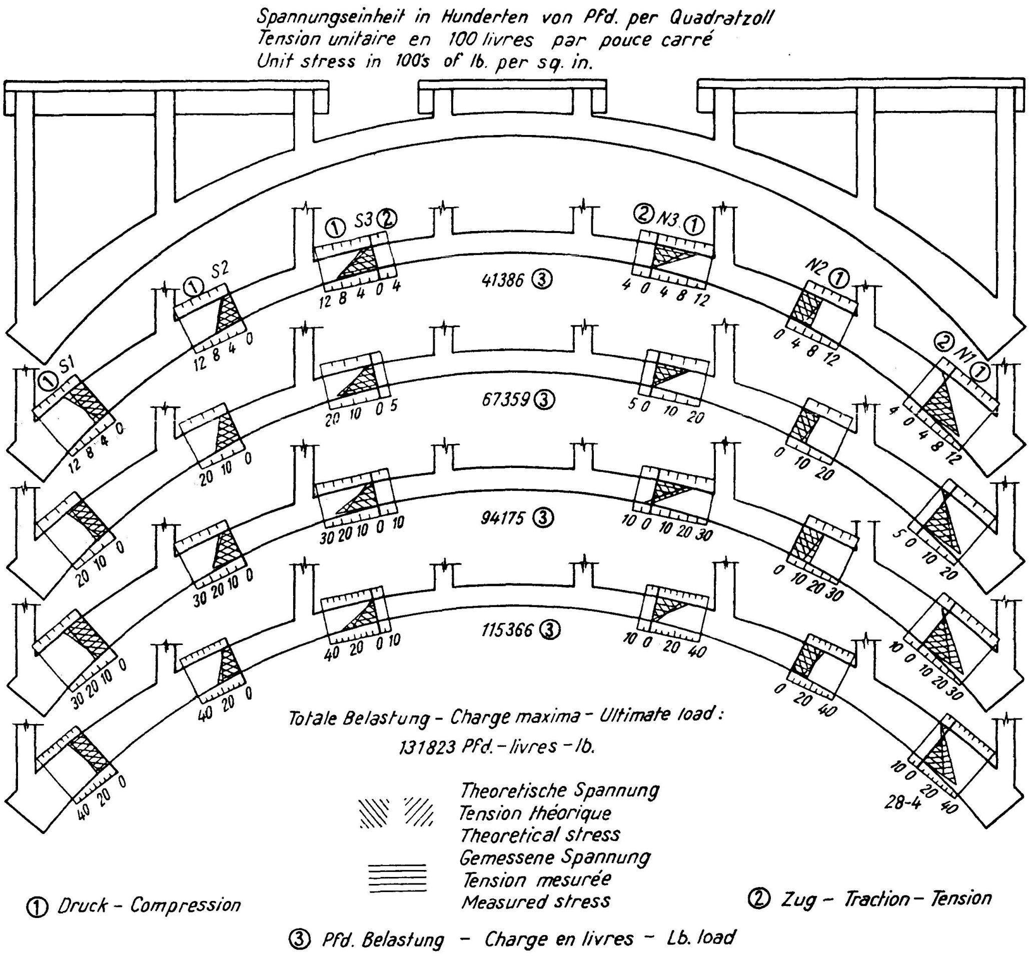


Fig. 17.

Beziehungen zwischen theoretischen und gemessenen Spannungen, Bogen 28—4. Die angegebene Belastung ist die totale Belastung des Bogens, Spannungsunterschied ergibt sich aus totaler Belastung weniger Anfangslast, 9993 Pfd. aus Eigengewicht des Bogens und der Apparatur.

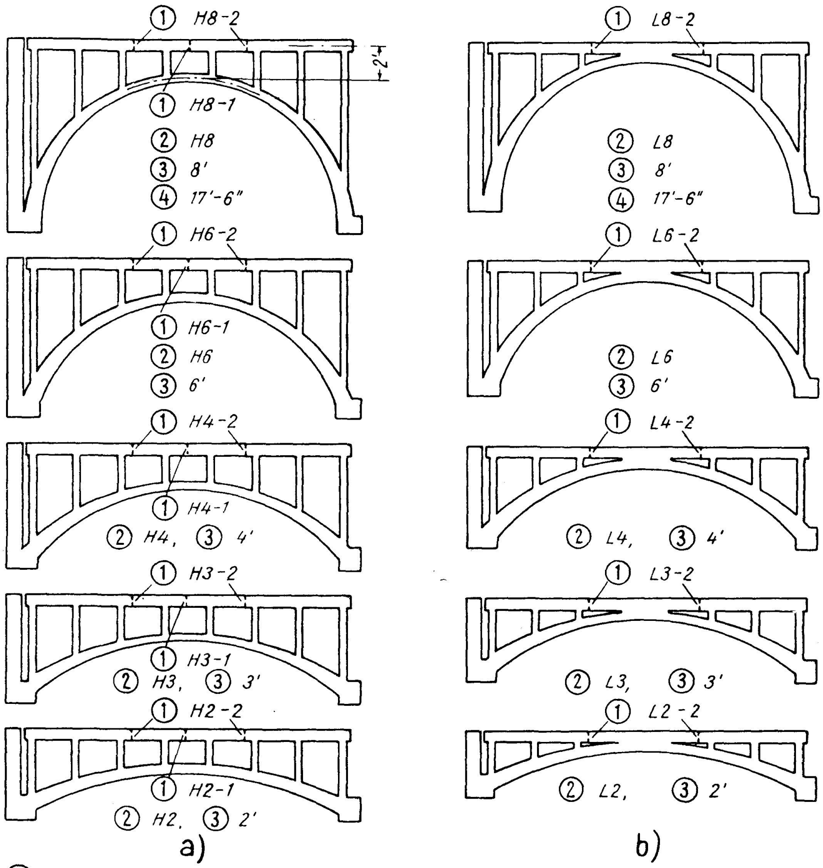
Relation entre la tension théorique et la tension mesurée, arc 28—4. La charge indiquée est la charge totale de la voûte. La différence de tension est due à la charge totale moins la charge initiale, 9993 lb, représentée par le poids propre de l'arc et les appareils de mesurage.

Relation Between Theoretical and Measured Stress, Arch 28—4. Load given is total load on arch. Stress increment is due to total load less initial load, 9993 lb., due to weight of arch and apparatus.

crown. This movement is so small that the expansion joints can be omitted without producing excessive flexure in the spandrel columns. The movement at the expansion joints is considerably more for the structure with high than it is for those with low decks.

The field observations and laboratory tests appear to justify the following conclusions relative to expansion joints in the deck of open-spandrel arch bridges.

For a structure with a deck so low as to be integral with the rib at the crown, intermediate expansion joints are not needed. If expansion joints are provided the temperature moment in the rib immediately below the joints will be increased and the temperature moment at the springing will be decreased. The expansion joints will also reduce the flexural resistance of the central portion of the structure.



① *Ausdehnungsfugen für – Joints de dilatation pour – Expansion joints for*
② *Reihe – Séries – Series* ③ *Pfeil – Flèche – Rise*
④ *Spannweite – Portée – Span*

Fig. 18.
Zelluloidmodelle zur Bestimmung des Temperatur-Bogenschubes in Bogen mit verschiedenen geometrischen Eigenschaften.
Modèles de celluloide pour la détermination des pressions dues à la température dans des arcs avec différentes propriétés géométriques.
Celluloid Models for Determining Temperature Thrust in Arches Having Various Geometrical Properties.

For a structure with a deck a considerable distance above the rib at the crown, intermediate expansion joints do not materially increase the temperature moment in the rib at sections directly under the expansion joints and they reduce the temperature moment at the springing. If expansion joints are provided a considerable movement will occur, but, the spandrel columns are so long that no general statement can be made as to whether or not preventing the movement will endanger the columns or deck, each bridge requiring a separate study.

There are practical objections to the use of expansion joints and they reduce the flexural strength of the central portion of the structure. It would appear, therefore, that the use of intermediate expansion joints should be limited to structures for which it can be demonstrated that their omission would be detrimental.

6. Laboratory Tests of Three-Span Arch Bridges With Decks on High Piers: Tests were made on three-span arch bridges with decks on high piers to determine the inflence ordinates for the reaction components at the springings of the ribs and to determine the load-carrying capacity of the structure (6). The general dimensions of one of the structures are shown in Fig. 20. The ribs and the piers are the same as for the three-span structures without deck, described in Section 2. Four types of decks were used: A high deck with intermediate expansion joints and a high deck without expansion joints, shown in Fig. 21; a low deck with and a low deck without expansion joints, shown in Fig. 22. The apparatus, the character of the tests and the methods of testing are the same as for the structure

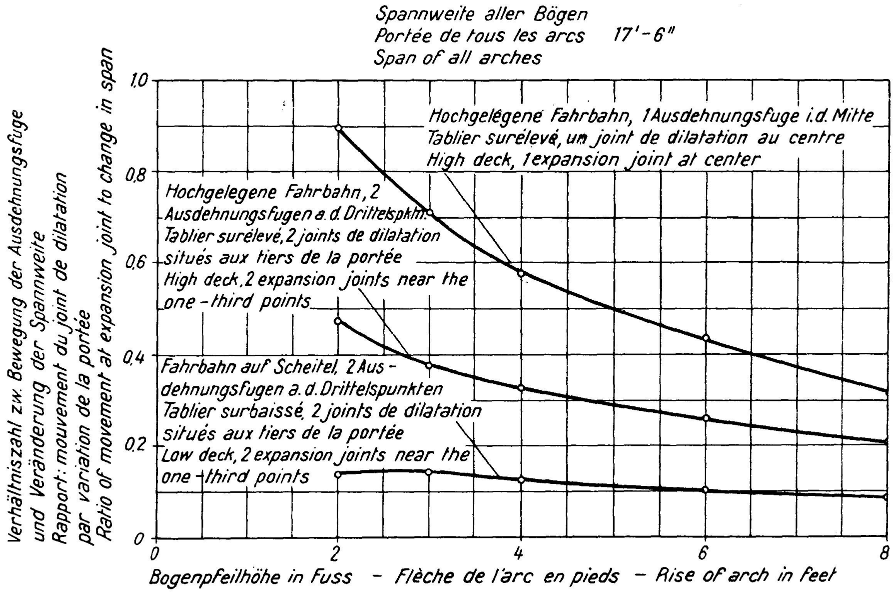


Fig. 19.

Beziehungen zwischen Bewegung der Ausdehnungsfuge und Pfeilhöhe des Bogens.
Relation entre le déplacement des joints de dilatation et la flèche de l'arc.
Relation Between Movement At Expansion Joint and Rise of Rib.

without deck, described briefly in Section 2 of this paper and presented in detail in the original report (2). The structures with a high deck were tested at pier heights of 20 ft., 15 ft. and 10 ft., those with a low deck at a pier height of 20 ft. only.

The superimposed design dead load, shown in Fig. 23, was on the arches during the tests to determine the elastic properties of the structure.

The influence lines for the reaction components at the springings of the center span for a structure with a low deck having intermediate expansion joints, are given in Fig. 24. Influence lines determined by several experimental methods are compared with each other and, for the middle span, with the influence lines for a single span with fixed ends. Diagrams similar to those of Fig. 24, for both ends of all spans and for all structures, are given in the original report (6).

The position of the thrust line due to the combined dead load and live load was determined by two methods. By the first method the changes in the components of the reactions at the pier bases and abutments accompanying the application of the dead and live loads were weighed and the thrust lines constructed from these measured forces. By the second method, the strain diagrams are constructed from the strain measured at the intrados and extrados at sections intermediate between the spandrel columns, and the thrust line is drawn through the center of the strain diagram. The thrust line determined from the measured reactions is for the structure as a whole whereas the centers of pressure are for the rib alone. The center of pressure was located only for those sections for which it falls within the kern. The thrust lines located by the two methods should coincide at the springings and in panels containing expansion joints.

The thrust lines determined from the measured forces are shown by the broken lines and the centers of pressure by the small circles of Figs. 25,

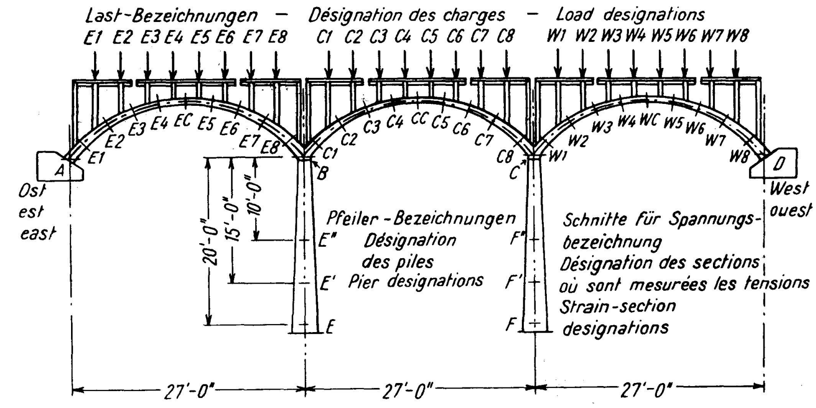


Fig. 20.

Bezeichnungen für dreifachen Bogen mit Fahrbahn. Spannungsbezeichnungen an Lastpunkten erhalten die gleiche Bezeichnung wie der benachbarte Schnitt, z. B. für Lastpunkt *W* 2 gilt Schnittbezeichnung *W* 2—3 usw.

La notation des sections sous un point de charge est donnée par les désignations des points de charge adjacents. Par ex. la section située sous la charge *W* 2 est désignée par *W* 2—3, etc.

Notation Used for Three-Span Structure with Deck. Notation for strain-section at a load-point has the adjacent strain-section designations. As, the strain-section at Load *W* 2 is section *W* 2—3, etc.

26 and 27. The thrust lines and pressure diagrams have been constructed for dead load, dead load plus one live load, dead load plus three live loads, and dead load plus five live loads for each structure. The magnitude and distribution of the load is given in Fig. 23 for the dead load and in Figs. 25, 26 and 27 for the live load. For Figs. 25 and 26, the diagrams in the upper part of the figures are for a live load producing a maximum stress at *B* of *BC*, the left springing of the center span; the diagrams in the lower part of the figure are for a live load producing a maximum stress at *C 4*. The diagrams of Fig. 27 are for a live load producing a maximum stress at *C 3*. The fact that, for the structures without intermediate expansion joints, the thrust lines for the structure as a whole has a greater eccentricity than the center of pressure for the rib alone at sections of maximum stress, indicates that the deck acts with the rib to resist the moment if there are no intermediate expansion joints. This participation of the deck is largely destroyed by the intermediate expansion joints.

The elaborate series that have been outlined above and which are described in full in the original report (6) appear to justify the following conclusions:

The influence ordinates for fixed-end reactions are quite different for a structure having a deck from those for one consisting of a rib without deck. The effect of the deck is to reduce the moment at the springing due to load, where it is all resisted by the rib, and to increase the moment over the middle of the span where the deck acts with the rib. The effect of the deck is to increase the temperature moment at the springing.

A deck without intermediate expansion joints increases the stiffness and the moment-resisting capacity of the central part of the structure. Inter-

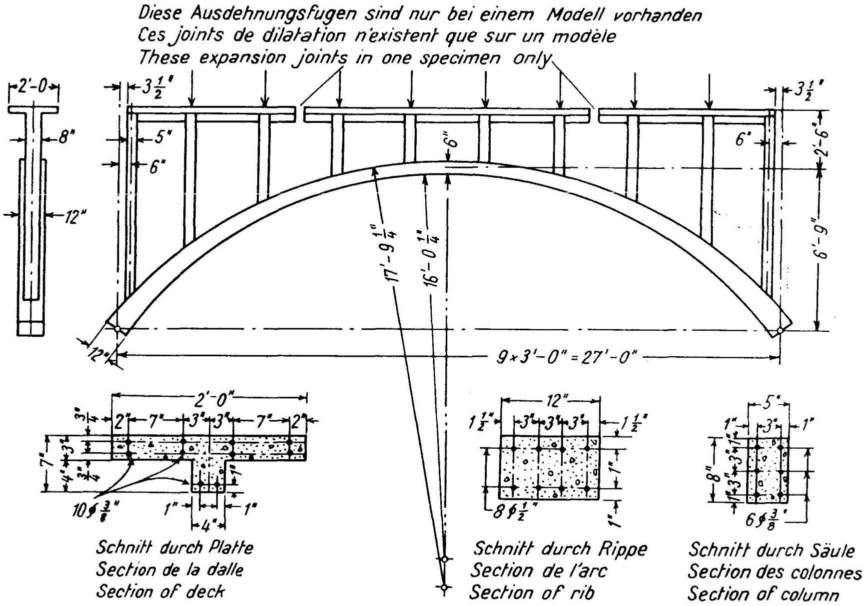


Fig. 21.
Einzelheiten von Bogen mit hochliegender Fahrbahn.
Détails des arcs à tablier surélevé.
Details of Span with High Deck.

mediate expansion joints reduce both of these effects. The ratio of the movement at each expansion joint to a change in span is about 0.23 for the structure with a high deck, and 0.11 for the one with a low deck.

For the structure with a high deck with intermediate expansion joints, the maximum unit compression due to design load was 36 per cent greater for the three-span structure on 20-ft. piers than for a single span with fixed ends. For the structure with a low deck with intermediate expansion joints the difference was 21 per cent. A considerable part of this difference was due to the difference in the dead-load stress, a function which would not have been influenced by the flexure of the piers if the specimen had been a symmetrical structure of homogeneous material, symmetrically loaded.

There is considerable evidence indicating that the dead-load stress in a multiple-span structure may exceed the corresponding stress in a similar single span having fixed ends.

The concrete in the arch developed approximately the same unit stress as the same concrete in 6-in. by 12-in. control cylinders.

7. Movement of Piers During Construction of Multiple-Span Arch Bridges: Multiple-span arch bridges are designed so that the dead-load thrusts from the two adjacent arches supported by an intermediate pier balance each other and there is, or should be, no considerable dead-load moment tending to overturn the pier after the bridge has been completed and all forms removed. But during the construction of the bridge, an arch on one side of a pier is poured before the one on the other side, forms are removed on one side before they are on the other, and other temporary unbalanced conditions exist. Observations (7) were made on five multiple-span arch bridges during construction to determine whether or not these temporary unbalanced conditions caused movements of the piers. The general characteristics of these bridges are as follows:

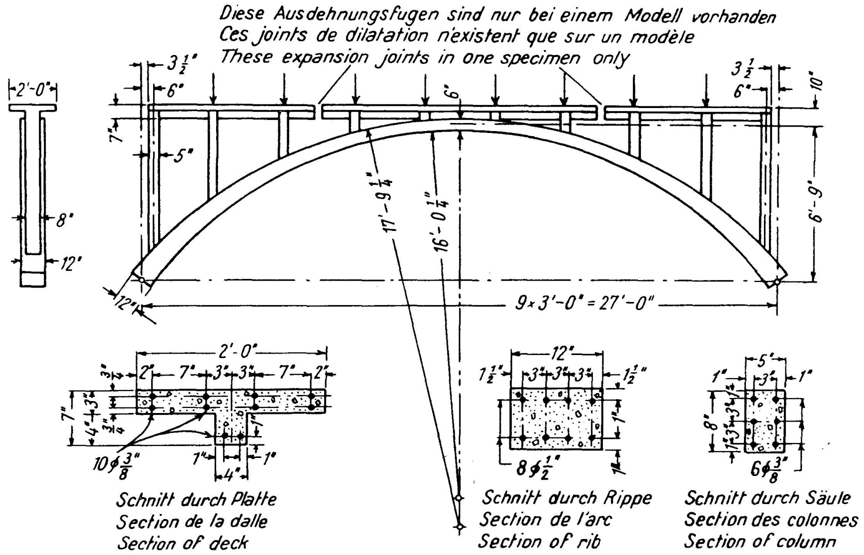


Fig. 22.
Einzelheiten von Bogen mit Fahrbahn auf Scheitel.
Détails des arcs à tablier surbaissé.
Details of Span with Low Deck.

South Bridge, Yorkville, Illinois: The South Bridge at Yorkville, Illinois, is a three-span, twin-rib, open-spandrel highway bridge, each arch having a span of 70 ft. and a rise of 10 ft. The piers rest on bedrock.

North Bridge, Yorkville, Illinois: The North Bridge at Yorkville, Illinois, is the same as the South Bridge except for the rise of the arch, which is 15 ft. 6 in.

Washington Street Bridge, Rockton, Illinois: The Washington Street Bridge at Rockton, Illinois, is an eight-span, twin-rib, open-spandrel highway bridge, each arch having a span of 65 ft. and a rise of 9 ft. 9 in. Three piers and one abutment rest on bedrock, the other piers and abutment are carried on piles.

Oliver Avenue Bridge, Indianapolis, Indiana: The Oliver Avenue Bridge at Indianapolis, Indiana, is an eight-span earth-filled skew barrel arch. The center arch, which is somewhat longer than the adjacent ones, has a span of 130-ft. and a rise of 20 ft. 7 in. The piers rest on piles driven into gravel, and make an angle of 83° 4′ with the centerline of the bridge.

Kentucky Avenue Bridge, Indianapolis, Indiana: The Kentucky Avenue Bridge is a seven-span earth-filled skew barrel arch. The center span, which is somewhat longer than the adjacent ones, has a span of 138 ft. and a rise of 22 ft. The piers are supported on piles driven into gravel, and make an angle of 43° 8′ with the centerline of the bridge.

The results of the observations on the Kentucky Avenue Bridge are showr in Figs. 28 and 29. The observations include movements of the piers as follows: settlement, rotation about a longitudinal axis, and horizontal movement normal to the axis of the piers at both ends. The algebraic difference between the transverse horizontal movement at the two ends is a

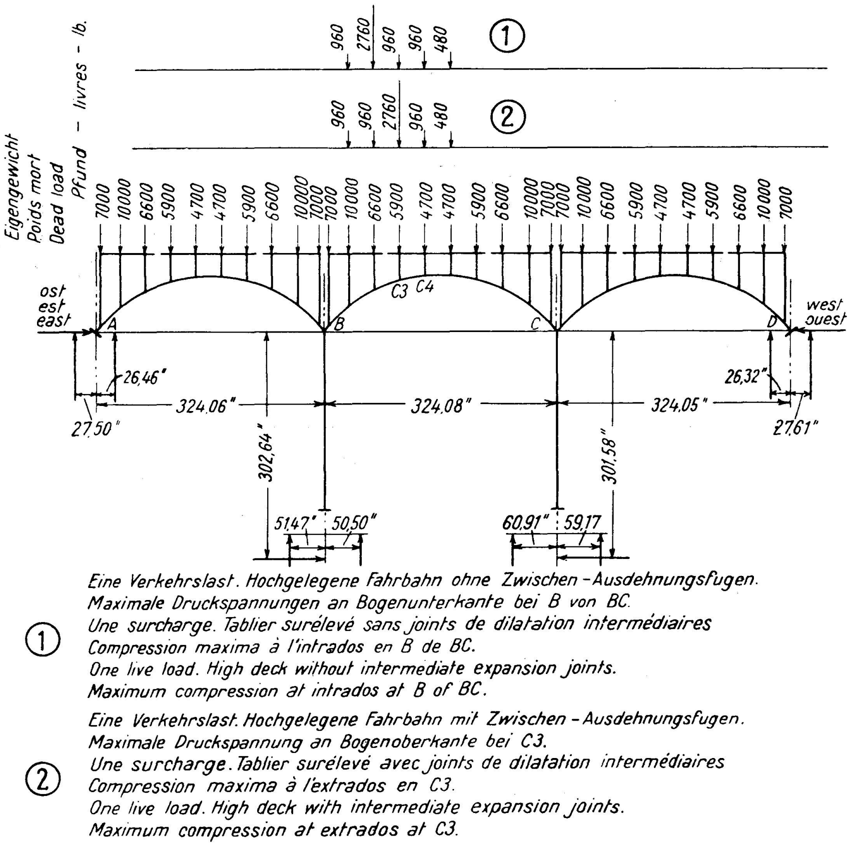


Fig. 23.
Belastungschema der Berechnung — Répartition du poids mort — Design Dead Load.

measure of the rotation of the piers about a vertical axis, a movement of particular interest for this bridge because of the combination of a large angle of skew, a wide bridge and pile supports for the piers.

The rotation of the piers about a horizontal longitudinal axis is shown in Fig. 28. The stage of construction at the time a set of observations was made is indicated by sketches, and the legend is given at the top of the figure just below the plan of the pier bases. The first readings were taken May 25, 1925, after the two spans at each end had been poured but before the shores had been removed. Observations were made at frequent intervals, as indicated at the right of the figure, until September 26 of the same year, at which time the bridge had all been poured, the earth fill placed and all the shores removed except in spans 4 and 5. In span 5 the original shores had been replaced by temporary ones.

The rotation of a pier is represented to scale by the horizontal deviation of the heavy broken vertical line from the corresponding light full vertical line through the center of the pier. The position of the broken line relative to the full line indicates the direction in which the top of the pier tipped.

The maximum rotation observed was of pier 3. On August 2 it was tipped to the left 0.00109 radians relative to its position May 5. The final rotation, September 26, was 0.00076. In general the piers tipped about 0.0005 radians during the period covered by the observations.

The transverse horizontal movement of the piers is shown in Fig. 29. The movement at the north end is shown by the lines composed of short dashes. The maximum movement recorded is for pier 1 on August 23 when the north end of the pier was 0.41 in. west of the position which it occupied June 1. The greatest rotation about a vertical axis was for the same pier on August 30 when the north end of the pier was west 0.31 in. and the south end was east 0.05 in., a difference of 0.36 in. in a pier length of approximately 1000 in. This corresponds to an angle of 0.00036 radians. The rotation is clockwise, indicating that the horizontal component of the thrust from the filled skew barrel arch is slightly greater at the obtuse than it is at the acute angle. But this difference in thrust produced only a very small rotation of the piers. The settlement varied from a trifle up to 1 in. for the various piers.

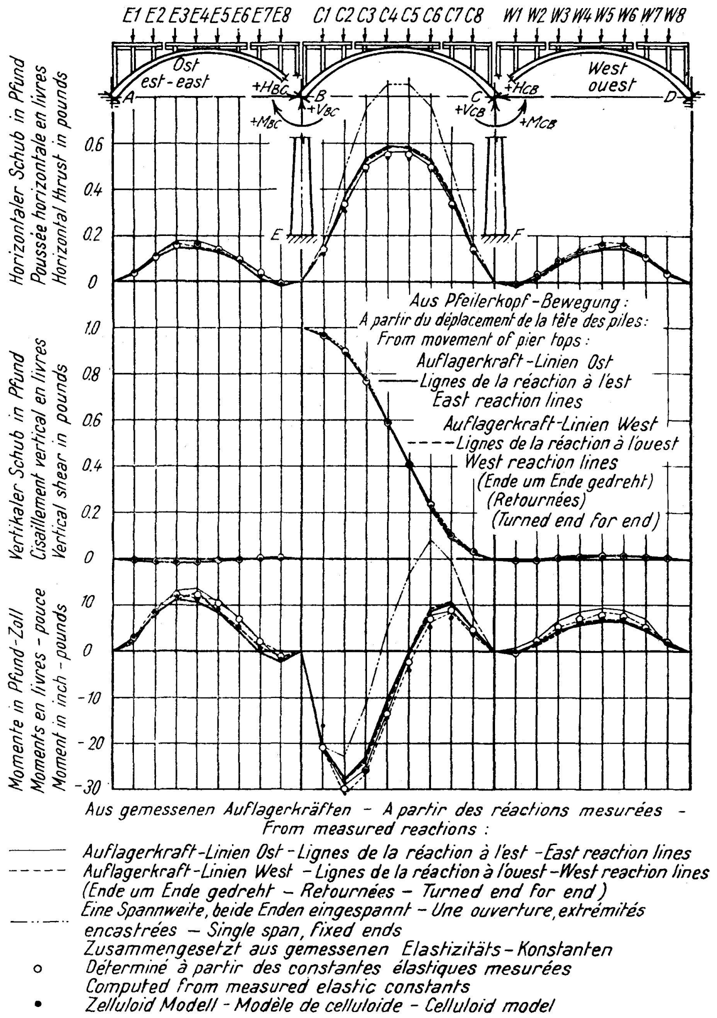


Fig. 24.

Einflußlinien für Einheitslasten für Pfeilerkopf-Auflagerkräfte der mittleren Spannweite. Aufbau mit Fahrbahn auf Scheitel und Zwischenausdehnungsfugen, 20-Fuß Pfeiler — Lignes d'influence pour charges unitaires pour les réactions à la tête des piles dans l'arc central. Tablier surélevé avec points de dilatation intermédiaire. Piles de 20 pieds — Influence Lines by Unit Loads for Center Span Pier-Top Reactions. Structure with Low Deck with Intermediate Expansion Joints. 20-Foot Piers.

Diagrams similar to Figs. 28 and 29 are given in the original report (7) for the other bridges under observation. The rotation of the piers of the other bridges about a horizontal axis did not differ greatly from that for the piers of the Kentucky Avenue Bridge except that, for the North Bridge at Yorkville, Illinois, one end span was poured six weeks before the adjacent span and the intervening pier rotated 0.0011 radians. After the adjacent pier had been poured there was a counterrevolution so that the final rotation of the pier after all spans had been poured and all shores removed was 0.0007 radians.

The observations on the five multiple-span bridges during construction appear to justify the following conclusions:

The movement of the piers, although small, produced an appreciable strain in the concrete. The strain often occurred when the concrete was green and did not occur instantaneously, but over a considerable period of time, so that, although the modulus of elasticity is not known, it was probably small, and may not have exceeded 1 000 000 lb. per sq. in. Even with this value of E the stress corresponding to the strain caused by the pier movements is appreciable.

The movements observed were so small that they could not be detected without the use of precision instruments, yet the stress resulting from these movements is considerable. It would not appear, therefore, to be too great a precaution to require that the piers be kept under observation with precision instruments during construction in order that any motion of the piers might be detected and steps taken to prevent its continuation. Since an unbalanced thrust from adjacent arches is the undesirable condition most likely to move a pier, and since the movement that this condition is most likely to produce is a rotation of the piers, it would appear that the most valuable single observation to make is the rotation of the pier about its horizontal longitudinal axis. This movement can be observed both accurately and easily by means of the level bar [4]).

What took place during the construction of five bridges is no assurance as to what will take place during the construction of some other bridge, but the observations on these bridges are reassuring. They indicate that pier movements during construction will not be excessive if proper precautions are taken. The stability of piers on piles driven into gravel is of particular interest.

8. Movement of Piers During the Life of a Multiple-Span Arch Bridge: Multiple-span reinforced concrete arch bridges are designed on the assumption that the bases of the piers are fixed [5]). If the bases of the piers move, stresses are introduced into the structure that are not considered by the designer. For this reason the use of multiple-span arches should be limited to sites at which solid foundation conditions exist. The piers of the Gilbert Street Bridge at Danville, Illinois (4) are supported on shale and the dead load produces an unbalanced thrust which caused the unit soil pressure under the pier bases to be greater on the side away from the long span than it is on the side toward the long span. Shale is plastic and flows under a continuous load. As a result the piers of this bridge gradually tipped outward from the center span until the tops of the piers came into contact with the floor of the bridge, which prevented further movement. Balanced dead-load thrusts are desirable for all piers supporting adjacent arches in a multiple-span series and are imperative unless the piers are on a foundation that will not flow.

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[4]) See page 10, reference 4 for a description of the level bar.

[5]) Sometimes the elastic deformation of the piers is neglected and the design is on the basis that the arch ribs are fixed at the springing.

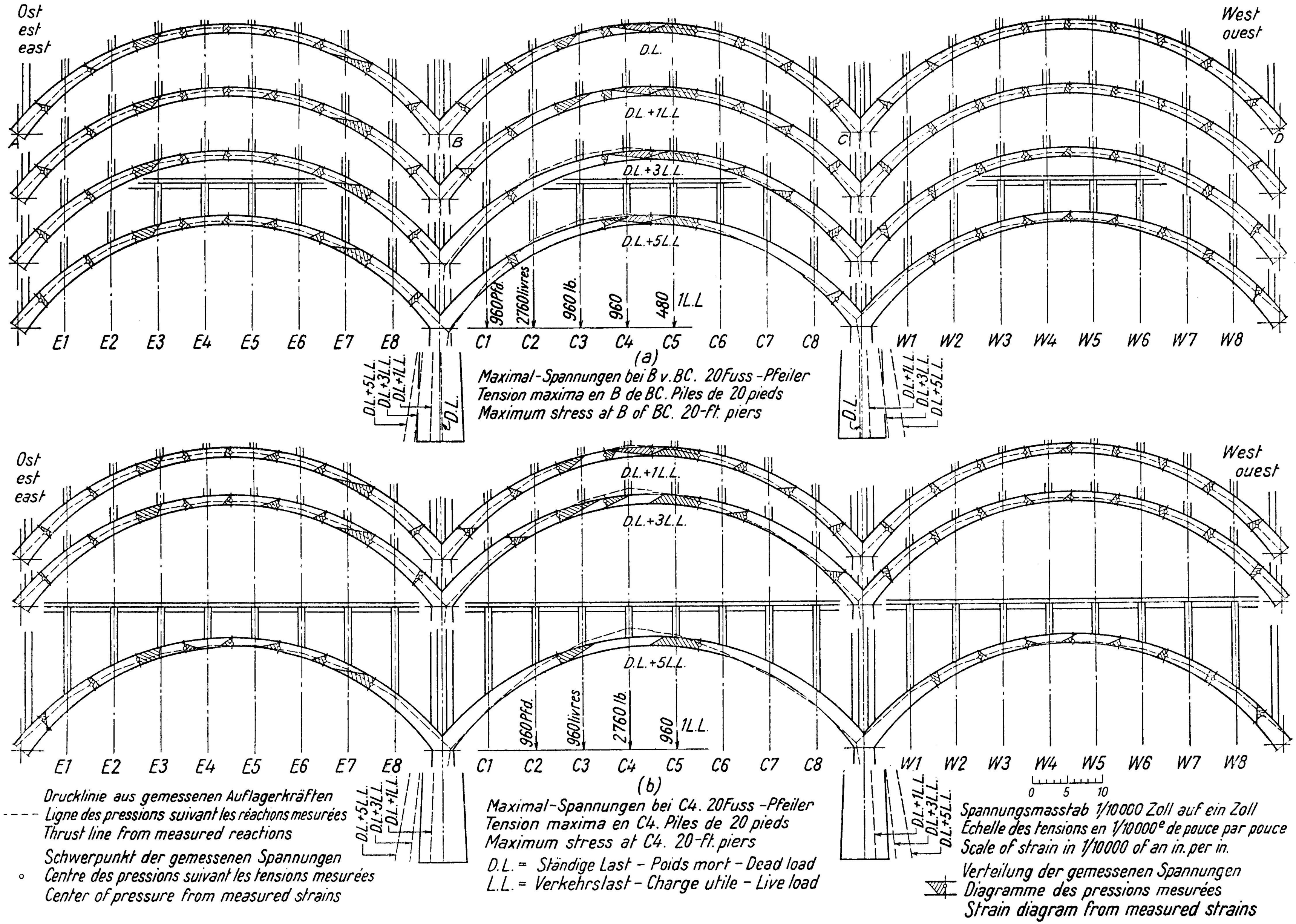

Fig. 25.

Drucklinien für Tragfähigkeitsversuch. Aufbau mit hochliegender Fahrbahn ohne Zwischenausdehnungsfugen, 20-Fuß Pfeiler.
Ligne de pression déterminée par l'essai de résistance. Tablier surélevé sans joints de dilatation intermédiaires. Piles de 20 pieds.
Thrust Line for Capacity Test. Structure with High Deck without Intermediate Expansion Joints. 20-Foot Piers.

9. Analysis of Multiple-Span Arch Bridges on Slender Piers: The action of a multiple-span series of arch ribs supported on high piers differs from that of a single span in that, for the latter, the ends of the arch are fixed, whereas, for the former, the fixed points are the outer ends of the end arches and the bases of the intermediate piers. A load on one arch adjacent to a pier, unless balanced by a similar load on the arch on the other side of the pier, causes the top of the pier to rotate and to move horizontally; and these movements introduce stresses in the arch rib.

An analysis of a multiple-span series of arch ribs takes into account the movement of the pier tops due to the deformation of the piers. Many methods of making the analysis have been devised, but they are similar in this respect. Each reaction component at the springings of the rib is considered as the resultant of two parts. One part is the fixed-end reaction for the arch span, and the other part is due to the movement of the pier tops. The first part is obtained from an analysis of a similar single-span arch rib with fixed ends. The second part is obtained by using some form of the elastic theory to determine: 1. The relation between the unit component movements (x and Θ) of the pier top, and the reaction components (H, V and M) which it is necessary to apply to the top of the pier to produce, separately, the components of movement. 2. The relation between the unit component movements (x and Θ) of one end of the arch with its other end fixed, and the reaction components (H, V and M) which it is necessary to apply to the ends of the arch to produce, separately, the two components of movement. The reaction components necessary to produce a unit moment (each component separately) of one end of a member when the other end is fixed, are called the elastic constants of the member.

If, for a multiple-span series of arch ribs on slender piers, the fixed-end reaction components for the arch span are known, and if the elastic constants of each arch and of each pier are known, then the reaction components at the ends of all spans can be computed, taking into consideration the elastic deformation of the piers [6]).

A multiple-span series of ribs without deck supported on slender piers can be analyzed by the all-algebraic method outlined above, but if, as is usually the case, the arch spans contain spandrel columns and deck the determination of the fixed-end reactions and the elastic constants by an all-algebraic method is difficult and is not usually attempted in the actual design of arch bridges. A combined Algebraic-and-Experimental Method is, however, feasible [7]). By this method, the fixed-end reaction components due to load and the elastic constants for the arch span are obtained from tests of elastic models. The elastic constants for the piers are determined either algebraically or by tests of elastic models. The various constants, all determined experimentally or part experimentally and part algebraically, are then used in the algebraic solution outlined above. The resulting analysis gives the reaction components at the springings of all arches and the tops of all piers. This method has an advantage over an allexperimental method in that the models are much smaller and the tests much simpler where the model is of

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[6]) This method is explained in detail in Trans., A. S. C. E., Vol. 100, page 1573.

[7]) This method was developed under the supervision of the author by Donald E. Larson, and is presented under the title, „A Study of Multiple-Span Arches"; a thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering in the Graduate School of the University of Illinois, 1930.

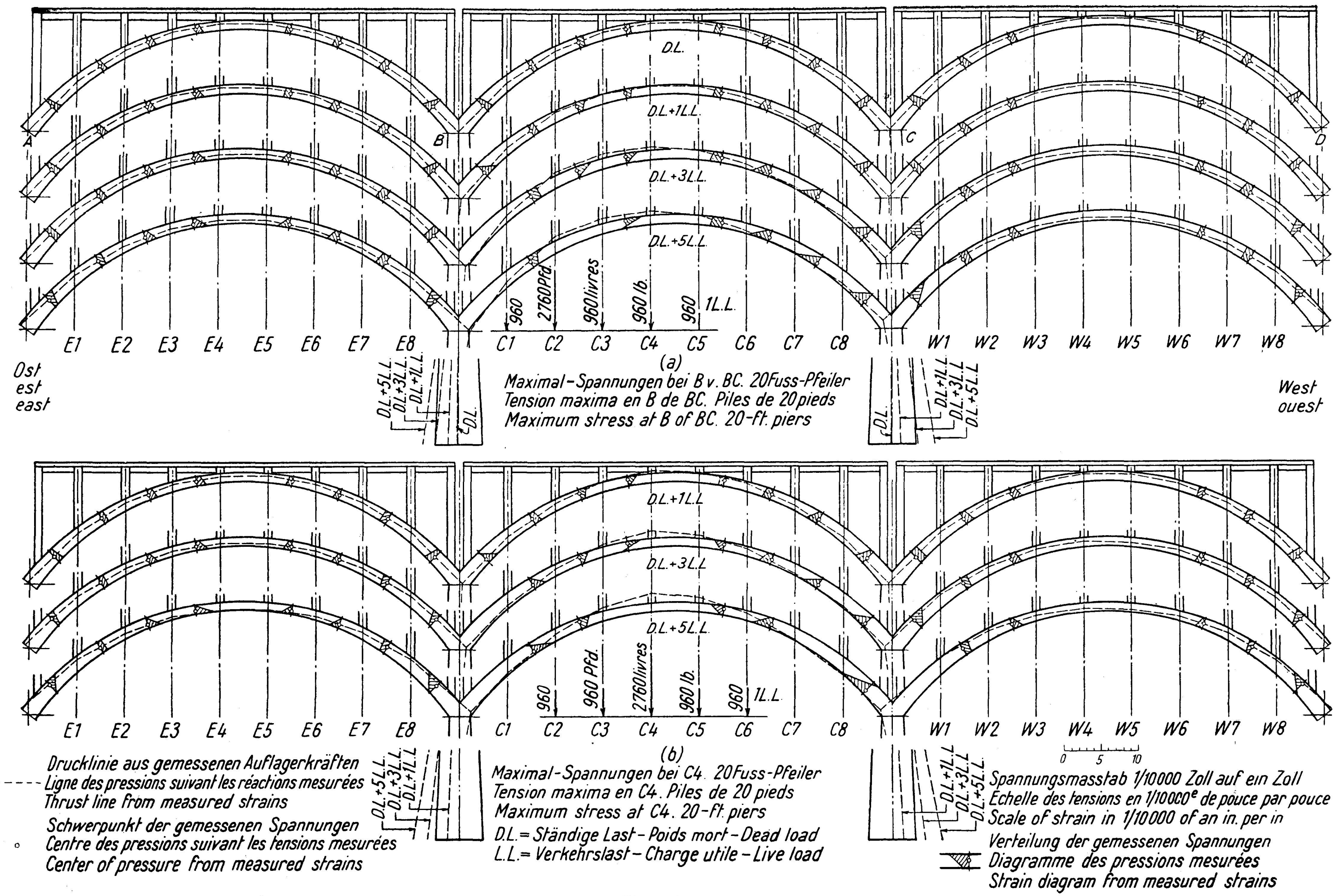

Fig. 26.

Drucklinien für Tragfähigkeitsversuch. Aufbau mit Fahrbahn auf Scheitel ohne Zwischenausdehnungsfugen. 21-Fuß Pfeiler.
Ligne de pression déterminée par l'essai de résistance. Tablier surbaissé sans joints de dilatation intermédiaires. Piles de 20 pieds.
Thrust Line for Capacity Test. Structure with Low Deck without Intermediate Expansion Joints. 20-Foot Piers.

a single span or a single pier than they would be if the model is of the structure as a whole. Moreover, a study [8]) has shown that a considerable error in the elastic constants of the arch span or of the piers produces a comparatively small error in the reactions at the ends of the spans of the multiple-span structure.

10. Influence of Time Yield in Concrete Upon the Stresses in a Single-Span Arch Bridge: The stresses in a statically indeterminate structure are of two kinds: 1. Load stresses, which are due to the direct effect of the loads that the structure carries, and 2. deformation stresses, which are due to a movement of the supports. Stresses essentially the same as deformation stresses are also produced if the lengths of certain members change, due to a change in temperature or other causes, even though the supports remained fixed.

The length of a reinforced concrete arch rib changes: 1. because of the immediate strain accompanying the application of a load, usually designated as rib shortening, 2. because of the time yield in the concrete, 3. because of temperature changes, and 4. because of the shrinkage in the concrete that takes place as the concrete dries. All changes in the length of the rib of an arch with fixed abutments change the shape of the rib, thereby producing flexural stresses similar to the flexural stresses that are produced by moving the abutments without changing the length of the rib. Since the stress is due to a change in shape and not due to a load, it is a deformation stress, and, since the changes, except rib shortening, take place slowly, it would appear possible that the time-yield property of the concrete might enable the rib to assume its new shape without incurring the stresses that would be produced if the same changes occurred quickly.

In view of this possibility, tests were made to determine the change in reactions at the springings of an arch due to the shrinkage of the concrete, due to the combined effect of shrinkage and time yield under dead load, and due to changing the span by an amount equivalent to a change in temperature of 100 degrees F. (9). In order to simulate the gradual temperature trends throughout a season, the change in span was made in five equal increments; each increment was equivalent to a change of 20 degrees F. and the changes were made at intervals of approximately 30 days. Readings were taken just before and after each change in span to determine the immediate change in the reactions. Subsequent readings were taken at intervals of approximately one week to determine the change in reactions due to time yield.

A series of tests was also made to determine the effect of the gradual yielding of the abutments upon the load-carrying capacity of the arch. The initial readings were taken when the design dead load was on the structure and when the abutments were in their normal position relative to each other. Subsequent readings were taken after each of a number of live loads had been added, and after each of a number of movements of the abutments. In general, a period of one week was allowed to elapse between successive abutment displacements to permit time yield of the concrete to occur.

The specimen was similar to the one shown in Fig. 22 except there were no intermediate expansion joints. Reactions were weighed with scales as in tests described in Section 1.

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[8]) Trans., A. S. C. E., Vol. 100, page 1577.

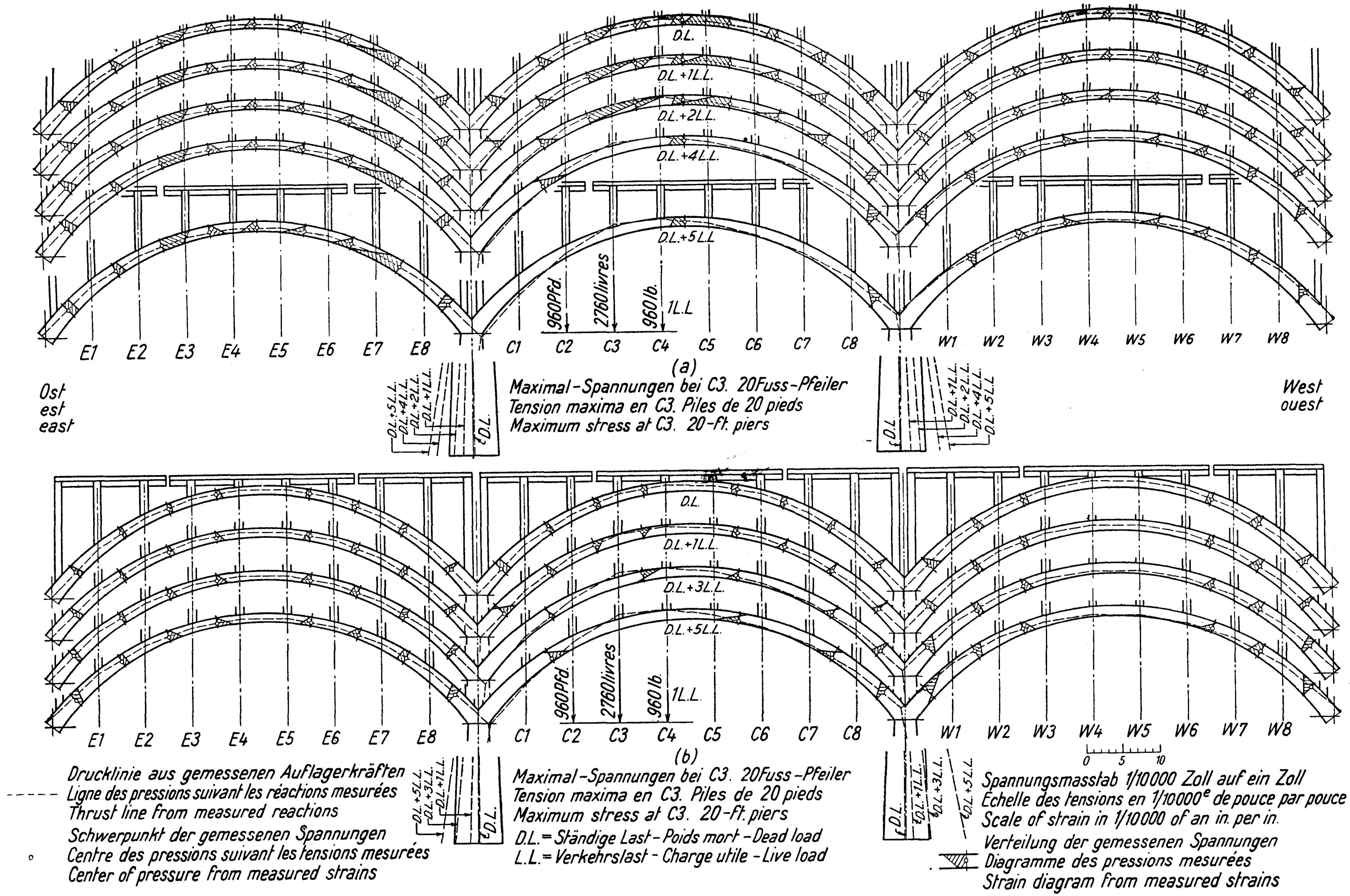

Fig. 27.

Drucklinien für Tragfähigkeitsversuch. Aufbauten mit hoch gelegener Fahrbahn auf Scheitel ohne Zwischenausdehnungsfugen. 20-Fuß Pfeiler.

Ligne de pression déterminée par l'essai de résistance. Tabliers surélevés et surbaissés sans joints de dilatation intermédiaires. Piles de 20 pieds.

Thrust Line for Capacity Test. Structures with High and Low Decks with Intermediate Expansion Joints. 20-Foot Piers.

Beginning when the structure was 16 days old and when it carried no load except its own weight, observations were made to determine the changes in the abutment reactions due to time yield and shrinkage. Observations were made at frequent intervals, and the abutments were adjusted carefully for position before readings were taken. The changes in the reactions during a period of 34 days are shown in the left-hand portion of Fig. 30. These diagrams indicate that, for the period of 34 days beginning when the arch was 16 days old, the horizontal thrust fell off 540 lb. and the moment fell off 23 600 in. lb. at the springings. The shrinkage of the concrete in the arch rib, as determined from strain-gage readings at sections midway between the spandrel columns, at the intrados and extrados, was 0.000092 in. per inch, the average of the values for all sections. This corresponds to a change in the length of the 324-in. span of 0.0298 in. Using this value as the change in span that was prevented, the observed changes in reactions are equivalent to a horizontal thrust of 18 100 lb., and a moment at the springing of 800 000 in. lb. per inch change in span. These observations cover a period from the time the arch was 16 days old until it was 50 days old. Other tests made to determine the immediate change in the reactions due to a unit change in span when the arch was nearly a year old, gave a change in thrust of 25 400 lb., and a change in moment of 1 313 000 in. lb. per inch change in span. The ratio of the latter value of the thrust to the former is 1.40, and the ratio of the latter value of the moment to the former is 1.64. The fact that these ratios exceed unity may have been due in part to the low value of the modulus of elasticity of the concrete during these tests when the concrete was green, or it may have been due to the fact that the time yield in the concrete reduced the flexural stress due to shrinkage. But, whatever the cause, the unit stress due to shrinkage was much less than the stress due to a sudden change in span equivalent to the unit shrinkage and based upon a value of E for the aged concrete.

The deflection of the arch rib due to shrinkage of the concrete is shown by the broken lines of Fig. 31.

After the tests to determine the effect of shrinkage had been made, the same structure was used in a series of tests to determine the effect of combined shrinkage and time yield of the concrete upon the reactions at the springings when the arch carried the design dead load. This superimposed dead load, the same as the dead load on one span of the three-span structure shown in Fig. 23, except that an additional load of 4000 lb. was added at points 4 and 5, was purposely chosen so as to produce a large moment at the springing.

The changes in the reactions at the springings that occurred while the dead load was on the structure are shown in Fig. 30, and the vertical movement of points on the arch axis is shown in Fig. 31. The application of the dead load changed the moment on the east abutment from — 5000 in. lb. to + 120 000[9]) in. lb., and on the west abutment from — 10 000 in. lb. to + 106 000 in. lb. The resulting change in moment at the springing, the average of the two ends, is 113 000 in. lb. The horizontal thrust at the springing increased from 2500 lb. to a little over 35 500 lb. The stress across the section at the springing, computed from the weighed reactions on the basis that concrete takes tension, varied from 44 lb. per sq. in. tension at the intrados to 620 lb. per sq. in. compression at the extrados.

[9]) A plus (+) moment is one that produces tension at the intrados.

The dead load remained on the arch for a period of 167 days, and observations were made at frequent intervals during this time.

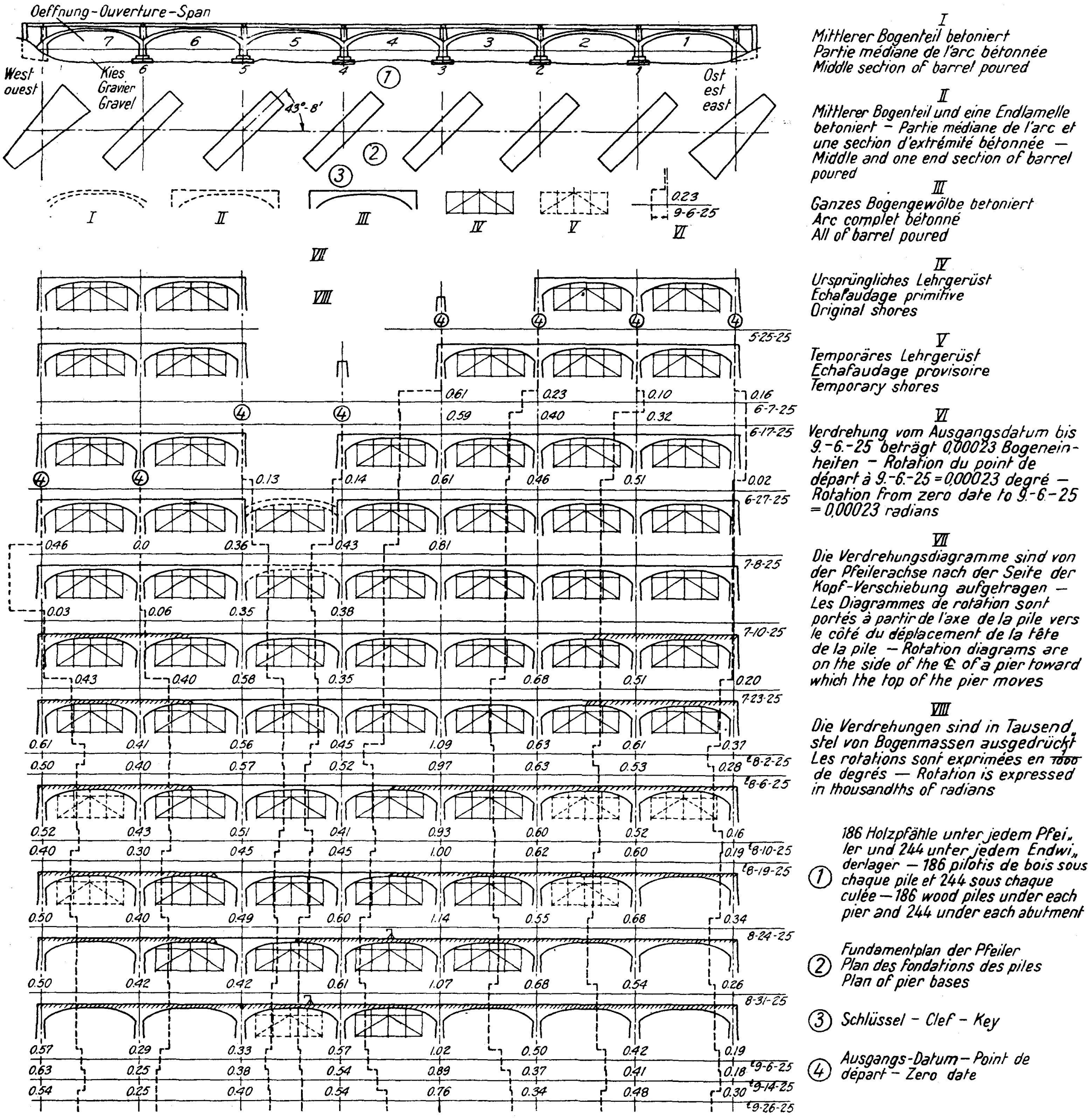

Fig. 28.

Verdrehung der Pfeiler während des Baues, Kentucky-Avenue Brücke, Indianapolis, Indiana.

Rotation des piles durant la construction du pont de l'avenue Kentucky à Indianapolis, Indiana.

Rotation of Piers During Construction, Kentucky Avenue Bridge, Indianapolis, Indiana.

The diagrams of Fig. 30 show that both the horizontal thrust and the moment at the springing decreased as the test continued. The question of interest is, are the reductions in the reactions as great as they would have been if the shortening of the arch axis had occurred in a few hours instead of extending over a period of several months?

The unit shortening of the arch axis as determined by strain readings at various sections was 0.00031 in. per inch; this is equivalent to a shortening of 0.100 in. for the 324-in. span. Tests indicate that a quick change in span of 0.10 in. without any other movement of the abutments and without any change in load, produces a change in moment at the springing of 131 300 in. lb. This is comparable to the reduction in the moment of 33 000 in. lb. which actually did take place as determined from the weighed reactions. Tests also indicate that a quick change in span of 0.10 in. produces a change in the horizontal thrust of 2540 lb. This is comparable to the reduction in thrust of 640 lb. which actually did take place. That is, the change in the length of the arch axis that occurred in a little less than six months, beginning when the arch was a little over one month old, caused a change in both the horizontal thrust and in the moment at the springing only one-fourth as great as would have been produced if the change in the length had taken place quickly. It is not clear whether the low value of the changes is due entirely to time yield or partly to time yield and partly to the fact that the value of E for the concrete was less at this time than later when the elastic constants were determined by a quick change in span. But the arch had been in a warm, dry atmosphere for more than a month when the tests of this series were begun. Moreover, the tests from which the change in thrust and moment due to a quick change in span were determined were made immediately after this series was completed. It would therefore appear safe to assume that E did not vary greatly between the time of the tests of this series and the time when the elastic constants were determined by a quick change in span.

After the dead-load tests had been completed the dead load was changed by removing 4000 lb. from each of load points 4 and 5. The resulting load is the same as the dead load for a single span shown in Fig. 23. A series of tests was then begun to determine the effect of time yield in concrete upon temperature stresses. Beginning with the normal value, the span was reduced 0.036 in., equivalent to a rise in temperature of 20 degrees F., at intervals of approximately one month until five reductions, equivalent to a rise in temperature of 100 degrees F., had been made. Readings were taken just before and after each change in span and at intermediate periods of approximately one week. The abutments were returned to their original position relative to each other, except for the change in span, before each set of readings.

The change in the reactions are shown in Fig. 32. The straight vertical portions of the diagrams represent the changes that occurred immediately when the span was changed, and the portions of the diagrams that slope downward to the right represent the changes that occurred between the changes in span.

The diagrams of Fig. 32 indicate that a quick change in span of 1 in. is accompanied by a change in the horizontal thrust of 25 400 lb. and a change in the moment at the springing of 1 313 000 in. lb. The total shortening of the span in the period of the test was 0.18 in., of which 0.02 in. may be considered as being due to the free contraction of the rib. The remainder, or 0.16 in., would produce flexure. If this shortening had taken place quickly it would, according to the values just given for unit change in span, have been accompanied by a change in horizontal thrust of 4060 lb., and a change in the moment at the springing of 210 000 in. lb. The actual change in the thrust was 2350 lb. and in the moment was 136 800 in. lb. That is, for this test, the effect of time yield in the concrete upon the horizontal reaction and the moment at the springing due to a change in span corresponding to a change in temperature of 100 degrees F. in a period of 230 days was as follows: The horizontal thrust was decreased 42 per cent and the moment

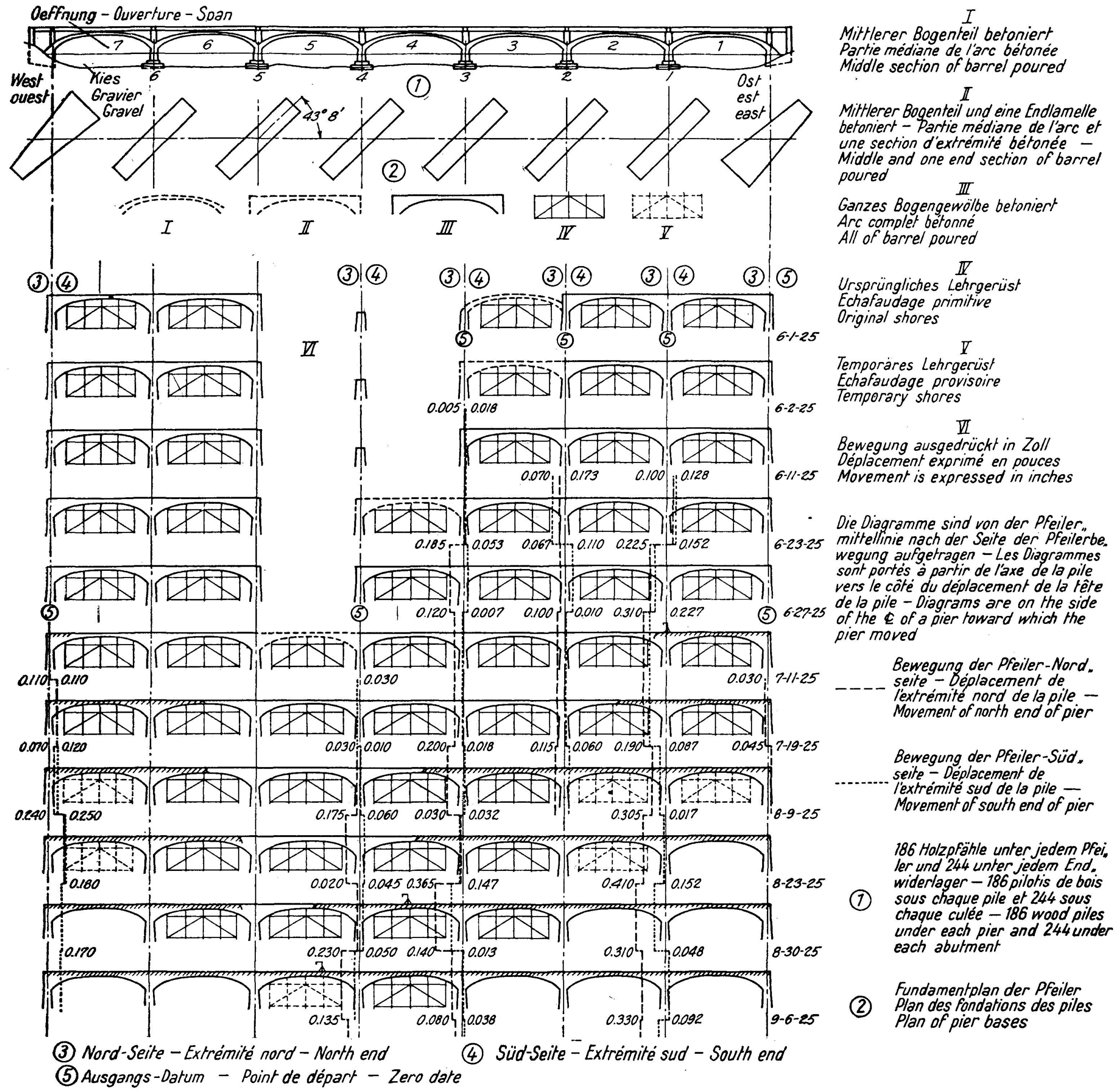

Fig. 29.

Horizontale Bewegung der Pfeiler während des Baues, Kentucky-Avenue Brücke, Indianapolis, Indiana.

Déplacement horizontal des piles durant la construction du pont de l'avenue Kentucky à Indianapolis, Indiana.

Horizontal Movement of Piers During Construction, Kentucky Avenue Bridge, Indianapolis, Indiana.

at the springing was decreased 35 per cent. The fact should be noted, however, that if time yield in concrete decreases with age and with repeated applications of load, then the fact that time yield caused an appreciable reduction of the deformation stresses in this arch does not necessarily mean that it would have an equal effect in an arch several years old.

A test was also made on the structure to determine the combined effect of increasing the live load and spreading the abutments. The arch was about 16 months old when the tests began, and intermediate expansion joints had been cut in the deck as shown in Fig. 22. The dead load was the same as the dead load on one span of the three-span series and shown in Fig. 23. The live load consisted of 960 lb. at load point 3 and 2760 lb. at load point 4. The two loads taken together were designated as one live load.

The initial readings were taken when the dead load was on the structure, and when the abutments were in their normal position relative to each other. The second set of readings was taken after one live load had been applied to the arch and the abutments had been returned to their normal position. The abutments were then allowed to spread 0.10 in. without rotation or vertical movement and without change in load. Readings were taken immediately after these changes had been made, and again at the end of the week. Changes were made in this manner, increasing the number of live loads and the spread of the abutments, until the arch carried six times the design live load and the abutments had been spread 2.0 in. Readings were taken just before and after each change, and again in seven days, and for some changes, a third set of readings was taken two weeks or more after the change in span or load. The reactions of the abutments were determined from the scale readings, and include the effect of the weight of the structure as well as the superimposed load.

The relation between time and the moment at the springing is shown in Fig. 33; and the relation between time and the moment at load point 4, the point of maximum unit stress, is shown in Fig. 34. In these figures the legend, 0.30 in. and 0.40 in., indicates that the abutments have been allowed to spread 0.30 in. and 0.40 in., respectively; the legend, 2 LL and 3 LL, indicates that the arch carried two times and three times the design live load, respectively.

If each increment of live load had produced the same moment and thrust as the first increment, then the application of six times the live load in addition to the design dead load and the weight of the structure would have produced a unit stress equal to 4535[10]) lb. per sq. in. If each 0.10 in. of spread had produced the same moment and thrust as the first increment, the spread of 2.0 in. would have produced a maximum unit stress at the section through load point 4 of 5075[10]) lb. per sq. in. This added to the hypothetical value of 4535 due to load would make a total of 9610 lb. per sq. in., a value greatly in excess of the strength of the concrete. Nevertheless the arch carried six times the live load for several days after the abutments had been spread 2.0 in. It is true, however, that there were several large cracks in the rib, deck, and columns, and the concrete in the rib at load point 4 was badly spalled. The behavior of the arch under these extreme conditions is of interest, primarily, as demonstrating the punishment that a concrete arch can sustain and still carry a large load.

The condition of the structure when carrying four times the design live load with the abutments spread 0.60 in. is of interest. The stress due to load, based upon the assumption that each live load produced the same increment

[10]) The unit stress was determined from the thrust and moment on the basis that concrete takes tension. The author realizes that this method is not satisfactory but, in his opinion, no other method is any more satisfactory for a badly cracked section.

of stress as the first live load, is 3640 lb. per sq. in.; and the stress due to the spread of the abutments, based upon a corresponding assumption, is 1520 lb. per sq. in., a total of 5160 lb. per sq. in., which is considerably in excess of the strength of the control cylinders. Nevertheless the structure was not seriously cracked.

The maximum stress at load point 4 as computed from the measured abutment reactions occurred when the abutments were spread 1 inch and

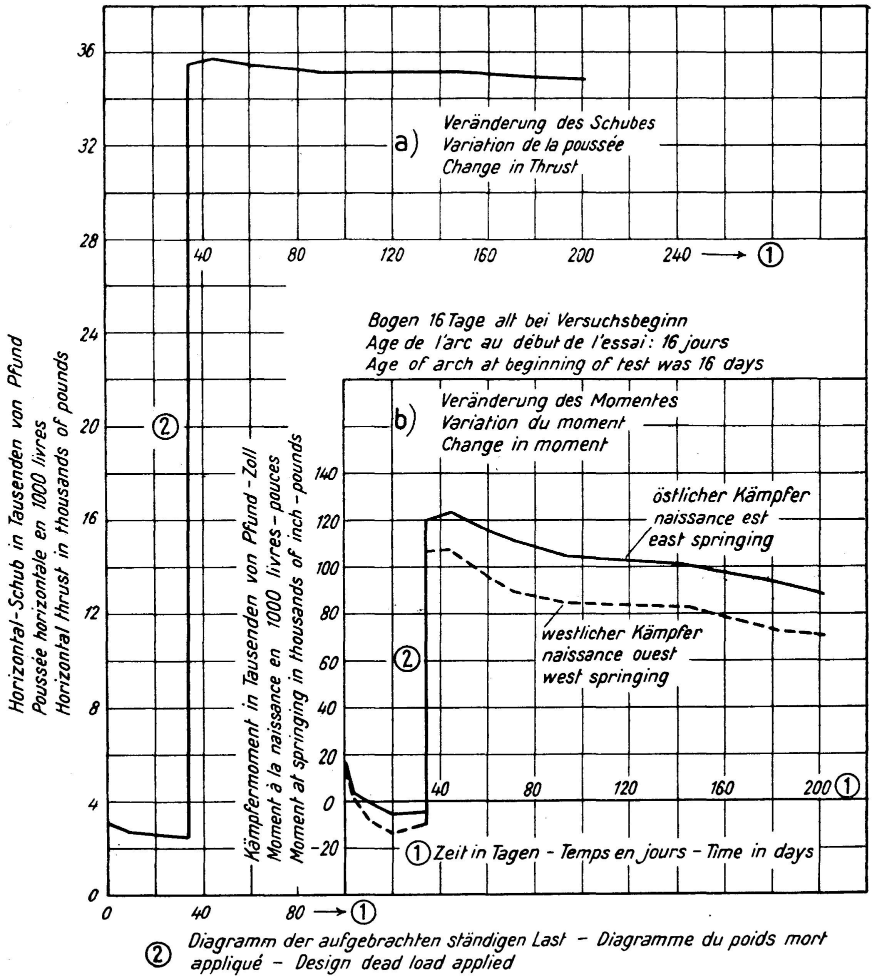

Fig. 30.

Veränderung der Auflagerkraft-Komponenten als Folge von Schwinden und Kriechen.
Variation des composantes de la réaction par suite du retrait et de la déformation lente.
Change in Reaction Components Accompanying Shrinkage and Time Yield.

the arch carried five times the live load in addition to the design dead load and the weight of the structure. The maximum unit stress under these conditions, computed from the measured reactions on the basis that concrete takes tension, was 4480 lb. per sq. in.

The strain in the concrete was measured on two gage lines at the intrados and two at the extrados on a section midway between each pair of adjacent load points. For most sections there were cracks in the concrete that made the strain gage readings of little value, but there were no cracks across the gage lines at section 4 in the panel just east of load point 4 at any time during the test, and the strains on these lines are reported.

Figure 35 shows the deformation in the concrete at section 4 during the 80-day period covered by the capacity test. The upper part of the figure shows the total deformation relative to conditions July 10, 1933; and the lower part the deformation that occurred during the capacity tests, from Oct. 16, 1934 to Jan. 4, 1935. The deformation at the beginning of the period was — 0.000536 in. per inch at the extrados and — 0.000567 in. per inch at the intrados. This deformation is due to shrinkage, dead load, and time yield. It is interesting to note that, on Oct. 16, when the arch was approximately 16 months old and carried the design dead load in addition to its own weight, the average unit deformation in the concrete at section 4 was 0.000552 and nearly uniform over the section. Assuming that the deformation in the steel was the same as in the adjacent concrete, the compression in the steel due to shrinkage, dead load and time yield was 16 500 lb. per sq. in.

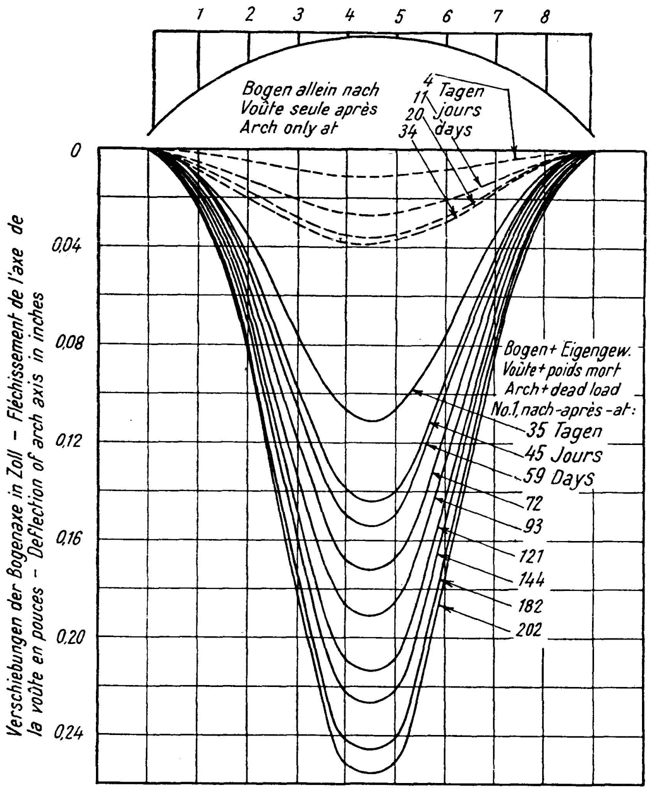

Fig. 31.

Verschiebung der Bogenaxe infolge Schwindens und Kriechens — Fléchissement de l'axe de la voûte par suite du retrait et de la déformation lente — Deflection of Arch Axis Accompanying Shrinkage and Time Yield.

A single test has little statistical significance and has value primarily in that it may indicate the manner in which a structure functions and direct attention to phenomena that affect its behavior. The fact that time yield in concrete quickly eliminates a large part of the stress in an arch that would otherwise accompany early volume changes (shrinkage, initial drop, in temperature, and time yield), has been anticipated by rational methods [12]). This test supports the results of analysis, and it would appear safe to conclude that a considerable part of the early volume changes have little effect upon the stresses in the concrete of an arch rib. Although time yield of concrete reduces deformation stress in the concrete of an arch, it increases the stress in the steel, a phenomenon that is recognized in the design of reinforced concrete columns. It is believed that the high compression in the steel is not a serious source of weakness, providing the rods are supported against buckling. If time yield in concrete diminishes with age and the number of applications of a load, it is possible that temperature stresses in the concrete of arches a few years old may not be greatly relieved by time yield.

[12]) „Plastic Flow in Concrete Arches", by Lorenz G. Straub, Assoc. Mem. A. S. C. E., Trans., A. S. C. E., Vol. 95, 1931, p. 613. „Plain and Reinforced Concrete Arches", by Charles S. Whitney, Proc., Am. Con. Inst., Vol. 28, 1932, p. 479.

The fact that the large gradual change in the span of the arch did not greatly affect its load-carrying capacity has been attributed to the fact that near the ultimate load a large gradual increase in strain is accompanied by a small increase in stress.

11. Relative Merits of High and Low Decks: A question pertaining to the design of multiple-span arch bridges which should be given more attention than it receives in the literature on arches, is the distribution of the vertical distance from the bases of the piers to the top of the roadway. The elevation of the roadway is usually fixed by the approaches, and that of the bases of the piers by the foundation conditions. Thus the difference between these two elevations is usually fixed within narrow limits, but the total vertical distance can be divided into three parts at the discretion of the designer. These parts are: 1. The height of the pier; 2. The rise of the rib; and 3. The distance of the deck above the rib at the crown.

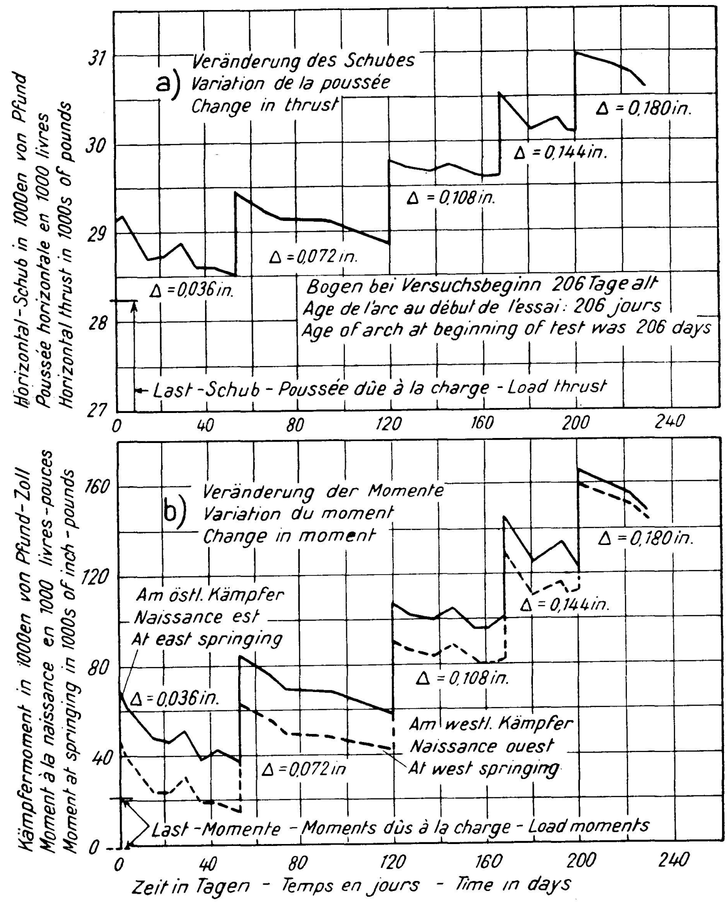

Fig. 32.

Veränderung der Auflagerkraft-Komponenten infolge Schwindens, Kriechens und aus Veränderung der Spannweite — Fléchissement de l'axe de la voûte par suite du retrait, de la déformation lente et de la variation de la portée — Change in Reaction Components Accompanying Shrinkage, Time Yield, and Changes in Span.

Insofar as the limited number of tests show, for a particular rib and a particular deck, the height of the deck above the rib does not materially affect the load-carrying capacity of the structure, although it does affect somewhat the stresses resulting from temperature change. Increasing the height of the deck above the crown of the rib also increases the horizontal movement that would take place at intermediate expansion joints if they are provided. This is compensated for, at least in part, by the greater flexibility of the longer spandrel columns. Therefore, it would appear that, structurally, the high deck has no appreciable advantage over the low deck, or the reverse.

If the total available vertical distance is small, the natural design is to make the abutments or piers as low as the flood channel will permit and make the deck so low as to be integral with the rib at the crown, thereby obtaining a maximum rise for the arch rib. If the total vertical distance is ample, or possibly excessive, the best distribution is not so easily apparent.

Increasing the flexibility of the piers of a multiple-span series increases the live-load stress in the rib, but a considerable increase in the flexibility causes a relatively small increase in the stress. Moreover, increasing the flexibility of the piers has a lesser effect on a rib with a large, than on one with a small, rise ratio. Increasing the height of the pier increases the flood

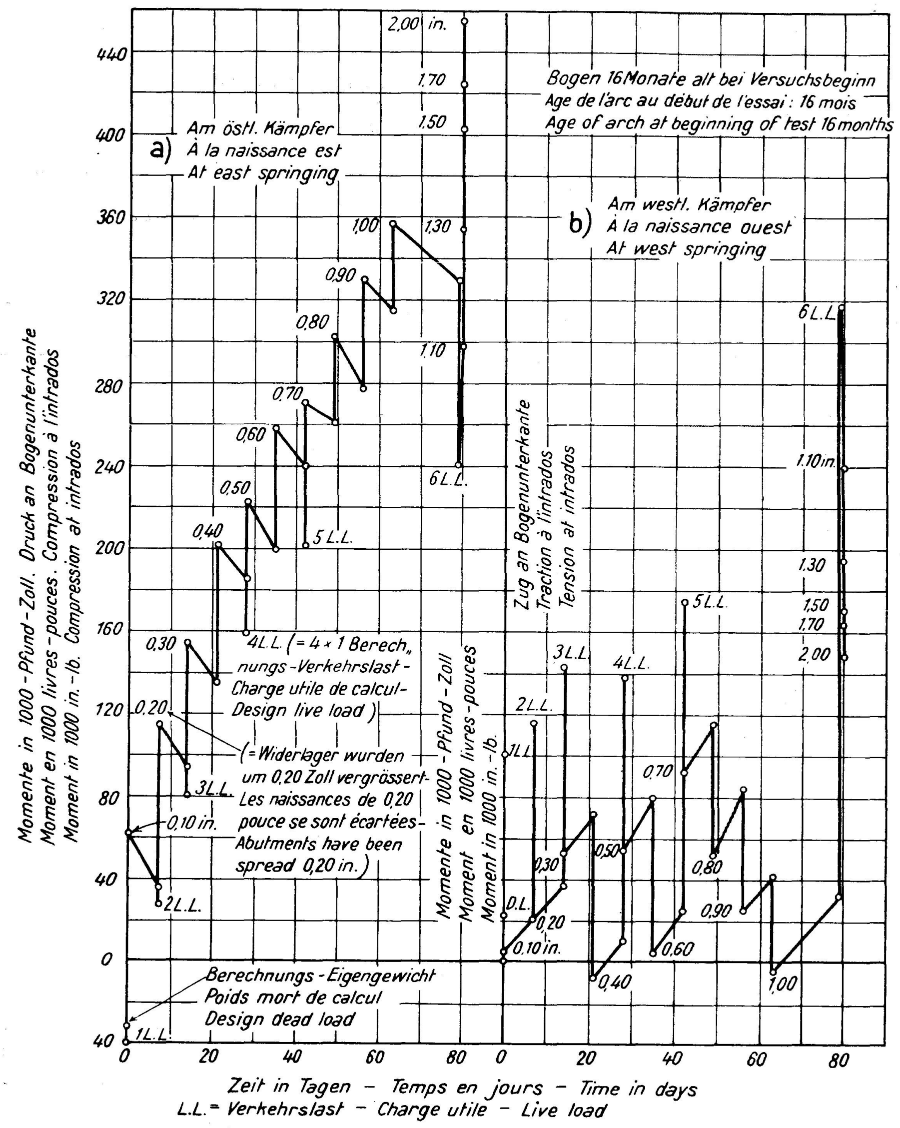

Fig. 33.
Beziehung zwischen Zeit und Moment am östlichen und westlichen Kämpfer.
Relation entre le temps et le moment aux naissances est et ouest.
Relation Between Time and Moment at East and West Springings.

channel. Increasing the rise ratio of the arch decreases the horizontal thrust on the piers due to load and decreases the temperature stress in the rib. It would appear, therefore, that in general there is no advantage in separating the rib from the deck at the crown as the vertical distance can be better disposed of by increasing the height of the pier and increasing the rise ratio of the arch rib. The use of a multiple-span series of flat arches on high piers would appear to be undesirable.

12. S t r e n g t h D e v e l o p e d b y C o n c r e t e i n a n A r c h R i b R e l a t i v e t o t h e S t r e n g t h D e v e l o p e d b y S i m i l a r C o n c r e t e i n

6-in. by 12-in. Control Cylinders: The strength that concrete will develop in an arch rib is of primary interest in determining the unit stress to be used in design. Control cylinders were made at the time the various arches were poured and the ultimate strength and modulus of elasticity of the concrete were determined from these specimens. The unit stress in the arch rib at the section of failure was also determined for many of the arches. The unit strength developed by the concrete in the arch and the strength of similar concrete in the control cylinders are compared in Table 1. It is to be noted that for the seven similar arches having a span of 17 ft. 6 in., the average strength of the concrete in the arches at the section of failure is

Table 1.
A Comparison of the Strength Developed by Concrete in an Arch Rib and the Strength Developed by Similar Concrete in 6-in. by 12-in. Control Cylinders.

Specimen No.	Figure No.	Unit Strength Developed by Concrete, lb. per sq. in.		E, 10^6, lb. per sq. in.	
		In Arch Rib	In Control Cylinders	Rib at Section of Failure	Control Cylinders
26—1 [13])	13	2479	2563	2.18	3.51
26—2	13	2428	2590	2.38	4.08
26—5	13	2394	2470	2.00	2.76
26—6	13	2487	2990	1.69	2.94
26—10	13	2307	1396	1.44	1.41
27—3	13	1716	3380	1.34	2.95
27—4	13	3670	3575	2.13	4.00
	Average	2500	2710	1.88	3.10
Single-Span	2	3674	3773	2.66	3.64
Three-Span	2	3812	3310	2.95	3 37
Three-Span	21	3200	3400		
Three-Span	22	4000	4280		
	Average	3670	3690		

2500 lb. per sq. in. and the average strength of similar concrete in the control cylinders is 2710 lb. per sq. in. This is not a true comparison because although the concrete that went into the control cylinders was the same as that which went into the arches, the concrete was not consolidated the same amount in the two instances. The arches broke at or near the crown where the concrete could be placed without much spading and where the hydraulic head was only a few inches. The concrete that went into the control cylinders was systematically rodded and the hydraulic head was about twice as great as for the arch ribs. The modulus of elasticity of the concrete in the rib was determined from longitudinal strain readings on the rib and the thrust in the rib computed from the measured reactions of the abutments. The modulus at the section of failure determined in this manner and the modulus as determined by tests of the control specimens, are compared in the last two columns of Table 1. The modulus in the rib is seen to be much less than the modulus in the control cylinders and it is believed that the somewhat lower strength of the concrete in the ribs can be attributed to this lack of

[13]) The arches of the 26 series had the same profile, the same mix and the same percentage of steel as arch 27—3 and 27—4. Ribs less than 6-in. in width were not included in this comparison.

consolidation and not to any difference in action between the rib and the cylinder.

The four arches having a 27-ft. span were poured after the others had been tested and an effort was made to consolidate the concrete over the central portion. But, even for these, the modulus was smaller over the central portion where the concrete is easily placed and where the hydraulic head is small than it was at the ends where the concrete had to be rodded to get it into place and where there was a considerably larger head. The average unit strength developed by the concrete in these ribs was very nearly the same as the average for the corresponding cylinders.

In view of these tests it appears safe to conclude that concrete in an arch rib will develop approximately the same strength as similar concrete in a 6-in. by 12-in. control cylinder.

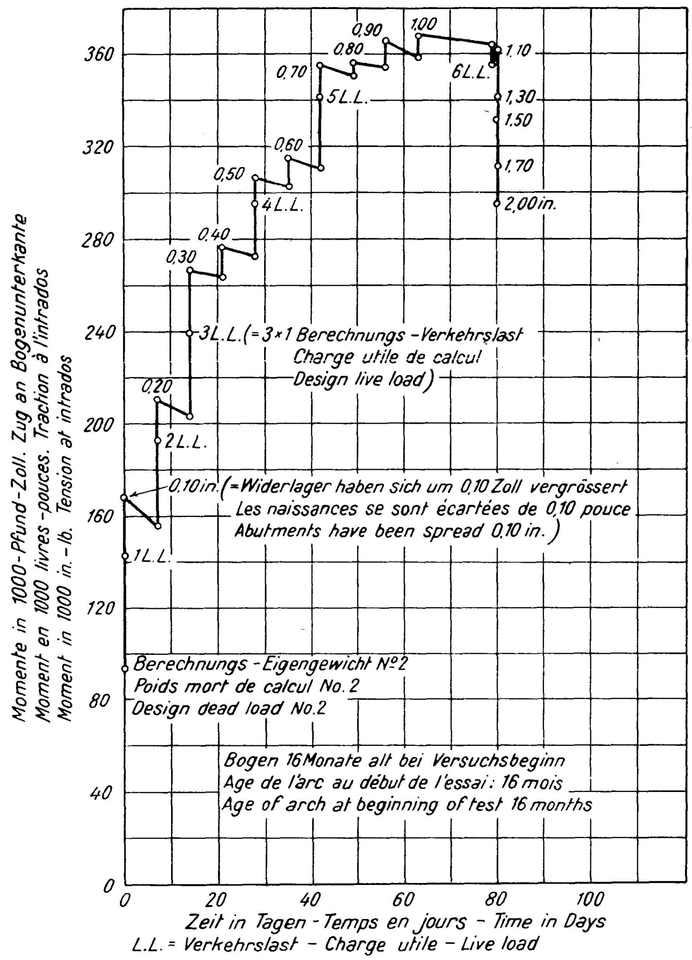

Fig. 34.

Beziehung zwischen Zeit und Moment für Lastpunkt 4 — Relation entre le temps et le moment au point de charge 4 — Relation Between Time and Moment at Load Point 4.

13. Summary of Conclusions: The following summary of conclusions is based upon the tests by the author outlined above and upon investigations made by other members of the Special Committee [14]) on Concrete and Reinforced Concrete Bridges of the American Society of Civil Engineers. The conclusions are taken from the Final Report (10) of this Committee.

1. The elastic theory based upon the usual assumptions is adequate for determining the thrust and moment in reinforced concrete arch systems without continuous superstructure, with sufficient accuracy for purposes of design. The usual assumptions referred to are:

a) A plane section before flexure remains a plane after bending.
b) The moment of inertia for purposes of determining thrust and moment may be calculated upon the assumption that concrete takes tension.
c) The modulus of elasticity of concrete may be assumed to have the same value at all stresses and sections.

2. A considerable accidental variation in the moment of inertia or in the modulus of elasticity will not materially affect the position or magnitude

[14]) See Section 14.

of the thrust due to loads in a single-span arch. This variation may be caused by nonhomogeneity of the material or by cracks on the tension side or the rib.

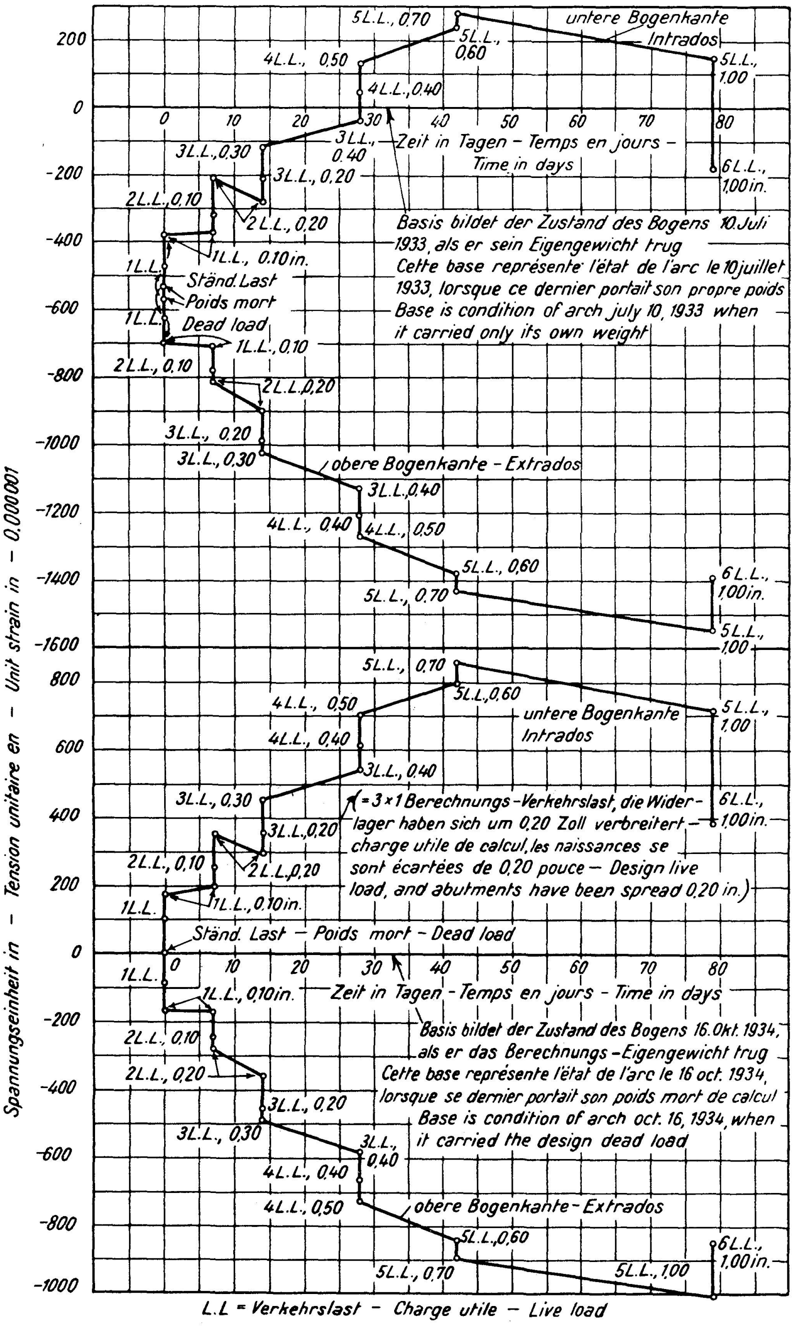

Fig. 35.

Beziehung zwischen Zeit und Spannung im Beton für Schnitt 4.
Relation entre le temps et la tension dans le béton à la section 4.
Relation Between Time and Strain in Concrete at Section 4.

3. Failure of a reinforced concrete arch rib is very improbable except as it may be subjected to a large moment. In such a contingency the failure is primarily a flexural failure.

4. The concrete in an arch rib will develop approximately the same ultimate strength as the same concrete in 6-in. by 12-in. cylinders.

5. If an arch rib is not subjected to lateral forces its strength is not affected by its slenderness, so long as the ratio of the unsupported length of the arch axis to width of the rib does not exceed 25 to 30.

6. In multiple-span arch systems the dead load unit stresses are sometimes greater than the corresponding unit stresses in a similar span having fixed ends, due to lack of homogeneity of the material and accidental variations in dimensions. This suggests that the design of multiple-span arch systems on slender piers should be more conservative than that of single-span arches.

7. The following method of analyzing the stresses in a single-span arch bridge with open spandrel and deck is suggested in the design of structures for which deck participation may be important:

a) Determine the dead load stresses analytically on the basis that the dead load is carried by the rib, unrestrained by the deck.
b) Determine the live load stresses by an experimental method using elastic models, considering the structure as a whole and not the rib alone. This is important because of the effect of the deck participation upon the stresses in the deck and columns rather than because of its effect upon the stresses in the rib.
c) Determine the shrinkage and temperature stresses by an experimental method using elastic models of the structure as a whole.

The theoretical analysis of an arch with open spandrel and deck is so complicated that few engineers have acquired the facility that justifies them in using it with confidence. Furthermore, the time required for such a theoretical analysis is excessive.

8. The following method of analyzing a multiple-span arch bridge with open spandrel and deck is suggested, in the design of structures in which deck participation may be important:

a) Determine the dead load stresses analytically on the basis that the dead load is carried by the rib unrestrained by the deck.
b) Determine the live-load reaction components at the springings by an algebraic analysis based upon experimentally determined elastic constants, or by an all-experimental method, considering the structure as a whole and not the ribs alone.
c) Determine the shrinkage and temperature reaction components at the springings from experimentally determined elastic constants, or by an all-experimental method.

The elastic constants may be determined from tests of an elastic model of a single span.

9. A deck without intermediate expansion joints, constructed so as to act with the rib in carrying loads, reduces the magnitude of the live load moment at the springing where the deck does not increase the strength of the structure; and it increases the moment over the central portion of the span where the strengthening effect of the deck is greatest. Intermediate expansion joints in the deck reduce these effects.

10. The moment at the springing due to shrinkage, time yield, temperature changes, and other causes equivalent to a change in span is greater for a structure having a continuous deck than for the same arch rib without a deck. Intermediate expansion joints in the deck may increase or may reduce this difference, depending upon the rise ratio of the rib and the proportions of the deck.

For an arch having a deck, the moment in the rib at a section near the crown and in a panel below an intermediate expansion joint in the deck, due to any causes equivalent to a change in span, will be greater than for a structure without expansion joints, and may be greater than for a rib without deck. If intermediate expansion joints are placed in the panel containing the intersection of the temperature thrust line and the axis of the arch, the temperature moment in the rib in the panel will be small.

The general use of intermediate expansion joints in the deck is not recommended, except where an analysis shows that they are needed and are not harmful.

11. The height of the deck above the rib of an arch bridge does not greatly influence the stresses in the rib due to loads; but increasing the height of the deck above the rib by reducing the rise of the arch greatly increases the stresses due to any cause equivalent to a change in span. From this it would follow that the rise of the rib should be made as great as the grade of the roadway will allow.

12. The elastic deformation of the piers in multiple-span structures increases the value of the live-load positive moment over the central portion of the span. For arches with a continuous deck, this increase in moment occurs over that portion of the structure which is strengthened by the deck.

The elastic deformation of the piers decreases the live-load negative moment and increases the live-load positive moment at the springing of the center span, and increases the negative moment at the end abutment. The deck does not assist the rib in resisting moment at these points.

13. The bases of the piers and abutments of arch bridges should be designed so that the time yield of the soil due to dead load pressure will not cause the piers to tip. The uniformity of pressure distribution that is necessary depends upon the character of the soil but, with materials like hardpan and shale, which are particularly susceptible to time yield, an uneven distribution may cause considerable rotation even if the maximum pressure is not greater than would be permitted with uniform distribution.

14. The piers and abutments of arch bridges should be kept under observation with precision instruments during the construction of the superstructure, in order that any excessive motion may be detected.

15. For skew barrel arches tests of three-dimensional models confirm the theory for computation of reaction components presented by J. Charles Rathbun, M. Am. Soc. C. E.

16. The length of the rib of a reinforced concrete arch shortens gradually after the concrete sets due to many causes, including initial drop in temperature, shrinkage, and time yield. The plastic flow of the concrete greatly reduces the flexural stress that would otherwise accompany this change in length. Plastic flow also reduces the stress that would otherwise accompany seasonal changes in temperature during the first year but possibly does not appreciably reduce temperature stresses later. It would appear, therefore, that if temperature stresses corresponding to the seasonal temperature range were taken into account in the design of an arch, the initial drop in temperature, shrinkage, and time yield might be neglected in determining stresses in the concrete. The initial drop in temperature, shrinkage, and time yield, do, however, cause an appreciable permanent vertical movement of the rib at the crown, especially for flat arches, which should be considered in the design and construction of the superstructure. They also produce a high

compressive stress in the steel, but this stress will not affect the load-carrying capacity of the arch if the steel is restrained against buckling.

Because, at stresses near the ultimate, a large increase in the strain of concrete is accompanied by only a very small increase in stress, it would appear that a considerable gradual movement of the abutments has very little effect upon the load-carrying capacity of an arch.

14. Acknowledgments: The investigations described in this paper were made by the author as a part of the work of the Committee on Concrete and Reinforced Concrete Arches of the American Society of Civil Engineers. This Committee was appointed by the President of the Society, on authorization of the Board of Direction, in May, 1923. The members invited to serve on this Committee were: Clyde T. Morris, Chairman, George E. Beggs, John R. Chamberlin, E. H. Harder, A. C. Janni and Wilbur M. Wilson. Mr. Chamberlin died December 15, 1925. The other members remained on the Committee until it was dismissed at its own request at the end of the year 1934. In the later years of its work, the Committee secured the active cooperation of the United States Bureau of Public Roads, and E. F. Kelley, Chief of the Division of Tests, and A. L. Gemeny, Senior Structural Engineer, sat with the Committee in its deliberations. The final report of the Committee is published in Transactions, A. S. C. E., Vol. 100, page 1429.

The author made a large number of tests for the Committee which were reported in detail in the bulletins of the University of Illinois, Engineering Experiment Station listed in the bibliography that follows.

The direct expenses of the tests were paid from funds contributed by the American Society of Civil Engineers, the Engineering Foundation, the United States Bureau of Public Roads and various companies that manufacture or sell bridge materials. The University of Illinois contributed the laboratory facilities and the services of the author to plan and supervise the work.

Bibliography.

All bulletins are University of Illinois, Engineering Experiment Station publications.

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2. Bulletin 269. Laboratory Tests of Three-Span Reinforced Concrete Arch Ribs on Slender Piers, Wilbur M. Wilson and Ralph W. Kluge.
3. Bulletin 203. Dependability of the Theory of Concrete Arches, Hardy Cross.
4. Bulletin 174. The Effect of Climatic Changes Upon a Multiple-Span Reinforced Concrete Arch Bridge, Wilbur M. Wilson.
5. Bulletin 226. Laboratory Tests of Reinforced Concrete Arches with Decks, Wilbur M. Wilson.
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10. Trans. A. S. C. E., Vol. 100, page 1431.
11. Trans. A. S. C. E., Vol. 103, page 62.

Contents.

Summary.

This paper is an abridged report of an extensive series of tests of reinforced concrete arch bridges. The investigations, which included laboratory tests of specially constructed experimental structures and field observations on actual bridges, pertain to such phases of design and construction as: Verification of the elastic theory as applied to single-span arch ribs and to multiple-span bridges consisting of three arch ribs on slender piers; the influence of various types of deck upon the action of the rib; the influence of intermediate expansion joints in the deck upon stresses in the rib; movement of piers during the construction of multiple-span arch bridges; effect of climatic changes upon a multiple-span arch bridge; influence of time yield in concrete upon the stresses in a single-span arch; and the strength developed by concrete in an arch rib relative to the strength of similar concrete developed in 6-in. by 12-in. control cylinders.

The summary of conclusions based upon tests by the author outlined above and upon investigations made by other members of the Special Committee on Concrete and Reinforced Concrete Arches of the American Society of Civil Engineers, is given in Section 13 of the paper.

Zusammenfassung.

Die vorliegende Arbeit gibt in abgekürzter Fassung die Resultate einer ausgedehnten Reihe von Versuchen mit Eisenbeton-Bogenbrücken.

Die Untersuchungen, teils an eigens hergestellten Laboratoriumsmodellen, teils an bestehenden Brücken vorgenommen, beziehen sich auf die Nachprüfung der Elastizitätstheorie bei Bogenrippen einer Spannweite und bei kontinuierlichen Bögen über drei Spannweiten auf schlanken Pfeilern, auf das Verhalten der Bogenrippen unter dem Einfluß verschiedenartiger Fahrbahnkonstruktionen, auf den Einfluß von Zwischen-Ausdehnungsfugen der Fahrbahnplatte auf die Spannungen in der Bogenrippe, auf Pfeilerbewegungen während des Baues kontinuierlicher Bogen, auf Einflüsse klimatischer Änderungen auf kontinuierliche Bogenbrücken, auf Einflüsse des

Kriechens des Betons auf die Spannungen in einer einfachen Bogenrippe, und auf die Festigkeit des Betons einer Bogenrippe verglichen mit der Festigkeit des gleichen Betons in Prüfzylindern von 6″ × 12″.

Eine Zusammenfassung von Schlußfolgerungen der oben erwähnten Versuche des Verfassers wie auch von andern Mitgliedern des Ausschusses für Beton und Eisenbetonbogen der American Society of Civil Engineers befindet sich in Abschnitt 13 der Abhandlung.

Résumé.

Dans le présent travail sont résumés les résultats d'une grande série d'essais sur ponts en arc de béton armé.

Les investigations ont été faites en partie sur des modèles de laboratoire et en partie sur des points existants. Elles concernent:

le contrôle de la théorie de l'élasticité sur des arcs d'une seule portée et sur des arcs continus sur trois ouvertures reposant sur des piles élancées;

le comportement des arcs avec tabliers de différents types;

l'influence des joints de dilatation intermédiaires du tablier sur les tensions dans l'arc;

le mouvement des piles durant la construction des arcs continus;

l'influence des variations de climat sur les ponts en arc continu;

l'influence de la déformation lente du béton sur les tensions dans un arc simple et

la résistance du béton d'un arc en comparaison avec la résistance du même béton en éprouvettes cylindriques de 6″ × 12″.

Un résumé des conclusions tirées des essais de l'auteur et d'autres membres du Comité pour les arcs en béton et béton armé de l'American Society of Civil Engineers est contenu dans la 13e partie de ce mémoire.