Vierendeel truss analysis using equivalent elastic systems

Autor(en): **Beaufoy, L.A. / [s.n.]**

Objekttyp: **Article**

Zeitschrift: **IABSE publications = Mémoires AIPC = IVBH Abhandlungen**

Band (Jahr): **11 (1951)**

PDF erstellt am: **22.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-11427

Vierendeel Truss Analysis Using Equivalent Elastic Systems

Berechnung von Vierendeelträgern mit Hilfe gleichwertiger elastischer Systeme

Calcul des poutres Vierendeel à l'aide de systèmes élastiques équivalents

Dr. L. A. Beaufoy, M. Sc. (Eng.), A.M.I.C.E., M. I. Mech. E., M. I. Struct. E., M. Am. Soc. C. E., Chartered Civil Engineer, London

Introduction

The present paper is an extension of the method described elsewhere[1]) of analysing continuous frames by the use of equivalent elastic systems. In the case of the continuous frame, where the feet of the columns are fixed, the displacements of the heads of the columns could be obtained as absolute displacements. The Vierendeel truss, however, is supported only at its ends and it follows that both ends of the intermediate posts will displace under load; relative, instead of absolute, displacements are therefore the feature of the Vierendeel truss. The method of determining these relative displacements is developed here as a step in the complete analysis of the truss.

Notation and Sign Convention

$E\,I$	denotes flexural rigidity.
f	denotes intensity of normal stress at point (x, y) on analogous column.
I	denotes a centroidal moment or product of inertia.
I_x'	$= I_x - (I_{xy}^2/I_y)$.
I_y'	$= I_y - (I_{xy}^2/I_x)$.
L	denotes length of a member.
m, M h, H v, V	denote moment, horizontal force, and vertical force respectively.
M_x'	$= M_x - (M_y\,I_{xy}/I_y)$.
M_y'	$= M_y - (M_x\,I_{xy}/I_x)$.

[1]) Beaufoy and Diwan, "Equivalent elastic systems in the analysis of continuous structures", Concrete and Constructional Engineering, Nov. and Dec. 1950.

P denotes normal elastic load on analogous column.
$\bar{S}$ denotes elastic area.
x, y denote horizontal and vertical co-ordinate distances respectively.
$\bar{x}, \bar{y}$ denote horizontal and vertical co-ordinates respectively of the elastic centre.
ϕ, Δ, λ denote rotational, horizontal, and vertical displacements respectively.
e as suffix, denotes that the quantity refers to a virtual equivalent system.
x, y, xy as suffixes, denote that the quantity is taken about the x-axis, the y-axis, or axes x, y.
ϕ, Δ, λ as suffixes, denote that the quantity is in respect of applied rotational, horizontal or vertical displacement respectively.

Sign Convention. Moments and rotational displacements are taken as positive when clockwise; horizontal and vertical forces, displacements, and co-ordinate distances as positive when measured to the right and upwards respectively.

The Single Closed Panel

Vierendeel trusses consist of connected rigid panels, usually quadrangular but occasionally triangular in shape. In any given case an individual closed panel such as $ABCD$ (Fig. 1) constitutes a group of members connected in series. These may be compounded into a single substitute system by successively applying the following equations[2])

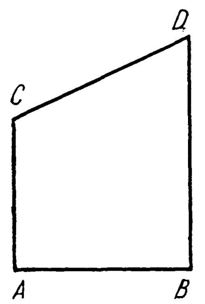

Fig. 1. A single closed panel

$$\bar{S} = \bar{S}_1 + \bar{S}_2 \tag{1a}$$
$$\bar{x} = (\bar{S}_1 \bar{x}_1 + \bar{S}_2 \bar{x}_2)/\bar{S} \tag{1b}$$
$$\bar{y} = (\bar{S}_1 \bar{y}_1 + \bar{S}_2 \bar{y}_2)/\bar{S} \tag{1c}$$
$$I_x = (I_{x_1} + I_{x_2}) + (\bar{S}_1 y_1^2 + \bar{S}_2 y_2^2) \tag{1d}$$
$$I_y = (I_{y_1} + I_{y_2}) + (\bar{S}_1 x_1^2 + \bar{S}_2 x_2^2) \tag{1e}$$
$$I_{xy} = (I_{xy_1} + I_{xy_2}) + (\bar{S}_1 x_1 y_1 + \bar{S}_2 x_2 y_2) \tag{1f}$$

which give the elastic constants for the resultant of any two members connected together in series, in terms of those for the two separate members. The elastic constants for panels consisting of curved members of variable cross section are obtained from the same equations and involve only slightly greater difficulty.

The actual bending-moment diagram for a closed panel carrying external loads may be obtained from the elastic constants by using the column analogy procedure[3]).

[2]) Ibid.

[3]) CROSS, "The column analogy", Bulletin No. 215, University of Illinois Engineering Experiment Station, 1930.

Relative Displacement of One Joint with respect to Another

In general, if $ABCD$ (Fig. 1) be any panel in a loaded Vierendeel truss, all joints will be subject to displacement; it will be convenient here to consider the relative displacement of any one joint, say C, with reference to any other, say A. The general case in which joints A and C are connected by a curved or polygonal member will be considered. Let A and C (Fig. 2a) be the positions of these points for the unloaded condition. Imagine a rigid arm AC' to be connected to A, so that C' coincides with C but is not connected to it. Under load, let joints A and C move to new positions A_1 and C_1 respectively, while the rigid arm AC' moves to $A_1 C_1'$. Then $C_1' C_1$ will be the relative translation vector of joint C with respect to joint A. If ϕ^C, Δ^C, λ^C and ϕ^A, Δ^A, λ^A are the absolute displacements of joints C and A respectively and ϕ^{CA}, Δ^{CA}, λ^{CA} are the relative displacements of joint C with respect to A, then it follows from the figure that:

$$\phi^{CA} = \phi^C - \phi^A \tag{2a}$$

$$\Delta^{CA} = (\Delta^C - \Delta^A) - y \cdot \phi^A \tag{2b}$$

$$\lambda^{CA} = (\lambda^C - \lambda^A) + x \cdot \phi^A \tag{2c}$$

where x, y are the co-ordinates of joint C with respect to joint A.

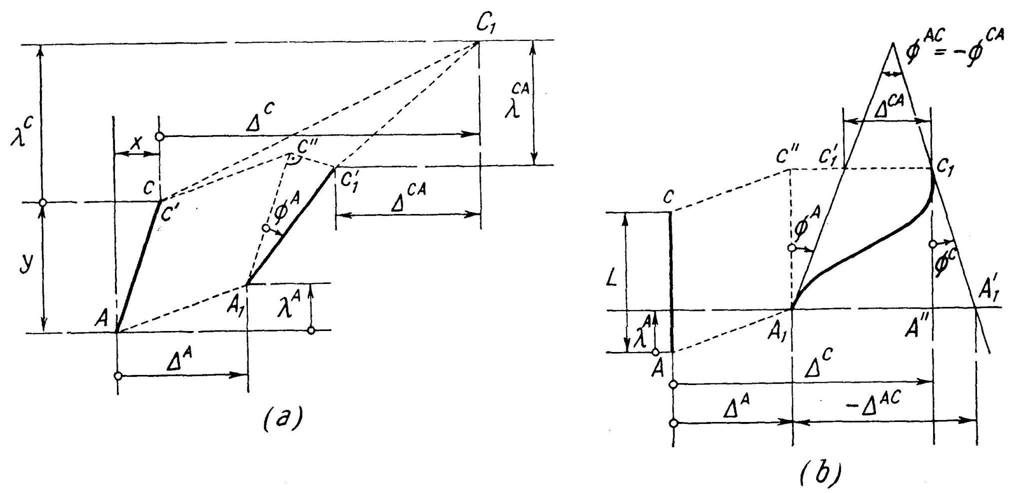

Fig. 2. Relative displacement of joint C with respect to joint A in a panel (Fig. 1) of a loaded Vierendeel truss: (a) general case; (b) particular case when joints A and C are connected by a straight member and axial deformations are neglected

Usually, in practice, joints A and C are connected by a straight member, and axial deformations of members may be disregarded. A special case of the above then arises, since only relative rotation and horizontal translation are possible. This case is shown in Fig. 2b, from which it will be clear that the relative displacements of joint C with respect to joint A are:

$$\phi^{CA} = \phi^{C} - \phi^{A} \tag{3a}$$
$$\Delta^{CA} = (\Delta^{C} - \Delta^{A}) - L \cdot \phi^{A} \tag{3b}$$
$$\Delta^{AC} = (\Delta^{A} - \Delta^{C}) + L \phi^{C}, \tag{3c}$$

where L is the length of the member AC.

The relative stiffness factors for a specified displacement, which may be rotation or translation, at joint C with respect to joint A are the moment and forces required at joint C to produce a unit relative displacement there, in the specified direction, with respect to joint A.

Virtual Equivalent System

If the panel $ABCD$ (Fig. 3a) is considered to have its joint A temporarily fixed, the actual stiffness factor for joint C may be obtained as the sum of the relevant stiffness factors for the ends C of CA and $CDBA$, added in parallel (Fig. 3b). Since joint A is really free, these actual stiffness factors are in fact the relative actual stiffness factors of joint C with respect to joint A. Then, if a single elastic system considered to be rigidly connected to joints A and C is so selected that the relative actual stiffness factors of joint C with respect to joint A for the system are the same as those for the panel, the relative elastic displacements of joint C with respect to joint A will also be identical for the system and the panel. Such a system, which is entirely virtual, may be used in replacement of all the real members of the panel combined; it will be called a virtual equivalent system.

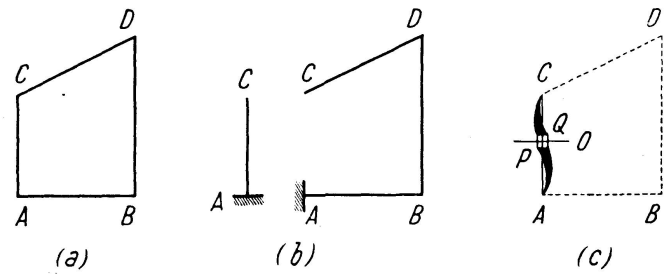

Fig. 3. Virtual equivalent system (c) for the single closed panel (a)

It is convenient to represent the virtual equivalent system diagrammatically as in $APQC$ (Fig. 3c), which replaces the panel $ABCD$ (shown here in dotted lines). The elastic area is represented by PQ and rigid arms PA and QC link it to joints A and C respectively; the elastic centre O of PQ is so located with respect to joints A and C that the relative displacements of these two points are the same for the virtual equivalent system as for the panel.

The elastic constants for the virtual equivalent system are obtained from Table I by inserting the appropriate values of the relative actual stiffness factors of joint C with respect to joint A. When, as is common in practice, joints A and C are connected by a straight upright member and axial deformations may be disregarded, I_y, I_{xy}, and $\bar{x}$ are all zero, and the elastic constants reduce to values for $\bar{S}$, $\bar{y}$, and I_x.

No moments or forces may be applied to a virtual equivalent system between its terminals; the only loading it may carry is that due to external moments or forces transmitted to it through its terminals.

Table I. Elastic Properties in Terms of Stiffness Factors

Elastic properties (1)	Nature of joint displacement			
	Rotation and translation in any direction (2)	Rotation and vertical translation only (3)	Rotation and horizontal translation only (4)	Rotation only (5)
$\bar{S}$	$\frac{A}{AM_\phi - BH_\phi + CV_\phi}$	$\frac{V_\lambda}{D}$	$\frac{H_\Delta}{E}$	$\frac{1}{M_\phi}$
$\bar{x}$	$\frac{C}{A}$	$\frac{V_\phi}{V_\lambda}$	0	0
$\bar{y}$	$\frac{B}{A}$	0	$\frac{H_\phi}{H_\Delta}$	0
I_x	$\frac{V_\lambda}{A}$	0	$\frac{1}{H_\Delta}$	0
I_y	$\frac{H_\Delta}{A}$	$\frac{1}{V_\lambda}$	0	0
I_{xy}	$\frac{H_\lambda}{A}$	0	0	0

$A = H_\Delta V_\lambda - H^2_\lambda$; $B = H_\phi V_\lambda - H_\lambda V_\phi$; $C = H_\phi H_\lambda - H_\Delta V_\phi$;
$D = M_\phi V_\lambda - V^2_\phi$; $E = M_\phi H_\Delta - H^2_\phi$

Partial Equivalent System

Any part of a Vierendeel truss may be reduced to a compound system composed of two real chord members plus one virtual equivalent system connected in series, the chord members constituting the upper and lower members in a panel and the virtual equivalent system replacing all other members to one side of the panel. Such a compound system will be called a partial equivalent system.

Consider the Vierendeel truss shown in Fig. 4a. Imagine the truss to be completely severed across section XX. Then the part of the frame to the right of the cut section can be reduced to the system shown in Fig. 4b, the virtual equivalent system replacing the seven members shown dotted to the right. The system $ABCD$ (Fig. 4b) constitutes a partial equivalent system, for which the elastic constants are readily obtained. If these values are inserted in the following equations[4])

$$m_{\phi} = \frac{1}{\bar{S}} + \frac{(\bar{y})^2}{I_x'} + \frac{(\bar{x})^2}{I_y'} - \frac{2\,\bar{x}\,\bar{y}\,I_{xy}}{I_x'\,I_y} \quad (4a)$$

$$h_{\phi} = m_{\Delta} = \frac{\bar{y}}{I_x'} - \frac{\bar{x}\,I_{xy}}{I_x'\,I_y} \quad (4b)$$

$$v_{\phi} = m_{\lambda} = \frac{\bar{y}\,I_{xy}}{I_y'\,I_x} - \frac{\bar{x}}{I_y'} \quad (4c)$$

$$h_{\Delta} = \frac{1}{I_x'} \quad (4d)$$

$$v_{\Delta} = h_{\lambda} = \frac{I_{xy}}{I_x'\,I_y} \quad (4e)$$

$$-\,v_{\lambda} = \frac{1}{I_y'} \quad (4f)$$

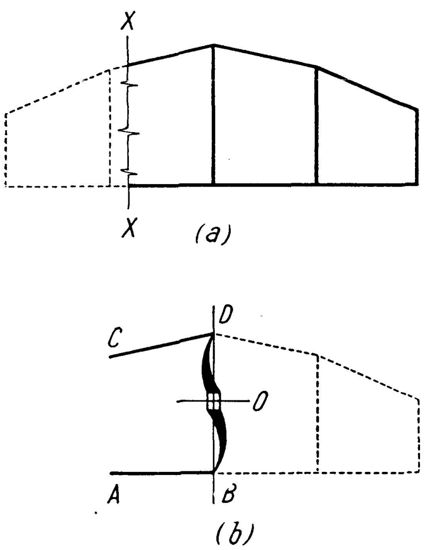

Fig. 4. Partial equivalent system (b) for the part of the Vierendeel truss to the right of section XX (a)

values of the relative stiffness factors for the end C with respect to end A will be obtained.

If a moment and forces equal to the relative actual stiffness factors for end C are applied there, end C by definition will undergo unit displacement. Transfer these forces to the elastic centre O of the virtual equivalent system, making the necessary corrections to moment, and apply the following equations[5])

$$\phi = m \cdot \bar{S} \quad (5a)$$

$$\Delta = h \cdot I_x - v\,I_{xy} - \phi\,\bar{y} \quad (5b)$$

$$\lambda = v \cdot I_y - h\,I_{xy} + \phi\,\bar{x}\,; \quad (5c)$$

the resulting displacements are the relative induced movements of joint C with respect to joint A.

If a moment and forces equal to the relative actual stiffness factors for end C (for $\Delta = 1.0$) are applied there, the resulting bending-moment diagram in the panel is obtained by statics, the part referring to the virtual equivalent system being imaginary. Similarly, a bending-moment diagram may be obtained for the case of $\phi = 1.0$, and these diagrams may be subsequently employed as the basis of determining the bending-moment diagram for the truss from the total relative joint displacements.

---

[4]) Beaufoy and Diwan, "Equivalent elastic systems in the analysis of continuous structures", Concrete and Constructional Engineering, Nov. 1950, Eqs. (7), p. 386.

[5]) Beaufoy and Diwan, "Analysis of continuous structures by the stiffness factors method", Quarterly Journal of Mechanics and Applied Mathematics, vol. II, pt. 3 (1949).

Equivalent Panel

The whole of a Vierendeel truss may be reduced either to (a) a single virtual equivalent system linked in series with two or more real members in series, or (b) two virtual equivalent systems linked in series with two real members (upper and lower chord members of the same panel). Such a system will be called an equivalent panel. The elastic constants of an equivalent panel may be obtained from equations (1).

The forces acting upon an equivalent panel are the external loads applied to the real members of the panel together with such other forces, which will here be called conjugate forces, as must act at the joints to replace the external forces acting on the replaced members in the virtual equivalent parts of the system.

This combination of forces sets up primary moments in the equivalent panel, which may be evaluated by column analogy. In doing this, loadings on parts of the analogous column which correspond to a virtual equivalent system are dealt with as follows. First, the part of the bending-moment diagram referring to the virtual equivalent system must be linear, since no loading may be applied to a virtual equivalent system except at its ends.

Then, the contribution of the virtual equivalent system to the flexure formula

$$f = \frac{P}{\overline{S}} + \frac{M_x'}{I_x'}\, y + \frac{M_y'}{I_y'}\, x$$

is as follows:

a) The elastic load P_e on the part of the analogous column corresponding to the virtual equivalent system is

$$P_e = M_e \cdot \overline{S}_e, \tag{6}$$

where M_e = ordinate of the bending-moment diagram at the elastic centre of the virtual equivalent system; $\overline{S}_e$ = elastic area of the virtual equivalent system.

b) Due to this elastic loading, the moment M_{xe} of the elastic loading on the virtual equivalent system about the x-axis is

$$M_{xe} = P_e \cdot \overline{y}_e + H_e \cdot I_{xe}, \tag{7}$$

where $\overline{y}_e = y$ co-ordinate of the elastic centre of the virtual equivalent system with respect to the elastic centre of the equivalent panel; $H_e = (dM/dy)$ at the elastic centre of the virtual equivalent system; I_{xe} = centroidal moment of inertia of the virtual equivalent system about the x-axis.

c) The moment M_{ye} of the elastic loading on the virtual equivalent system about the y-axis is

$$M_{ye} = P_e \cdot \overline{x}_e, \tag{8}$$

where $\overline{x}_e = x$ co-ordinate of the elastic centre of the virtual equivalent system with respect to the elastic centre of the equivalent panel.

When the above are known, the diagram of primary moments may be obtained as for a single panel.

Considering the virtual equivalent system on one side of the equivalent panel, the relative displacements of the upper and lower joints which constitute its terminals are given by equations (5). Hence, by proportion from the bending-moment diagram caused by unit relative displacement, the moments induced in the panel adjacent to the equivalent panel are found.

Method of Analysis

The steps involved in the analysis of a Vierendeel truss using equivalent elastic systems are as follows:

Structural Analysis

1. For each of the partial equivalent systems, determination of the bending-moment diagram corresponding to unit relative displacement imposed at the cut ends.
2. Evaluation of the elastic constants for the virtual equivalent systems for successive groupings of panels working from left to right, and also from right to left if the truss is unsymmetrical. In the case of a symmetrical truss the two sets of evaluations will be similar.
3. Evaluation of the elastic constants for each of the equivalent panels.

Stress Analysis

4. Evaluation of the conjugate forces acting on each equivalent panel.
5. Determination of the primary moments due to the conjugate forces and applied loadings on each of the equivalent panels.
6. Determination of all the moments induced both to the right and to the left of each of the equivalent panels.
7. Summation of the primary moments and the induced moments to obtain the total bending-moment diagram.

Example

The method will be illustrated by reference to the four-panel Vierendeel truss shown in Fig. 5 [6]).

Structural Analysis. Step (1)

Working from right to left, the partial equivalent system composed of members $GIJH$ is first considered. The $\bar{S}$ (or L/I) values for bars GI, IJ, JH are respectively 1.6, $1.\dot{3}$, and 2.0. Combine these three bars in series

[6]) See Rathbun and Cunningham, "Continuous frame analysis by elastic support action", Proc. Am. Soc. C. E., April 1947, p. 439 et seq.

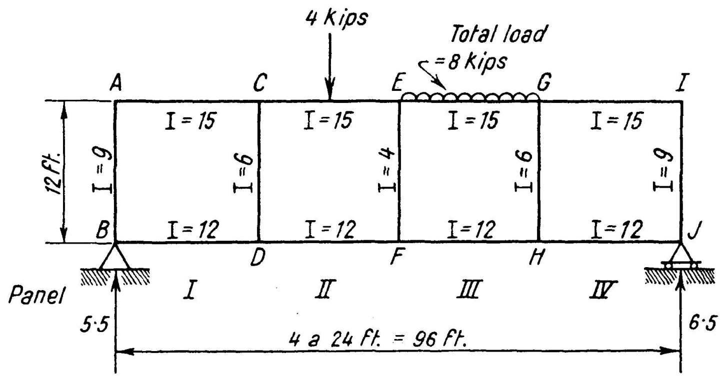

Fig. 5. Four-panel Vierendeel truss

(equation (1a)) to obtain the $\bar{S}$ value for the partial equivalent system, viz., 4.933. Take moments about the axes of symmetry of the panel; then, from equation (1b), $\bar{x} = 3.25$; from equation (1c), $\bar{y} = -0.486$. The centroidal values of the moments and product of inertia follow. From equation (1d),

$$I_x = 1.6\,(6.486)^2 + 2\,(5.514)^2 + 1.\dot{3}\left[\frac{(12)^2}{12} + (0.486)^2\right] = 144.1$$

From equation (1e),

$$I_y = 3.6\left[\frac{(24)^2}{12} + (3.25)^2\right] + 1.\dot{3}\,(8.75)^2 = 312.5$$

From equation (1f),

$$I_{xy} = 1.\dot{3}\,(0.486\cdot 8.75) + 2\,(3.25\cdot 5.514) - 1.6\,(3.25\cdot 6.486) = 7.77$$

Hence,

$$I_x' = 144.1 - 0.2 = 143.9$$
$$I_y' = 312.5 - 0.4 = 312.1$$

Assuming the continuous system $GIJH$ temporarily fixed at end H, the next thing is to find, from equations (4), the moment m_Δ and the horizontal and vertical forces h_Δ and v_Δ respectively required at G to cause unit horizontal translation there. Thus

$$h_\Delta = \frac{1}{143.9} = 0.00695$$

$$v_\Delta = \frac{7.77}{143.9\cdot 312.5} = 0.000173$$

$$m_\Delta = -\frac{0.486}{143.9} - \frac{3.25\cdot 7.77}{143.9\cdot 312.5} = -0.0478$$

The bending-moment diagram corresponding to these forces is as shown in Fig. 6b, where tension is plotted on the outside.

Similarly it can be shown from equations (4) that to cause a unit rotation at G, it is necessary to apply at G a moment m_ϕ and horizontal and vertical forces h_ϕ and v_ϕ respectively having the following values:

$$m_{\phi} = 1.276;\ h_{\phi} = -0.0478;\ v_{\phi} = -0.054.$$

The corresponding bending-moment diagram is given in Fig. 6a.

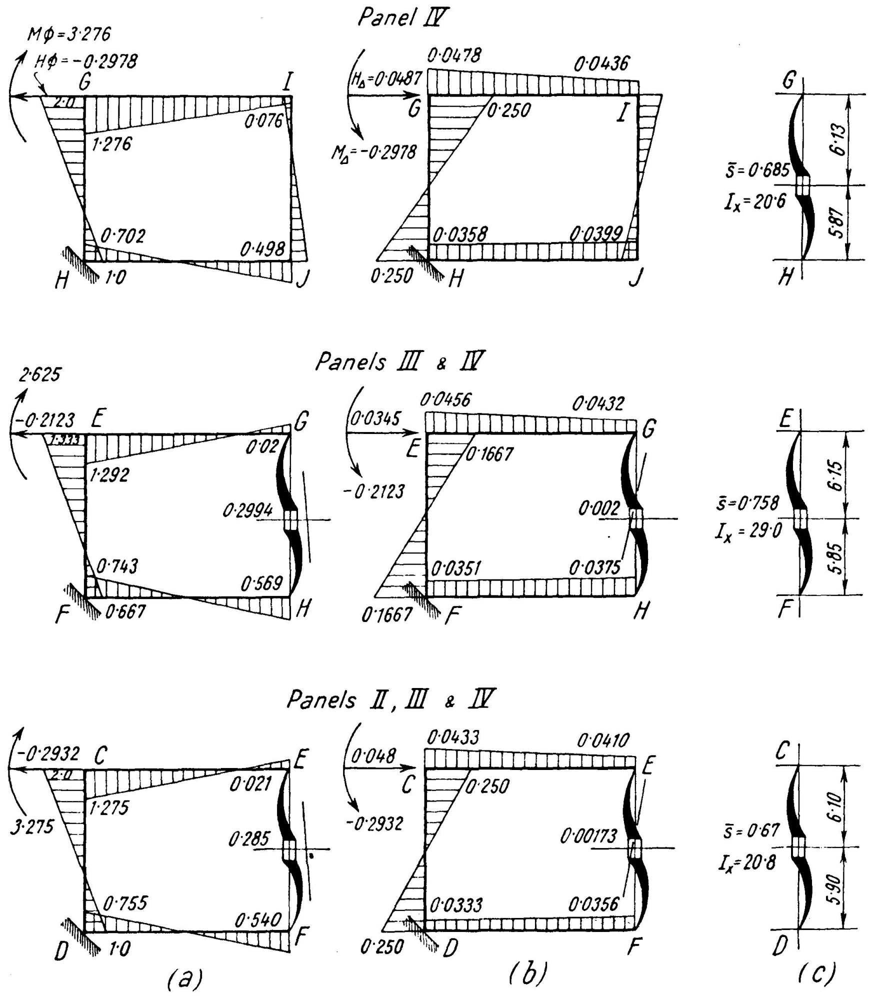

Fig. 6. (a) Bending-moment diagram producing unit relative rotation between upper and lower joints on left-hand side and relative actual stiffness factors of upper joint; (b) as for (a) but in respect of unit relative horizontal translation; (c) elastic constants for corresponding virtual equivalent systems (Steps 1 and 2)

Step (2). For the post GH, the elastic area $\bar{S}$ is L/I or 12/6, i.e., 2. Hence, if the end H is considered temporarily fixed, equations (4) can be used to find the forces needed at G to produce given unit movements there. Thus, from equation (4a), $m_{\phi}=4/\bar{S}=2$; from equation (4b), $h_{\phi}=-6/(2\cdot 12)=-0.25$; from equation (4d), $h_{\Delta}=1/143.9=0.0418$.

Add the system $GIJH$ in parallel to the post GH to form the closed panel IV. Then, the relative actual stiffness factors of joint G with respect to joint H are

$$\begin{aligned} M_{\phi} &= 1.276+2.0 &&= 3.276 \\ H_{\phi} &= -0.0478-0.25 &&= -0.2978 \\ H_{\Delta} &= 0.00695+0.0418 &&= 0.04865 \end{aligned}$$

Table I now gives the elastic constants for the virtual equivalent system replacing the bars in panel IV as follows:

$$\bar{S} = \frac{0.04865}{3.276 \cdot .04865 - (.2978)^2} = 0.685$$

$$\bar{y} = -0.2978/0.04865 = -6.13$$

$$I_x = 1/0.04865 = 20.65$$

so that the four bars of this panel could be replaced by an elastic area 0.685 with an I_x value 20.65 located with respect to joints G and H as shown in Fig. 6c and considered connected to these joints by rigid arms. Fig. 6 also shows the results of similarly combining panels III and IV and panels II, III and IV.

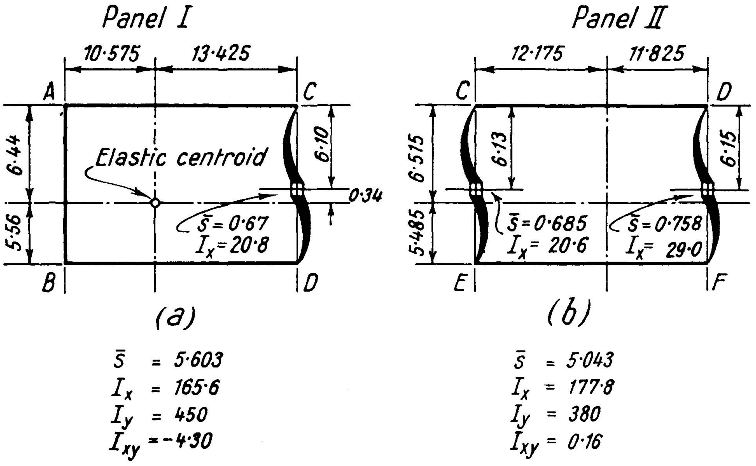

Fig. 7. Elastic constants for the equivalent panels (Step 3)

Step (3). These virtual equivalent systems are combined as in Fig. 7 to get the elastic constants of the equivalent panels. The equivalent panels III and IV are respectively the same as equivalent panels II and I, but are on the opposite hand; the signs of the I_{xy} values will therefore be reversed.

The above completes the structural analysis, which relates solely to the elastic properties of the framework.

Stress Analysis. Step (4)

For a particular loading, the first item required is the conjugate forces acting on the equivalent panels (Fig. 8), which can be written down from considerations of statics.

Step (5). The primary moments in, say, panel II are determined from the statical bending-moment diagram for the panel, which is as shown in Fig. 8; it is drawn on the tension side. Using the column analogy sign convention, viz., that tension on the inside/outside means positive/negative loading on the analogous column, the elastic loads P and their signs will be as shown, so that:

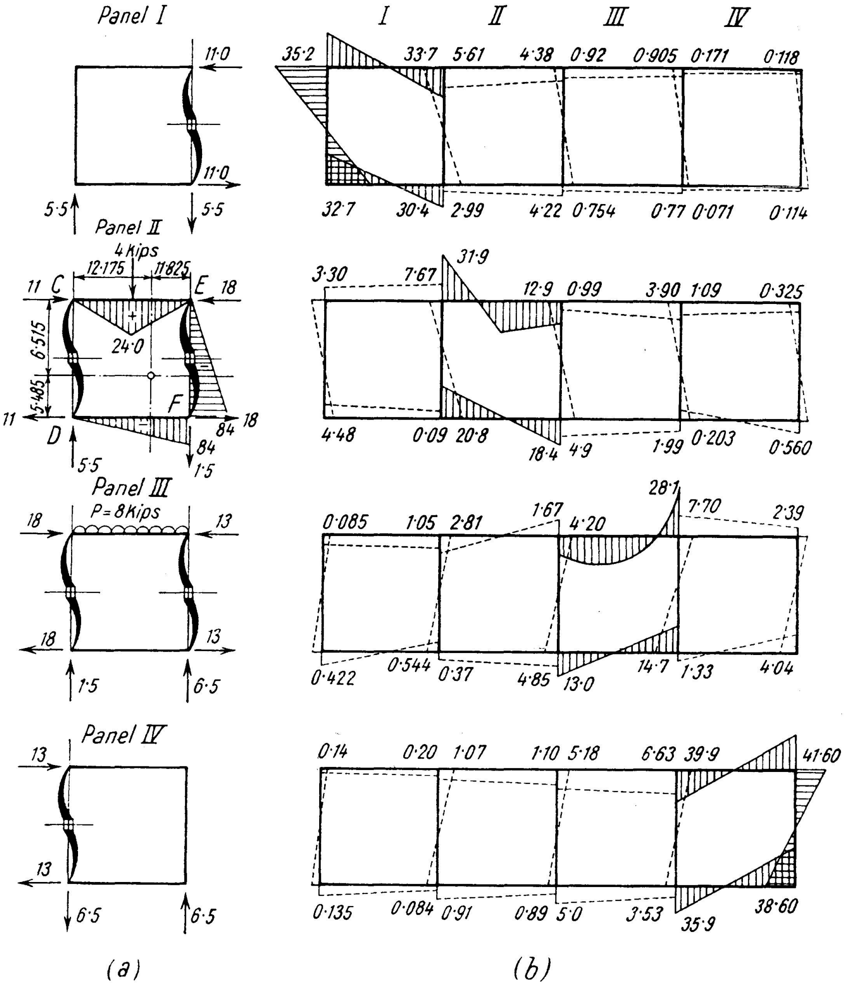

Fig. 8. (a) Conjugate forces and applied loadings on the equivalent panels; (b) primary moments due to conjugate forces and applied loadings on the equivalent panels (firm lines) and induced moments in other panels (dotted lines) (Steps 4—6)

For CE: $P = 1.6 \cdot 24/2 = +19.2$
For DF: $P = -2 \cdot 84/2 = -84.0$
For EF: $P_e = M_e \cdot \bar{S}_e = -43.05 \cdot 0.758 = -32.6$
Total elastic load P for the panel -97.4

The moments M_x and M_y of these loadings are

$$M_x = (19.2 \cdot 6.515) - 84\,(-5.485) - 32.6 \cdot 0.365 + \frac{84 \cdot 29}{12} = 777$$

$$M_y = 19.2\,(-0.175) - 84 \cdot 3.825 - 32.6 \cdot 11.825 = -710.$$

Corrections for dissymmetry are small and values of M_x' and M_y' may be taken as 777 and –710 respectively.

In the flexure formula

$$f = \frac{P}{\bar{S}} + \frac{M_x'}{I_x'}\,y + \frac{M_y'}{I_y'} \cdot x$$

the values of $P/\bar{S}$, M_x'/I_x' and M_y'/I_y' are respectively the moment M, the horizontal force H, and the vertical force V acting at the elastic centre of the panel, so that

$$f = M + H \cdot y + V \cdot x.$$

In the present example,

$$\begin{aligned} M &= -97.4/5.043 = -19.4 \\ H &= 777/177.85 = 4.39 \\ V &= -710/380 = -1.875 \end{aligned}$$

so that, for instance at C

$$f_c = -19.4 + 4.39 \cdot 6.515 - 1.875(-12.175) = 31.9.$$

Now, the moment M_c at C is the statical moment there minus f_c, so that

$$M_c = 0 - 31.9 = -31.9,$$

where the negative sign implies tension on the outside.

Moments at E, D, and F are similarly obtained and the diagram of primary moments drawn as shown in firm lines in Fig. 8b.

Step (6). The moments induced in the other panels by these primary moments in panel II are found as follows. First, the relative angular displacement of E with respect to F is, from equation (5a),

$$\phi = m_e \cdot \bar{S}_e = 3.15 \cdot 0.758 = 2.38.$$

The sign of ϕ depends on the sign of m_e, which, taken by proportion between the values 12.9 at E and 18.4 at F, is 3.15, producing tension on the outside. If the end F of the virtual equivalent system EF is considered fixed, tension on the outside means a contra-clockwise moment at the elastic centre, and this according to our sign convention is negative, so that $\phi = -2.38$.

Next, the relative horizontal displacement is, from equation (5b),

$$\Delta = h_e \cdot I_{xe} - \phi \cdot \bar{y} = -\frac{(12.9 + 18.4)}{12} \cdot 29 - (-2.38)(-6.15) = -88.7.$$

Here, h_e must be negative (i.e., to the left) to produce the kind of bending moment obtaining in this case.

Finally, apply these values of ϕ and Δ to the bending-moment diagrams previously obtained (Figs. 6a and 6b) for unit values of ϕ and Δ. Thus, at E in panel III the induced moment is $(-2.38 \cdot 1.292) + [-88.7(-0.0456)]$, viz., 0.99 (see Fig. 8b). Similarly, moments induced elsewhere throughout the truss may be found. Fig. 8b summarises the primary moments in each equivalent panel and the moments induced in other panels.

Step (7). When the respective totals are added together, the final bending-moment diagram (Fig. 9) may be drawn.

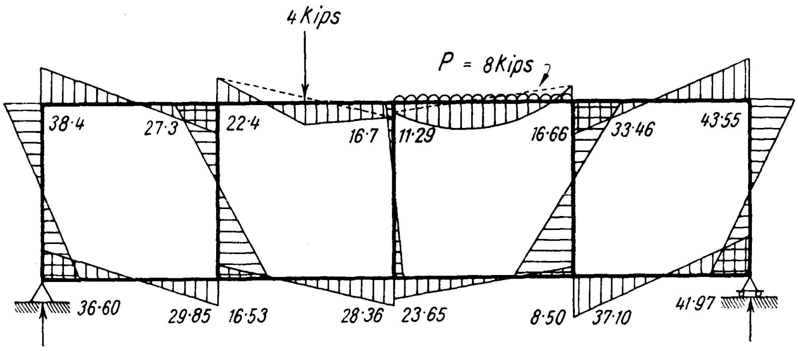

Fig. 9. Final bending-moment diagram (Step 7)

Acknowledgements

Acknowledgements are due to Dr. A. F. S. Diwan for doing the computations and Mr. C. C. Brearley for preparing the drawings.

Conclusions

By the method described, a solution of the "exact" type is obtained, which may be applied to the general case of a Vierendeel truss of any given proportions loaded in any manner. Simultaneous equations are avoided, and computation by slide rule is possible. Since the solution is obtained in two parts, one of which is concerned only with the geometrical properties of the unloaded truss, the effect of different loadings can be evaluated with relatively little additional effort.

Summary

Any Vierendeel truss may be reduced to an equivalent closed panel based on any one of the panels of the Vierendeel truss and composed of the real upper and lower chord members in that panel linked on the left with a virtual equivalent system combining all the real members on the left and on the right with a similar system combining all the real members on the right. Such an equivalent panel is subject to the external loading carried by the real members plus forces at the joints in replacement of the external forces acting on the replaced members in the virtual equivalent systems. It is shown how the primary moments set up in the equivalent panel for this combination of forces may be evaluated. Considering the virtual equivalent system on one side of the equivalent panel, the relative displacements of the upper and lower joints which constitute its terminals are found and so, by proportion from the bending-moment diagram caused by unit relative displacement, the moments induced in the panel adjacent to the equivalent panel are also found. In this way the moments induced in all members of the truss both to the right and to the left of the equivalent panel are determined.

If this process is repeated in respect of each equivalent panel (of which there are as many as there are real panels in the truss) the total bending-moment diagram is obtained from a summation of all primary and induced moments. Only simple arithmetical operations are involved throughout the process, which avoids the use of simultaneous equations. Thus slide rule calculation is made possible without sacrifice of accuracy. The resulting solution is of the "exact" type and is not one of convergence or successive corrections. An important feature of the analysis is that as it is obtained in two parts, of which one is concerned only with the geometrical properties of the unloaded truss, the effect of a number of different loadings can be evaluated with relatively little additional effort.

A numerical application is given to the case of a four-panel Vierendeel truss with intra-panel loading.

Zusammenfassung

Jeder Vierendeelträger läßt sich auf einen gleichwertigen geschlossenen Rahmen zurückführen, der irgend ein Feld des Trägers zur Grundlage hat. Dieser Rahmen besteht aus den wirklichen Ober- und Untergurtstäben des Feldes, an die links und rechts je ein gedachtes gleichwertiges System, das alle wirklichen Stäbe auf der linken resp. rechten Seite vereinigt, gelenkig angeschlossen ist. Ein solches gleichwertiges Rahmenfeld ist durch äußere, von seinen wirklichen Stäben aufgenommene Lasten und durch Gelenkkräfte, die an Stelle der äußeren Kräfte an den ersetzten Stäben des gedachten gleichwertigen Systems wirken, beansprucht. Es wird gezeigt, wie die im gleichwertigen Rahmen auftretenden primären Momente für diese Kombinationen von Kräften bestimmt werden können. Durch Betrachtung des gedachten gleichwertigen Systems auf einer Seite des gleichwertigen Rahmens werden die relativen Verschiebungen seiner Endpunkte, d. h. des oberen und unteren Gelenks ermittelt und damit, durch Vergleich mit der Momentenfläche infolge einer relativen Einheitsverschiebung, auch die Momente gefunden, die in die Nachbarfelder des gleichwertigen Systems übertragen werden. Auf diese Weise werden alle in die Stäbe auf beiden Seiten des gleichwertigen Rahmens übertragenen Momente bestimmt.

Wenn dieses Vorgehen in bezug auf alle gleichwertigen Rahmen wiederholt wird (von denen es so viele gibt wie der Träger wirkliche Felder zählt), erhält man die vollständige Momentenfläche aus der Überlagerung aller primären und übertragenen Momente. Das Verfahren wendet nur einfache arithmetische Operationen an und vermeidet die Aufstellung von Gleichungssystemen. So kann mit dem Rechenschieber gearbeitet werden, ohne daß die Genauigkeit darunter leiden würde. Das Ergebnis stellt eine „exakte" Lösung dar und beruht nicht auf Konvergenz oder sukzessiven Verbesserungen. Ein wichtiges

Merkmal des Berechnungsresultates besteht darin, daß — nachdem es in zwei Anteilen erhalten wird, von denen einer sich nur auf die geometrische Form des unbelasteten Trägers bezieht — der Einfluß einer Anzahl verschiedener Lasten mit verhältnismäßig geringer zusätzlicher Arbeit ermittelt werden kann.

Ein numerisches Beispiel behandelt den Fall eines vierfeldrigen Vierendeelträgers mit direkter Belastung des Obergurtes über die Felder.

Résumé

Toute poutre Vierendeel peut être ramenée à un cadre fermé équivalent, ayant comme base l'une des travées de la poutre. Ce cadre est constitué par les barres réelles de membrure supérieure et de membrure inférieure de la travée, sur lesquelles est raccordé, à droite et à gauche, un système supposé équivalent et qui réunit toutes les barres réelles du côté gauche et du côté droit. Un tel cadre élémentaire équivalent est sollicité par des charges extérieures absorbées par ces barres réelles et par des efforts d'articulation qui agissent à la place des efforts extérieurs sur les barres substituées du système supposé équivalent.

L'auteur montre comment les moments primaires qui apparaissent dans le cadre équivalent peuvent être déterminés pour ces combinaisons d'efforts. En considérant le système supposé équivalent sur l'un des côtés du cadre équivalent, on détermine les déformations relatives de ses points extrêmes, c'est-à-dire de l'articulation supérieure et de l'articulation inférieure, par comparaison avec l'aire des moments résultants d'une déformation unitaire relative; on trouve également les moments qui sont transmis dans les panneaux voisins du système équivalent. On peut ainsi déterminer tous les moments qui sont transmis dans les barres des deux côtés du cadre équivalent.

En répétant ce processus par rapport à tous les cadres élémentaires équivalents (dont il existe un nombre égal au nombre des panneaux réels de la poutre), on obtient l'aire totale des moments à partir de la superposition de tous les moments primaires et transmis. Le processus ne fait intervenir que des opérations arithmétiques simples et évite l'établissement de systèmes d'équations. On peut, par suite, travailler avec la règle à calcul sans que la précision ait à en souffrir: le résultat fournit une solution „exacte" et ne repose ni sur une convergence, ni sur des améliorations successives. L'une des caractéristiques essentielles de cette méthode de calcul est de permettre la détermination de l'influence d'un certain nombre de charges différentes, au prix d'un travail supplémentaire relativement faible, en procédant d'ailleurs en deux étapes dont l'une ne porte que sur la forme géométrique de la poutre non chargée.

L'auteur expose, à titre d'exemple numérique, le cas d'une poutre Vierendeel à quatre panneaux, avec charge directe de la membrure supérieure sur les différentes travées.