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Analysis of Continuous Conical Shells of Rotational Symmetry by the Method of Successive Approximations

Investigations sur les voûtes minces coniques continues de section symétrique, à l'aide de la méthode des approximations successives.

Untersuchungen über durchlaufende konische Schalen von symmetrischem Querschnitt mit Hilfe von schrittweise abgestuften Näherungsberechnungen

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Synopsis

The object of this paper is to present a method of successive approximations for the solution of rotationally symmetrical reinforced concrete conical shells continuous at the boundary.

No claim is made on the advantages of this procedure over the conventional ones except that it might appeal to some engineers familiar with the moment distribution method.

This analysis is based on simplifying assumptions and therefore its application has definite limitations.

Introduction

In this short paper no derivations of formulas were deemed necessary as the basic theory can be found in various books on shells[1]).

The final design values for thin shells are algebraic summations of the so-called "membrane" and "bending" solutions. The membrane shell is a shell with negligible transverse bending stiffness. Modern thin shells tend to support the load by stresses acting in their own plane unless the boundary conditions can

[1]) See "Theory of Plates and Shells" by S. TIMOSHENKO, McGraw-Hill, and "Statik und Dynamik der Schalen" by W. FLÜGGE, Edwards Bros., Ann Arbor.

not be satisfied otherwise than by transverse bending. The damping action of the shell on boundary disturbance is considerable and therefore the effects of boundary forces diminish rapidly with distance from the edge. The damping action increases with the decreasing shell thickness.

The main object of this work is to present a method of solution of continuity of the conical shell by an iteration procedure. The final continuity solution is composed of three parts as shown in the numerated items below:

1. Analysis of shells under loading with no joint deflection.
2. Investigation of joint deflection on joint reaction.
3. Adding the two solutions in proper proportions to satisfy the conditions of joint support.

Membrane Solution

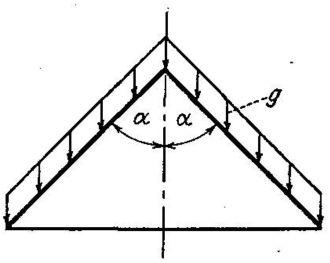


Fig. 1

Dead Load (Fig. 1)

Meridional force $N_y = -\frac{g\,y}{2\cos\alpha}$;

Hoop force $N_\theta = -g\,y \sin\alpha \tan\alpha$

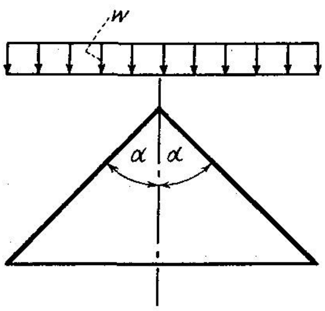


Fig. 2

Live Load (Fig. 2)

$$N_y = -\frac{w\,y}{2}\tan\alpha;\; N_\theta = -w\,y \sin^2\alpha \tan\alpha$$

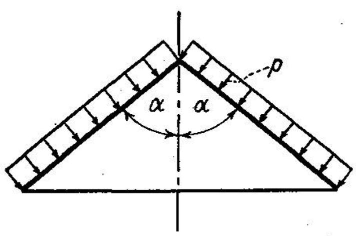


Fig. 3

Normal Load (Fig. 3)

$$N_y = -\frac{p\,y}{2}\tan\alpha;\; N_\theta = -p\,y \tan\alpha$$

Hydrostatic Load on Erect Shells (Fig. 4)

$$N_y = -\frac{\rho y^2}{3}\sin\alpha\,;\; N_\theta = -\rho y^2 \sin\alpha$$

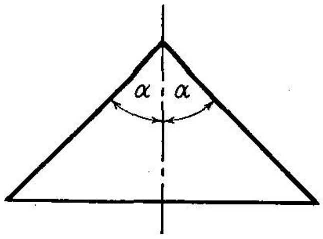


Fig. 4

Hydrostatic Load on Inverted Shells (Fig. 5)

$$N_y = \frac{\rho y}{2}\left(L - \frac{2}{3}y\right)\tan\alpha\,;\; N_\theta = \rho y\,(L-y)\tan\alpha$$

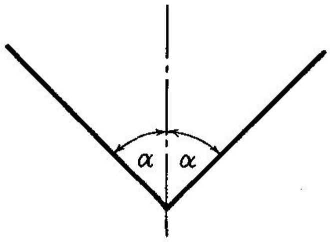


Fig. 5

Bending Solution

Stiffness

The term stiffness defines the moment necessary to produce a specified rotation of the edge of the shell while preventing the deflection of the boundary. (Fig. 6).

a) Stiffness $S_L = M_L = -\dfrac{d^3\gamma^2}{3\sqrt{2}\,\beta_L}\,\theta\,E$

Change of Reaction due to Change of Moment

As in the case of a beam on two supports a change in support moment brings about a change in support reaction when the edge of the shell has to remain stationary (Fig. 6). For conical shells

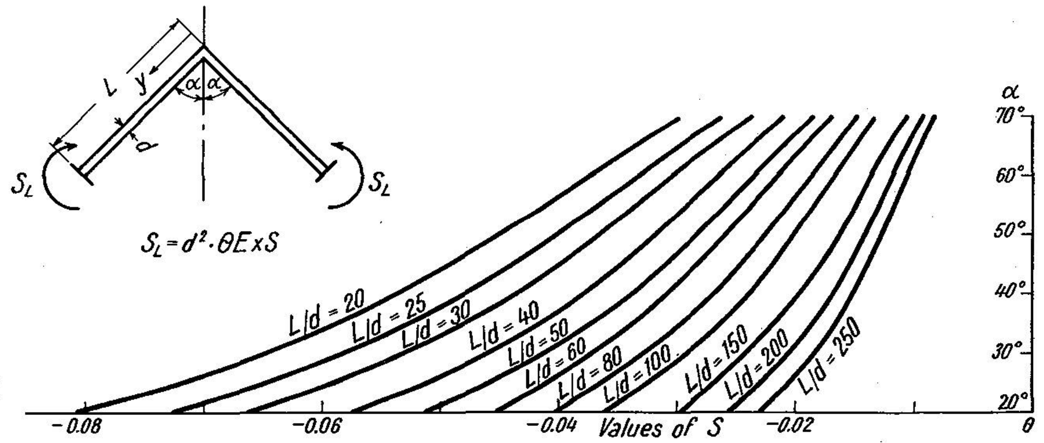


Fig. 6a. Stiffness of Conical Shell

$$\text{b)} \qquad \Delta H_L = \left[-\frac{\sqrt{2}\,\gamma^2}{\cos\alpha\,\beta_L} \right] \Delta M_L$$

The direction of reaction can be found by inspection as it should oppose the horizontal deflection of the boundary, which the moment M_L tends to give to the shell.

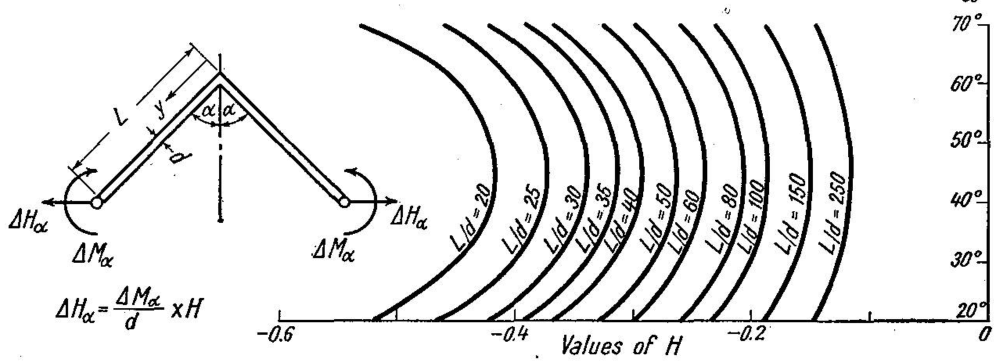


Fig. 6b. Change of Reaction due to Change of Moment

Fixed-End Moment and Reaction due to the Horizontal Deflection "Δ_F"

This solution is necessary in the second step in order to determine the boundary moment induced by a certain horizontal radial deflection (Fig. 7).

$$\text{a)} \quad M_F = \left[-\frac{d}{L} \, \frac{1}{\gamma^2 \sin\alpha \tan\alpha} \right] \Delta_F E$$

$$\text{b)} \quad \mathrm{H}_F = \left[\frac{d}{L} \, \frac{2\sqrt{2}}{\beta_L \sin^2\alpha} \right] \Delta_F E$$

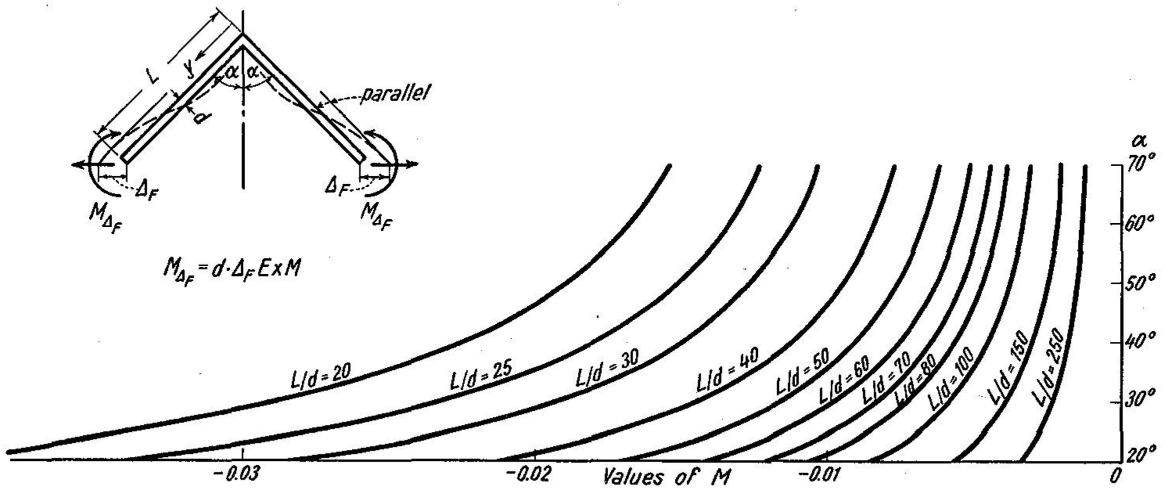


Fig. 7a. Fixed End Moment due to Δ_F

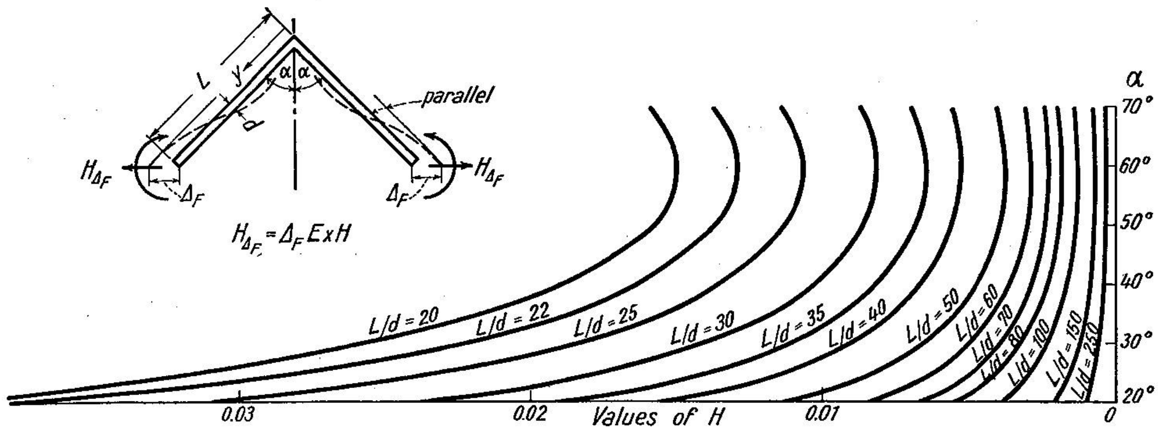


Fig. 7b. Fixed End Reaction due to Δ_F

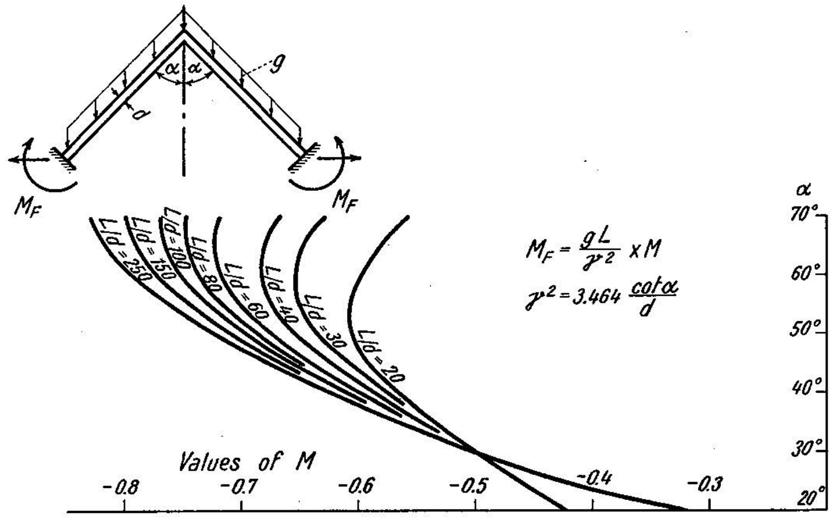


Fig. 8a. Fixed End Moment — Dead Load

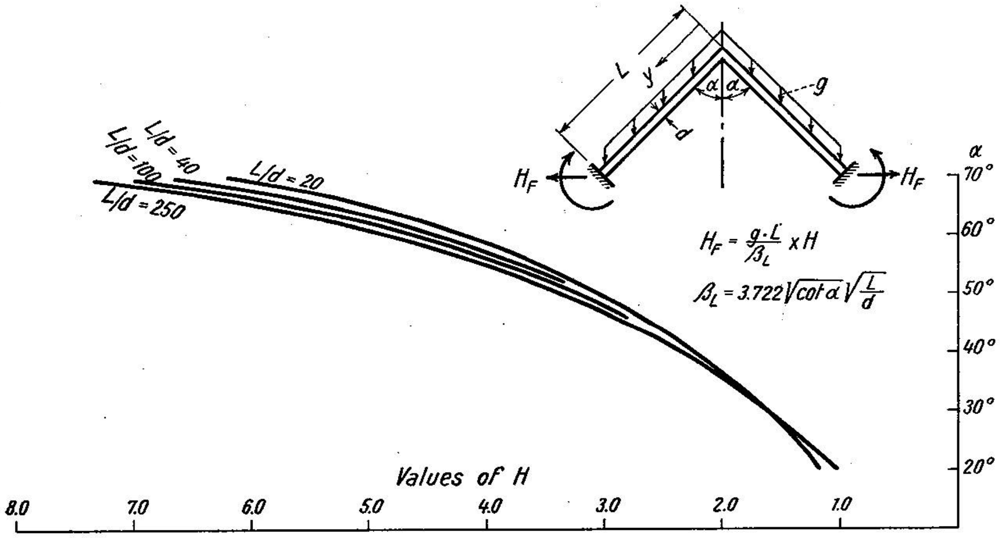


Fig. 8b. Fixed End Reaction — Dead Load

Dead Load (Fig. 8)

$$\text{a)}\quad M_F = \frac{g\,L}{\gamma^2}\left\{-\left[\sin\alpha + \frac{d^2\gamma^4}{6\sqrt{2}\,\beta_L}\,\frac{\tan\alpha}{\cos\alpha}\,(1-4\sin^2\alpha)\right]\right\}$$

$$\text{b)}\quad H_F = \frac{g\,L}{\beta_L}\left\{\tan\alpha\left[2\sqrt{2} + \frac{d^2\gamma^4}{6\,\beta_L\cos^2\alpha}\,(1-4\sin^2\alpha)\right]\right\}$$

Live Load (Fig. 9)

$$\text{a)}\quad M_F = \frac{w\,L}{\gamma^2}\left\{-\left[\sin^2\alpha - \frac{d^2\gamma^4}{3\sqrt{2}\,\beta_L}\,\tan^2\alpha\left(2\sin^2\alpha - \frac{1}{2}\right)\right]\right\}$$

$$\text{b)}\quad H_F = \frac{w\,L}{\beta_L}\left\{\frac{1}{\cos\alpha}\left[2\sqrt{2}\sin^2\alpha - \frac{d^2\gamma^4}{3\,\beta_L}\,\tan^2\alpha\left(2\sin^2\alpha - \frac{1}{2}\right)\right]\right\}$$

Fig. 9a. Fixed End Moment — Live Load

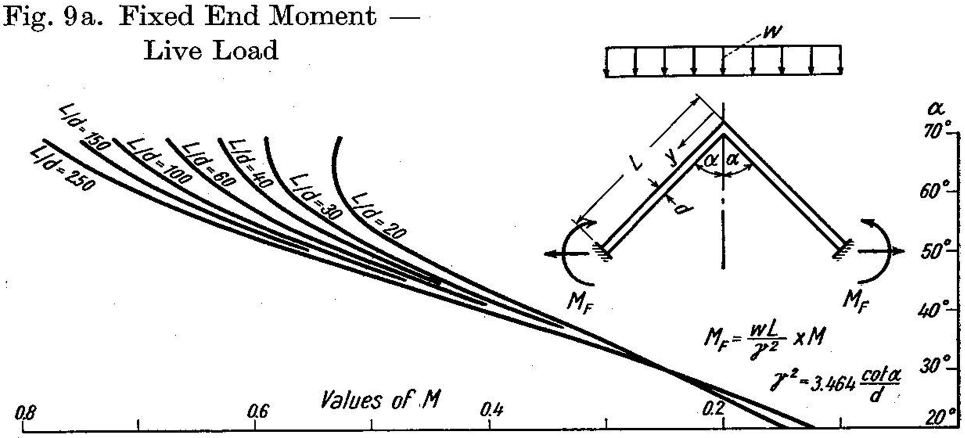
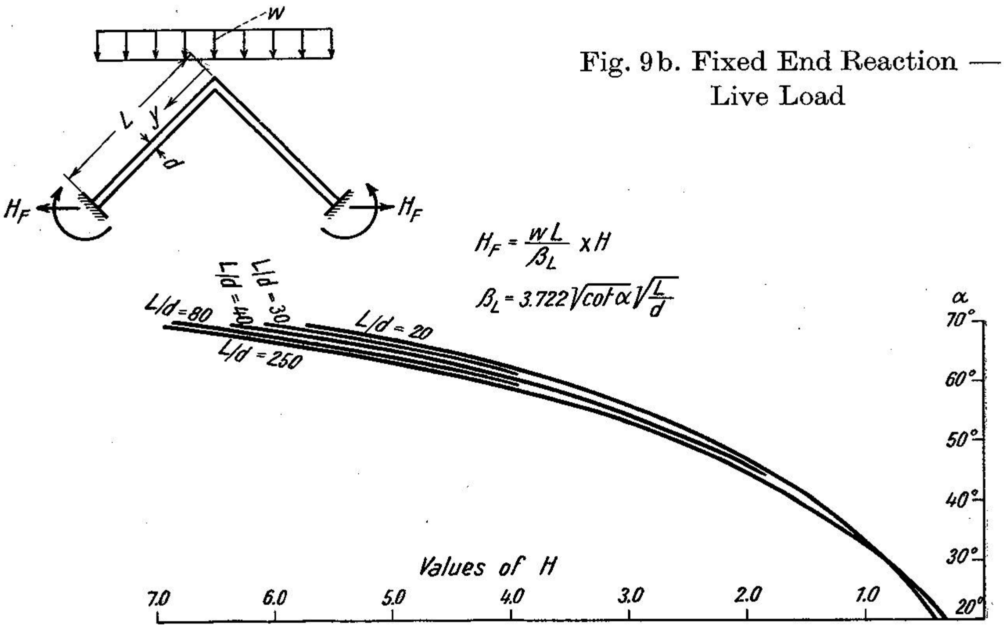


Fig. 9b. Fixed End Reaction — Live Load



Normal Load (Fig. 10)

$$\text{a)}\quad M_F = \frac{p\,L}{\gamma^2}\left\{-\left[1-\frac{1}{2\sqrt{2}}\,\frac{d^2\gamma^4}{\beta_L}\tan^2\alpha\right]\right\}$$

$$\text{b)}\quad H_F = \frac{p\,L}{\beta_L}\left\{\frac{1}{\cos\alpha}\left[2\sqrt{2}-\frac{1}{2}\,\frac{d^2\gamma^4}{\beta_L}\tan^2\alpha\right]\right\}$$

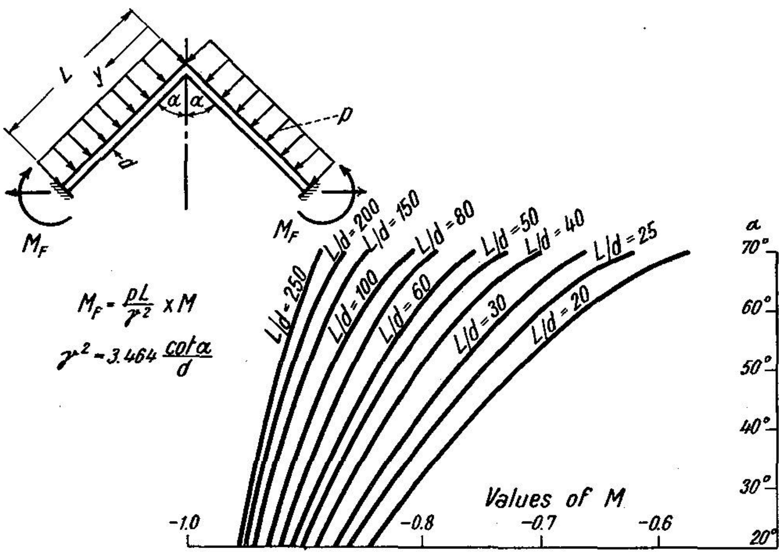


Fig. 10a. Fixed End Moment — Normal Load

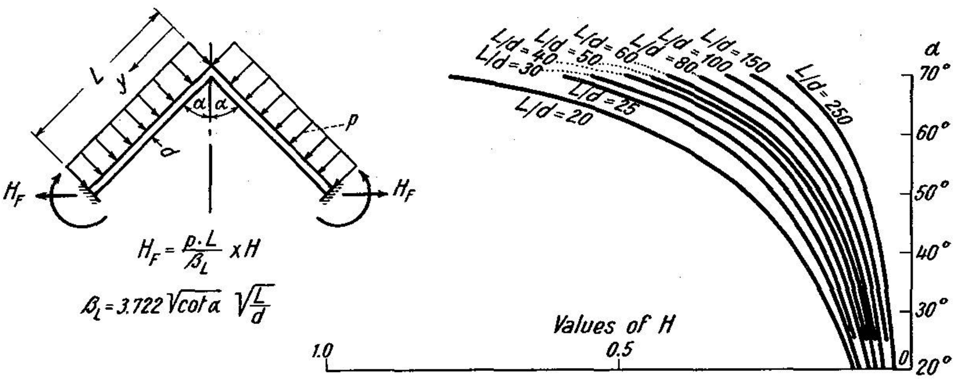


Fig. 10b. Fixed End Reaction — Normal Load

Hydrostatic Load on Erect Shells (Fig. 11)

$$\text{a)}\quad M_F = \frac{\rho\,L^2}{\gamma^2}\left\{-\left[\cos\alpha-\frac{8}{9}\,\frac{d^2\gamma^4}{\sqrt{2}\,\beta_L}\sin\alpha\tan\alpha\right]\right\}$$

$$\text{b)}\quad H_F = \frac{\rho\,L^2}{\beta_L}\left[2\sqrt{2}-\frac{8}{9}\,\frac{d^2\gamma^4}{\beta_L}\tan^2\alpha\right]$$

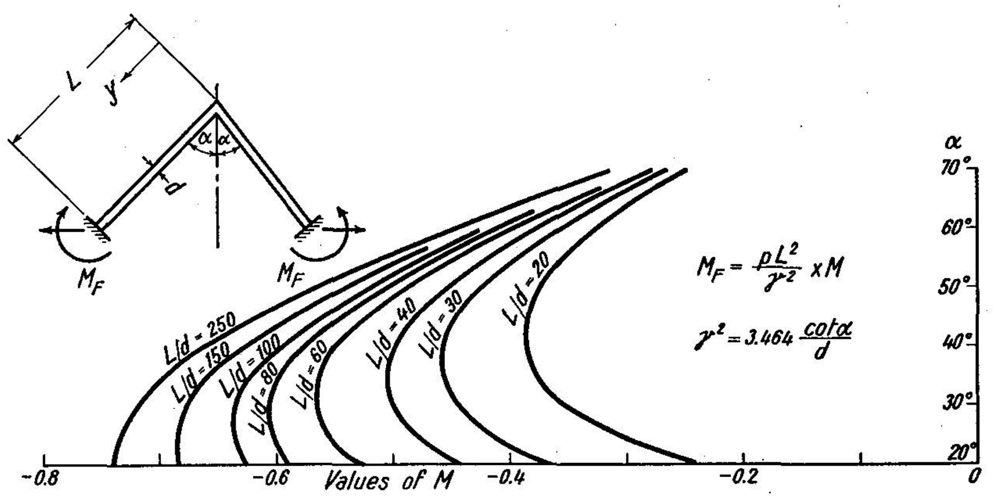


Fig. 11a. Fixed End Moment — Hydrostatic Load

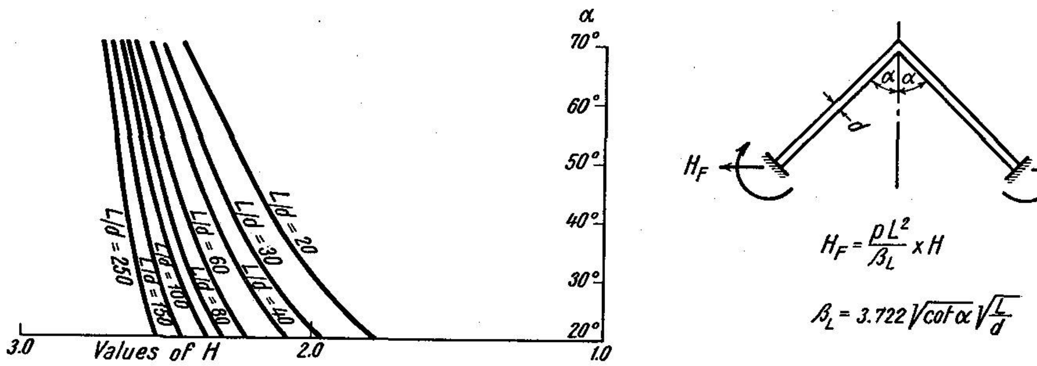


Fig. 11b. Fixed End Reaction — Hydrostatic Load

Hydrostatic Load on Inverted Shells (Fig. 12)

a) $$M_F = \frac{\rho L^2}{\gamma^2}\left[\frac{7}{18}\,\frac{d^2\gamma^4}{\sqrt{2}\,\beta_L}\tan^2\alpha\right]$$

b) $$H_F = \frac{\rho L^2}{\beta_L}\left[-\frac{7}{18}\,\frac{d^2\gamma^4}{\beta_L\cos\alpha}\tan^2\alpha\right]$$

where

$$\gamma^2 = 3.464\cot\alpha/d,\ \beta_y = 3.722\sqrt{\cot\alpha}\ \sqrt{y/d}$$

These formulas should not be applied to shells with $\alpha \leqq 20°$ and when $2\gamma\sqrt{y} < 10$ as then the approximations made at the derivation of these simple expressions introduce too large errors even for practical applications.

All the forces shown in figures are positive. Negative forces act in the opposite directions. The values for hydrostatic loadings should be taken with the precaution that they are more indicative than true as here the simplifications used in the derivations introduce sizeable errors in the final results. Conical

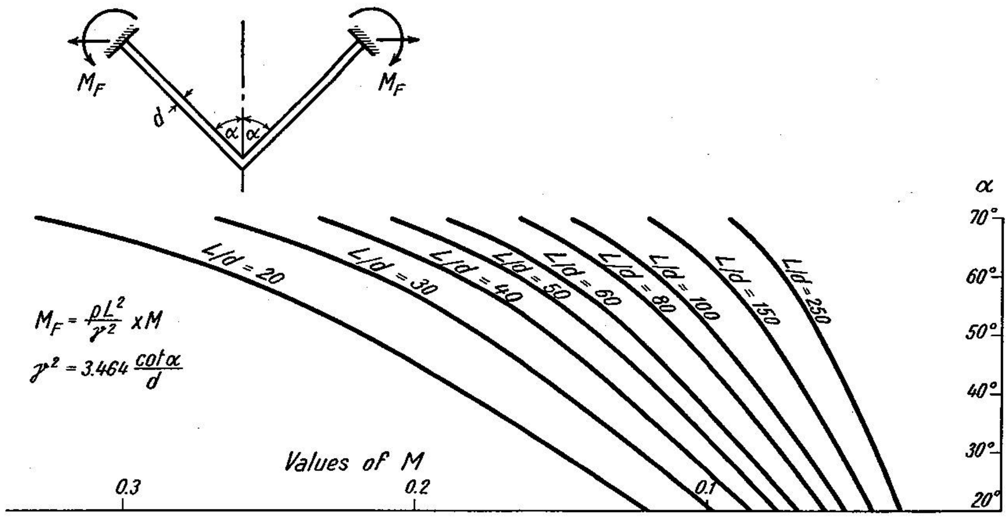


Fig. 12a. Fixed End Moment — Hydrostatic Load

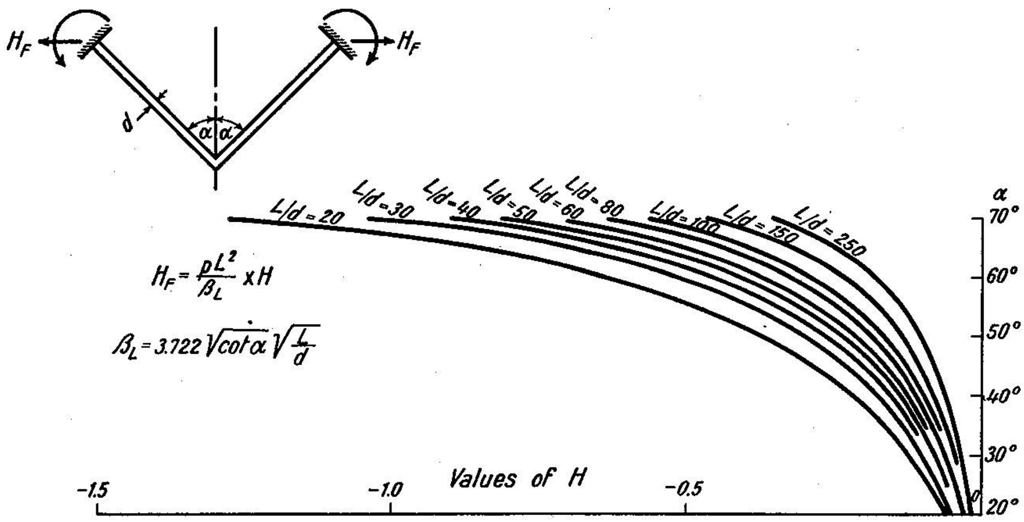


Fig. 12b. Fixed End Reaction — Hydrostatic Load

shells with apex angle "α" more than 65° can be analysed as circular plates with better agreement with the accurate solution than can be expected with the values derived here. It should be noted that the method developed in this paper can be used for conical shells with small openings at the apex as the perturbation of the inside hole has negligible influence at the outer boundary. Of course the membrane forces have to correspond to the shell with the hole.

Application

A cylindrical tank shown in Fig. 13 b is covered by a conical shell roof of dead weight $g = 0.1$ k/ft^2 of roof area. Solve the problem of continuity.

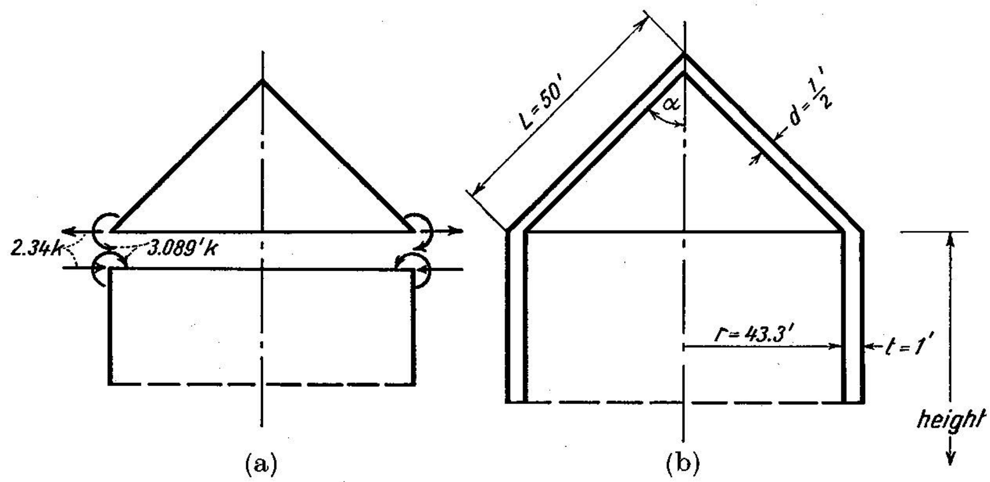


Fig. 13

Conical Shell:

$N_y = -5k$; $\alpha = 60°$; $L = 50'$; $d = 0.5'$; $\beta_L = 28.3$; $\gamma^2 = 4.0$
$M_F = -0.939'k$; $H_F = 0.812k$; $S_L = -0.00415\,\theta E$; $H_L = -0.4\,M_L$
$M_F = -0.00165\,\Delta_F E$; $H_F = 0.00133\,\Delta_F E$.

Cylindrical Shell:

For cylindrical shells with $\lambda \cdot$ (height) > 6 can be considered as semi-infinite, that is while analyzing one end of the shell the other end perturbation effect can be neglected.

Then its stiffness becomes

$$S_0 = -\frac{\lambda t^3}{6}\,\theta E = -0.0333\,\theta E$$

The change of reaction due to change of moment of the cylinder is

$$\Delta H_0 = -\lambda \Delta M_0 = -0.2\,\Delta M_0$$

where

$$\lambda = \frac{\sqrt{3}}{\sqrt{r t}} = 0.2$$

The boundary deflection of the cylindrical shell gives

$$M_{\Delta F} = -\frac{\lambda t^3}{6}\,\Delta_F E = 0.00667\,\Delta_F E; \quad H_{\Delta F} = 0.00267\,\Delta_F E$$

Distribution Factors:

Cone:	$0.00415\,\theta E$	0.111
Cylinder:	$0.03333\,\theta E$	0.889
	$0.03748\,\theta E$	1.000

First Step:

3.387	0.111	0.889
	Cylinder	Cone
	−0.938	
	0.104	0.834
	−0.834	0.834

H			
Cone:	$F.E.$	$\overleftarrow{0.808}$	
	ΔH_L	$\overrightarrow{0.042}$	$= 0.4\,(0.104)$
			$0.766 \leftarrow$
Cylinder:	$F.E.$	$\overrightarrow{4.320}$	$= (N_y \sin\alpha)$
	ΔH_0	$\overleftarrow{0.167}$	$= 0.834\,(0.2)$
			$4.153 \rightarrow$
			$3.387^k \rightarrow$

Second Step:

Take $\Delta_F E = 1000'/k$

3.331	0.111	0.889
	Cylinder	Cone
	−1.650	6.667
	−0.557	−4.460
	−2.207	2.207

H			
Cone:	$F.E.$	$\overleftarrow{1.330}$	
	ΔH_L	$\overleftarrow{0.223}$	$= 0.4\,(0.557)$
			$1.553 \leftarrow$
Cylinder:	$F.E.$	$\overleftarrow{2.670}$	
	ΔH_0	$\overrightarrow{0.892}$	$= 0.2\,(4.460)$
			$1.778 \leftarrow$
			$\overleftarrow{3.331}^k$

Third Step:

Equation of equilibrium for exterior reaction at the joint is

$$3.387 - \frac{3.331}{1000}\,\Delta_F E = 0$$

Multiplier: $\Delta_F E = 1.02\ (1000)$.

Final joint moment:

$$\text{Reactions:}\quad \left.\begin{aligned} M &= -0.834 + (2.207)(1.02) = -3.089'\,k \\ H_{cone} &= 0.766 + 1.570 = \overleftarrow{2.336}\,k \\ H_{cylinder} &= 4.153 - 1.813 = \overrightarrow{2.340}\,k \end{aligned}\right\}\ \text{Fig. 13a}$$

Note that these reactions are brought into action at the joint in order to reproduce continuity between the shells. The membrane force $N_y \sin\alpha$ has to be added to these reactions to obtain the actual reaction acting between the shells, for instance

$$H^*_{cone} = 4.320 - 2.336 = -\overrightarrow{1.984}\,k.$$

The bending moments are not appreciably changed by the membrane stresses as indicated indirectly in Introduction, therefore membrane bending is neglected.

The meridional moment in the conical shell can be obtained from

$$m_y = H_L \frac{\sqrt{2}\cos\alpha}{\gamma} \frac{L}{\sqrt{y}} \sqrt[4]{\frac{L}{y}}\, e^{-\left(\frac{\beta_L-\beta_y}{\sqrt{2}}\right)} \sin\left(\frac{\beta_L-\beta_y}{\sqrt{2}}\right) + $$

$$+ M_L \sqrt{2} \sqrt{\frac{L}{y}} \sqrt[4]{\frac{L}{y}}\, e^{-\left(\frac{\beta_L-\beta_y}{\sqrt{2}}\right)} \cos\left(\frac{\beta_L-\beta_y}{\sqrt{2}} - \frac{\pi}{4}\right)$$

The sectional hoop forces are obtainable from

$$n_\theta = N_\theta + H_L \left[\frac{\sin\alpha}{\sqrt{2}} \left(\frac{\beta_L{}^2}{\beta_y}\right) \sqrt{\frac{\beta_L}{\beta_y}}\, e^{-\left(\frac{\beta_L-\beta_y}{\sqrt{2}}\right)} \cos\left(\frac{\beta_L-\beta_y}{\sqrt{2}}\right)\right] -$$

$$- M_L \left[\gamma^2 \tan\alpha \left(\frac{\beta_L}{\beta_y}\right)^{\frac{3}{2}} e^{-\left(\frac{\beta_L-\beta_y}{\sqrt{2}}\right)} \sin\left(\frac{\beta_L-\beta_y}{\sqrt{2}} - \frac{\pi}{4}\right)\right]$$

Meridian force can be obtained from the following equation

$$n_y = N_y + H_L \left[\sqrt{2} \left(\frac{\beta_L}{\beta_y}\right)^{\frac{5}{2}} \sin\alpha\; e^{-\left(\frac{\beta_L-\beta_y}{\sqrt{2}}\right)} \sin\left(\frac{\beta_L-\beta_y}{\sqrt{2}}\right)\right] -$$

$$- M_y \left[2\sqrt{2}\,\gamma^2 \tan\alpha \left(\frac{\beta_L}{\beta_y}\right)^{\frac{3}{2}} e^{-\left(\frac{\beta_L-\beta_y}{\sqrt{2}}\right)} \sin\left(\frac{\beta_L-\beta_y}{\sqrt{2}}\right)\right]$$

The hoop force acts in the horizontal plane in the direction of the hoop circle.

Summary

A method of successive approximations is presented in this paper for the analysis of rotationally symmetrical continuous reinforced concrete conical shells with constant thickness. The shells of that type are used for roofs or bottoms of tanks and bins.

This development is of an approximate nature and therefore its application is subjected to definite limitations. It should be observed that shells with small apex openings can be analysed by the same method. Also shells with prestressed edge member can be analysed by the presented procedure. A set of design graphs are included to reduce the numerical work of the designer.

The writer owes a debt to Paul Chenea, professor of engineering mechanics at Purdue University, Dr. Lawrence C. Maugh and Dr. Bruce G. Johnston, professors of Civil Engineering at University of Michigan for valuable advice given on the subject of stress analysis.

Résumé

L'auteur expose une méthode pour le calcul par approximations successives des voûtes minces coniques, symétriques et continues, d'épaisseur constante en béton armé. Ces voûtes se prêtent à la constitution des couvertures ou des fonds de cuves et autres réservoirs.

Cette méthode est sujette à certaines limitations d'emploi, étant donné qu'il s'agit d'un calcul par approximations successives. Il est à noter que les voûtes minces présentant de petites ouvertures au sommet peuvent être calculées par la même méthode, de même que les voûtes minces qui comportent des éléments de retombée précontraints.

L'auteur joint à son rapport un certain nombre de tableaux graphiques, destinés à faciliter les calculs.

Zusammenfassung

Es wird eine Methode schrittweiser Näherungsberechnung für symmetrische, konische, durchlaufende Eisenbetonschalen von gleichbleibender Wandstärke dargestellt. Die Schalen dieser Art eignen sich als Decken oder Böden von Tanks und sonstigen Behältern.

Da es sich bei der beschriebenen Methode um eine Näherungsberechnung handelt, sind ihrer Anwendung bestimmte Grenzen gesetzt. Es ist zu beachten, daß Schalen mit kleinen Scheitel-Öffnungen mit der gleichen Methode berechnet werden können, ebenso Schalen mit vorgespannten Randgliedern.

Es werden eine Anzahl graphischer Tabellen beigefügt, damit die Berechnung erleichtert werden kann.

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