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Redistribution of Moments due to Creep in Prestressed Concrete Beams, Placed Freely on Supports and Afterwards Made Continuous

Rédistribution par le fluage des moments fléchissants dans les poutres précontraintes, simplement appuyées et rendues continues ultérieurement

Momentenumlagerung durch Kriechen in Spannbetonträgern, ohne Einspannung auf die Lager gestellt und nachher kontinuierlich gemacht

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Introduction

Prestressed concrete beams, which are continuous over several supports, are much used for bridges. Often such a continuous beam is composed of prefabricated prestressed beams, one span long, placed freely on the supports. Afterwards these beams are connected to form a continuous beam.

For the changes of the moments, which may occur due to creep after the beam has been made continuous a computation based on a formula of Habel is considered.

According to this formula these changes are a function of the creep factor φ. The time does not appear in the formula. It is therefore unnecessary to know φ as a function of the time. For φ experimentally determined values may be used.

An experiment has been started. The results of the first 1000 days agree reasonably with the theory. This experiment is still carried on.

Theoretical principles

If there were no creep the moments in a beam composed of a number ol simply supported beams, which are made continuous, would always be equaf to the moments caused by weight and prestressing in the original freely supported beams plus the moments caused in the continuous beam by the loads that are applied after the continuity has been effected. As creep does occur, however, these moments as a rule will change.

The results of the experiment described in this publication are compared with the following theory. This theory cannot be more than a *rough approximation* for the complicated natural phenomenon. The principles have been formulated and used by other investigators (see literature).

These principles are:

1. When applying a load there occurs an immediate, so called elastic, deformation.

2. The elastic deformation is assumed to be proportional to the stress and independent of the age of the concrete (the period of hardening excluded). The first assumption is sufficiently accurate for its purpose. As E (Young's modulus) increases in the course of time, the second assumption calls for special consideration.

3. If the applied load is permanent the deformation increases with time. This additional deformation is called creep.

4. The creep factor

$$\varphi = \frac{\text{deformation by creep}}{\text{elastic deformation}}$$

is independent of the stress σ. This implies that the total strain

$$\epsilon = \frac{\sigma}{E}(1+\varphi).$$

5. The principle of *Whitney*. Relations between φ and the time t are shown in Fig. 1. The instant of the placing of the concrete is chosen for the time

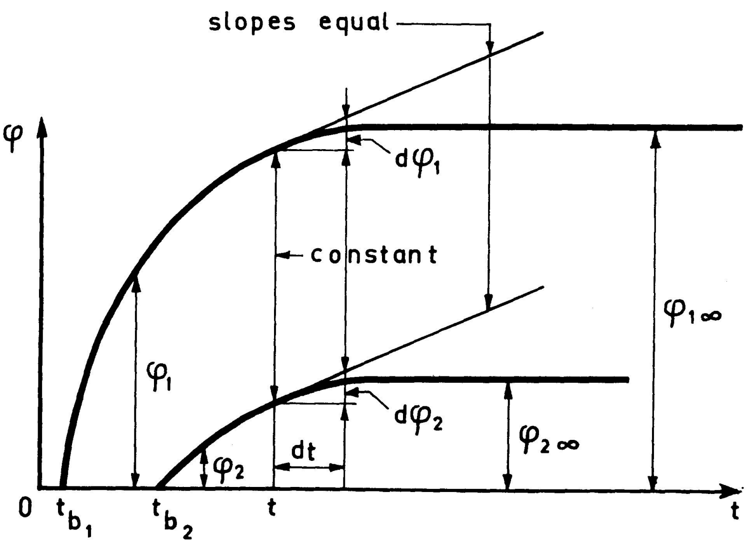


Fig. 1.

origin. For two different instances of applying the load (t_{b1} and t_{b2}) the values for φ (φ_1 and φ_2) are plotted. According to the principle of Whitney the difference between the values of φ_1 and φ_2, for the same value of t is constant (independent of t). This implies

$$\frac{d\,\varphi_1}{d\,t} = \frac{d\,\varphi_2}{d\,t}$$

and also that the creep deformation caused by a load during a period is independent of the loads that have acted before the beginning of that period.

This principle of Whitney is the only relation between φ and time, that is used for deriving the following formula for the changes of the moments of the beams. It is therefore unnecessary to know φ as a function of t.

For φ experimentally determined values may be used. For the determination of the final moments, which occur after the creep is finished, the only value for φ required is that of φ_∞.

Relaxation

If a load is applied to concrete and deformation by creep is prevented, the stress will decrease in the course of time.

This phenomenon is called relaxation and may be considered as the result of simultaneously occurring creep deformation caused by the stress and elastic deformation caused by the change of the stress. The resulting deformation will be equal to zero. By considering the deformation, during an infinitely small time element, the following formula is derived.

$$\sigma = \sigma_b\, e^{-\varphi} = \frac{\sigma b}{2{,}718^{\varphi}} \qquad \text{([1], pg. 106).}$$

σ_b is the stress at the beginning of the considered period, σ the stress at the end of this period and φ the factor for the creep that would be caused by a constant load during the period.

Gradual deformation and change of stress during the creep period also may be considered as the result of simultaneously occurring creep deformation caused by the stress and elastic deformation caused by the change of the stress.

Theoretical Moments in Beams

For the sake of simplicity the beam is supposed to be made of an infinite number of equal spans, every span consisting of a prismatic element prestressed with a parabolic tendon with its anchorage in the axis (fig. 2). The conclusions however, have a more general validity.

In the center of the span the distance from the tendon to this axis $= f$.
The weight of the beam per unit of length $= q$. The tendon force $= k$.
For the freely supported beams the moment in the concrete section equals M free dead load plus M free tendon.

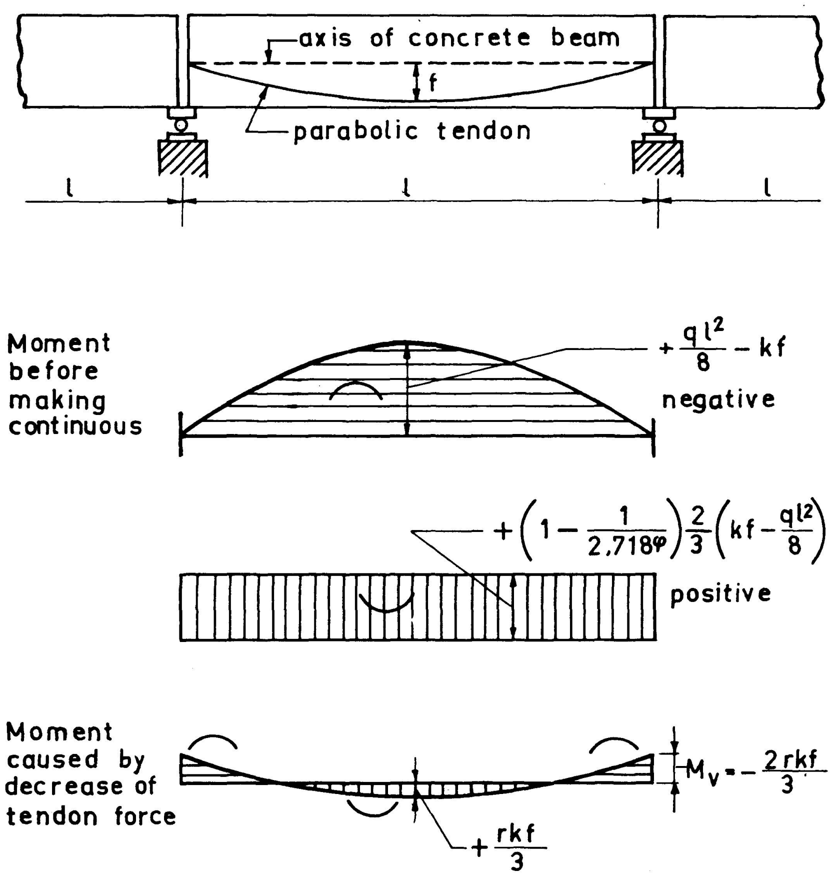


Fig. 2.

The first term is the moment due to weight (positive), the second term is the moment due to the tendon force (negative). Whenever the second term is bigger than the first the resultant moment will be negative. The beam will bend upward and the ends of the beam will rotate.

These rotations are caused by immediately occurring elastic deformation and by creep. If the beam were *not* made continuous and no new load were applied, the angle of rotation by creep would develop to φ times the angle of elastic rotation. This conclusion is based on the assumption that the tendon force remains constant. The influence of the reduction of the tendon force by shrinkage, creep of the concrete, and relaxation of the steel will be considered later.

By making the beams continuous, the rotations of the ends of the beams, that would occur due to creep, are prevented. This causes positive moments above the supports, indicated by M, which develop during the creep period from zero to their final value.

After the beam is made continuous, the change of the angle of rotation of the ends of the beam, which equals zero, may be considered as the result of creep caused by the weight, the tendon force and M and the elastic deformation caused by the increase of M.

By considering the rotation during an infinitely small time element the formula of Habel ([3] and [4] pg. 762) is derived.

$$M = (1-e^{-\varphi})\, M_c = \left(1-\frac{1}{2{,}718^{\varphi}}\right) M_c.$$

φ is the factor for the creep in the period which begins when the beam is made continuous and ends at the instant for which M is determined. M_c is the moment above the support that would occur if a continuous beam were loaded by weight and tendon forces.

Some values for M computed with this formula are given below.

$$\begin{array}{ll} \varphi = 0 & M = 0 \\ \varphi = 1 & M = 0{,}63\ Mc \\ \varphi = 2 & M = 0{,}86\ Mc \\ \varphi = 3 & M = 0{,}95\ Mc \\ \varphi = 4 & M = 0{,}98\ Mc \end{array}$$

For $\varphi = 4$ the limit $M = Mc$ would be nearly reached. The real values for M will, however be smaller because the length of the tendon will decrease due to creep of the concrete. This will cause a decrease of the tendon force which will cause a negative moment above the support and a decrease of M.

If the beam of fig. 2 is freely supported, the moment line for weight and tendon force will be parabolic and the moment in the centre of the span will be

$$+\frac{q\,l^2}{8} - k\,f.$$

M_c will be equal to the moment above the support that would be caused by weight, and tendon force in a continuous beam and so be equal to

$$-\frac{2}{3}\left(+\frac{q\,l^2}{8} - k\,f\right) = +\frac{2}{3}\left(k\,f - \frac{q\,l^2}{8}\right).$$

The positive rectangular moment area due to $M c$ is equal to minus the parabolic moment area caused by weight and tendon force in a freely supported beam. If the tendon force had remained constant, the moment over the support would have been

$$\left(1-\frac{1}{2{,}718^{\varphi}}\right) M_c = \left(1-\frac{1}{2{,}718^{\varphi}}\right)\frac{2}{3}\left(k\,f - \frac{q\,l^2}{8}\right).$$

Permanent or temporary loads, which are applied after the beam has been made continuous, will not influence the additional moment due to creep, because the elastic deformation and the creep deformation are both assumed proportional to the stress.

This implies that the stresses caused by such a load will always be in accordance with the theory of elasticity, which is based on this proportionality. The moments caused by such a load may therefore be computed according to the theory of elasticity for a continuous beam. The reduction of the tendon force is to be considered as such a load. This reduction may be caused by:

1. Relaxation of the steel of the tendon. A value may be determined with data for the type of steel.

2. Shrinkage of the concrete which causes shortening of the tendon and therefore elastic reduction of the tendon force.

3. Creep of the concrete which also shortens the tendon. This shortening is equal to that of the concrete fibers along the tendon. The stresses in these fibers may be computed from the compression force and the moment in the concrete beam. For computing this compression force and moment accurately, the reduction of tendon force has, however, to be known. This problem can be solved by successive approximation which is rather complicated. The simpler method of making a reasonable assumption for the reduction of the tendon force will in most cases be sufficiently accurate for practical design.

The reduction of the tendon force is indicated by $r\,k$. It will cause a parabolic moment line in the continuous beam with a value at the support

$$M_v = -\frac{2\,r\,k\,f}{3} \qquad \text{(negative)}$$

and in the center of the span

$$+\frac{r\,k\,f}{3} \qquad \text{(positive)}$$

see fig. 2.

The final moments, that will occur in the concrete beam after the creep period is finished, will be obtained by superposition of the three moment lines shown in fig. 2 and the moments caused by dead or live loads that might be applied after the beam is made continuous. These moments caused by dead or live loads will be in accordance with the theory of elasticity for a continuous beam.

Up till now it has been assumed that the modulus of elasticity E is constant. In reality E increases with time. For the computation of φ by the formula

$$\varphi = \frac{\text{creep deformation}}{\text{elastic deformation}} = E\,\frac{\text{creep deformation}}{\text{length} \times \text{stress}},$$

it seems reasonable to use the average value of E for the period between the instant that the beam is made continuous and the instant for which φ is to be determined.

Experiment

For two prestressed concrete beams, the changes in the moments caused by creep have been measured. These beams, indicated by A_I and A_{II}, are identical and made according to the schema of fig. 3. Each beam is supported

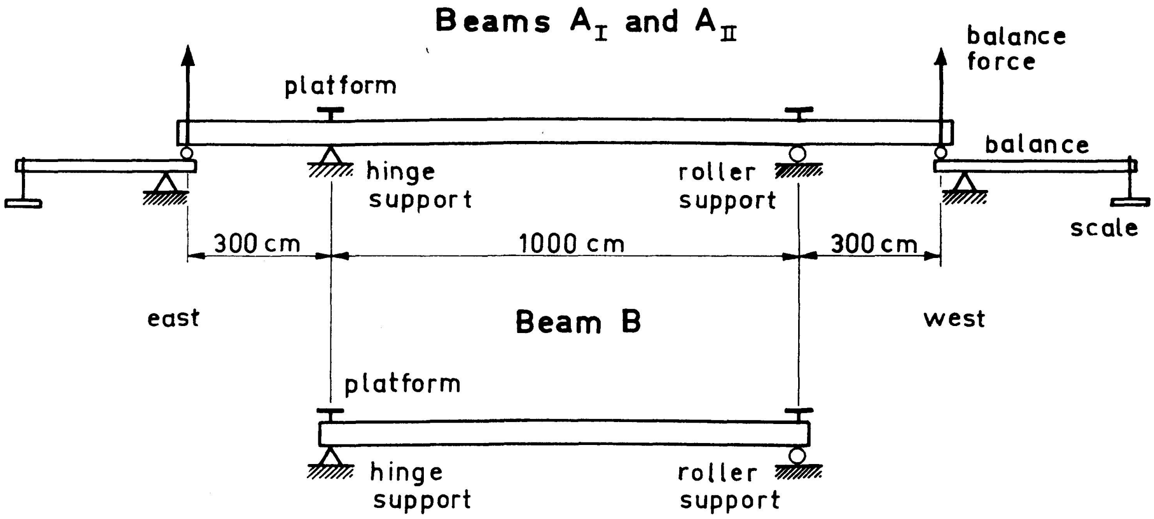
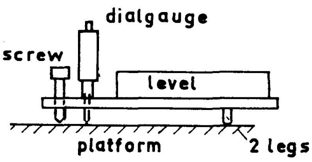


Fig. 3. Scheme.

on a hinge and a roller. The span equals 10 m. Vertical upward forces, which are provided by balances, act on the cantilevered ends, at 3 m from the supports. By placing weights on the scales of the balances the magnitude of these forces can be regulated. From these balance forces the moments above the supports can be computed. Above each support a steel platform is fastened to the beam with an adjustable connection. The upper surface of this platform is accurately plane and can be put horizontal with a level. It is also possible to measure the slope of this platform with a slope meter (fig. 4).

Fig. 4. Slope Meter.



This slope meter consists of a steel table supported by a screw and two legs. On the table a level and a dial gage are fastened. After placing the table on the platform the screw is used to adjust the table until the level is horizontal. Now the slope of the platform may be measured by reading the dial gage. From these slopes the rotation angles of the beams at the supports are derived. With this method of measuring slopes the desirable accuracy can easely be obtained.

At the beginning of the considered creep period the platforms were fixed horizontally with the adjustable connections. If the balance forces remain constant the creep will cause rotation of the beam above the supports and the platforms will follow that rotation. By placing weights on the scales of the balances, as soon as small deviations from the horizontal position of the platforms had occurred, the rotation of the beam above the supports was practically prevented. The beam could therefore be considered as being completely clamped at the supports. The clamping moment is equal to the total change of the balance force multiplied by 3 m (distance balance force to support).

The beams have a square section of 44×44 cm (fig. 5). The reinforcement of mild steel consists of 6 longitudinal bars of 12 mm diameter and some stirrups. These stirrups will not be considered in the computations. The prestress is furnished by 4 Dywidag bars of 26 mm diameter, which are each stressed with an initial force of 30 000 kg. These bars are located outside the beam. Their position in relation to the beam is maintained by guides. These

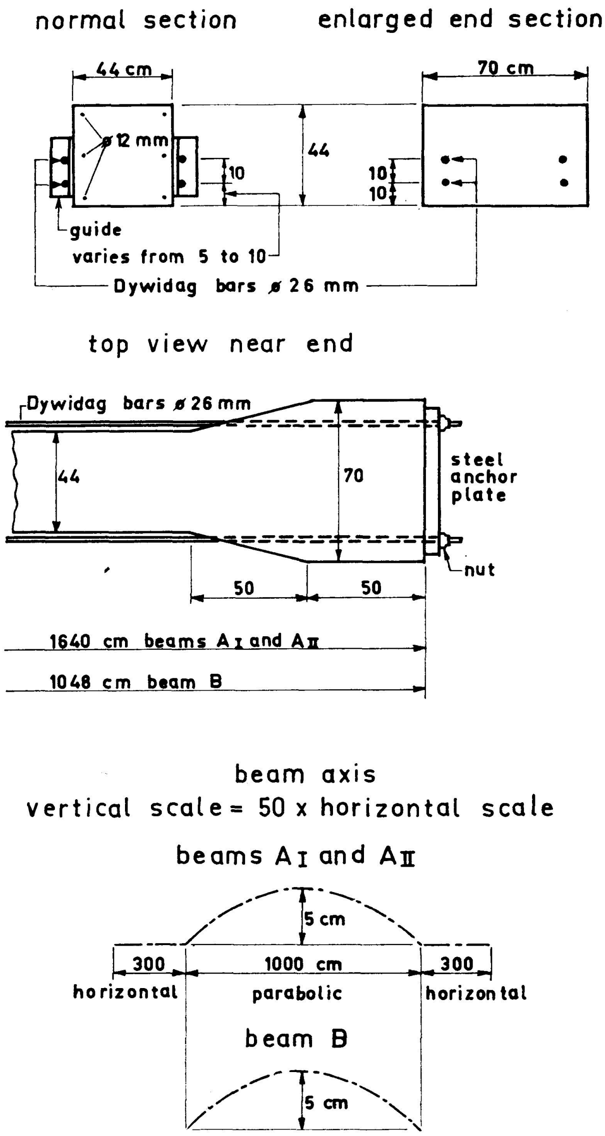


Fig. 5. Beam Details.

are angles fastened to the beam. In these angles are holes through which the bars can slide. The guides make the bars follow the vertical displacements of the beam but do not transfer any horizontal forces of importance to the bars.

In order to transfer the bar forces to the concrete, the ends of the beam are provided with steel anchor plates and the concrete section near these ends is enlarged. In this enlarged section are holes through which the bars can slide. The bar forces are transferred by nuts to the anchor plate.

The part of the beam axis between the supports is parabolic with a camber of 5 cm. The cantilever parts of the beam are straight and horizontal. The Dywidag bars are horizontal over their entire length. This implies that the vertical distances of the axis of the bars to the axis of the beam vary for the lower bars from 12 cm at the supports to 17 cm in the centre of the span and for the upper bars from 2 cm at the supports to 7 cm in the centre of the span.

The moment line caused by the bar forces in the concrete beam is parabolic between the supports. The moments at the supports, however, are not equal to zero as is the case with the corresponding moment line of fig. 2. This does not change the principle of the computation.

For comparison a third beam, indicated by B, was made (fig. 3 and 5). This beam also has a span of 10 m. Because it is not provided with balances the cantilever ends are short. For the rest beam B is identical with the beams A_I and A_{II}.

Beam B is freely supported. Above the supports platforms are fastened for measuring the rotation with the slope meter of fig. 4.

The beams are located in a space under the viaduct at Krimpen on the Ijssel. This space is enclosed by walls and covered by a reinforced concrete floor. It is dark. It was damp when the experiment started. During some periods in spring and summer air was blown in from the outside by a ventilator, and this reduced the dampness. Fig. 6 shows curves for the temperature and the relative humidity.

On the 26th June, 1958, the concrete of the beams was placed in situ. The mix used was 100 kg high strength Portland cement, 178 kg sand and 328 kg gravel. The modulus of finesse of the sand and gravel mixture was 5,4, the slump 2 cm and the water cement ratio 0,37. The concrete was vibrated with a needle of 45 mm diameter. The compression strength is given in table I.

For beams A_I and A_{II} the Dywidag bars were stressed and the forms removed on the 16th, July, 1958.

Table I. Compression Strength of 20 cm Cubes

age in days	7	14	20	28	196	365
kg/cm²	398 409	444 454	495 478	542 532	610 574	680 673

On the 17th of July 1958 the platforms above the supports of beams A_I and A_{II} were fixed horizontally. At that time the balance forces were so regulated, that the moments above the supports in the beam, which consist of the reinforced concrete and the Dywidag bars, were zero. For beam B on that same date the Dywidag bars were stressed and the forms removed.

After the experiment had been started, as mentioned before, the following routine work was done at regular times. The platforms of the beams A_I and A_{II} were brought back in the horizontal position by correcting the weights on the scales and the slopes of the platforms of beam B were measured. This routine work has in the beginning been done twice a day, later once a day, 3 times a week, 2 times a week, once a week and is still going on about 2 times a month. The intention is to continue this routine work. The results of the observations during the first 1000 days are shown in fig. 7. The moments at the supports (clamping moments) of the beams A_I and A_{II} are computed from the weights on the scales. The rotations of beam B are derived from the measured slopes of the platforms and are based on a rotation equal to zero before the stressing and removal of the forms.

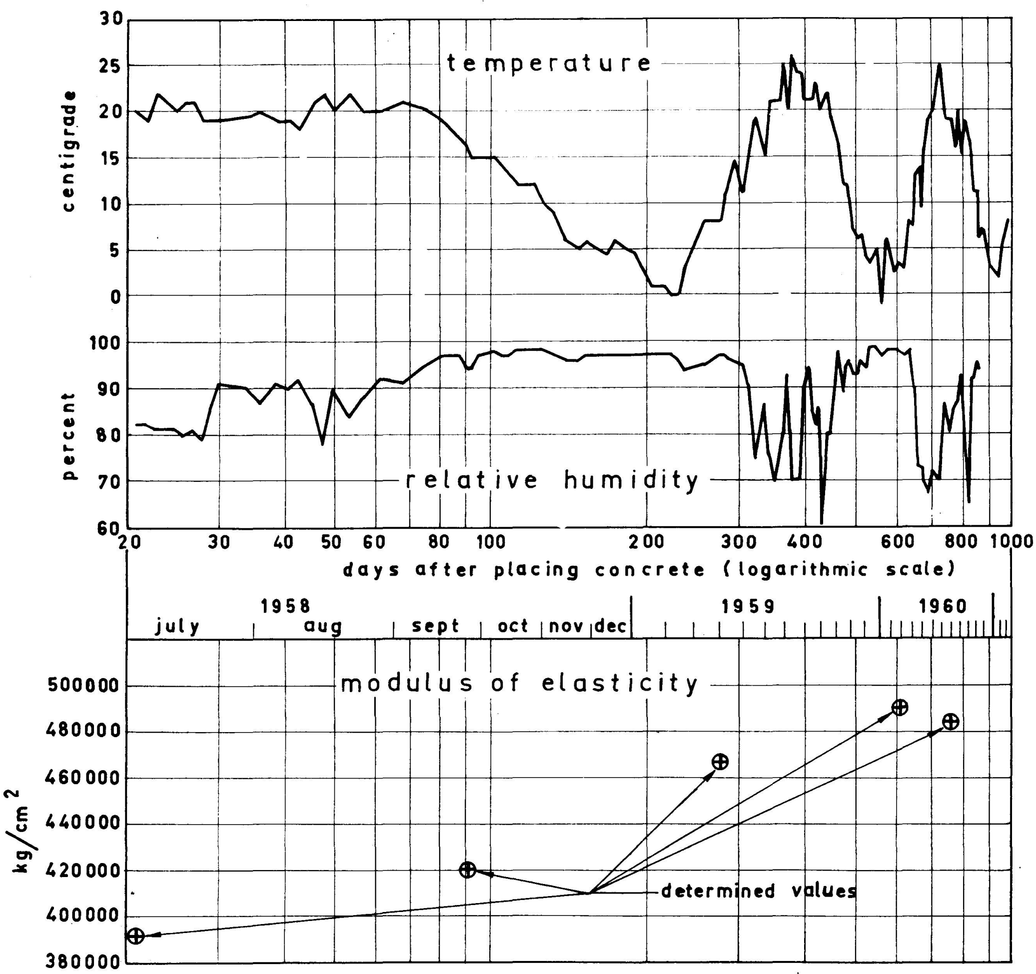


Fig. 6.

Fig. 7 shows that the character of the clamping moment lines for the beams A_I and A_{II} roughly agrees with that of the rotation lines for beam B. During some periods in spring or summer the clamping moments have decreased which is in a lesser degree the case with the rotations. An explanation

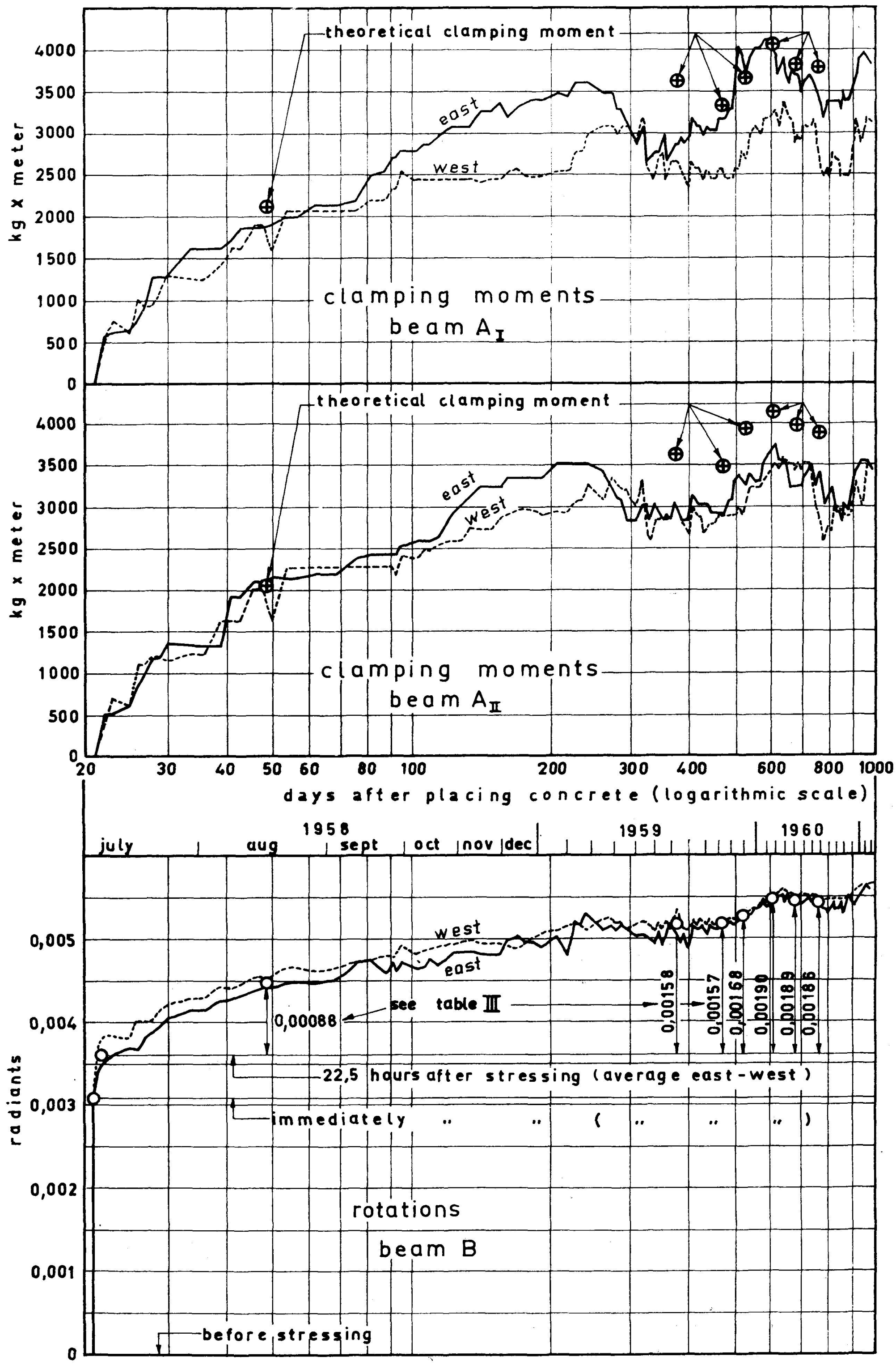


Fig. 7.

may be that the seasonal changes of the humidity (Fig. 6) causes swelling and shrinkage of the concrete, especially of the upper part of beams A_I and A_{II} on which condensation water was dripping from the floor above. Also the swelling of the concrete may have caused the temporary increase of the forces in the Dywidag bars (table II), which may have caused some increase of moments and rotations.

Table II. Forces in Dywidag Bars in kg

beam		16–7 1958	17–7 1958	18–7 1958	21–7 1958	14–8 1958	10–7 1959	8–10 1959	7–12 1959	24–2 1960	6–5 1960	26–7 1960
A_I	upper bars	30000	28200		28000	27200	26600	25900	26100	26500	26000	25000
		30000	27700		27600	27100	26400	25800	26000	26600	26100	25100
	lower bars	30000	28900		28700	28300	27500	26800	27000	27700	27300	26700
		30000	29300		29200	29200	28400	27000	27200	27900	27100	26600
A_{II}	upper bars	30000	28600		28100	27300	26200	25900	25800	26500	26200	25200
		30000	28300		28000	27200	26500	26000	26200	26500	26300	25300
	lower bars	30000	29900		29600	29000	28000	27500	27400	28100	27700	26800
		30000	29200		29000	28200	27500	26800	28000	27500	26900	26400
B	upper bars		30000	28000	27800	26900	26100	25700	25100	26100	26100	25100
			30000	28100	27900	27100	25800	25400	25100	25800	25900	24800
	lower bars		30000	28500	28200	27900	26500	26200	26100	26600	26600	25500
			30000	29200	28700	27900	26500	26400	26000	26900	26800	25900

In the diagrams for A_I and A_{II} of fig. 7 the theoretical clamping moments determined by the following method are also plotted.

The forces in the Dywidag bars were measured one day after stressing and later at various dates as shown in table II. This is done by determining the force required to pull the bar loose from the anchorage. As the bars have threaded ends the jacks that were used for stressing, can be employed. These jacks are provided with a dynamometer with electrical resistance strain gages.

The modulus of elasticity is determined by introducing moments of 5250 kg m at the supports of beam A_{II} by loading the scales. Without delay the rotations of the platform are measured, and the moments removed again. This short-time loading does not seem to influence the creeping-test to any extent. The rotations by loading were somewhat bigger than those by unloading. The average difference was 2,7%. The modulus of elasticity is computed from the average rotation. This experiment has been done 4 times (24th September, 1958, 31st March, 1959, 25th February, 1960, and 25th July, 1960).

A value for the modulus of elasticity has also been computed from the

Table III. Data for Theoretical Determination of Clamping Moments

Date for which clamping moment is determined		14–8 1958	10–7 1959	8–10 1959	7–12 1959	24–2 1960	6–5 1960	26–7 1960	
Average modulus of elasticity for considered creeping period		398000	446000	454000	458000	463000	466000	468000	kg/cm^2
Rotation of beam B during considered creeping period		0.00088	0.00158	0.00157	0.00168	0.00190	0.00189	0.00186	radians
Elastic rotation of beam B by decrease of forces in Dywidag bars		0.00016	0.00034	0.00037	0.00041	0.00030	0.00030	0.00044	radians
Rotation of beam B by creep		0.00104	0.00192	0.00194	0.00209	0.00220	0.00219	0.00230	radians
φ		0.356	0.765	0.796	0.868	0.906	0.910	0.991	
M_c	A_I	+7770	+7770	+7770	+7770	+7770	+7770	+7770	kg m
	A_{II}	+8290	+8290	+8290	+8290	+8290	+8290	+8290	kg m
$\left(1-\frac{1}{2.718^{\varphi}}\right) M_c$	A_I	+2330	+4150	+4260	+4510	+4630	+4640	+4890	kg m
	A_{II}	+2480	+4440	+4550	+4810	+4930	+4950	+5200	kg m
M_v = clamping moment by decrease of forces in Dywidag bars	A_I	− 200	− 520	− 920	− 840	− 560	− 810	−1090	kg m
	A_{II}	− 440	− 820	−1060	− 880	− 790	− 970	−1310	kg m
Theoretical clamping moment	A_I	+2130	+3630	+3340	+3670	+4070	+3830	+3800	kg m
	A_{II}	+2040	+3620	+3490	+3930	+4140	+3980	+3890	kg m
Theor. clamping moment ÷ M_c	A_I	0.27	0.47	0.43	0.47	0.52	0.49	0.49	
	A_{II}	0.25	0.44	0.42	0.47	0.50	0.48	0.47	

rotations, which have been measured for each beam as soon as possible after stressing and removal of forms (on July 16th or 17th, 1958).

All these determined values for E are plotted on the diagram of fig. 6. For computing the average modulus of elasticity for the considered creeping period noted in table III, the values shown in fig. 6 have been plotted on another diagram (not shown in this publication), with an ordinary time scale (not logarithmic).

At one day after stressing the rotations were considerably increased. The average increase was 15% of the rotations, which were measured immediately after stressing.

The platforms of beam A_I were fixed horizontally 22 hours after the stressing of that beam. For beam A_{II} this time was 23 hours. For this reason the comparable creep period for beam B is assumed to have started $(22+23):2 = 22{,}5$ hours after the stressing of beam B.

According to the principle of Whitney, explained before in this publication, the creep, that a load will cause during a given period, is dependent on the age of the concrete during the period and independent of the time that the load has been acting before the beginning of the period. This implies that for comparing the behaviour of the beams the age of the concrete should be considered and not the time of stressing.

The concrete of all the beams was placed on the same day but the stressing of beams A_I and A_{II} happened one day before the stressing of beam B. For this reason the method used for comparing the behaviour of the beams is *not* in agreement with this principle of Whitney.

For all the beams, however, considerable increases of deformation during the first hours after stressing have been observed, which are much bigger than could be expected according to this principle. The deformations that have occurred more than 22,5 hours after stressing might be in reasonable agreement with the principle.

Comparison of the Theory with the Results of the Experiment

The clamping moments, which should occur according to the theory in the beams A_I and A_{II} on various dates, have been computed (see table III) and plotted on the diagram of fig. 7. On these dates the forces in the Dywidag bars have been measured (see table II).

The creep factor φ is computed from the rotations of beam B during the preceding creep period which started 22,5 hour after stressing. This is the average of the rotations at both supports and is given in fig. 7 and table III. During the development of these rotations the forces in the Dywidag bars have decreased. These decreases are the differences between the bar forces measured at one day after stressing and those on the considered dates. The measured rotation is the rotation by creep of the concrete (table III) minus the rotation by elastic deformation of the concrete caused by the decrease of the bar forces (table III). To prove this the creep period is considered to be devided into an infinite number of infinitely short time elements. The moments due to bar forces and weight of the beam, which are acting at the beginning of a time element, will, during that element, cause rotation by creep. The moments due to the decrease of the bar forces, that occur during that element, will cause elastic rotation.

The rotation by creep caused by this decrease of bar forces during the element will be infinitely small of higher order (infinitely small bar forces working during an infinitely short time) and therefore negligible.

Adding (integrating) all these rotations for the entire creep period will show that the resulting rotation is equal to the rotation due to creep minus the elastic rotation caused by the decrease of bar forces. The elastic rotation caused by the decrease of bar forces is computed from the measured bar forces and the average modulus of elasticity during the creep period noted in table III. By adding this elastic rotation to the resulting rotation found in fig. 7 the rotation by creep is obtained (table III). By assuming that this rotation by creep is caused by the weight of the beam and the average bar forces during the creep period the factor φ is computed. The value of E used in the formula.

$$\varphi = E\frac{\text{creep deformation}}{\text{length} \times \text{stress}}$$

is the average modulus of elasticity during the creep period.

After φ has been determined (table III) the theoretical clamping moments for the beams A_I and A_{II} (table III and fig. 7) were computed. Therefore it was necessary to compute first the clamping moment Mc (table III) that would be caused in a completely clamped beam by the loads (bar forces one day after stressing and weight) that were acting on the beam at the instant of fixing the platforms horizontally. If the bar forces had remained constant during the creep period, the clamping moment would have been.

$$\left(1-\frac{1}{2{,}718^{\varphi}}\right)M_c \qquad \text{(table III)}.$$

This moment still has to be decreased by the clamping moment M_v (table III) that occurs in a completely clamped beam loaded by the decrease of the bar forces. The resulting theoretical clamping moment is

$$\left(1-\frac{1}{2{,}718^{\varphi}}\right)M_c+M_v.$$

M_c is here positive and M_v negative.

Fig. 7 shows that the theoretical clamping moments are mostly somewhat bigger than those determined from the balance forces. They are more in accordance with the biggest moments due to balance forces that have occurred in the preceding periods. For judging the suitability of this theory for design, it might be observed that it is important to know the biggest moments that might occur.

For practical design of beams the computation of M_c is rather simple. M_c is always bigger than the extreme value that the clamping moment can reach. If the stress in the prestressed steel remained unchanged, the value of M_c would be nearly reached in case of a big creep factor. This big creep factor will cause, however, a considerable shortening of the concrete and hence of the steel, which will cause a reduction of the steel stress. This reduction of the steel stress will cause a decrease of the clamping moment.

For the tested beams A_I and A_{II} the proportion theoretical clamping moment divided by M_c is given in table III. The proportion of the clamping moments caused by balance forces to M_c may easely be found from the diagrams of fig. 7.

The values for φ (table III) are rather small. This may be explained by the high humidity of the surrounding air (fig. 6) and the high quality of the concrete (table I).

The age of the concrete at che beginning of the creep period must also have influenced the creep factor. This age was 22 days.

It should be observed that for rather small values of φ the clamping moment is already considerable (about half M_c).

Creep and Relaxation

During the considered creep period the moments in beam B did not change much while considerable rotation was going on. In the beams A_I and A_{II}, however, important changes occurred in the moments during this period while the rotation angles above the supports have been kept constant. The changes of beam B are mainly creep (deformation by constant stress). The changes in the part between the supports of the beams A_I and A_{II} are mainly relaxation (decrease of stress by unchanged form). At the beginning of the creep period the moments in the concrete of this part of A_I and A_{II} were in equilibrium with the moments caused by bar forces and weight. Because the moments in the concrete decrease by relaxation, the clamping moments are required to restore the equilibrium and to prevent rotation.

The *theoretical* computation of the clamping moments for the beams A_I and A_{II}, which is based on the observed deformations of beam B, gives results that agree reasonably with the biggest moments determined from the *balance forces* for beams A_I and A_{II} (fig. 7). Therefore it appears that the theoretical principle used, that relaxation is to be considered as a combination of creep and elastic deformation, gives usable results for this case.

Some Details and Corrections of the Experiment and the Computations

The drawing that has been followed for making the beams agreed with fig. 5. After the beams had been made the dimensions of the concrete section and the height of the Dywidag bars in relation to this section were measured. For the computations these measured dimensions, which mostly deviated some millimeters from the dimensions of the drawing, were used.

The weight of a beam is determined by lifting it from one of its supports by a jack provided with a load cell.

This has been done for each support of A_I and A_{II}.

The vertical displacement of the centre of each beam (upward deflection) is also measured. These deflections were in good agreement with the measured rotations. The method of measuring deflections is, however, considered less accurate than that of measuring the rotations. The computations have therefore been based on the measured rotations. The measured deflections have not been used in the computations, but have been considered as a rough check for the measuring of the rotations.

The part of the moment that is taken by the longitudinal reinforcement of mild steel is accounted for in the computations. The modulus of elasticity of mild steel is assumed to be 2 100 000 kg/cm². This correction is applied in order to compare theory and experimental results as accurately as possible. For practical design the relatively small influence of the mild steel may mostly be overlooked.

For determining the modulus of elasticity by adding weights on the scales of A_{II} and then removing these weights again, the increase of the forces in the Dywidag bars, which are caused by the measured rotation, are taken into account.

The modulus of elasticity of the Dywidag bars is assumed to be 2 070 000 kg/cm².

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Literature

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Summary

If prestressed concrete beams are placed freely on supports and afterwards made continuous, the moments in these beams may change due to creep. To investigate these changes of moments an experiment has been started for which three prestressed concrete beams have been made. Each beam is supported on a hinge and a roller. For two of these beams (A_I and A_{II}) the angles of rotation above the supports were kept constant by changing the weights on the scales of balances. The moments that developed by creep above the supports (clamping moments) have been computed from the weights on the scales.

The third beam (B) remained freely supported. The rotations above the supports, caused by creep, were measured and used for computing the creep-factor φ.

During the first 1000 days of the experiment the clamping moments computed from the weights on the scales were about equal to or somewhat smaller than the theoretically derived value.

$$(1-2{,}718^{-\varphi})\, M_c + M_v.$$

M_c is the moment at the support that would occur if the beam were completely clamped and then loaded by its weight and the forces of the prestressed steel, which existed at the instant of clamping. M_v is the change of the clamping moment, which will be caused in a completely clamped beam by the decrease of stress in the prestressed steel.

Résumé

Quand des poutres précontraintes sont d'abord simplement appuyées et rendues continues par la suite, la distribution des moments fléchissants est modifiée par le fluage. Afin d'examiner cette redistribution, des essais ont été effectués sur trois poutres. Chaque poutre est supportée par une articulation et un rouleau. Pour deux de ces poutres (A_I et A_{II}), l'angle des tangentes à la ligne élastique au-dessus des appuis est maintenu constant en modifiant les poids sur les plateaux des balances; les moments causés par cette variation des poids sont égaux aux moments causés par le fluage en cas d'encastrement.

La troisième poutre (B) reste simplement appuyée. Les rotations au-dessus des appuis, provoquées par le fluage, sont mesurées afin de déterminer le coefficient de fluage.

Pendant les premiers mille jours des essais, les moments sur appuis, calculés à l'aide des poids sur les balances, étaient à peu près égaux ou un peu inférieurs aux valeurs théoriques

$$(1-2{,}718^{-\varphi})\, M_c + M_v.$$

M_c désigne le moment sur appui de la poutre parfaitement encastrée, chargée de son poids propre et de l'effort de précontrainte existant à l'instant où l'encastrement est réalisé.

M_v désigne le moment d'encastrement provoqué, dans une poutre parfaitement encastrée, par la perte de précontrainte dans les fils d'acier.

Zusammenfassung

Wenn vorgefertigte Spannbetonträger vorerst frei aufgelagert und später durchlaufend gemacht werden, verlagern sich die Momente in diesem Träger infolge des Kriechens.

Um diese Verlagerung zu untersuchen, wurden Versuche mit 3 vorgespannten Betonträgern gemacht.

Jeder Träger wurde auf einem Gelenk und auf einer Rolle aufgelagert. An zweien dieser Träger ($A_I + A_{II}$) wurden die Auflagerdrehwinkel konstant gehalten, indem die Gewichte auf den Waagschalen geändert wurden. Die Momente, welche durch Kriechen über den Auflagern entstehen, sind gleich den Momenten infolge der Änderung dieser Gewichte.

Der dritte Träger (B) blieb einfach aufgelagert. Die infolge Kriechens entstandenen Auflagerdrehwinkel wurden gemessen und daraus das Kriechmaß φ bestimmt.

Während der ersten 1000 Tage waren die nach den aufgelegten Gewichten gerechneten Einspannmomente ungefähr gleich oder wenig kleiner als die theoretischen Werte.

$$(1-2{,}718^{-\varphi})\,M_c + M_v.$$

M_c bedeutet das Einspannmoment des vollkommen eingespannten Balkens infolge seines Eigengewichtes und der im Zeitpunt der Einspannung wirkenden Vorspannkraft. M_v bedeutet die Änderung des Einspannmomentes in einem vollkommen eingespannten Träger infolge Abnahme der Vorspannkraft im Spannglied.

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