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A General Analysis of Prestressed Nets

Etude générale des réseaux de câbles précontraints

Eine allgemeine Untersuchung vorgespannter Netzwerke

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Introduction

Several studies [1, 2] have recently been published on the analysis of two-directional prestressed networks of cables, based on the following assumptions:

a) The cables in each direction lie in vertical parallel planes, with the two sets of planes usually perpendicular to each other.
b) Horizontal displacements are small and may, at least tentatively, be neglected.
c) The behaviour of the net may, also tentatively, be assumed linear.

The first assumption imposes a serious limitation on the applicability of the theory, but the error introduced by the other two assumptions may subsequently be corrected by iteration.

It should be stressed, however, that the effects of assumptions (b) and (c) may be separated from each other. Linear behaviour may, as a matter of fact, occur in practice even in non-linear systems, under infinitesimal load. There is, however, a difference between conventional trusses and prestressed nets. In conventional systems, linear behaviour is defined as the case where the change in geometry is so small as to have a negligible effect on the stresses. In prestressed nets the situation is different: prestress forces and shape are interdependent so as to satisfy the equilibrium conditions. Any displacement upsets the equilibrium and thus affects the bearing capacity of the net. Moreover, two-directional nets cannot be solved without considering the effect of the change in geometry, while in three-directional nets the effect of deformation on stresses depends on a dimensionless ratio between the rigidity of the net and the prestress forces [3]. The effect of deformation already sets in under infinitesimal load [3], i.e. in the linear state.

Separation of the effects of displacements on stresses and non-linearity makes possible the analysis presented in this paper. *The only assumption made in the following is that of linear behaviour, with the error involved subsequently corrected by iteration.* The net may be of any shape (some types of nets are shown in Figs. 1—5), and may even contain compression members (Fig. 5) using frictionless ball joints. The analysis is based on the "strain method", with the three components of displacements u_i in the x_i directions considered as unknowns, and the stresses expressed as a function of the displacements. This approach necessitates solution of a large number of simultaneous equations, which calls for the use of electronic computers. Recourse to matrix algebra facilitates the task of programming.

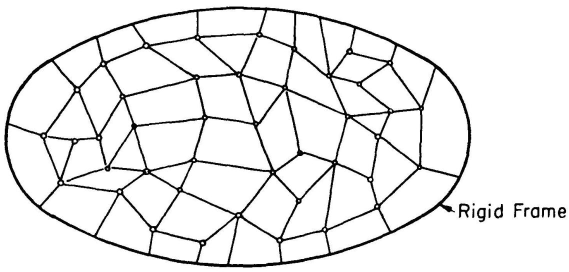

Fig. 1. An irregular-shaped net.

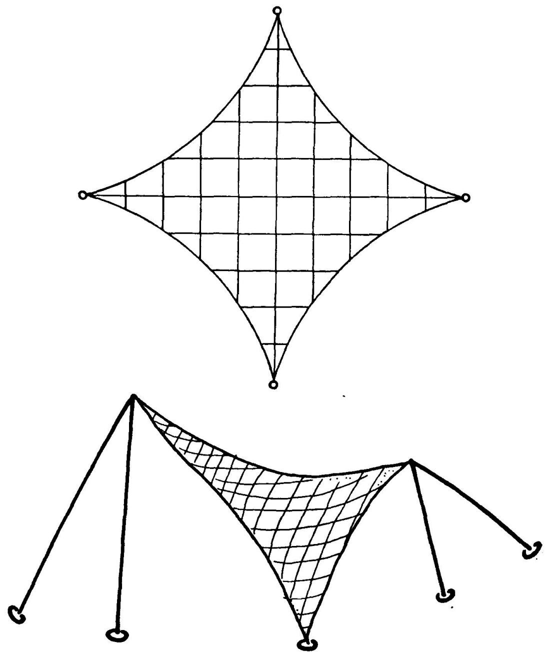

Fig. 2. A net bounded by main prestressed cables.

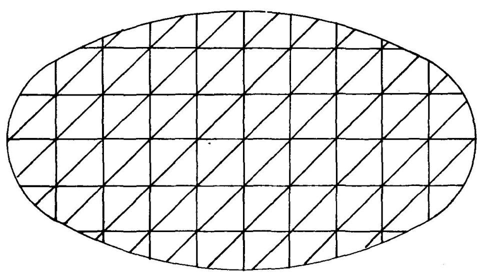

Fig. 4. A three-directional net.

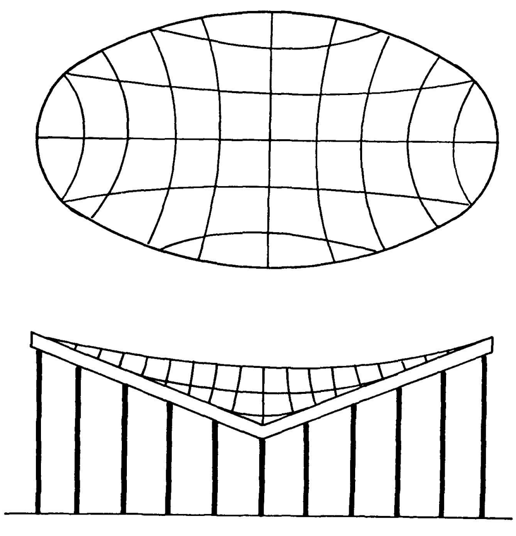

Fig. 3. A net of cables forming geodesic lines of the surface.

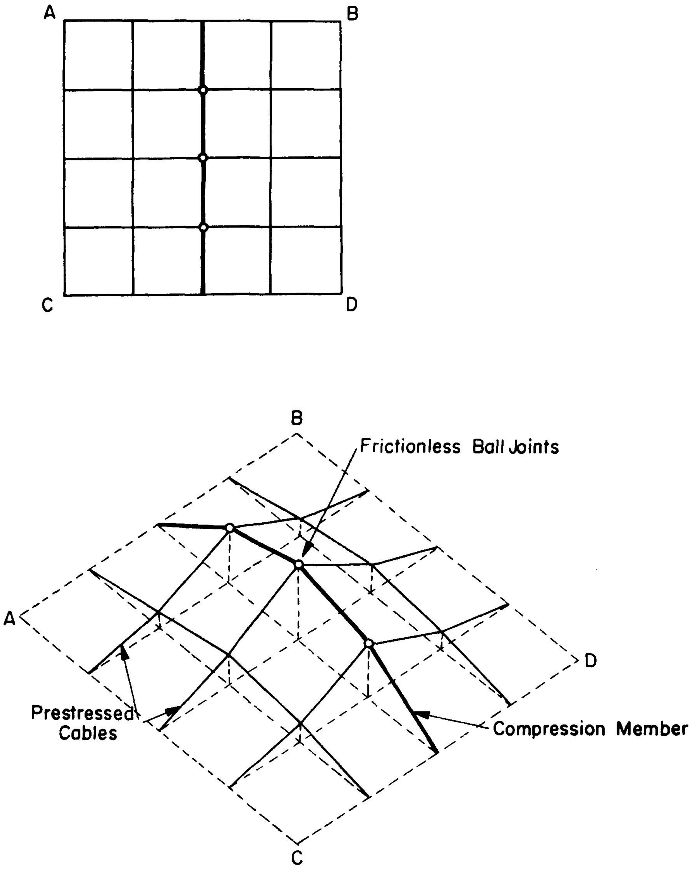

Fig. 5. A net including compression members.

The Unloaded Net

Let the pretension in a section $a-b$ be denoted by T_{ab} (compression assumed as negative), and the direction angle between the bar and the x_i axis by $\alpha_{i,ab}$. The force component acting through section ab on joint a is:

$$T_{ab} \cos \alpha_{i;ab}. \tag{1}$$

Assuming a weightless net, the conditions of equilibrium at joint a are:

$$\sum_b T_{ab} \cos \alpha_{i;ab} = 0, \tag{2}$$

where $\sum_b$ signifies inclusion of all joints b connected to joint a. Eqs. (2) refer to all except the anchoring joints. The ordinates of the anchoring joints are known and determined by architectural or other considerations. Thus, in Fig. 1 all inner joints are included, while in Fig. 2 all except the four anchoring joints are considered. (In the following, the number of considered joints will be denoted by J.)

It is seen from Eq. (2) that the prestress forces and geometry of the net are interdependent. Normally the shape of the net is not random as in Fig. 1, but shows a certain regularity.

The case of cables lying in two sets of vertical parallel planes was discussed in [1, 2, 3, 4]. In this case Eq. (2) is automatically satisfied in two directions by the condition of constant horizontal component of pretension in each cable, equilibrium being required in the third direction only. In other words, there are only J unknowns and J equations in this case.

Another case is of cables forming geodesics of the surface. This case was mentioned in [3] and a more detailed paper on the subject is in preparation.

In general, geodesic cables are obtained by assuming the tension T as constant along each cable. A particular case is when the tension T is equal throughout all the cables. This case is of extreme importance for roofs formed of strips of cloth.

However, the following discussion refers to the general case.

The Loaded Net

After loading, the net undergoes deformation and the stresses in the cables vary. Let the component of displacements be denoted by u_i. The squared length $(l+\Delta l)$ of section $a b$ after deformation is:

$$(l+\Delta l)^2_{ab} = \sum_i (x_{i;b}+u_{i;b}-x_{i;a}-u_{i;a})^2 \tag{3}$$

while before loading, the squared length of $a b$ was:

$$l^2 = \sum_i (x_{i;b}-x_{i;a})^2 . \tag{4}$$

Expanding Eq. (3), subtracting Eq. (4) from it and rearranging, we obtain, after omitting second-order terms (linearity assumption) the known formula:

$$\Delta l = \sum_i (u_{i;b}-u_{i;a}) \cos \alpha_{i;ab} . \tag{5}$$

However,

$$\Delta l = \frac{\Delta T l}{EA} + \mu l \Delta t , \tag{6}$$

where:
T = change in tension,
A = cross section of cable or bar,
E = Young's modulus of elasticity,
μ = coefficient of thermal expansion,
t = change in temperature.

Substituting Eq. (5) in Eq. (6), the change in tension is expressed in terms of displacements.

The equilibrium condition under load is, instead of Eq. (2):

$$\sum_b (T+\Delta T) \cos (\alpha+\Delta \alpha)_i + P_i = 0 . \tag{7}$$

P_i being the load component in the x_i direction. The new direction cosine will now be expressed in terms of the displacements:

$$\cos (\alpha+\Delta \alpha)_i = \frac{x_{i;b}+u_{i;b}-x_{i;a}-u_{i;a}}{l_{ab}+\Delta l_{ab}} = \left[\cos \alpha_i + \frac{u_{i;b}-u_{i;s}}{l_{ab}}\right] : \left[1+\frac{\Delta l_{ab}}{l_{ab}}\right] . \tag{8}$$

This equation is simplified by using the approximation:

$$\frac{C+D}{1+\epsilon} = (C+D)(1-\epsilon) = C+D-C\epsilon, \tag{9}$$

where D and ϵ are small numbers, which is the case for small loads (linearity assumption). Eq. (8) thus becomes (after substituting Eq. (5)):

$$\cos(\alpha+\Delta\alpha)_{i;ab} = \cos\alpha_i + \frac{u_{i;b}-u_{i;a}}{l_{ab}} - \frac{\cos\alpha_{i;ab}}{l_{ab}} \sum_j (u_{j;b}-u_{j;a})\cos\alpha_{j;ab}. \tag{10}$$

Expanding Eq. (7), subtracting from it Eq. (2), and omitting second-order terms, we obtain:

$$\sum_b [\Delta T\cos\alpha_i + T\{\cos(\alpha+\Delta\alpha)_i - \cos\alpha_i\}] + P_i = 0 \tag{11}$$

and substituting T from Eqs. (5, 6) and the new direction cosines from Eq. (10), with no temperature change assumed for simplicity.

$$\sum_b \Big[\Big\{\frac{EA_{ab}}{l_{ab}}\cos\alpha_{i;ab}\sum_j (u_{j;b}-u_{j;a})\cos\alpha_{j;ab}\Big\} + \frac{T_{ab}}{l_{ab}}\{u_{i;b}-u_{i;a}-\cos\alpha_{i;ab}\sum_j (u_{j;b}-u_{j;a})\cos\alpha_{j;ab}\}\Big] + P_i = 0. \tag{12}$$

This equation should, of course, be expanded so as to adapt it for use with an electronic computer.

Eq. (12) comprises $3J$ linear equations, with J counted as defined earlier; when the frame (e.g. Fig. 1) undergoes considerable deformation, additional equations for these joints are written [3]. In matrix form we have:

$$\boldsymbol{A}\boldsymbol{U} = -\boldsymbol{P} \tag{13}$$

or, after inversion:

$$\boldsymbol{U}_i = -\boldsymbol{A}^{-1}\boldsymbol{P}_i. \tag{14}$$

The matrix $\boldsymbol{A}^{-1}$ represents the influence coefficients for the displacements u_i at any joint as a function of the load components P_i in the x_i directions.

Correction for Non-linearity

As explained earlier, Eq. (14) describes the true state under infinitesimal load. The exact behaviour under higher load is determined by iteration, as will be described later. This procedure calls for a vast amount of computational work, especially if several states of loading are considered. Yet, when an electronic computer is used programming is easy due to the repetitive character of the procedure.

For the iterative process, only a real state of loading should be considered, not influence coefficients.

The displacement vector u_i is computed tentatively according to the linearity assumption (Eq. 14).

The computed displacements u_i are not exact and the errors Δu_i depend on the loads and on specific conditions. It should be borne in mind that the correction of the vector u_i is valid only for the specific combination of load on all joints. The correction is computed as follows:

First the residuals R are computed by using the values of u_i' obtained by Eq. (14):

$$R_i = \sum_b (T + \Delta T) \cos(\alpha + \Delta \alpha)_i + P_i. \tag{15}$$

The residuals are considered as unbalanced load R. Whenever the effect of non-linearity is small, which is the general case, the matrix $\boldsymbol{A}^{-1}$ may be assumed valid, and the change Δu_i of u_i' is:

$$\Delta \boldsymbol{U}_i = -\boldsymbol{A}^{-1} \boldsymbol{R} \tag{16}$$

and the new displacements are $u_i = u_i' + \Delta u_i$.

These values may be used again to calculate R, the iteration being discontinued when R becomes negligible. The rate of convergence depends on the error introduced by assuming the matrix $\boldsymbol{A}^{-1}$ for higher loads.

Alternatively, a different procedure may be followed, using corrected coordinates each time. The corrected coordinate $x'_{i;a}$ equals the previous one $x_{i;a}$ plus its displacement found by solving Eq. (12), i.e.:

$$x_i' = x_i + u_i. \tag{17}$$

These coordinates serve for computing the new section lengths, and hence their elongations, the new tension forces T'_{ab} and the new direction cosines $\cos \alpha'$. Eq. (15) in the new notation becomes:

$$R_i = \sum_b T' \cos \alpha_i' + P_i. \tag{18}$$

Evidently, if the exact values of u_i are used, the residual R is zero.

$$\sum_b T' \cos \alpha_i' + P_i = 0. \tag{19}$$

Substituting the new values T' and α' in Eq. (7):

$$\sum_b (T' + \Delta T') \cos(\alpha' + \Delta \alpha')_i + P_i = 0. \tag{20}$$

Rewriting in analogy to Eq. (11), we obtain:

$$\sum_b [\Delta T' \cos \alpha_i' + T' \{\cos(\alpha' + \Delta \alpha')_i - \cos \alpha_i'\}] + (P_i + \sum_b T' \cos \alpha_i') = 0. \tag{21}$$

Using the same considerations as before, an equation similar to Eq. (12) is obtained. In matrix form:

$$\boldsymbol{A}' \boldsymbol{U} + (P_i + \sum_b T' \cos \alpha_i') = 0 \tag{22}$$

or

$$\boldsymbol{A}' \boldsymbol{U} = -(P_i + \sum_b T' \cos \alpha_i'). \tag{23}$$

Substituting Eq. (18):

$$\boldsymbol{A}' \boldsymbol{U} = -\boldsymbol{R}, \tag{24}$$

where A' is the matrix formed by the corrected coordinates.

For the exact solution

$$\boldsymbol{A'}\,\boldsymbol{U} = 0\,. \tag{25}$$

Eq. (25) is analogous to Eq. (12) and represents a linear solution in the vicinity of the load vector $\boldsymbol{P}$.

Had the exact values u_i been used to calculate A', solution of Eq. (24) would have yielded the exact incremental displacements $\Delta u_i'$. As this is not the case, iteration is continued until $\boldsymbol{R}$ has become negligible.

The similarity beween Eq. (25) and Eq. (13) permits generalisation of the procedure and the use of the same computer programme both for the linear solution and the iteration. In all cases, the solution is carried out for residual loads R_i [Eq. (25)] and obviously, because of Eq. (2), $R_i = P_i$ in the first step. Moreover, the same programme may be used to check the initial shape of the unloaded net, by substituting $P_i = 0$ in Eq. (18). Alternatively, if the initial section lengths and rigidity parameters are known, the exact shape may be obtained by assuming approximate coordinates, again substituting $P_i = 0$ in Eq. (18) and solving by iteration until R approaches zero.

In certain cases, e.g. when non-linearity is considerable or when buckling is expected, the solution may be obtained through raising the loads by increments, as explained below.

Buckling Phenomena

The possibility of drawing the non-linearity curve of deflection permits the study of buckling phenomena. The buckling process in this case refers to the structure as a whole, not to single members which can be analysed by known formulas for column buckling.

The system shown in Fig. 5 may serve as an example of a structure with buckling tendency. This system may be subjected to any mode of loading, using given constant increments. In other words, the load ratio remains constant throughout the process.

Fig. 6 shows, to a distorted scale, the displacement $u_{i;a}$ versus the load vector P. The tangent curve at zero load corresponds to the linear solution [Eq. (14)].

For infinitesimal load, the linear solution coincides with the true one. For the load vector P_I the linear solution yields displacements u_i' of r'. The correction $r r'$ is computed by iteration and the true values $u_{i;I}$ are determined.

The load is now increased to P_{II}. R_i of Eq. (24) will equal the correct incremental load. Using the corrected ordinates for P_I, a solution is obtained for points on the tangents to the curves through r. Thus point s' will be obtained instead of the true solution s, while point s'' would have been obtained had the matrix $\boldsymbol{A}$ (zero load) been used. By the iteration process explained

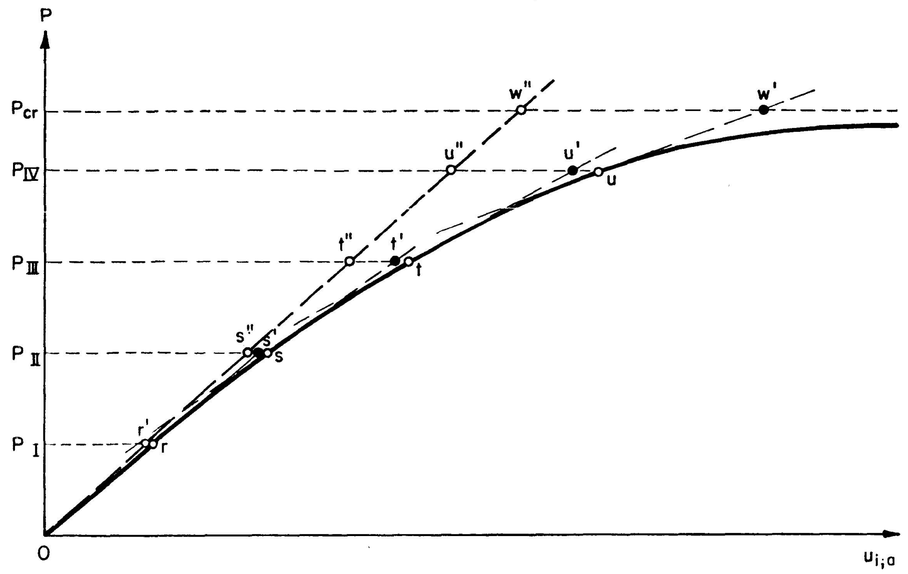

Fig. 6. Displacement curve $U_{i,a}$ versus load vector P.

above, point s is computed, and the matrix A' corrected. Increasing the load to P_{III} yields point t', etc. When the slope of the curve approaches zero (i.e. infinite displacements for small increments of load), buckling has set in. This situation is arrived at when the determinant of A' is zero. At the present stage of knowledge it is not certain whether the determinant changes sign for loads above the buckling limit. A check should therefore be provided in the computer for such a change, along with the other possibilities. In that case, the load should be reduced in steps equal to half the increments, until the sign is changed again. The load is then again increased in squared halves of the initial increments, etc., until the desired accuracy has been obtained.

Remarks

a) At the stage of buckling, infinite displacements will occur simultaneously at most joints of the system.
b) For the study of buckling, combinations of loads yielding large displacements should be assumed.
c) Systems without tendency to buckling will have a convex curve of displacements, i. e. the non-linearity effect represents "strengthening" of the structure and reduction of its deflections.

Summary and Conclusions

The theory presented is general and covers all types of nets and trusses. Two-directional nets or un-prestressed trusses (determinate or indeterminate) may be considered as special cases.

Note that for $T=0$ Eq. (12) is same as in reference 5.

The number of equations to be solved is $3J$, irrespective of the degree of indeterminacy. It should be borne in mind, however, that only systems with a limited number of joints, about 35 (100 equations), are capable of convenient solution with the largest modern computers. This is less than the number of joints in actual roofs; still, a good insight into the behaviour of any system can be obtained by analysing a sparser net instead of the real one.

Notation

a	Joint for which an equilibrium equation is written.
A	Cross section of cable or bar.
b	All joints connected to joint a.
E	Young's modulus of elasticity.
J	Number of considered joints for equilibrium equations.
l	Length of section.
P	Load.
P_i	Component of load in x_i direction.
$P_I; P_{II}; P_{III}$	Vector loads of same mode.
R_i	Residual load.
x_i	Coordinates.
Δt	Change of temperature.
T	Tension in cable.
u_i	Displacement component in i direction.
$\alpha_{i;ab}$	Angle between section ab and axis x_i.
μ	Coefficient of thermal expansion.

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Summary

A general analysis of prestressed spatial nets, based on the strain (or rather displacement) method is presented. The method is general and applies to any net, such as three- or multi-directional nets, nets in which the cables form geodesic lines of the surface, etc. Conventional space trusses or plane trusses may be considered as a particular case of nets with zero prestress. The theory is tentatively based on the assumption of linearity, with the error involved subsequently corrected by iteration.

Matrix algebra and simultaneous linear equations are used, so that the programming for an electronic computer is not too cumbersome and the manipulation and solution of a large number of equations do not present any difficulty.

Résumé

L'auteur présente une étude générale des réseaux spatiaux de câbles précontraints, basée sur la méthode des déformations (ou plutôt des déplacements). La méthode est générale et s'applique à n'importe quel type de réseau tels que, par exemple, des réseaux à trois ou plusieurs directions ou des réseaux dans lesquels les câbles forment les lignes géodésiques d'une surface, etc. Les fermes à treillis classiques planes ou dans l'espace peuvent être considérées comme des cas particuliers de ces réseaux, avec une précontrainte nulle. La théorie est établie dans l'hypothèse d'un comportement linéaire, les erreurs qui en résultent étant ensuite corrigées par approximations successives.

Le calcul matriciel et les systèmes d'équations linéaires simultanées sont traités de telle sorte que la programmation pour des calculatrices électroniques puisse s'effectuer sans de trop grandes difficultés et que la transformation et la résolution d'un grand nombre d'équations puissent s'effectuer facilement.

Zusammenfassung

Auf Grund der Deformationsmethode (oder eher einer Verschiebungsmethode) wird eine allgemeine Untersuchung vorgespannter, räumlicher Netzwerke dargelegt. Die Methode ist allgemein und auf irgendwelche Netzwerke anwendbar, wie z. B. Netze, welche auf 3 oder mehreren Richtungen aufgebaut sind oder in welchen die Kabel geodätischen Linien der Fläche folgen usw. Die klassischen räumlichen oder ebenen Fachwerke können als Spezialfälle von Netzwerken ohne Vorspannung betrachtet werden. Die Theorie basiert auf der Annahme der Linearität, wobei der daraus resultierende Fehler nachträglich iterativ verbessert wird.

Die Matrizenrechnung und lineare Gleichungssysteme werden so angewandt, daß die Programmierung für elektronische Rechengeräte nicht zu umständlich wird und daß die Umformung und Lösung einer großen Zahl von Gleichungen keine Schwierigkeiten mit sich bringt.