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Response of Portal Frames to Dynamic Loading

Comportement des portiques soumis à des charges dynamiques

Portalrahmen unter dynamischer Belastung

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In this paper the response of a portal frame to each a force and a mass moving with constant velocity across the top member is found. The problem is solved by expanding the deflection of each member of the frame in a series of products of the eigen functions (or shape functions for the frame) and generalized time functions. The kinetic and potential energies of the system (frame and mass) are then expressed in terms of the deflection and its derivatives. From these expressions a set of nonhomogeneous simultaneous differential equations in the generalized time functions is obtained by using Hamilton's principle of Least Action. The first two equations in this set are solved to give the response in the first and second modes of the frame. Numerical results found for several different values of velocity of the mass are presented in graphical form at the end of the paper. Numerical results for a load are readily obtained from the computational procedures and are included for comparison. In addition, charts useful in determining the two lowest fundamental frequencies of the frame are presented.

Analytical Development of the Problem

Fig. 1 shows a symmetric portal frame with a mass moving across the frame with a velocity v. The column members of the frame are built in at the bottom and the joints at B and C are rigid. The lengths of the members AB, BC are L_C, L_B, their stiffness EI_C, EI_B respectively. Each member of the frame is uniform and made of the same material.

The analysis is based on the following basic assumptions.

1. The structure is assumed to be a continuous system with distributed mass.
2. Material of the frame is assumed to obey Hooke's Law.
3. Damping effects are assumed to be negligible.
4. Each member of the frame is straight, has uniform cross section and mass.
5. Effect of shearing stresses and axial stresses have been neglected.
6. Effect of rotatory inertia is neglected.
7. The joints are rigid and the fixture at the base is perfect.
8. Vibrations in the plane of the frame only are considered.

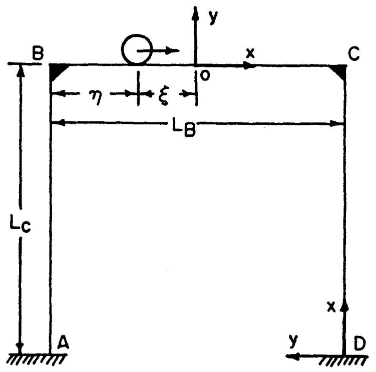

Fig. 1.

The initial problem is that of determining the natural modes and natural frequencies for the frame. For convenience the co-ordinate system shown in Fig. 1 has been chosen. It will therefore be sufficient to treat only one half of the frame in computing the eigen values and eigen functions.

The equation of motion for each member of the frame when it is vibrating freely may be represented by:

$$E\,I\,y^{\mathrm{IV}} + \rho\,A\,\ddot{y} = 0\,, \tag{1}$$

where ρ = mass density.

A = area of cross section of the member.
I = moment of inertia of the cross section about the neutral axis.
y = deflection, E = Young's modulus of elasticity.

Primes indicate derivatives with respect to x, and dots indicate derivatives with respect to time.

Under free oscillations, the frame vibrates harmonically with frequency ω such that $y = X(x)\sin\omega t$. Substituting this into Eq. (1) gives the generating equation for the eigen functions.

$$E\,I\,X^{\mathrm{IV}}(x) - \rho\,A\,\omega^2\,X(x) = 0\,. \tag{2}$$

The solution of this equation is:

$$\begin{aligned} X(x) &= A(\cosh\lambda x + \cos\lambda x) + B(\cosh\lambda x - \cos\lambda x) \\ &\quad + C(\sinh\lambda x + \sin\lambda x) + D(\sinh\lambda x - \sin\lambda x)\,. \end{aligned}$$

Where $$\lambda^4 = \frac{\rho A \omega^2}{E I}.$$

An equation and corresponding solution of this kind will exist for each member of the frame. By forcing the functions to satisfy the geometrical restraints on each member, a set of linear, homogeneous, algebraic equations in the integration constants is obtained. The number of equations will be equal to the number of unknowns. A nontrivial solution for these constants requires that the determinant formed by these coefficients on the unknowns be zero. This characteristic determinant will define an infinite number of admissible values for λ and are designated by λ_m and the corresponding functions by X_m where $m = 1, 2, 3, \ldots \infty$. X_m will denote the eigen functions for the column. A similar expression designated by Z_m will describe the eigen functions for the beam.

$$\begin{aligned} X_m(x) &= A_m(\cosh\lambda_m^C x + \cos\lambda_m^C x) + B_m(\cosh\lambda_m^C x - \cos\lambda_m^C x) \\ &\quad + C_m(\sinh\lambda_m^C x + \sin\lambda_m^C x) + D_m(\sinh\lambda_m^C x - \sin\lambda_m^C x), \\ Z_m(x) &= E_m(\cosh\lambda_m^B x + \cos\lambda_m^B x) + F_m(\cosh\lambda_m^B x - \cos\lambda_m^B x) \\ &\quad + G_m(\sinh\lambda_m^B x + \sin\lambda_m^B x) + H_m(\sinh\lambda_m^B x - \sin\lambda_m^B x). \end{aligned}$$

The frame can vibrate symmetrically and asymmetrically as shown in Figs. 2a and 2b. The constraints imposed on $X_m(x)$ and $Z_m(x)$ will be different

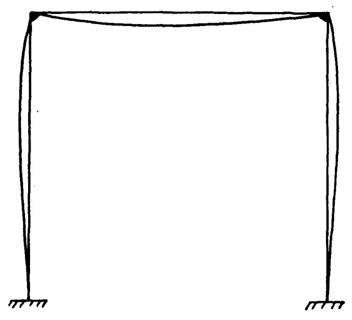

Fig. 2a.

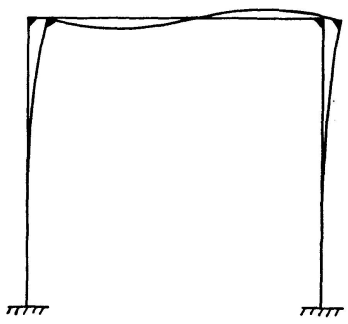

Fig. 2b.

for the two different modes. This leads to some complications on the solution since the two different forms will lead to two sets of eigen values and thus to two sets of eigen functions.

For the symmetric mode the following conditions in $X_m(x)$ and $Z_m(x)$ apply:

$$\begin{aligned} &X_m(0) = X_m'(0) = X_m(L_C) = 0, && Z_m\left(\frac{L_B}{2}\right) = Z_m'(0) = Z_m'''(0) = 0, \\ &X_m'(L_C) = Z_m'\left(\frac{L_B}{2}\right), && E I_C X_m''(L_C) + E I_B Z_m''\left(\frac{L_B}{2}\right) = 0. \end{aligned} \qquad (3)$$

These conditions of restraint yield the following characteristic determinant:

$$\begin{vmatrix} 0 & \left(\operatorname{Cosh}\frac{K\alpha}{2}+\operatorname{Cos}\frac{K\alpha}{2}\right) & 2\operatorname{Cosh}\frac{K\alpha}{2} \\ \frac{2(1-\operatorname{Cosh}\alpha\operatorname{Cos}\alpha)}{(\operatorname{Cosh}\alpha-\operatorname{Cos}\alpha)} & -\frac{\lambda_B}{\lambda_C}\left(\operatorname{Sinh}\frac{K\alpha}{2}-\operatorname{Sin}\frac{K\alpha}{2}\right) & -\frac{2\lambda_B}{\lambda_C}\operatorname{Sinh}\frac{K\alpha}{2} \\ \frac{2\operatorname{Cosh}\alpha\operatorname{Sin}\alpha-2\operatorname{Sinh}\alpha\operatorname{Cos}\alpha}{(\operatorname{Cosh}\alpha-\operatorname{Cos}\alpha)} & \frac{I_B\lambda_B^2}{I_C\lambda_C^2}\left(\operatorname{Cosh}\frac{K\alpha}{2}-\operatorname{Cos}\frac{K\alpha}{2}\right) & \frac{2I_B\lambda_B^2}{I_C\lambda_C^2}\operatorname{Cosh}\frac{K\alpha}{2} \end{vmatrix}$$

Note: $\alpha = \lambda_C L_C = L_C\sqrt[4]{\frac{\rho A_C \omega^2}{E I_C}}$; $K\alpha = K\lambda_C L_C = \lambda_B L_B$.

A similar characteristic determinant for the asymmetric mode is generated by using the appropriate boundary and continuity conditions as follows:

$$X_m(0) = X_m'(0) = 0, \qquad Z_m(0) = Z_m\left(\frac{L_B}{2}\right) = Z_m''(0) = 0,$$

$$X_m'(L_C) = Z_m'\left(\frac{L_B}{2}\right), \qquad E I_C X_m''(L_C) + E I_C Z_m''\left(\frac{L_B}{2}\right) = 0, \tag{4}$$

$$E I_C X_m'''(L_C) = \frac{M}{2}\ddot{X}_m(L_C), \quad (M = \text{mass of beam}).$$

The last equation arises from equilibrating the horizontal inertial force of the beam due to rigid body translation.

The corresponding characteristic determinant is

$$\begin{vmatrix} (\operatorname{Sinh}\alpha+\operatorname{Sin}\alpha) & (\operatorname{Cosh}\alpha-\operatorname{Cos}\alpha) & \frac{\lambda_B}{\lambda_C}\left(\operatorname{Coth}\frac{K\alpha}{2}\operatorname{Sin}\frac{K\alpha}{2}-\operatorname{Cos}\frac{K\alpha}{2}\right) \\ (\operatorname{Cosh}\alpha+\operatorname{Cos}\alpha) & (\operatorname{Sinh}\alpha+\operatorname{Sin}\alpha) & -\frac{\lambda_B^2}{\lambda_C^2}\frac{I_B}{I_C}2\operatorname{Sin}\frac{K\alpha}{2} \\ \left\{\begin{matrix}(\operatorname{Sinh}\alpha-\operatorname{Sin}\alpha)+ \\ \frac{\alpha L_B A_B}{2 L_C A_C}(\operatorname{Cosh}\alpha-\operatorname{Cos}\alpha)\end{matrix}\right\} & \left\{\begin{matrix}(\operatorname{Cosh}\alpha+\operatorname{Cos}\alpha)+ \\ \frac{\alpha L_B A_B}{2 L_C A_C}(\operatorname{Sinh}\alpha-\operatorname{Sin}\alpha)\end{matrix}\right\} & 0 \end{vmatrix}$$

The characteristic determinants have been solved for α on a high speed digital computer by using the "Interval Halving Method". The characteristic value of α will depend on the ratios λ_B/λ_C, I_B/I_C and the value of K. These are expressed in terms of the ratios of (a) radius of gyration (R_C/R_B), (b) cross sectional areas (A_C/A_B) and (c) lengths L_C/L_B, of column to beam.

$$\frac{\lambda_B}{\lambda_C} = \sqrt{\frac{R_C}{R_B}}, \qquad \frac{I_B\lambda_B^2}{I_C\lambda_C^2} = \frac{R_B}{R_C}\frac{A_B}{A_C}, \qquad K = \sqrt{\frac{R_C}{R_B}}\frac{L_B}{L_C}.$$

Frequency charts have been prepared for several values of these parameters and are given in Figs. 3 through 8. They are constructed to enable one to determine the fundamental frequencies in the symmetric and asymmetric modes for known values of R_C/R_B, L_C/L_B, and A_C/A_B. An example will illustrate the use of these charts. Let it be required to find α in the symmetric

Sym. Mode $A_C/A_B=1$

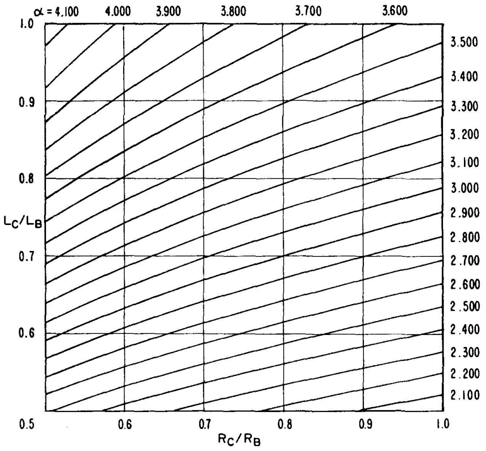

Fig. 3.

Sym. Mode $A_C/A_B=2/3$

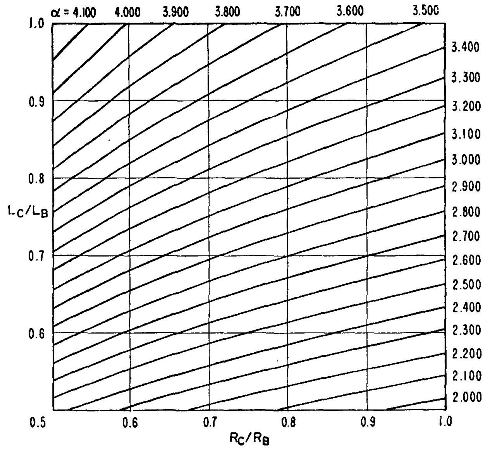

Fig. 4.

Sym. Mode $A_C/A_B=1/3$

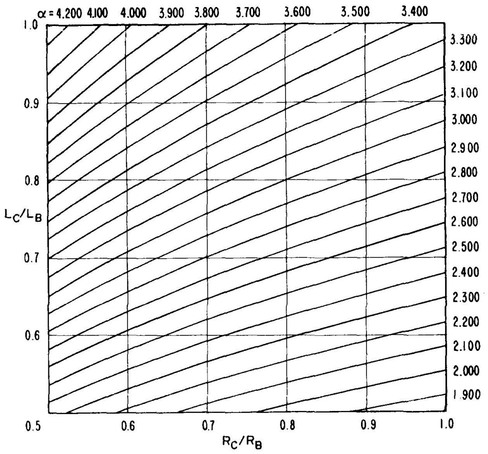

Fig. 5.

Asym. Mode $A_C/A_B=1$

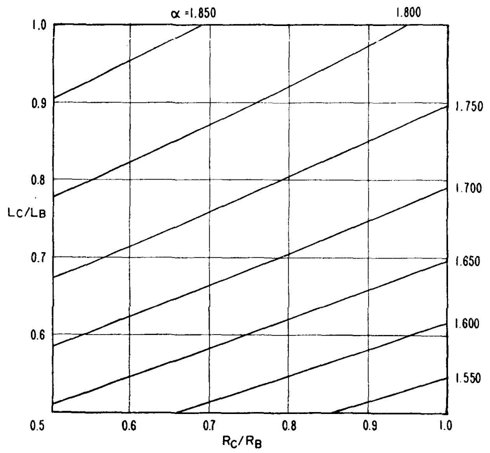

Fig. 6.

Asym. Mode $A_C/A_B=2/3$

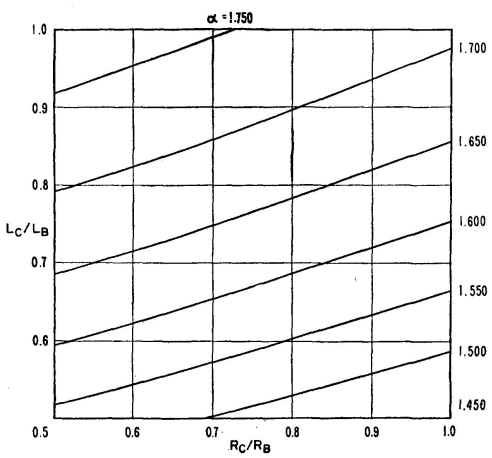

Fig. 7.

Asym. Mode $A_C/A_B=1/3$

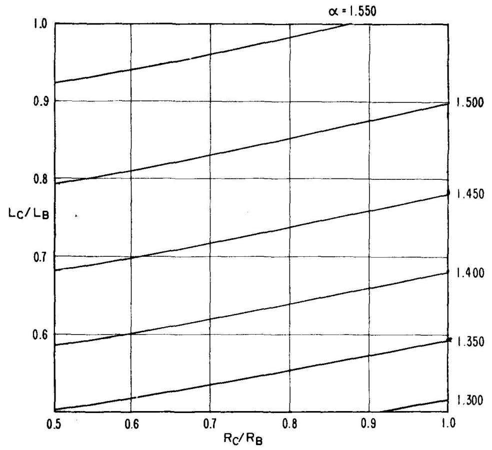

Fig. 8.

mode for the set $A_C/A_B = 0.75$, $R_C/R_B = 0.725$, $L_C/L_B = 0.80$. From Fig. 4, the following values are obtained for $A_C/A_B = 2/3$

α	L_C/L_B	R_C/R_B
3.2	0.7900	0.725
3.3	0.8225	0.725

Interpolating between these values, a value of $\alpha = 3.231$ is obtained for $L_C/L_B = 0.8$. Similarly from Fig. 3 the following values are obtained

α	L_C/L_B	R_C/R_B
3.3	0.79625	0.725
3.4	0.83000	0.725

Interpolating between these values, a value of $\alpha = 3.311$ is obtained for $L_C/L_B = 0.8$.

A subsequent interpolation on A_C/A_B gives the required value of α

A_C/A_B	L_C/L_B	R_C/R_B	α
0.666	0.8	0.725	3.231
1.000	0.8	0.725	3.311
0.750	0.8	0.725	3.251

Shape Functions

The shape functions for the column and beam are obtained to within one arbitrary constant from the boundary conditions given in Eqs. [3], [4]. For the symmetric mode they are given by Eqs. [6] and for the asymmetric mode by Eqs. (5). These are represented graphically in Fig. 9.

$$\begin{aligned} X_m(x) &= \mu_1^m\{\beta_9(\operatorname{Cosh}\lambda_m^C x - \operatorname{Cos}\lambda_m^C x) - (\operatorname{Sinh}\lambda_m^C x - \operatorname{Sin}\lambda_m^C x)\}, \\ Z_m(x) &= (\operatorname{Sinh}\lambda_m^B x - \operatorname{Sin}\lambda_m^B x) - \frac{\beta_{61}^m}{\beta_{51}^m}(\operatorname{Sinh}\lambda_m^B x + \operatorname{Sin}\lambda_m^B x), \end{aligned} \tag{5}$$

$$\begin{aligned} X_m(x) &= \mu_2^m(\operatorname{Cosh}\lambda_m^C x - \operatorname{Cos}\lambda_m^C x) - \frac{\beta_4^m}{\beta_2^m}(\operatorname{Sinh}\lambda_m^C x - \operatorname{Sin}\lambda_m^C x), \\ Z_m(x) &= (\operatorname{Cosh}\lambda_m^B x + \operatorname{Cos}\lambda_m^B x) - \frac{\beta_7^m}{\beta_8^m}(\operatorname{Cosh}\lambda_m^B x - \operatorname{Cos}\lambda_m^B x). \end{aligned} \tag{6}$$

The subscript m indicates the mode and the constants are given below. α_m is a symmetric mode eigen value and α_m', the asymmetric mode eigen value.

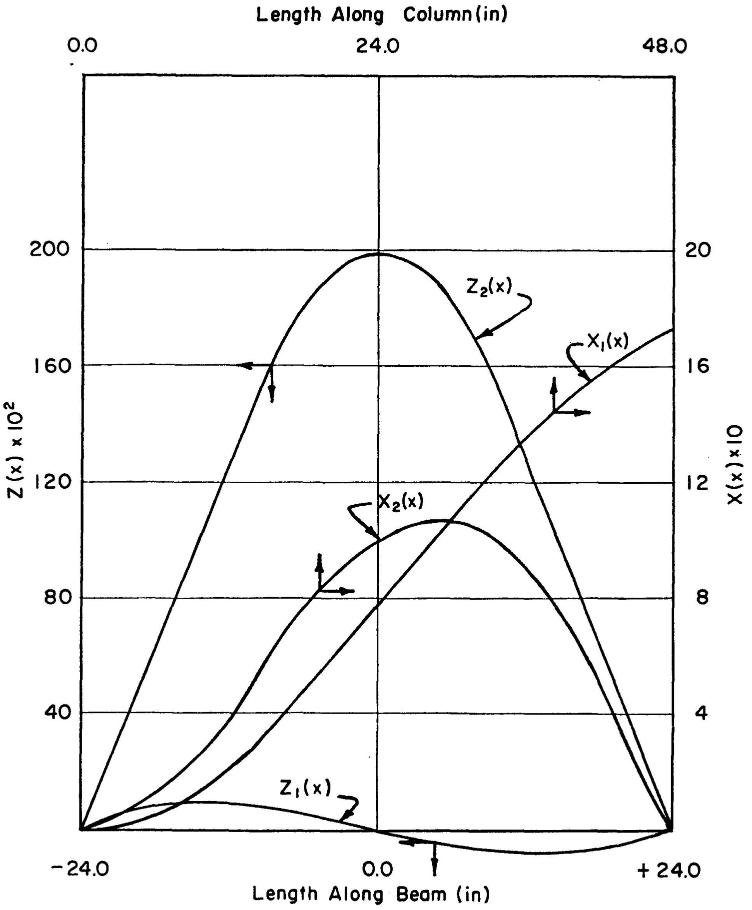

Fig. 9.

$$\beta_1^m = \operatorname{Sinh} \alpha_m + \operatorname{Sin} \alpha_m, \qquad \beta_{11}^m = \operatorname{Sinh} \alpha_m' + \operatorname{Sin} \alpha_m',$$

$$\beta_2^m = \operatorname{Sinh} \alpha_m - \operatorname{Sin} \alpha_m, \qquad \beta_{21}^m = \operatorname{Sinh} \alpha_m' - \operatorname{Sin} \alpha_m',$$

$$\beta_3^m = \operatorname{Cosh} \alpha_m + \operatorname{Cos} \alpha_m, \qquad \beta_{31}^m = \operatorname{Cosh} \alpha_m' + \operatorname{Cos} \alpha_m',$$

$$\beta_4^m = \operatorname{Cosh} \alpha_m - \operatorname{Cos} \alpha_m, \qquad \beta_{41}^m = \operatorname{Cosh} \alpha_m' - \operatorname{Cos} \alpha_m',$$

$$\beta_5^m = \operatorname{Sinh} \frac{K\alpha_m}{2} + \operatorname{Sin} \frac{K\alpha_m}{2}, \qquad \beta_{51}^m = \operatorname{Sinh} \frac{K\alpha_m'}{2} + \operatorname{Sin} \frac{K\alpha_m'}{2},$$

$$\beta_6^m = \operatorname{Sinh} \frac{K\alpha_m}{2} - \operatorname{Sin} \frac{K\alpha_m}{2}, \qquad \beta_{61}^m = \operatorname{Sinh} \frac{K\alpha_m'}{2} - \operatorname{Sin} \frac{K\alpha_m'}{2},$$

$$\beta_7^m = \operatorname{Cosh} \frac{K\alpha_m}{2} + \operatorname{Cos} \frac{K\alpha_m}{2}, \qquad \beta_{71}^m = \operatorname{Cosh} \frac{K\alpha_m'}{2} + \operatorname{Cos} \frac{K\alpha_m'}{2},$$

$$\beta_8^m = \operatorname{Cosh} \frac{K\alpha_m}{2} - \operatorname{Cos} \frac{K\alpha_m}{2}, \qquad \beta_{81}^m = \operatorname{Cosh} \frac{K\alpha_m'}{2} - \operatorname{Cos} \frac{K\alpha_m'}{2},$$

$$\beta_9^m = \frac{2\dfrac{L_C}{L_B}\dfrac{A_C}{A_B}\beta_{31}^m + \alpha'\beta_{21}}{2\dfrac{L_C}{L_B}\dfrac{A_C}{A_B}\beta_{21}^m + \alpha'\beta_{41}},$$

$$\mu_1^m = \frac{R_B}{R_C}\,\frac{\beta_{51}^{m^2} - \beta_{61}^{m^2}}{\beta_{51}^m(\beta_{11}^m - \beta_{31}^m\beta_9^m)}, \qquad \mu_2^m = K\frac{L_C}{L_B}\,\frac{\beta_2^m}{\beta_8^m}\left\{\frac{\beta_6^m\beta_8^m - \beta_5^m\beta_7^m}{\beta_1^m\beta_2^m - \beta_4^{m^2}}\right\}.$$

Determination of Response for the Moving Load

Use is made of the Hamilton's principle of least action to find the equations of behavior under a moving load. From these expressions the kinetic and potential energies in the system when a load rolls over the frame are constructed. The displacement of each member of the frame is defined by an infinite series of products of shape functions and generalized time functions as shown below.

$$y_B = \sum_1^\infty Z_m(x)\, q_m(t), \qquad y_C = \sum_1^\infty X_m(x)\, q_m(t).$$

The kinetic and potential energies of the frame written in terms of these desplacements have been derived to be as follows. (m_1 = mass of moving load.)

$$T = \tfrac{1}{2} m_1 \dot{\xi}^2 + \tfrac{1}{2} m_1 \{\sum_1^\infty Z_m(\xi)\, \dot{q}_m(t)\}^2 + \frac{\rho A_B}{2} \int_{L_B/2}^{+L_B/2} \{\sum_1^\infty Z_m(x)\, \dot{q}_m(t)\}^2 d x$$
$$+ \rho A_C \int_0^{L_C} \{\sum_1^\infty X_m(x)\, \dot{q}_m(t)\}^2 d x + M \{\sum_1^\infty X_m(L_C)\, \dot{q}_m(t)\}^2,$$

where the first two terms are the kinetic energy of the moving mass, the two integral terms represent the total kinetic energy of the frame and the last term represents the translational kinetic energy of the beam, when the frame vibrates in the asymmetric mode. This last term is zero for the symmetric modes. Since $q_m(t)$ are functions of time only, we may write the above expression for T as

$$T = \tfrac{1}{2} m_1 \{\dot{\xi}^2 + \sum_m \sum_n Z_m(\xi)\, Z_n(\xi)\, \dot{q}_m(t)\, \dot{q}_n(t)\}$$
$$+ \tfrac{1}{2} \rho A_B \sum_m \sum_n \dot{q}_m(t)\, \dot{q}_n(t) \int_{-L_B/2}^{L_B/2} Z_m(x)\, Z_n(x)\, d x$$
$$+ \rho A_C \sum_m \sum_n \dot{q}_m(t)\, \dot{q}_n(t) \int_0^{L_C} X_m(x)\, X_n(x)\, d x$$
$$+ \frac{M}{2} \sum_m \sum_n X_m(L_C)\, X_n(L_C)\, \dot{q}_m(t)\, \dot{q}_n(t).$$

The potential energy in the frame may be expressed as

$$V = E I_C \int_0^{L_C} \{\sum_m X''_m(x)\, q_m(t)\}^2 d x + E I_B \int_{-L_B/2}^{+L_B/2} \{\sum_m Z''_m(x)\, q_m(t)\}^2$$
$$+ m_1 g \sum_m Z_m(\xi)\, q_m(t) - \int_0^{L_B/2+\xi} F(\eta)\, d\eta.$$

The first two terms refer to the total potential energy stored in the columns and the beam respectively, the third term refers to the loss in potential energy of the mass and is positive because Z_m is measured positive upward. The last term refers to the work done by the driving force in moving through a distance

$L_B/2+\xi$, which is equal to the negative of the potential energy given up by the driving force.

$$V = E I_C \sum_m \sum_n q_m(t)\, q_n(t) \int_0^{L_C} X''_m(x)\, X''_n(x)\, dx$$
$$+ E I_B \sum_m \sum_n q_m(t)\, g_n(t) \int_{-L_B/2}^{L_B/2} Z''_m(x)\, Z''_n(x)\, dx$$
$$+ m_1 g \sum_m Z_m(\xi)\, q_m(t) - \int_0^{L_B/2+\xi} F(\eta)\, d\eta .$$

Evaluation of the terms in the kinetic and potential energy requires the computation of the following integrals.

$$\int_0^{L_C} X_m(x)\, X_n(x)\, dx ; \qquad \int_{-L_B/2}^{L_B/2} Z_m(x)\, Z_n(x)\, dx ;$$
$$\int_0^{L_C} X''_m(x)\, X''_n(x)\, dx ; \qquad \int_{-L_B/2}^{L_B/2} Z''_m(x)\, Z''_n(x)\, dx .$$

For purpose of brevity these integrals are designated by the constants R_m, J_m, H_m, K_m respectively. Each of these has been found for the steel frame solved in this paper. The integrals involving the shape functions of the frame obey the familiar properties of orthogonality. As such they are zero for all values of $m \neq n$, when the entire frame is considered.

T and V may now be written in terms of these constants as follows:

$$T = \tfrac{1}{2} m_1 \{\dot{\xi}^2 + \sum \sum Z_m(\xi)\, Z_n(\xi)\, \dot{q}_m(t)\, \dot{q}_n(t)\}$$
$$+ \rho A_B \sum_m J_m \dot{q}_m^2(t) + \rho A_C \sum R_m \dot{q}_m^2(t) + \frac{M}{2} \sum_m \sum_n X_m(L_C)\, X_n(L_C)\, \dot{q}_m(t)\, \dot{q}_n(t),$$
$$V = E I_C \sum_m H_m q_m^2(t) + E I_B \sum K_m q_m^2(t) + m_1 g \sum Z_m(\xi)\, q_m(t) - \int_0^{L_B/2+\xi} F(\eta)\, d\eta .$$

It may be noted that the response to a moving force may be determined if m_1 is set to zero in T.

By appropriate algebra values for J_m, R_m, H_m, and K_m can be found in terms of the boundary values alone [10]. The constants were evaluated on the IBM Computer using the values of X_m and Z_m at 0, L_C and 0, $L_B/2$ respectively.

Governing Equations of Motion

The principle of least action requires that the $\int_{t_1}^{t_2} L\, dt$ is a minimum, where $L = T - V$. The Euler equations which L has to satisfy in order that the above integral should be a minimum is:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_m} - \frac{\partial L}{\partial q_m} = 0 .$$

This yields the following differential equation in the generalized time functions.

$$m_1\{Z_m(\xi)[\sum_n \dot{Z}_n(\xi)\dot{q}_n(t)+\sum_n Z_n(\xi)\ddot{q}_n(t)]+\dot{Z}_m(\xi)\sum_n Z_n(\xi)\dot{q}_n(t)\}$$
$$+2\rho A_B\sum_n J_{mn}\ddot{q}_n(t)+2\rho A_C\sum_n R_{mn}\ddot{q}_n(t)+M\sum_n X_m(L_C)X_n(L_C)\ddot{q}_m(t)$$
$$+2EI_C\sum_n H_{mn}q_m(t)+2EI_B\sum_n K_{mn}q_m(t)+m_1 g Z_m(\xi)=0.$$

Considering the first two modes only the resulting differential equations are as follows:

$$a_1\ddot{q}_1+a_2\ddot{q}_2+a_3\{2\dot{q}_1\dot{Z}_1(\xi)Z_1(\xi)+\dot{q}_2[\dot{Z}_1(\xi)Z_2(\xi)+Z_1(\xi)\dot{Z}_2(\xi)]+gZ_1(\xi)\}+a_4q_1=0,$$
$$a_5\ddot{q}_2+a_2\ddot{q}_1+a_3\{2\dot{q}_2\dot{Z}_2(\xi)Z_2(\xi)+\dot{q}_1[\dot{Z}_2(\xi)Z_1(\xi)+Z_2(\xi)\dot{Z}_1(\xi)]+gZ_2(\xi)\}+a_6q_2=0,$$

Where

$$a_1=a_3Z_1^2(\xi)+2\frac{J_1}{L_B}+2\left\{\frac{R_1A_C}{L_BA_B}\right\}+X_1^2(L_C),$$
$$a_2=a_3Z_1(\xi)Z_2(\xi),$$
$$a_3=\frac{m_1}{M},$$
$$a_4=\frac{2\omega_1^2}{K\alpha_1^4}\left\{K\frac{L_C^4}{L_B}\frac{A_C}{A_B}H_1+\frac{L_B^3}{K^3}K_1\right\},$$
$$a_5=a_3Z_2^2(\xi)+2\frac{J_2}{L_B}+2\frac{R_2A_C}{L_BA_B},$$
$$a_6=\frac{2\omega_2^2}{K\alpha_2^4}\left\{K\frac{L_C^4A_CH_2}{L_BA_B}+\frac{L_B^3K_2}{K^3}\right\}.$$

The solution of these simultaneous differential equations was obtained by using the modified Euler's numerical procedure.

Conclusions

In this paper the response of a symmetric portal frame in the first two fundamental modes is found for a load which moves with constant velocity over the top member of the frame. The moving load is considered with and without inertia and a comparison of responses has been made between the two cases. The first fundamental mode for the frame is an asymmetric mode and the second a symmetric mode. Results have been presented to enable one to find the behavior of a wide class of frames under a moving force or a moving mass. The curves in Figs. 3 to 8 inclusive give the frequency parameter α for the first two modes. When the frequency parameter for a frame is known the critical velocity of the load can be found. The critical velocity is defined as the velocity which would cause the load to move across the frame in a time interval equal to the natural period. Clearly a different critical velocity will exist for each modal shape.

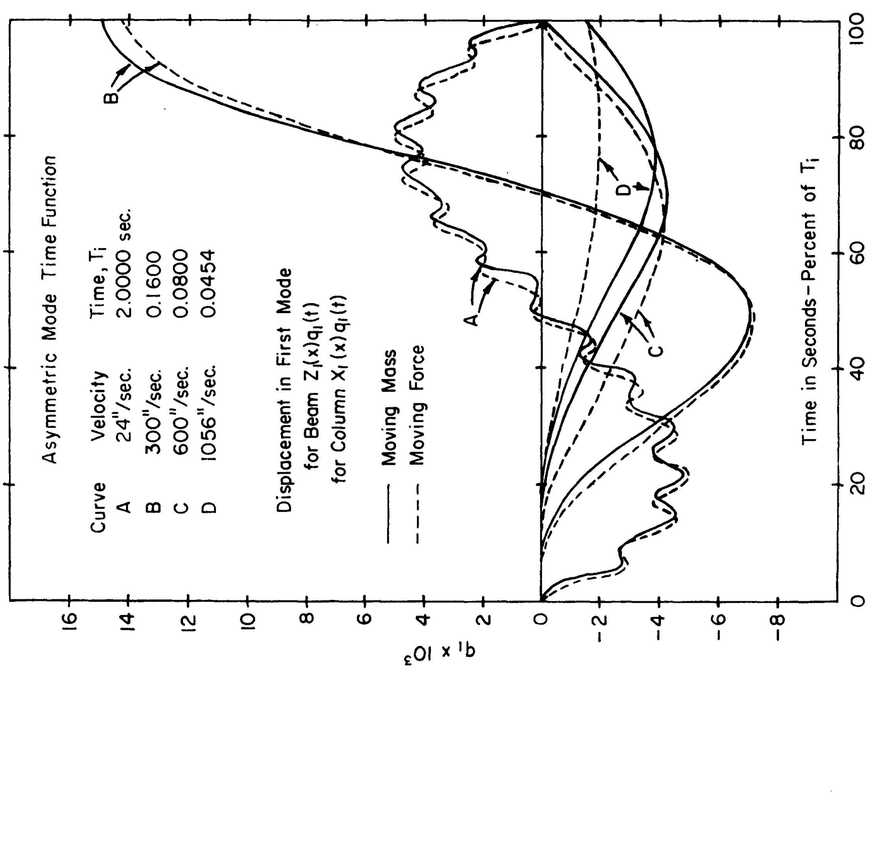
Asymmetric Mode Time Function
Curve Velocity Time, Ti
A 24"/sec. 2.0000 sec.
B 300"/sec. 0.1600
C 600"/sec. 0.0800
D 1056"/sec. 0.0454
Displacement in First Mode
for Beam Z1(x)q1(t)
for Column X1(x)q1(t)
Moving Mass
Moving Force
Time in Seconds - Percent of Ti
q1 x 10^3
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
0
20
40
60
80
100
A
B
C
D

Fig. 10.

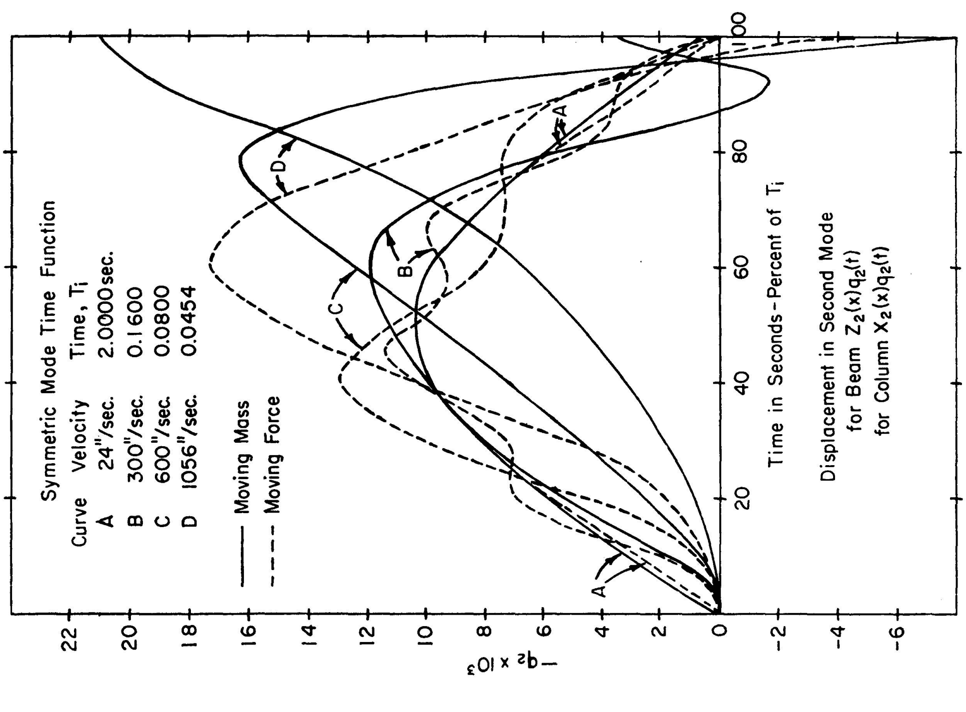
Symmetric Mode Time Function
Curve Velocity Time, Ti
A 24"/sec. 2.0000 sec.
B 300"/sec. 0.1600
C 600"/sec. 0.0800
D 1056"/sec. 0.0454
Moving Mass
Moving Force
Time in Seconds - Percent of Ti
Displacement in Second Mode
for Beam Z2(x)q2(t)
for Column X2(x)q2(t)
-q2 x 10^3
22
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
20
40
60
80
100
A
B
C
D

Fig. 11.

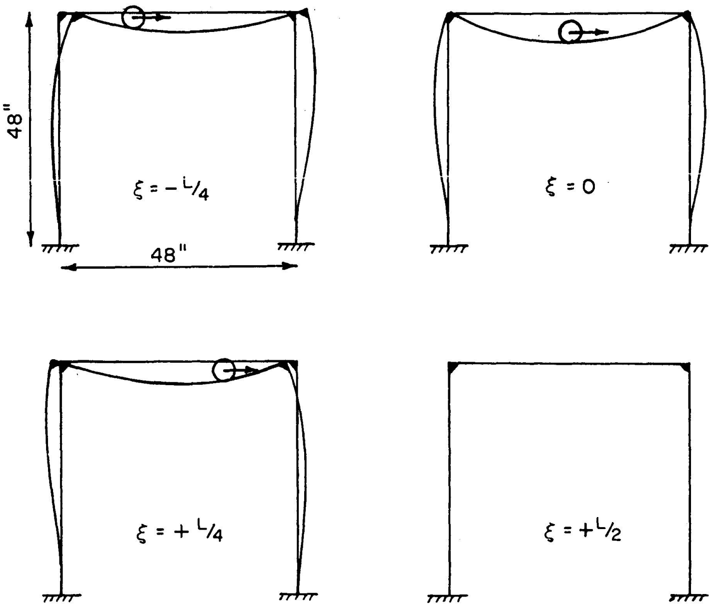

Fig. 12.

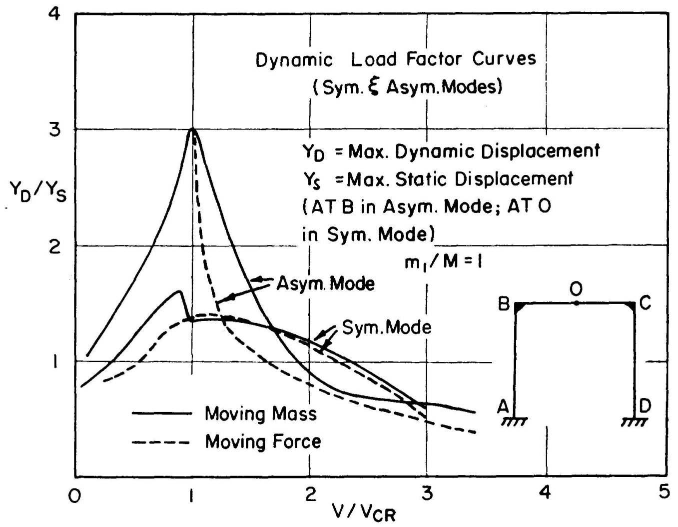

Fig. 13.

Fig. 13 gives a dynamic load factor that can be used to compute maximum displacements of the frame under the moving load. These curves are entered with a ratio of the actual velocity of the load to the critical velocity. The dynamic load factor is given as a ratio of maximum dynamic displacement to maximum static displacement. Dynamic bending moments in the frame may be computed by multiplying the static bending moment by the load factors in Fig. 13. However, it is noted that the response given in these curves is for the first two modes only and is therefore only a first approximation. Predicted values of dynamic displacement at the center of the span are known to be low. This is shown by the load factor curve in the range below $V/V_{cr} < 0.3$ where the value of the dynamic displacement is less than the corresponding static displacement. Preliminary computations show that this deficiency is almost completely corrected by including the next symmetric mode which has a frequency not far removed from the first symmetric mode. Work is underway to extend the computations to include the higher modes.

For the purpose of comparing the response for a force and a mass, numerical results for a steel frame $4' \times 4'$ are presented. The cross sectional dimension of each member is $1'' \times \frac{1}{2}''$ and the frame is loaded by a dimensionless unit load. The time functions q_1 and q_2 are shown in Figs. 10 and 11 for representative velocities. In each figure the time functions are plotted for both the moving force and the moving mass to facilitate comparison of the response. In the asymmetric mode the displacements do not differ significantly for the two cases of loading until the velocity of the load exceeds the critical velocity. Appreciable differences do occur for high velocities as can be seen by a comparison of curves C in Fig. 10. In the symmetric mode pronounced differences occur between the two cases even for relatively low velocities. This is clearly demonstrated in Fig. 11 which shows the response in the second mode.

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Summary

In this paper the dynamic response of a class of Portal frames to each a force and a mass moving with a constant velocity across the top member is found. Natural frequencies and the corresponding natural modal shapes of the frame are found for the two lowest modes by solving the free vibration problem of the frame. The forced behaviour of the frame is found by assuming that the response can be suitably approximated by the first two modal shapes for the frame. Equations for generalized time functions are generated by using Hamilton's principle of least action. The time functions are computed by using a suitable numerical integration procedure.

Results have been obtained for a large class of portal frames that can be described by appropriate ratios relating the geometry of the upright members to those of the transverse member.

Résumé

Les auteurs étudient le comportement dynamique de portiques simples soumis à une force ou à une masse se déplaçant à une vitesse uniforme sur la traverse. Ils établissent les équations des vibrations naturelles et les fréquences correspondantes pour les deux modes fondamentaux, en résolvant le problème de la vibration libre du portique. Pour les vibrations forcées, on suppose que le comportement du portique est déterminé avec une approximation suffisante au moyen des deux premières formes de vibration. Les équations pour les fonctions généralisées du temps sont fondées sur le principe de moindre action de Hamilton; ces fonctions sont calculées par une méthode appropriée d'intégration numérique.

On a obtenu des résultats pour une vaste catégorie de portiques pouvant être définie par des rapports appropriés entre les dimensions des montants et celles de la traverse.

Zusammenfassung

Im vorliegenden Aufsatz wird eine Reihe von Portalrahmen hinsichtlich der dynamischen Einwirkung von je einer Kraft und einer Masse untersucht, die sich mit konstanter Geschwindigkeit entlang des obersten Rahmengliedes bewegt. Aus den Gleichungen für die freie Schwingung des Rahmens werden die Eigenfrequenzen und die entsprechenden Schwingungsformen des Rahmens für die ersten zwei Grundschwingungen abgeleitet. Zur Lösung der erzwungenen Schwingung des Rahmens wird angenommen, daß die Schwingungsform durch die ersten zwei Grundschwingungen angenähert werden kann. Gleichungen für Zeitkoordinaten werden mit Hilfe des Hamiltonschen Prinzips der kleinsten Wirkung entwickelt. Zur Berechnung der Zeitkoordinaten wird ein numerisches Integrationsverfahren verwendet.

Ergebnisse werden für eine große Anzahl von Portalrahmen erzielt, die durch geeignete Formparameter beschrieben werden.