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Autor: Fujio, Takeaki / Ohmura, Hiroshi / Naruoka, Masao

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On the Rapid and Rational Calculation of Bending Moment of Skew Girder Bridges

Méthode rationnelle pour le calcul des moments fléchissants dans les ponts biais

Eine rationelle Methode zur Berechnung der Biegemomente bei schiefen Trägerbrücken

TAKEAKI FUJIO
Bridge Engineer, Sakurada Machine Industry Company Ltd., Tokyo, Japan

HIROSHI OHMURA
Professor of Civil Engineering, Kobe University, Kobe, Japan

MASAO NARUOKA
Professor of Civil Engineering, Nagoya University, Nagoya, Japan

Introduction

It has been well known that the research works by F. LEONHARDT, Y. GUYON-CH. MASSONNET and H. HOMBERG are very useful for the design of right grillage-right girder bridges. On the contrary, there are few works on the analysis of skew girder bridges. A work by T. Y. CHEN et al. is based on the theory of an isotropic continuous slab supported by flexible girders.

Y. GUYON and CH. MASSONNET assumed the right grillage-right girder bridge to be an orthogonal anisotropic (orthotropic) rectangular plate. From the same reasoning, the right (skew) grillage-skew girder bridge can be assumed to be an orthogonal (skew) anitotropic-parallelogram plate. The numerical analysis of orthotropic parallelogram plate was studied by H. OHMURA and M. NARUOKA, and the result was applied to the design of the Junshin Bridge. The load tests have proved that the method is useful for the analysis of right grillage-skew girder bridges. Next, the numerical analysis of the skew anisotropic-parallelogram plate was studied by O. FUKUCHI and M. NARUOKA, and the method has been proved to be a powerful tool to analize the skew grillage-skew girder bridges by the model test and the load tests at the Torisu Bridge.

In the design of skew girder bridges, it is first necessary to estimate the bending moment of the girder by a simple, but rational formula, and then to design the cross section of each girder. That is, we must know the value of factor k of the following formula:

$$M_p = k M_b,$$

where, M_p = the bending moment in the longitudinal direction which is calculated by the theory of plates;

M_b = the bending moment of a single girder per unit width which is calculated by the elementary beam theory.

This paper describes how to obtain the value of factor k and how to use the factor in the design of skew girder bridges.

Part I

1. Numerical Analysis of Orthotropic Parallelogram Plate

The outline of the numerical analysis of orthotropic parallelogram plates was published in references [1] and [2]. The plate is covered by a skew network and the finite difference equations are given for nine different kinds of network points, that is, a) general interior points, b), c) interior points near the left (right) simple support, d) interior points near the edge girders, e), f) interior points near the sharp (blunt) corners, g) general edge points, h), i) edge points on the sharp (blunt) corners. The element of the stiffness matrix is a function of the following parameters:

$$A = \frac{K^2}{\alpha}, \qquad B = K \tan\varphi, \qquad J = \frac{E\,I_r}{\lambda_y B_x}.$$
$$\kappa = \frac{H}{(B_x B_y)^{1/2}}, \qquad \left(\alpha = \left(\frac{B_y}{B_x}\right)^{1/2}, \quad K = \frac{\lambda_y}{\lambda_x}\right). \tag{1}$$

The following procedures have been programmed for Nagoya University Digital Computer NEAC 2203:

a) Calculating 29 coefficients involved in the element of stiffness matrix.
b) Completing the elements of stiffness matrix.
c) Calculating matrix inversion, that is, flexibility matrix — influence coefficients of deflection.
d) Calculating the influence coefficients of the bending moment in the longitudinal direction.

For the various combinations of $A = 1, 4, 9, 16$; $B = 0.5, 1.0, 1.5, 2.0$; $\kappa = 0$, $1/3, 1/2, 2/3, 1.0$ under the assumption of $J = 0$, the influence coefficients of the bending moment in the longitudinal direction at each network point have been obtained for the skew network shown in Fig. 1.

2. Determination of Factor k

Using the influence coefficients of bending moment in the longitudinal direction at the panel points 3, 8, 13 and 18, the plate bending moment (M_p)

has been calculated when the plate is subjected to the uniform and line loads specified in the Japan Standard Specification for Steel Highway Bridges. Thus, the plate bending moment in the longitudinal direction is obtained. On the other hand, the bending moment at midspan of a single girder per unit of girder spacing (M_b) can be calculated by the elementary beam theory. These two bending moments can be combined by the following equation:

$$M_p = k\,M_b. \tag{2}$$

As the result of complicated calculations, the following formula has been given for the factor k:

$$k = -m\sqrt[4]{\frac{B_y}{B_x}}\tan\varphi + n. \tag{3}$$

The coefficients m and n are given as functions of κ in Fig. 2 for uniform load and in Fig. 3 for line load.

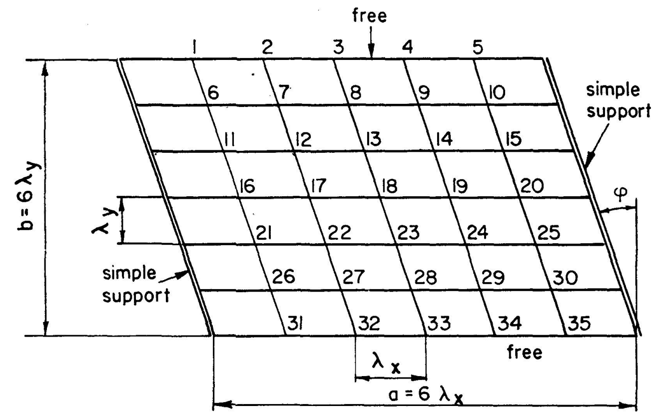

Fig. 1. Skew network for analyzing orthotropic parallelogram plate.

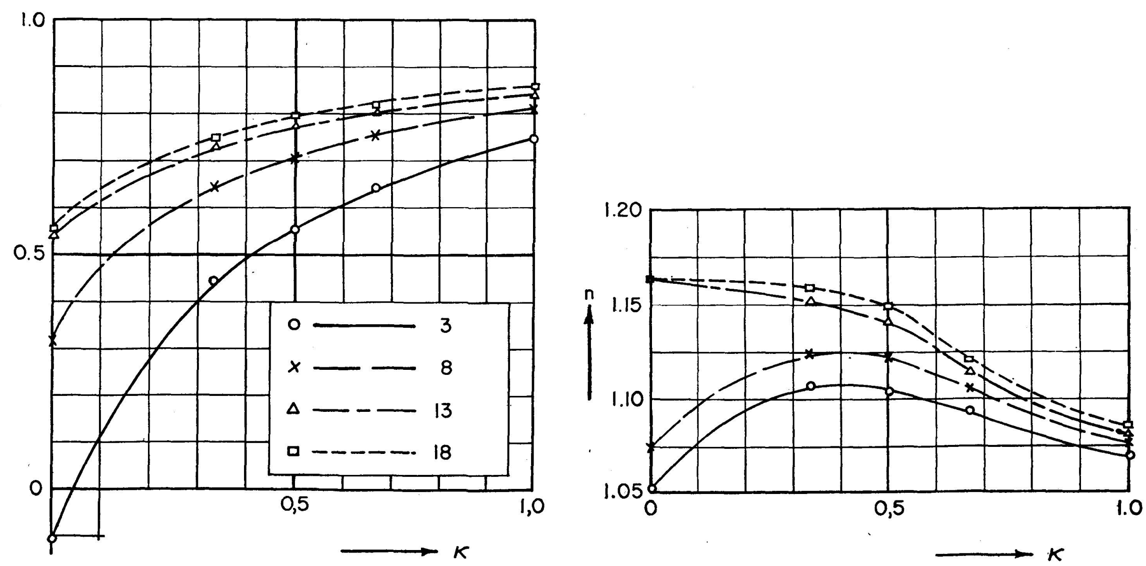

Fig. 2. Coefficients m, n — Torsional parameter κ curves for uniform load.

3. Numerical Example No. 1

Let us consider a practical application of formulae (2) and (3). The Junshin Bridge is a right grillage-skew girder bridge designed by the theory of orthotropic parallelogram plates, and its details are described in Ref. [2]. The bridge is assumed to be an orthotropic parallelogram plate and is covered by a 8×6 skew network. See Fig. 4. The example of application of formulae (2) and (3) will be shown in Table 1.

Using the influence coefficients of longitudinal bending moment at network

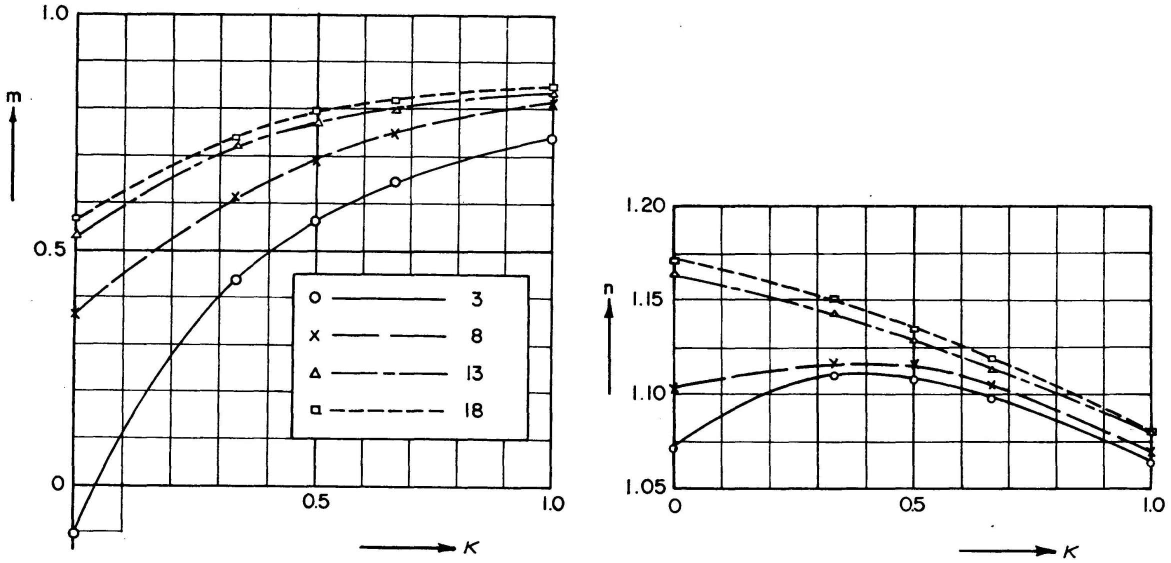

Fig. 3. Coefficients m, n — Torsional parameter κ curves for line load.

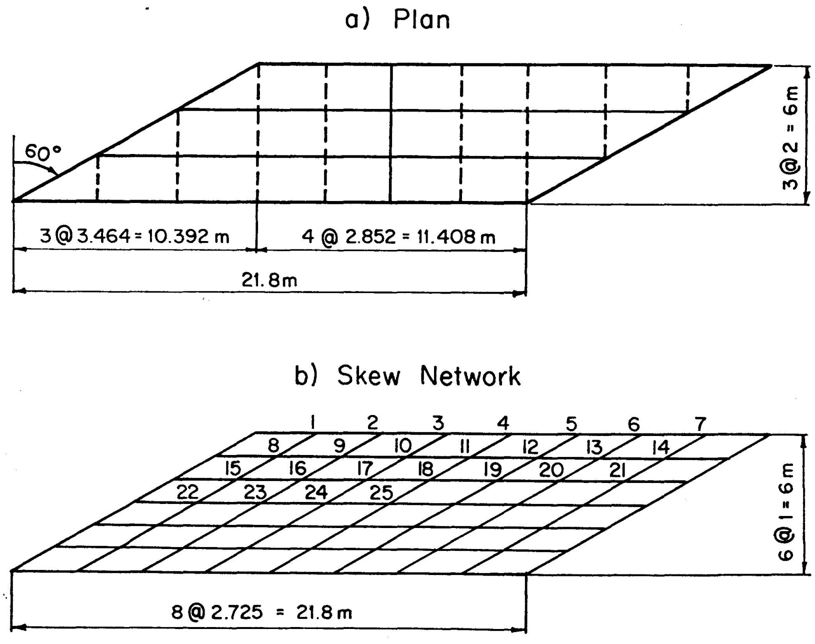

Fig. 4. The Junshin Bridge.

point 18 for the case of torsional parameter $\kappa = 0$ and 0.26, the bending moment at midspan of the inner girder is calculated by $M_p \times 2.0$ as shown in line (3) of Table 1.

Table 1. Calculated results on right grillage-skew girder bridge

The details of the Junshin Bridge: span = 21.8 m, effective width = 7.5 m, skew angle = 60 degrees ($\tan \varphi = 1.732$), number of main girder = 4, girder spacing = 2.0 m, stiffness parameter $\alpha = \sqrt{0.116} = 0.341$

1	load		uniform	line	uniform	line
2	torsional parameter		0	0	0.26	0.26
3	longitudinal bending moment at midspan = $M_p \times$ girder spacing		44.5	58.7	30.1	38.5
4	calculated	m	0.533	0.544	0.680	0.700
5		n	1.164	1.164	1.145	1.156
6		$k = -m \sqrt[4]{B_y/B_x} \tan \varphi + n$	0.849	0.843	0.743	0.743
7		$M_b \times$ girder spacing (tm)	53.2	69.7	41.6	54.5
8		(7) $\times k$ (tm)	45.2	58.8	30.9	40.5

The coefficients m and n for $\kappa = 0$ and 0.26 are obtained for uniform and line loads, as shown in lines (4) and (5), respectively. The values of factor κ are calculated by formula (3) as shown in line (6) of Table 1. The bending moment at midspan of a single girder is given in line (7) of Table 1. Good agreements are obtained from the comparison between lines (3) and (8) in Table 1.

Part II

1. *Numerical Analysis of Skew Anisotropic Parallelogram Plate*

The numerical analysis of skew anisotropic parallelogram plate was published in Ref. [3] and [4]. The plate is covered by a skew network (see Fig. 5), and nine different kinds of the finite difference equations are given for nine kinds of network points. The elements of stiffness matrix are functions of six parameters, as follows:

$$\alpha = \sqrt{\frac{B_y}{B_x}}, \qquad \kappa = \frac{2\,K_{xy}}{\sqrt{B_x B_y}}, \qquad J = \frac{E\,I_r}{\lambda_y\,B_x}, \qquad (4)$$
$$K = \frac{\lambda_y}{\lambda_x}, \qquad s = \sin\varphi, \qquad c = \cos\varphi.$$

The analytical procedures from a) to d) mentioned in Chapter 1 of Part I have been programmed for NEAC 2203 for skew anisotropic parallelogram plates.

After the calculation of various combinations of the parameters, the influence coefficients of the bending moment in the longitudinal direction have been obtained for each network point.

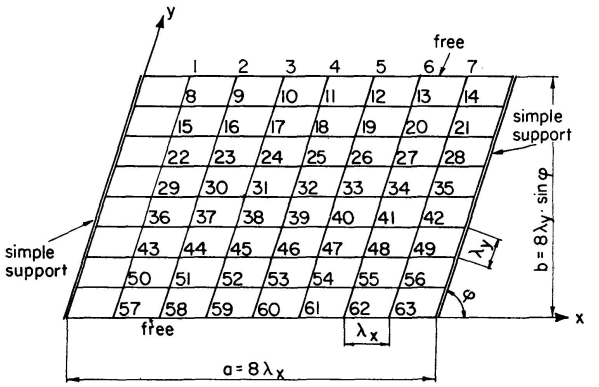

Fig. 5. Skew network for analyzing skew anisotropic parallelogram plate.

2. *Determination of Factor k*

Using the influence coefficients of the longitudinal bending moment at the panel points 11 and 32, the plate bending moment has been calculated under the uniform and line loads specified in the Japan Standard Specification for Steel Highway Bridges. On the other hand, the bending moment at midspan of a single girder per unit of girder spacing under the same loading conditions is calculated by the elementary beam theory. These two bending moments result in the following formula:

$$M_p = k\,M_b. \tag{5}$$

As the result of a complicated calculation, the following formula is obtained:

$$k = -A\beta + B, \qquad \beta = \left(\frac{b}{a}\right)\left(\frac{B_y}{B_x}\right)^{1/4}, \tag{6}$$

where, coefficient A and B can be obtained from Figs. 6 and 7 for arbitrary skew angle θ and for the specified values of $\kappa = 0$, 0.5 and 1.0.

When it is necessary to get A and B for the special value of κ other than 0, 0.5 and 1.0, the interpolation formula based on these three specified values is used.

3. *Numerical Example No. 2*

Let us consider the practical application of formulae (5) and (6) to the skew grillage-skew girder bridges. The Ono Bridge and the Hanazono Bridge shown in Figs. 8 and 9 are now under designing. The dimensions of these two bridges are shown in lines (1) to (5) of Table 2. The skew anisotropic-parallelogram plates corresponding to these two skew girder bridges are covered by 10×7 and 8×11 skew networks, respectively. The influence coefficients of the longi-

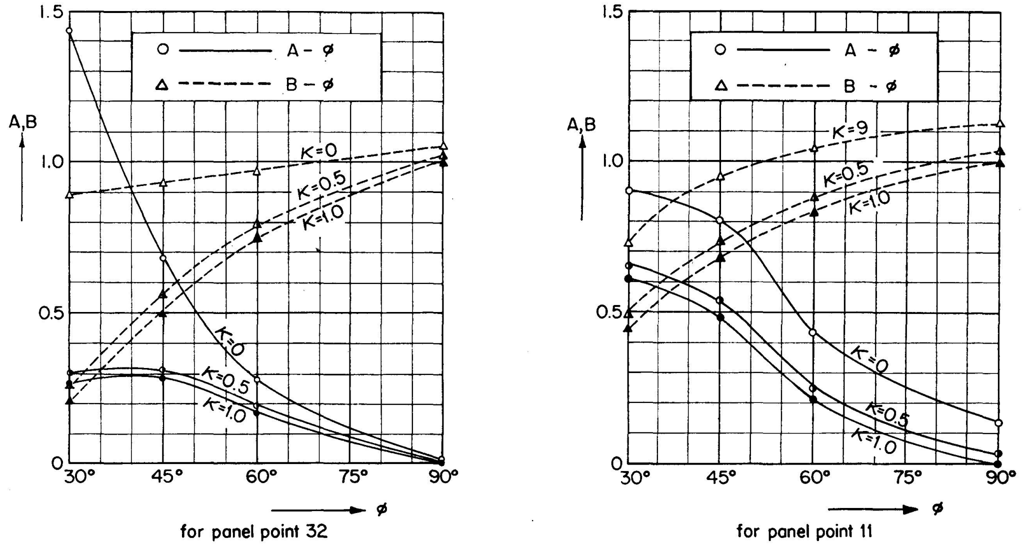

Fig. 6. Coefficients A, B — Skew angle φ curve for uniform load.

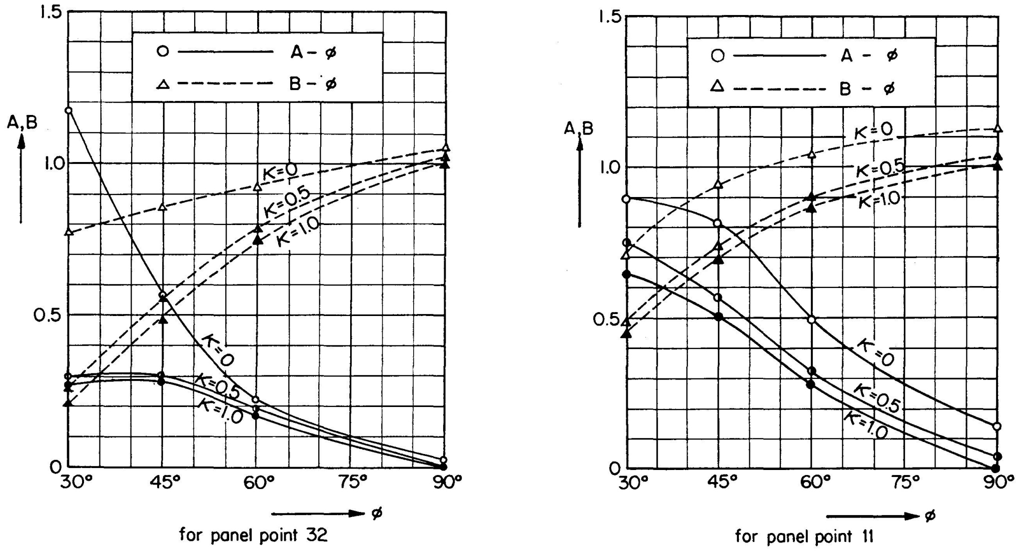

Fig. 7. Coefficients A, B — Skew angle φ curve for line load.

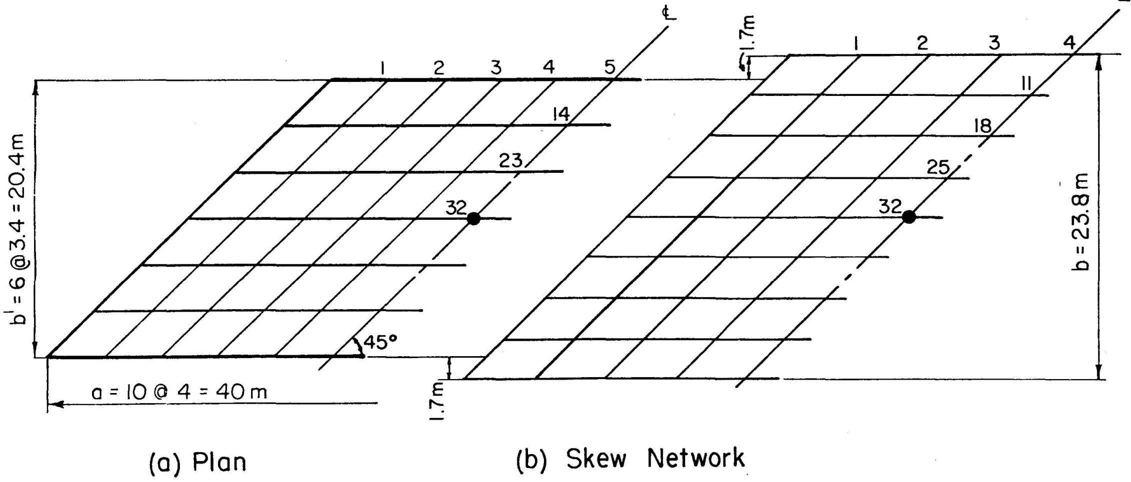

Fig. 8. The Ono Bridge.

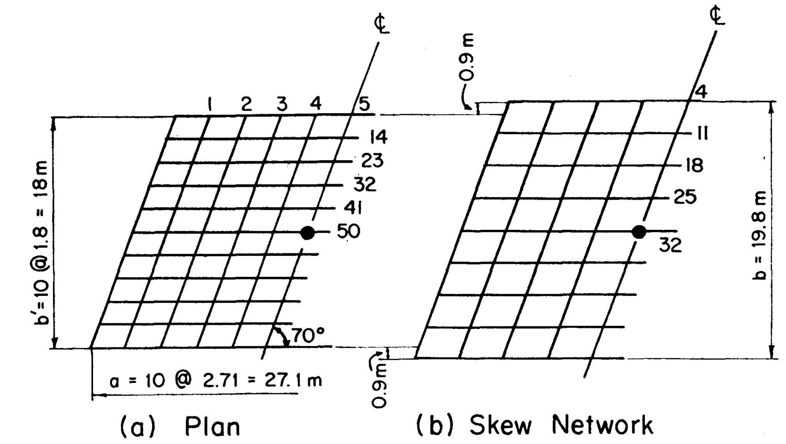

Fig. 9. The Hanazono Bridge.

Table 2. Calculated results on skew grillage-skew girder bridges

			Ono Bridge		Hanazono Bridge	
1	span		40.0 m		27.1 m	
2	effective width		20.4 m		18.0 m	
3	skew angle φ		45 degrees		70 degrees	
4	number of main girders		7		11	
5	girder spacing		3.4 m		1.8 m	
6	stiffness ratio		0.2213		0.2193	
7	torsional parameter		0		0.25	
8	load		uniform	line	uniform	line
9	analytical value of k		0.723	0.678	0.857	0.862
10	calculated	b used in Eq. (6)	23.8 m		19.8 m	
11	calculated	β	0.280		0.342	
12	calculated	A	0.675	0.565	0.139	0.133
13	calculated	B	0.930	0.855	0.925	0.925
14	calculated	$k=-A\beta+B$	0.741	0.697	0.877	0.880

tudinal bending moment at network point 32 for the Ono Bridge and at network point 50 for the Hanazono Bridge are calculated automatically by NEAC 2203, and the longitudinal bending moment at midspan of the inner central girder are obtained for line and uniform loads. The ratio of M_p to M_b are shown in line (9) of Table 2.

If the bridge widths are taken as shown in Figs. 6 and 7 and line (10) of Table 2, the parameter b can be calculated as given in line (11). The values of coefficients A and B are obtained directly for the case of skew angle $\varphi=45$ degrees and $\kappa=0$ from Figs. 4 and 5 for the Ono Bridge. For the Hanazono Bridge, the values of A and B for the case of skew angle $\theta=70$ degrees and $\kappa=0.25$ can be interpolated from three specified values (A, B) for ($\varphi=70$ degrees, $\kappa=0$), ($\varphi=70$ degrees, $\kappa=0.5$), ($\varphi=70$ degrees, $\kappa=1.0$), and are obtained as shown in lines (12) and (13), respectively. Comparison shows a fairly good agreement between lines (9) and (14).

Part III. Calculation of Bending Moment of Skew Girder Bridges

As can be understood from the above two parts, the bending moment at midspan of inner girders of skew girder bridges can be calculated by the following procedures:

1. Assume or calculate the stiffness ratios (stiffness parameters) and torsional parameters for right (skew) grillage-skew girder bridges, respectively.
2. Find the coefficients (m, n) from Figs. 2 and 3 or (A, B) from Figs. 6 and 7.
3. Calculate the value of factor k by Eq. (3) or (6).
4. Calculate the bending moment at midspan of a single girder by the elementary beam theory.
5. Find the bending moment at midspan of inner girders of skew girder bridge which is assumed to be a skew plate by Eq. (2) or (5).
6. Design the cross section of inner girders by means of the total bending moment (bending moment calculated by the elementary beam theory for the pre-dead load, plus, the bending moment for after-dead load, live uniform and line loads calculated as above).
7. Design the cross section of outer girders appropriately.
8. Find the equivalent moment of inertia of inner and outer girders, and prepare the values of various parameters necessary to the numerical analysis of parallelogram plates.
9. Follow the procedures programmed and stored in the computer.
10. Repeat the steps (1) to (9) until the assumed and calculated moment of inertia show a satisfactorily agreement.

The calculated bending moment thus involves the stiffness parameter, the torsional parameter and also the skew angle parameter. It is, therefore, more rational than those calculated by F. Leonhardt's method (which considers only the stiffness parameter, and neglects the torsional and skew angle parameters), by Y. Guyon-Ch. Massonnet's method (which neglects the skew angle parameter) and by the elementary beam theory (which neglects these three parameters).

Conclusion

Based on the numerical analysis of orthogonal (skew) anisotropic-parallelogram plates, the authors proposed the formulae (2, 3) and (5, 6) and Figs. 2, 3 and 6, 7 for the right (skew) grillage-skew girder bridges to calculate the longitudinal bending moment at midspan of inner central girders. The factor k involved in Eqs. (3) and (6) is a function of the stiffness and torsional parameters similar to those defined by GUYON-MASSONNET and also of the skew angle parameter. The bending moment thus calculated may be rational compared with those calculated by neglecting the skew angle.

The detailed description of the derivation of formulae and figures is omitted on account of space, but there may be no obstacle to the use of these equations and figures.

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Summary

Based on the numerical analysis of orthogonal (skew) anisotropic-parallelogram plates, the authors present a rapid and rational method for the calculation of the bending moments of skew girder bridges.

Résumé

Les auteurs présentent un procédé rapide et rationnel pour la détermination des moments fléchissants dans les ponts biais; la base en est l'étude numérique des dalles biaises orthotropes, en forme de parallélogramme.

Zusammenfassung

Basierend auf der numerischen Untersuchung von schiefwinklig gelagerten orthotropen Parallelogramm-Platten entwickeln die Autoren eine rasche und rationelle Methode zur Berechnung der Biegemomente bei schiefen Trägerbrücken.