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Plastic Design in Britain

Le calcul en plasticité en Grande-Bretagne

Das Traglastverfahren in Großbritannien

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Each theoretical advance in the development of plastic theory for steel framed structures has been checked by experiment, on models and on full-scale buildings [1, 2]. Experimental and theoretical results have in turn been applied to the practical design of actual structures [3, 4, 5, 6, 7, 8 and 9]. Fig. 1 shows the latest stage (1965) in the rebuilding programme at the Engineering Laboratories, Cambridge University; the four-storey four-bay frames form a fully rigid site-welded structure in high yield steel (BS. 968). This design

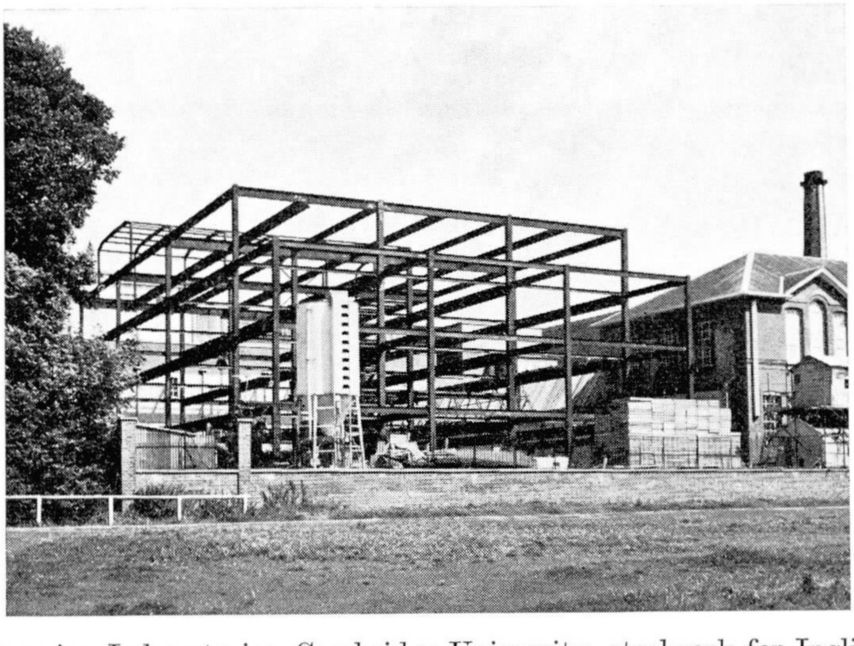

Fig. 1. Engineering Laboratories, Cambridge University, steelwork for Inglis "A". Consulting Engineers: J. F. Baker and Associates; R. T. James and Partners.

was prepared in accordance with the recent recommendations [10] of a Joint Committee on the design of tall steel buildings braced against wind, and these recommendations form the first design code published in England using plastic design rules. (Plastic design has been permitted since the 1948 revision of BS. 449, but no rules are given.)

The simplicity of the Joint Committee's design rules stems from an exceptionally clear appreciation of the process of engineering design, as well as from the simplicity inherent in plastic design methods. There are essentially two stages in the design of a steel frame: a) The determination of a set of structural forces in equilibrium with the given loading, and b), the design of an individual structural element (beam, column, connexion) to carry those forces. The two stages cannot in fact always be separated, but they are logically distinct. For exemple, the Joint Committee's recommendation is that each beam should be designed separately on the basis of plastic theory; the bending moment diagram for a beam is therefore determined at the same time that the beam is designed.

By contrast, the bending moments in the columns are determined by considering a limited substitute frame, for which a prescribed loading pattern is taken to lead to the worst conditions. Each column length is then checked for stability under these conditions, but this checking process is kept distinct from the problem of analysis of the structure.

In a sense, the whole art of structural design consists in the determination of a reasonable set of internal forces which can be used safely to find the sizes of the members. The Joint Committee, in making their design rules, nowhere pretend that they are calculating the *actual* forces in the frame. Indeed, such a calculation, even if it were possible, would be rendered meaningless by the accidental imperfections to which a complex structure is subject. This conclusion, of paramount importance for the development of a rational design method, was only reached after half a century of experience with steel structures.

And the conclusion was only believed when the Steel Structures Research Committee [1, 11] published the results of its tests, the first of any scale ever to be undertaken, on existing steel-framed buildings. It is no exaggeration to say that these results bore virtually no relation to the conventional calculations on which the buildings had been designed. The bending moment diagrams had, of course, the expected *shape* (i. e. parabolic for uniformly distributed loads), but the base-lines of the diagrams could not be predicted. That is, there was no correlation between the values of the redundancies as determined by experiment and as determined by calculation.

Now redundancies are calculated by the introduction into the structural analysis of *compatibility* conditions, sometimes in the form of boundary conditions (no settlement of supports, fixed-ended beams, and so on) and sometimes as internal constraints (e. g. perfectly rigid connexions or connexions of known flexibility). These compatibility conditions are notoriously difficult to satisfy in practice; moreover, slight departures from perfection can have large effects on the values of the redundancies.

Although the Steel Structures Research Committee produced design rules taking account as far as possible of the real behaviour of a steel frame, they realized that in practice actual behaviour was so variable that no further progress would seem to be possible along conventional lines. It was at this point that, the Committee having been disbanded, a start was made on the development of plastic theory as a design tool. This research has led to the widespread use of plastic theory in the structural industry, but the basic ideas of the Steel Structures Research Committee have not been forgotten. On the one hand concepts of structural analysis, like the limited substitute frame used by the Joint Committee, have been taken over almost unchanged; on the other, the fundamental lessons of experimentation in the laboratory and on real structures have been well learned.

It is important to realize that plastic theory is a method for assigning reasonable values to the redundancies so that the designer has a set of equilibrium bending moments on which he can base his design. This point is sometimes obscured by the fact that plastic theory can also predict, with very great accurary, the load at which a frame will collapse. But the occupant, and to some extent the designer, is not primarily interested in the *collapse* load, but in the behaviour of the structure under *working* load. From this point of view, plastic theory may be considered as just one of the alternative ways of constructing a reasonable working-load bending moment diagram; conventional elastic theory is another way.

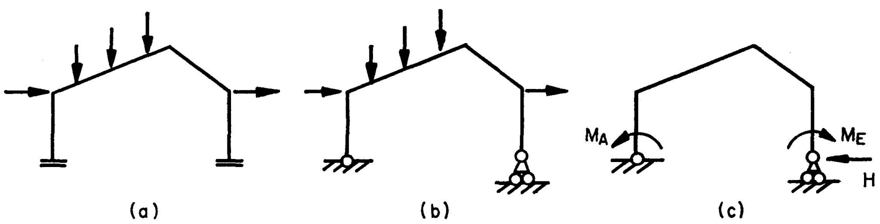

Fig. 2. Redundant portal frame.

The operation of plastic theory can be illustrated by discussion of a simple design problem. Suppose the design is required for a portal frame with "fixed" feet, say of pitched-roof type, Fig. 2a). Such a frame has three redundancies, and can be made statically determinate by, for example, pinning both feet and allowing the feet to spread, Fig. 2b). The redundancies may be chosen as M_A, M_E, and H, Fig. 2c). Now an *elastic* solution requires the introduction of the three conditions that the feet do not spread or rotate; a set of values of the redundancies can then be found, and a corresponding bending moment diagram drawn.

It will have been necessary to assign relative stiffnesses to the members of the frame in order to calculate the elastic solution; suppose, for the sake of illustration, that a uniform frame has been chosen. The largest bending moment can now be determined, and a section assigned to the frame such that the permitted stress, say 10.5 tons/in^2 if the yield stress is 16 tons/in^2, is not exceeded. The elastic design of the whole frame is based, therefore, on the attainment of the permitted stress at just one cross-section.

Starting from the elastic solution, the largest bending moment can be reduced if, for example, the value of H (Fig. 2c)) is changed. But this reduction in the largest bending moment will be accompanied by an increase in bending moment at some other section; the largest reduction will occur when the largest stress in the frame is attained at two sections simultaneously, and this result can be achieved by varying the value of H alone.

Since the frame under discussion has, in fact, *three* redundancies, it is possible to adjust the values of these redundancies so that the maximum stress in the frame is attained at *four* sections simultaneously, and the maximum stress will then have its smallest value for the given loads. If, then, a new section is chosen for the design so that the permitted stress is just reached, that design will be the most economical.

The process of adjusting the values of the redundancies so that there is an equalization of bending moments can be thought of in several ways. There is no need to start from the elastic solution and the simplicity of the design compared with conventional elastic procedure is evident; effectively, four linear simultaneous equations are used to determine the three values of the redundancies plus the maximum bending moment, instead of elastic compatibility conditions introduced into second-order differential equations of bending. But both methods lead to a set of bending moments on which a design may be based; the elastic solution is less economical because only one cross-section is working at the permitted maximum stress.

Neither the elastic solution, nor the "equalized" solution (which is, of course, the "plastic" solution), represent the true state of the structure under working load. In practice, the supports of the portal frame will settle, spread, and rotate, making nonsense of the elastic design calculations; for the plastic design, the bending moments are adjusted frankly to give the most economical distribution. But *either* calculation leads to a *safe* design; one of the fundamental theorems of plastic theory [2, 12] states that a frame designed on the basis of an arbitrarily assumed distribution of bending moments in equilibrium with the applied loads cannot possibly collapse under any other distribution of equilibrium bending moments. In other words, the actual behaviour of the frame (i. e. settlement of supports, defects in fitting, and so on) cannot affect its safety. Thus conventional *elastic* calculations lead to an uneconomical design for a ductile structure, but that design can be shown by *plastic* theory to be satisfactory in the sense of being safe.

Perhaps the easiest way, and certainly the traditional way, of visualizing the process of equalization of moments is to consider the frame subject hypothetically to gradually increasing loads. Whatever the actual behaviour, there will be a cross-section of maximum stress, and yield will be reached at that cross-section when the load level reaches a certain value. A plastic hinge is then formed, and the familiar process ensues; with the moment remaining constant at the full plastic value at the hinge already formed, a second hinge forms if the load is further increased, and then a third. For the design example of Fig. 2, a fourth hinge ensures a mechanism of collapse, and the limit of the process has been reached.

At this limit, there are four cross-sections where the stress is 16 tons/in^2, or rather, effectively 18.4 tons/in^2, to allow for the usual shape factor (1.15) for I-sections. If the whole bending moment diagram is scaled in the ratio $18.4/10.5 = 1.75$, no cross-section will have a stress exceeding 10.5 tons/in^2, and the usual load factor used in plastic design 1.75, has been derived.

This mechanistic approach to the problem is the one often used by designers [2, 12, 13, 14, 15], although the equilibrium approach is useful for simple frames [16]. The mechanisms of collapse are very easily observed in practice, both in the laboratory and in tests on real structures, and the accuracy with which plastic theory predicts collapse loads has led designers to have great confidence in its use.

It has been implicit in the whole of the discussion so far that strength of the frame is the overriding design criterion, and that deflexions are small and that individual elements, or the structure as a whole, do not become unstable. Before discussing these alternative design criteria, mention may be made of

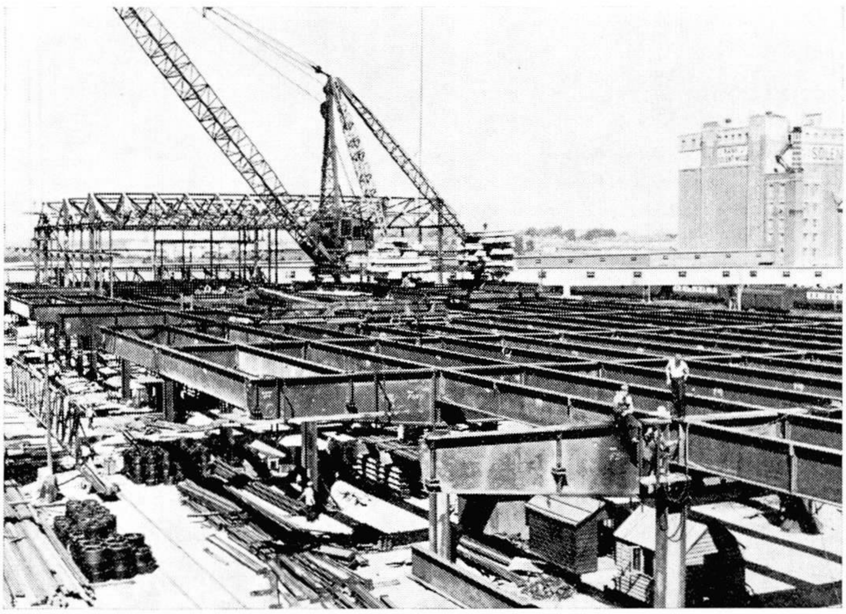

Fig. 3. Terminal building, Southampton Docks. Consulting Engineers: Scott and Wilson, Kirkpatrick and Partners.

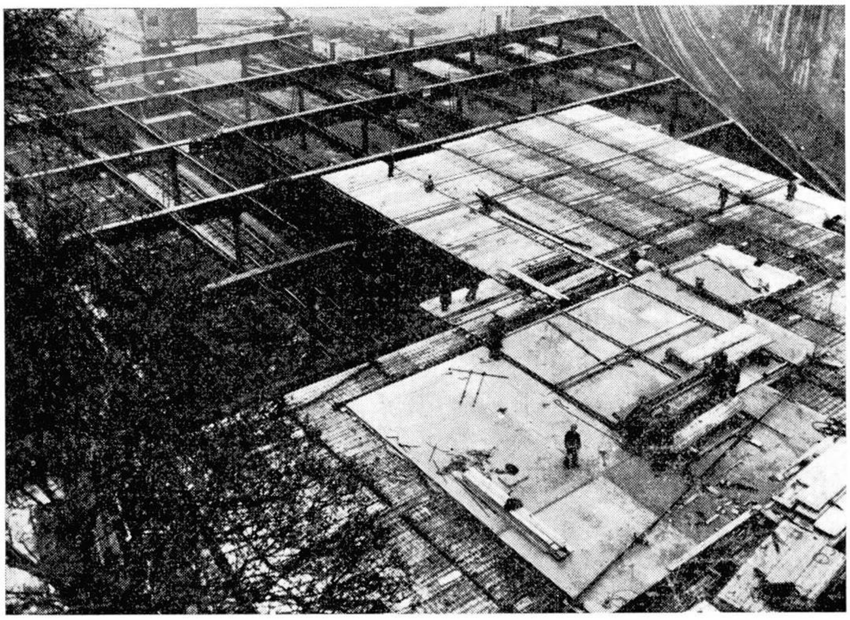

Fig. 4. Raft for air terminal, Cromwell Road, London. Engineer: C. E. Dunton, Chief Civil Engineer, London Transport.

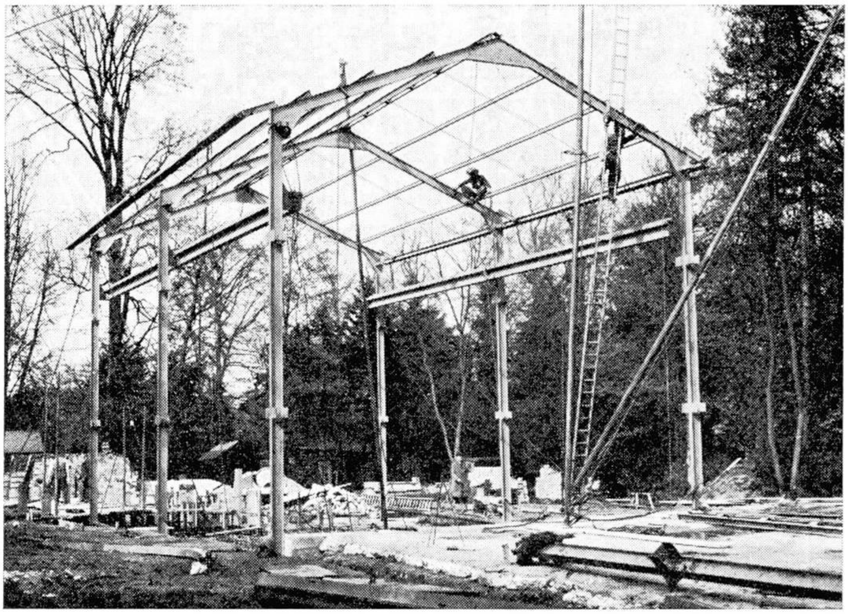

Fig. 5. Fatigue Laboratory, British Welding Research Association. Consulting Engineers: J. F. Baker and Associates; W. S. Atkins and Partners.

some practical applications of plastic theory to structures where strength is all-important.

A simple and pure form of plastic design may be made for the grillage of beams. The heavy plate girders of Fig. 3 form part of Transit Shed 102 at Southampton, on which working load tests were made [8, 9]. Fig. 4 shows the platform for supporting the Air Terminal in Cromwell Road, London, and was designed to carry any arrangement of vehicles and three-storey buildings.

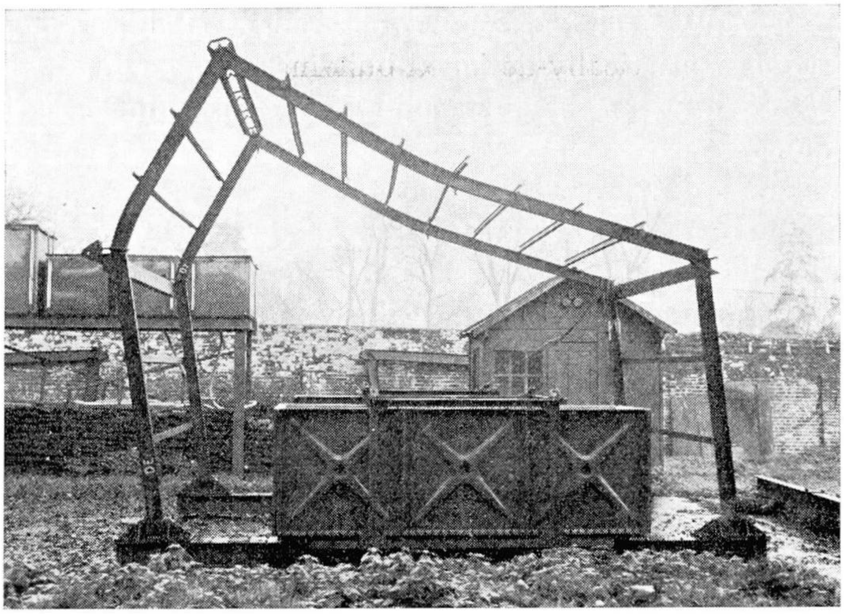

Fig. 6. Full-scale test on portal frame.

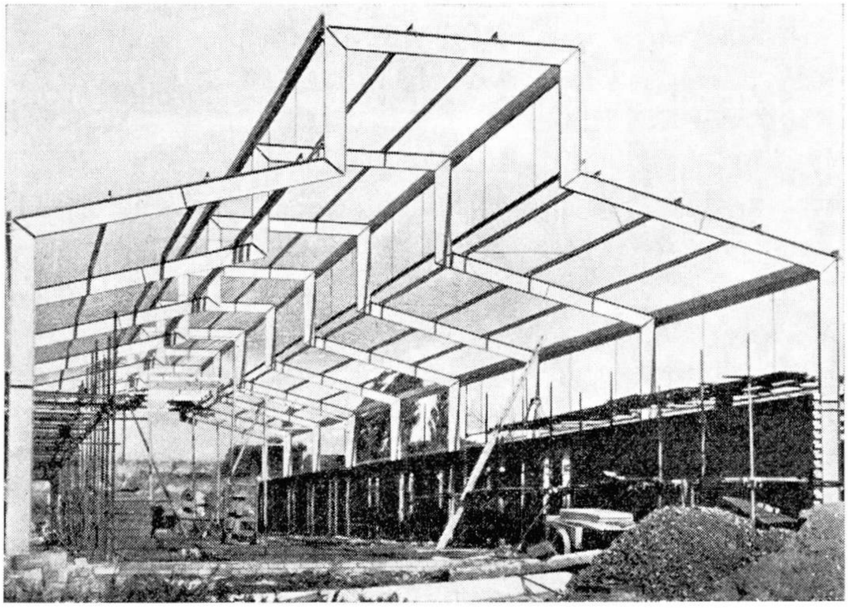

Fig. 7. Laboratory for Guest, Keen and Nettlefold Ltd. Consulting Engineers: J. F. Baker and Associates.

This grillage is irregular since column spacings were dictated by the underground running tracks over which the platform was constructed. These two grillage structures, the first welded and the second bolted full-strength, achieved economies, both in design time and in material, over conventional elastic designs.

The design of single-storey industrial buildings, of the type shown in Fig. 5, is now commonplace. This particular building, the fatigue laboratory for the

Research Station at Abington for the British Welding Research Association, was one of the first plastically-designed buildings to be erected; the Research Station was the scene of the first full-scale tests to destruction of portal frames (Fig. 6).

The frames in Fig. 7 are a slight variant on the normal pitched roof construction; the monitor roof is incorporated in the main framing. The calculation of such frames by plastic methods is no more difficult than that of normal frames. Similarly, multi-bay frames [16] present no difficulties; indeed, the use of plastic theory enables an insight to be obtained into the fundamental behaviour of such frames that cannot be gained from an elastic analysis. Realizing that spread at eaves level is the essential weakness of multi-bay frames, a design was proposed in which the external columns were made effectively rigid by additional bracing. A warehouse, Fig. 8, covering 600 ft. × 200 ft. was constructed using this principle, and showed remarkable economies. Each frame consisted of twelve 50 ft. bays, "arching" between the two strong columns 600 ft. apart.

All these designs were based upon strength considerations, but checks had to be made on deflexions and on column stability. For the average portal frame in steel to BS 15 deflexions are usually not critical, but this situation has been changed recently by the introduction of high yield steel (BS 968), whose use permits portal frames to be made of lighter sections. Such lighter sections have greater flexibility, and for some portal frames deflexions may become of critical importance. It is possible to make small modifications to simple plastic theory to allow the effect of deflexions to enter the analysis [17], and these modifications also serve to give the designer an estimate of the magnitude of the effect.

Similarly, elastic-plastic methods have been developed [2] for checking the stability of individual column lengths under known end conditions. This work is still incomplete, but the latest stage is the publication [18] of column design curves covering a wide range of practical cases.

Much heavier steelwork was used for the ship fitting-out shop shown in Fig. 9. The portal frames have 76 ft. span and are 60 ft. high; they carry two 50-ton cranes. A span of 76 ft. is approaching the largest that can be covered economically by a flat-roofed frame, but pitched-roof frames have been used for buildings of over double this span.

Plastic theory is slowly being applied to multi-storey buildings, and mention has already been made of the Joint Committee's proposals for buildings braced against wind [10]. The unbraced building has also been discussed [19, 20, 21], and a start has been made on the practical design of multi-storey frames. Fig. 10 shows the steelwork for the centre wing of the Engineering Laboratories at Cambridge [3], and Fig. 1 the newest extension at the same Laboratories [5]. Both buildings were site-welded.

An interesting development is the application of plastic theory to composite

Fig. 8. Factory for W. C. Jones Ltd., Manchester. Consulting Engineers: J. F. Baker and Associates.

Fig. 9. Fitting-out shop for Joseph L. Thompson and Sons, Shipbuilders, Sunderland. Consulting Engineers: J. F. Baker and Associates; W. H. S. Tripp and Partners.

concrete steel frames [22, 23]. There are certain difficulties which have not yet been overcome, but it was possible to design the north wing of the Engineering Laboratories, Fig. 11, as a plastic composite frame [4]. Full strength bolted connexions were used in this design.

Plastic theory, because of its essential simplicity and rationality, and because of the consequent economies in material, time, and money, has come

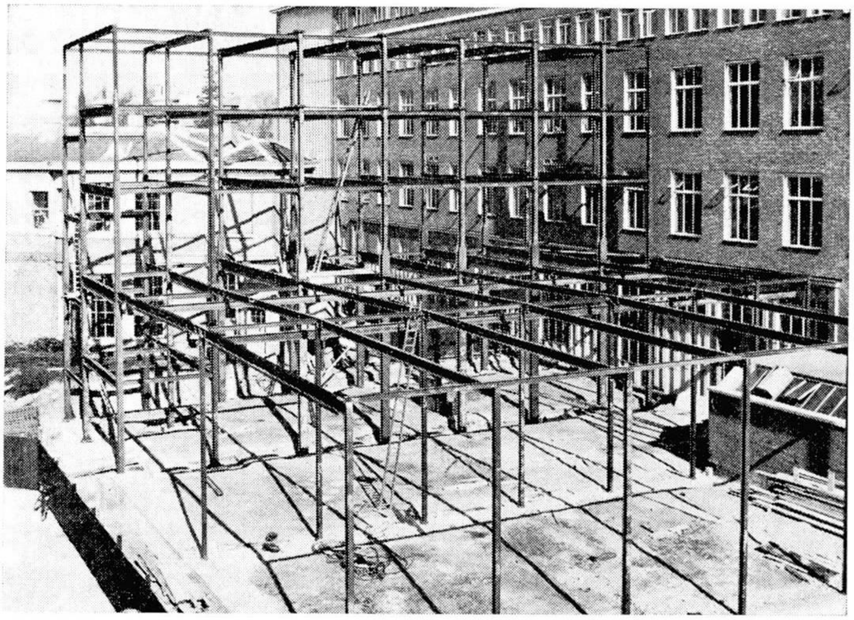

Fig. 10. Engineering Laboratories, Cambridge University: Centre wing. Consulting Engineers: J. F. Baker and Associates; R. T. James and Partners.

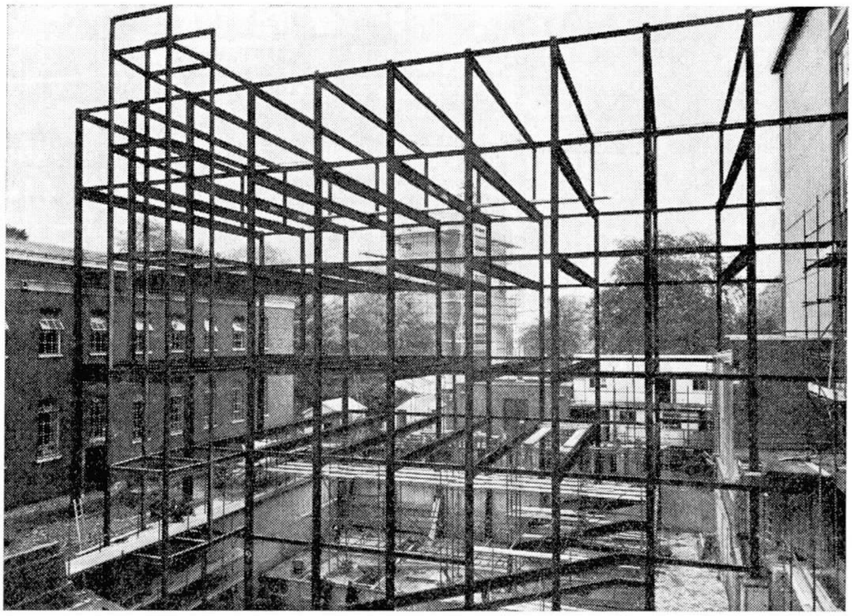

Fig. 11. Engineering Laboratories, Cambridge University: North Wing. Consulting Engineers: J. F. Baker and Associates; R. T. James and Partners.

to be an accepted technique in the design office. Not only can conventional structures be designed with greater assurance; the use of plastic theory gives the designer a new insight into the behaviour of conventional structures, and enables him to design with confidence structures of unconventional form.

References

1. BAKER J. F.: The Steel Skeleton. Vol. 1: Elastic behabiour and design. Cambridge University Press 1954.
2. BAKER J. F., HORNE M. R., HEYMAN J.: The Steel Skeleton. Vol. II: Plastic behaviour and design. Cambridge University Press 1956.
3. HORNE M. R.: Plastic design of four-storey steel frames. The Engineer, Vol. 206, 1958.
4. JOHNSON R. P., FINLINSON J. C. H., HEYMAN J.: A plastic composite design. Proc. Instn. Civ. Engrs. vol. 32, 1965.
5. HEYMAN J., FINLINSON J. C. H., JOHNSON R. P.: Inglis "A": A fully-rigid multi-storey welded steel frame. The Structural Engineer, vol. 44, 1966.
6. HEYMAN J.: Plastic design of new laboratory block for BWRA. Brit. Weld. Journ. Vol. 4, 1957.
7. BAKER SIR JOHN, HORNE M. R., JOHNSON L. G.: Use of plastic theory in the design of a fabricating shop. Proc. Instn. Civ. Engrs, vol. 20, 1961.
8. BOWEN F. M., RANGER B. E. S.: Steelwork in Transit Shed 102 Southampton Docks. The Structural Engineer, vol. 34, 1956.
9. RANGER B. E. S., HEYMAN J.: Full-scale loading tests on a welded plate-girder floor system. Proc. Conference on the correlation between calculated and observed stresses and displacements in structures. Instn. Civ. Engrs., Preliminary volume 1955, Final volume 1956.
10. Joint Committee of the Institution of Structural Engineers and the Institute of Welding. Fully rigid multi-storey welded steel frames. The Institution of Structural Engineers, Dec. 1964.
11. Steel Structures Research Committee. First Report 1931, Second Report 1934, Final Report 1936. London, HMSO.
12. HEYMAN J.: Beams and Framed Structures. Oxford, Pergamon, 1964.
13. NEAL B. G.: The plastic methods of structural analysis. 2nd Edition, London, Chapman and Hall, 1963.
14. SYMONDS P. S., NEAL B. G.: The calculation of failure loads on plane frames under arbitrary loading programmes. Proc. Instn. Civ. Engrs., vol. 35, 1951.
15. NEAL B. G., SYMONDS P. S.: A method for calculating the failure load for a framed structure subjected to fluctuating loads. Proc. Instn. Civ. Engrs., vol. 35, 1951.
16. HEYMAN J.: Plastic design of portal frames. Cambridge University Press 1957.
17. BAKER SIR JOHN: Plastic design in steel to BS 968. British Constructional Steelwork Association Publication No. 21, 1963.
18. HORNE M. R.: The plastic design of columns. British Constructional Steelwork Association Publication No. 23, 1964.
19. BAKER SIR JOHN: The design of high buildings. Symposium on the design of high buildings, Hong Kong University Press, 1963.
20. HEYMAN J.: The plastic design of tall steel buildings. Symposium on the design of high buildings, Hong Kong University Press, 1963.
21. WOOD R. H.: The stability of tall buildings. Proc. Instn. Civ. Engrs., vol. 11, 1958.
22. BARNARD P. R., JOHNSON R. P.: The ultimate strength of composite beams. Proc. Inst. Civ. Engrs., vol. 32, 1965.
23. BARNARD P. R., JOHNSON R. P.: The plastic behaviour of continuous composite beams. Proc. Instn. Civ. Engrs., vol. 32, 1965.

Summary

The basic principles of plastic design are examined in relation to the historical development of the steel frame. Examples are given of the application of the theory to the practical design of such structures.

Résumé

Les auteurs examinent les principes fondamentaux du calcul en plasticité, en relation avec le développement historique des ossatures métalliques. Ils donnent des exemples de l'application de la théorie à l'étude pratique de ces ossatures.

Zusammenfassung

Die Grundsätze des Traglastverfahrens werden im Zusammenhang mit der geschichtlichen Entwicklung der Stahlrahmen untersucht. Beispiele für die Anwendung der Theorie auf die praktische Berechnung solcher Tragwerke werden angegeben.