Zeitschrift: IABSE publications = Mémoires AIPC = IVBH Abhandlungen

Band: 27 (1967)

Artikel: Free Vibration of Beam and Slab Bridges

Autor: Iyengar, K.T. Sundara Raja / Iyengar, R. Narayana

DOI: https://doi.org/10.5169/seals-21539

Free Vibration of Beam and Slab Bridges

Oscillations libres des ponts à dalle nervurée

Freie Schwingung von Balken- und Plattenbrücken

K. T. SUNDARA RAJA IYENGAR
Dr. Ing., Professor, Department of Civil Engineering, Indian Institute of Science, Bangalore-12

R. NARAYANA IYENGAR
M. Sc. (Engg.), Research Scholar, Department of Civil Engineering, Indian Institute of Science, Bangalore-12

A clear understanding of the dynamic behaviour of a bridge is an essential pre-requisite for an efficient design and construction. But the dynamic analysis is relatively more complex than the static analysis and perhaps due to this reason the dynamic behaviour has not been studied thoroughly in the past. For an easy and comprehensive theoretical analysis certain simplifying assumptions are necessary at one stage or the other, but how far the results of such an analysis correspond with the actual behaviour is of utmost importance. For instance, the analysis of INGLIS [1], MISE and KUNII [2], in which the bridge is assumed to behave like a beam is applicable only for bridges with long and heavy girders as are usual in railway plate girder bridge practice. The above beam theory if applied to beam and slab highway bridges can account only for the beam type modes in which the plate action can be neglected. But for the calculation of response under moving loads or earthquake forces a complete and accurate knowledge of the free vibration characteristics including both the modal shapes and the natural frequencies is essential. For this it is clear that only a theory which can account for all types of modes is appropriate.

The physical model which corresponds to the beam and slab bridge is a thin plate with stiffeners, the edges perpendicular to the direction of traffic being simply supported and the other two edges remaining free or elastically supported. Though classical methods are available for analysing stiffened plates, the mathematical details of such procedures are so complex that they are of restricted application in practice. Many types of assumptions have been made to simplify the analysis under statical loading by various investigators, the most popular one being that of replacing the system by an equivalent orthotropic plate [3, 4]. NARUOKA and YONEZAWA [5], YAMADA and VELETSOS [6] have applied the orthotropic plate theory for studying the free vibration of beam and slab bridges.

NARUOKA and YONEZAWA have solved the differential equation of free vibration of an orthotropic plate with two opposite edges simply supported and with the other two either free or elastically supported. Characteristic equations have been derived for two values of the torsional parameter $\mu \left(= \frac{H}{\sqrt{D_x D_y}}\right)$ viz. 0 and 1. $\mu = 0$ implies $H = D_1 + 2 D_{xy} = 0$. This is possible only when both D_1 and D_{xy} vanish independently. This point has been overlooked by them and hence the equations given for the case of $\mu = 0$ are incorrect. Further it has been concluded that the fundamental mode shape in the lateral direction has either one or two nodal lines and that the beam theory fails to give an estimate of the fundamental mode frequency. This is also not correct. YAMADA and VELETSOS have studied the vibration of I-beam bridges by the Rayleigh-Ritz method. The structure has been considered as a plate supported on five identical beams of which two are at the edges. The free vibration equation of an orthotropic plate has also been solved by them for studying the bridge vibration problem using the orthotropic plate theory. A few comparisons have been made between the two approaches by HUANG and WALKER [7] for five and nine beam bridges having beams along the edges also.

SUNDARA RAJA IYENGAR and JAGADISH [8] have studied the free vibration of beam and slab bridges by expanding the deflection function in terms of the unstiffened plate-eigen functions. The torsion of the beams has been neglected in this analysis. JAGADISH [9] recently has analysed the same problem by the Rayleigh-Ritz method using the plate-eigen functions which satisfy all the boundary conditions exactly. The torsional rigidity of the beams has also been considered in this investigation. In both the above investigations the bridges are assumed to have only longitudinal beams.

Though the method of eigen functions or the Rayleigh-Ritz method are more exact than the orthotropic plate theory, they are tedious in their mathematical details and not suitable for a general presentation so as to be useful for ready reference. But the results of an exact analysis are indispensable for studying the usefulness and limitations of an approximate theory such as the orthotropic plate theory. In the present work the orthotropic plate theory has been utilised for studying the free vibration characteristics of single span beam and slab bridges. The accuracy of the orthotropic plate theory has been demonstrated by comparing the results with those of an exact theory. Results of experimental work conducted to verify the theory have also been reported.

Analysis

The differential equation governing the free vibrations of an orthotropic plate is (Fig. 1)

$$D_x \frac{\partial^4 W}{\partial x^4} + 2\,(D_1 + 2\,D_{xy}) \frac{\partial^4 W}{\partial x^2\,\partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} = -\bar{\rho}\,\frac{\partial^2 W}{\partial t^2}. \qquad (1)$$

Where $\bar{\rho}$ = mass per unit area of the plate.

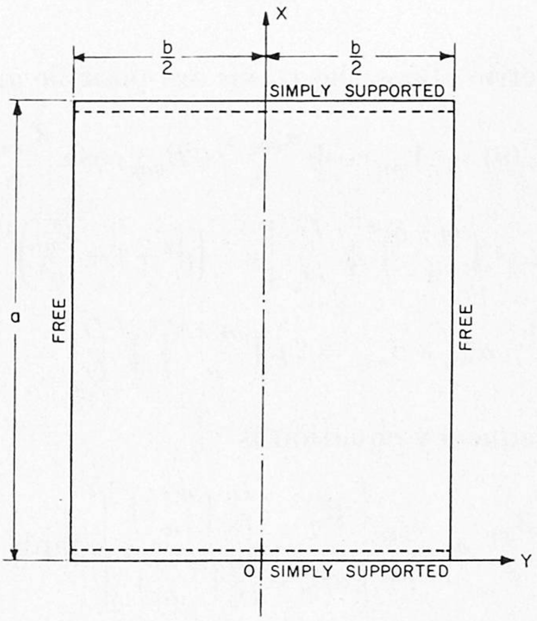


Fig. 1.

The boundary conditions simulating the bridge boundary conditions are

1. $$W = \frac{\partial^2 W}{\partial x^2} = 0 \quad \text{at } x = 0, a, \qquad (2)$$

2. $$\left(\frac{\partial^2 W}{\partial y^2} + \frac{D_1}{D_y}\frac{\partial^2 W}{\partial x^2}\right) = \left(\frac{\partial^3 W}{\partial y^3} + \frac{2\,H - D_1}{D_y}\frac{\partial^3 W}{\partial x^2\,\partial y}\right) = 0 \quad \text{at } y = \pm\frac{b}{2}. \qquad (3)$$

The solution of equation (1) can be taken as

$$W_{mn}(x, y) = Y_{mn} \sin\frac{m\,\pi\,x}{a} \sin p_{mn}\,t. \qquad (4)$$

This reduces equation (1) into

$$\frac{d^4 Y_{mn}}{dy^4} - \frac{2\,H}{D_y}\frac{m^2\,\pi^2}{a^2}\frac{d^2 Y_{mn}}{dy^2} + \left(\frac{D_x}{D_y}\frac{m^4\,\pi^4}{a^4} - \frac{\bar{\rho}\,p_{mn}^2}{D_y}\right) Y_{mn} = 0, \qquad (5)$$

where $H = D_1 + 2\,D_{xy}$.

A complete solution of the problem of free vibration is the one which satisfies both the differential equation (1) and the boundary conditions (2) and (3). The satisfaction of the boundary conditions yields characteristic equations, the real roots of which lead to the natural frequencies. Hence the existence of real roots of the different characteristic equations is essential for the plate to execute normal modes, defined by $Y_{mn}(y) \sin \frac{m \pi x}{a}$. The physically possible normal mode solutions along with the corresponding frequency equations for different cases are given in the following.

Case (A) $H = D_1 + 2 D_{xy} \neq 0$.

$$1. \qquad \mu > \left(\mu^2 - 1 + \frac{\lambda_{mn}^2}{\pi^4}\right)^{1/2}, \qquad \lambda_{mn} = \left(\frac{\bar{\rho}\, p_{mn}^2\, a^4}{D_x\, m^4}\right)^{1/2} < \pi^2,$$

where $\mu = H / \sqrt{D_x D_y}$.

Only modes symmetric about the x-axis are possible and they are given by

$$Y_{mn}(y) = A_{mn} \cosh \frac{\alpha_{mn}\, y}{b} + B_{mn} \cosh \frac{\beta_{mn}\, y}{b}, \tag{6}$$

where
$$\beta_{mn}^2 = \left(\frac{m \pi b}{a}\right)^2 \sqrt{\frac{D_x}{D_y}} \left[\mu - \left(\mu^2 - 1 + \frac{\lambda_{mn}^2}{\pi^4}\right)^{1/2}\right].$$

$$\alpha_{mn}^2 + \beta_{mn}^2 = 2\mu \left(\frac{m \pi b}{a}\right)^2 \sqrt{\frac{D_x}{D_y}}.$$

The transcendental frequency equation is

$$\tanh \frac{\beta_{mn}}{2} = \frac{\alpha_{mn}}{\beta_{mn}} \left[\frac{\beta_{mn}^2 - \frac{D_1}{D_y}\left(\frac{m \pi b}{a}\right)^2}{\alpha_{mn}^2 - \frac{D_1}{D_y}\left(\frac{m \pi b}{a}\right)^2}\right]^2 \tanh \frac{\alpha_{mn}}{2}. \tag{7}$$

The natural frequency parameter λ_{mn} is given by

$$\lambda_{mn} = \left[\pi^4 + \left(\beta_{mn} \frac{a}{m b}\right)^4 \frac{D_y}{D_x} - 2 \mu \pi^2 \left(\beta_{mn} \frac{a}{m b}\right)^2 \sqrt{\frac{D_y}{D_x}}\right]^{1/2}. \tag{8}$$

The corresponding eigen function $Y_{mn}(y)$ may be taken as

$$Y_{mn}(y) = \frac{\cosh \frac{\alpha_{mn} y}{b}}{\cosh \frac{\alpha_{mn}}{2}} - \frac{\alpha_{mn}^2 - \frac{D_1}{D_y}\left(\frac{m \pi b}{a}\right)^2}{\beta_{mn}^2 - \frac{D_1}{D_y}\left(\frac{m \pi b}{a}\right)^2} \, \frac{\cosh \frac{\beta_{mn} y}{b}}{\cosh \frac{\beta_{mn}}{2}}. \tag{9}$$

Antisymmetric modes do not exist since the corresponding transcendental frequency equation does not have real roots.

$$2. \qquad \mu = \left(\mu^2 - 1 + \frac{\lambda_{mn}^2}{\pi^4}\right)^{1/2}, \qquad \lambda_{mn} = \pi^2.$$

Only symmetric modes are possible, with

$$Y_{mn}(y) = 1, \tag{10}$$

$$D_1 = 0, \tag{11}$$

$$3. \qquad \mu < \left(\mu^2 - 1 + \frac{\lambda_{mn}^2}{\pi^4}\right)^{1/2}, \qquad \lambda_{mn} > \pi^2.$$

Symmetric Modes

The frequency equation is

$$\tan\frac{\beta_{mn}}{2} = -\frac{\alpha_{mn}}{\beta_{mn}}\left[\frac{\beta_{mn}^2 + \frac{D_1}{D_y}\left(\frac{m\,\pi\,b}{a}\right)^2}{\alpha_{mn}^2 - \frac{D_1}{D_y}\left(\frac{m\,\pi\,b}{a}\right)^2}\right]^2 \tanh\frac{\alpha_{mn}}{2}. \tag{12}$$

The eigen function is

$$Y_{mn}(y) = \frac{\cosh\frac{\alpha_{mn}\,y}{b}}{\cosh\frac{\alpha_{mn}}{2}} + \frac{\alpha_{mn}^2 - \frac{D_1}{D_y}\left(\frac{m\,\pi\,b}{a}\right)^2}{\beta_{mn}^2 + \frac{D_1}{D_y}\left(\frac{m\,\pi\,b}{a}\right)^2}\,\frac{\cos\frac{\beta_{mn}\,y}{b}}{\cos\frac{\beta_{mn}}{2}}. \tag{13}$$

Antisymmetric Modes

The frequency equation is

$$\tan\frac{\beta_{mn}}{2} = \frac{\beta_{mn}}{\alpha_{mn}}\left[\frac{\alpha_{mn}^2 - \frac{D_1}{D_y}\left(\frac{m\,\pi\,b}{a}\right)^2}{\beta_{mn}^2 + \frac{D_1}{D_y}\left(\frac{m\,\pi\,b}{a}\right)^2}\right]^2 \tanh\frac{\alpha_{mn}}{2}. \tag{14}$$

The eigen function is

$$Y_{mn}(y) = \frac{\sinh\frac{\alpha_{mn}\,y}{b}}{\sinh\frac{\alpha_{mn}}{2}} + \frac{\alpha_{mn}^2 - \frac{D_1}{D_y}\left(\frac{m\,\pi\,b}{a}\right)^2}{\beta_{mn}^2 + \frac{D_1}{D_y}\left(\frac{m\,\pi\,b}{a}\right)^2}\,\frac{\sin\frac{\beta_{mn}\,y}{b}}{\sin\frac{\beta_{mn}}{2}}, \tag{15}$$

where
$$\beta_{mn}^2 = \left(\frac{m\,\pi\,b}{a^2}\right)^2\sqrt{\frac{D_x}{D_y}}\left[-\mu + \left(\mu^2 - 1 + \frac{\lambda_{mn}^2}{\pi^4}\right)^{1/2}\right],$$

$$\alpha_{mn}^2 - \beta_{mn}^2 = 2\,\mu\left(\frac{m\,\pi\,b}{a^2}\right)^2\sqrt{\frac{D_x}{D_y}}.$$

The frequency parameter λ_{mn} is given by

$$\lambda_{mn} = \left[\pi^4 + \left(\beta_{mn}\frac{a}{m\,b}\right)^4\frac{D_y}{D_x} + 2\,\mu\,\pi^2\left(\beta_{mn}\frac{a}{m\,b}\right)^2\sqrt{\frac{D_y}{D_x}}\right]^{1/2}. \tag{16}$$

Case (B) $H = D_1 + 2\,D_{xy} = 0$.

In this case the differential equation (5) for $Y_{mn}(y)$ becomes

$$\frac{d^4\,Y_{mn}}{dy^4} + \left(\frac{D_x}{D_y}\,\frac{m^4\,\pi^4}{a^4} - \frac{\bar{\rho}\,p_{mn}^2}{D_y}\right)Y_{mn} = 0. \tag{17}$$

For H to vanish both D_1 and D_{xy} must vanish independently since each is positive. Hence the boundary conditions at $y = \pm\frac{b}{2}$ for this case become

$$\frac{d^2\,Y_{mn}}{dy^2} = \frac{d^3\,Y_{mn}}{dy^3} = 0. \tag{18}$$

1. $\lambda_{mn} = \pi^2$

$$\text{Symmetric modes:} \qquad Y_{mn}(y) = 1\,. \tag{19}$$

$$\text{Antisymmetric modes:} \quad Y_{mn}(y) = 2\sqrt{3}\,\frac{y}{b}\,. \tag{20}$$

2. $\lambda_{mn} > \pi^2$

For symmetric modes the characteristic equation and the eigen function are

$$\tan\frac{\alpha_{mn}}{2} = -\tanh\frac{\alpha_{mn}}{2} \tag{21}$$

and

$$Y_{mn}(y) = \frac{\cosh\frac{\alpha_{mn}\,y}{b}}{\cosh\frac{\alpha_{mn}}{2}} + \frac{\cos\frac{\alpha_{mn}\,y}{b}}{\cos\frac{\alpha_{mn}}{2}}\,. \tag{22}$$

For antisymmetric modes the characteristic equation and the eigen function are

$$\tan\frac{\alpha_{mn}}{2} = \tanh\frac{\alpha_{mn}}{2} \tag{23}$$

and

$$Y_{mn}(y) = \frac{\sinh\frac{\alpha_{mn}\,y}{b}}{\sinh\frac{\alpha_{mn}}{2}} + \frac{\sin\frac{\alpha_{mn}\,y}{b}}{\sin\frac{\alpha_{mn}}{2}}\,, \tag{24}$$

where

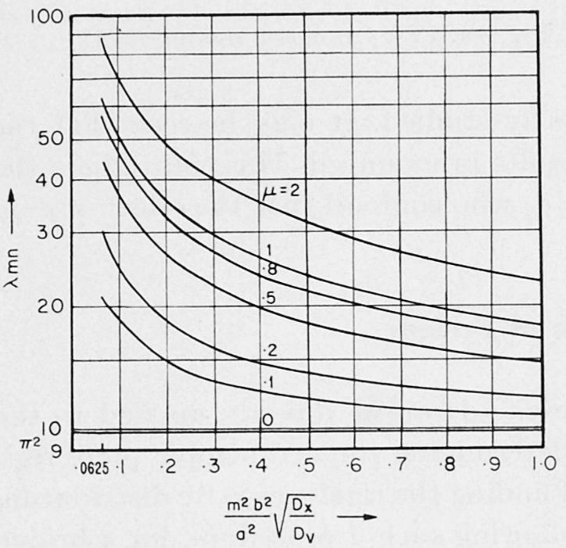
$$\alpha_{mn} = \left(\frac{m\,\pi\,b}{a^2}\right)^2 \sqrt{\frac{D_x}{D_y}} \left(\frac{\lambda_{mn}^2}{\pi^4} - 1\right)^{1/2}.$$

Discussion on the Solutions

In case (A) when $D_1 \neq 0$ the characteristic roots β_{mn} and transverse modes Y_{mn} are given by equations (7) and (9) respectively. In this case the transcendental equation (7) has only one root corresponding to $n=1$ associated with each 'm'. When $D_1 = 0$ the frequency parameter is $\lambda_{mn} = \pi^2$ for any m and $n=1$. The corresponding modal pattern Y_{mn} is a rigid displacement given by equation (10). It is interesting to note that this modal shape is the same as the fundamental rigid body mode of a free-free beam and in such modes the plate behaves as a beam simply supported at $x=0$ and $x=a$. In fact $\lambda_{mn} = \pi^2$ is exactly the frequency parameter of a beam with both the edges simply supported and with flexural rigidity D_x. Even when D_1 is not equal to zero the value of λ_{mn} for modes with $n=1$ does not differ very much from π^2 and also the modal shape given by (9) is almost a rigid body displacement. The higher symmetric and antisymmetric modes are covered by equations (12) through (16). When $H=0$, it is found that in the lateral direction both symmetric and antisymmetric modes exist with the same frequency parameter $\lambda_{mn} = \pi^2$. The symmetric mode shape is a rigid displacement as in equation (10) and the antisymmetric shape is a rigid body rotation about the x-axis.

Presentation of Results

The different transcendental equations have been solved in detail and the results presented for various values of $\frac{a}{b}$, μ, $\frac{D_x}{D_y}$ elsewhere [10]. Here only the values of λ_{mn} for three symmetric modes (any 'm' and $n=2, 3, 4$) and three antisymmetric modes (any 'm' and $n=1, 2, 3$) have been shown in figures 2, 3 and 4. The value of λ_{mn} for any 'm' and $n=1$ in the symmetric case may be taken equal to π^2 safely. For the fundamental mode (i. e. $m=1, n=1$) OEHLER [11] has verified this by conducting experiments on existing bridges. The curves given refer only to the case of $\frac{D_1}{D_y}=0$, since it was found that the effect of this parameter on λ_{mn} is negligible. Moreover the effect of Poisson's ratio to which the constant D_1 is closely related is usually neglected in practice [4]. From figures 2, 3



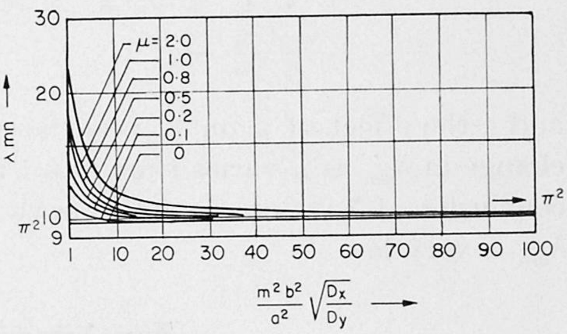
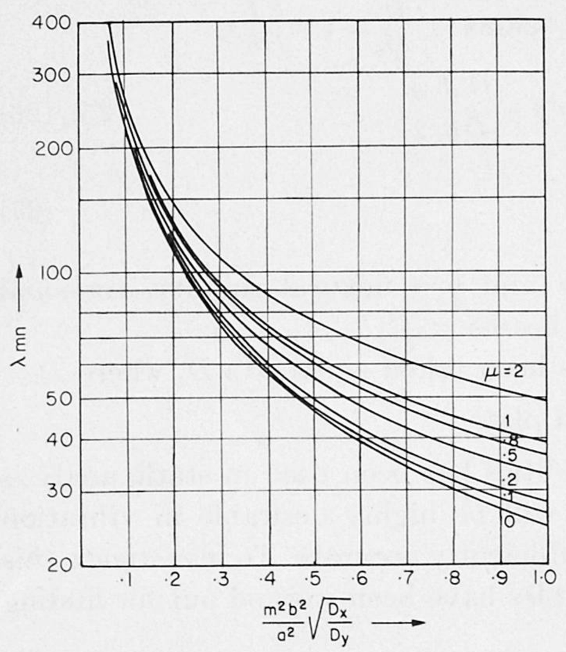


Fig. 2b. I Antisymmetric mode, $n=1$.

Fig. 2a. II Symmetric mode, $n=2$.



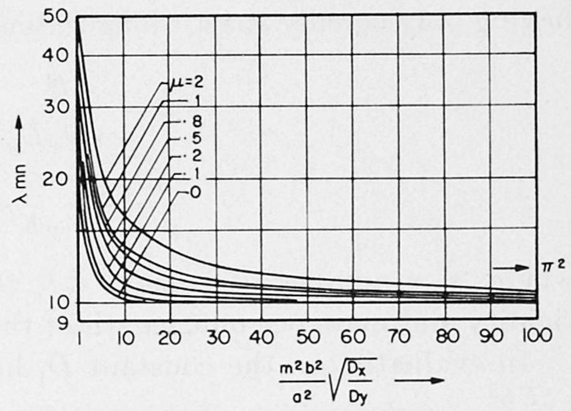
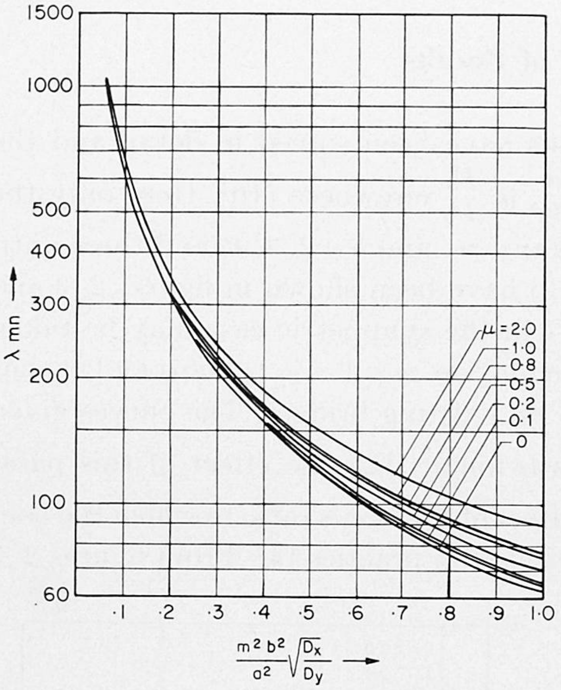


Fig. 3b. II Antisymmetric mode, $n=2$.

Fig. 3a. III Symmetric mode, $n=3$.



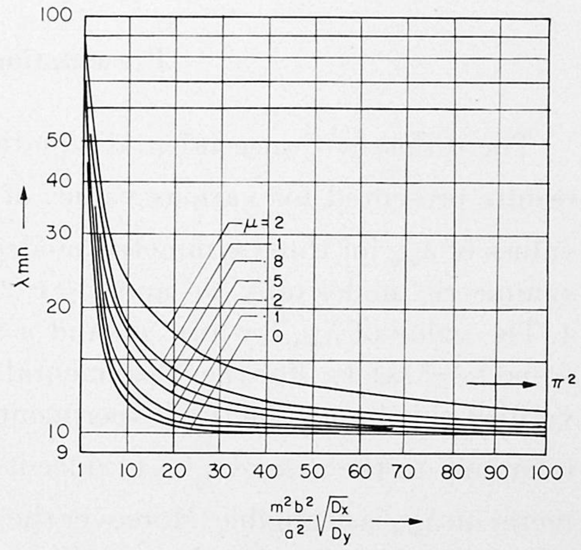


Fig. 4b. III Antisymmetric mode, $n = 3$.

Fig. 4a. IV Symmetric mode, $n = 4$.

and 4 the effect of μ on λ_{mn} can be clearly studied. It may be seen that the change in λ_{mn} as μ varies from 0 to 1 is quite pronounced. This contradicts the conclusion of NARUOKA and YONEZAWA [5] who contend that the effect of μ on λ_{mn} is very low.

Free Vibration of Bridges

The analysis and results hitherto presenetd can be directly applied to the vibration of beam and slab bridges by determining the orthotropic plate rigidities and the mass. The usual method of finding the rigidities is by distributing the stiffness of the beams uniformly. Following such a procedure, for a bridge having only equally spaced longitudinal beams — $\frac{D_x}{D_y} = 1 + \frac{EI}{Db} s$,

$$\mu = \frac{H}{\sqrt{D_x D_y}} = 1 + \frac{GJ}{Db}\frac{s}{2}, \qquad (25), (26)$$

$$\frac{\bar{\rho}}{\rho} = 1 + \frac{\gamma}{\rho b} s, \qquad (27)$$

where 's' = number of beams, EI, GJ and γ = flexural rigidity, torsional rigidity and mass per unit length of the beams.

In evaluating μ, the constant D_1 has been taken equal to νD, where $D = \frac{E h^3}{12(1-\nu)^2}$ is the rigidity of the unstiffened plate.

The above method of finding the rigidities has been used in static analysis with success. Such a simple procedure will be highly desirable in vibration studies also, if the results obtained are sufficiently accurate. To investigate this point, seven beam and slab bridge examples have been worked out for finding the natural frequencies and the modal shapes. The orthotropic plates have been defined as explained above. The sectional properties of the bridges are shown in table 1. It is assumed that the bridges have only longitudinal beams and also

Table 1. Sectional properties of bridges

Bridges	$\frac{a}{b}$	No. of beams	$\frac{EI}{Db}$	$\frac{GJ}{Db}$	$\frac{\gamma}{\rho b}$
A	1	3	37.63	1.340	0.200
B	1	4	28.21	0.716	0.150
C	1	5	28.21	0.716	0.150
D	2	4	28.21	0.716	0.150
E	2	5	76.11	1.241	0.225
F	4	3	37.63	1.340	0.200
G	4	4	28.21	0.716	0.150

that the effect of Poisson's ratio may be neglected. The results obtained by the present theory have been compared in table 2 with those obtained by JAGADISH [9] who has solved the same problems by the Rayleigh-Ritz method using plate eigen functions. The modal patterns have been compared for three bridges in figures 5, 6 and 7. From table 2 it may be seen that the natural frequency parameters predicted by the orthotropic plate theory compare well with the exact values, particularly when the number of beams is more than three. This is expected since with larger number of beams the actual structure corresponds more with the equivalent orthotropic plate.

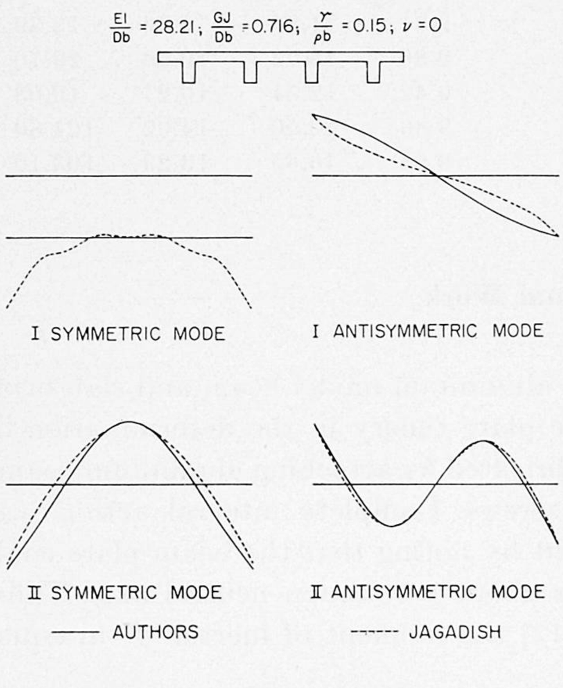


Fig. 5. Modal Shapes of Bridge B. $a/b = 1$, $m = 1$.

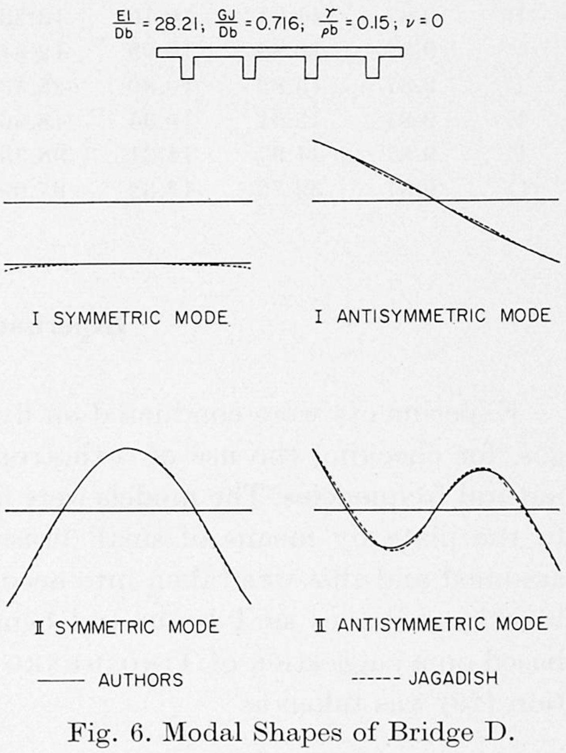


Fig. 6. Modal Shapes of Bridge D. $a/b = 2$, $m = 1$.

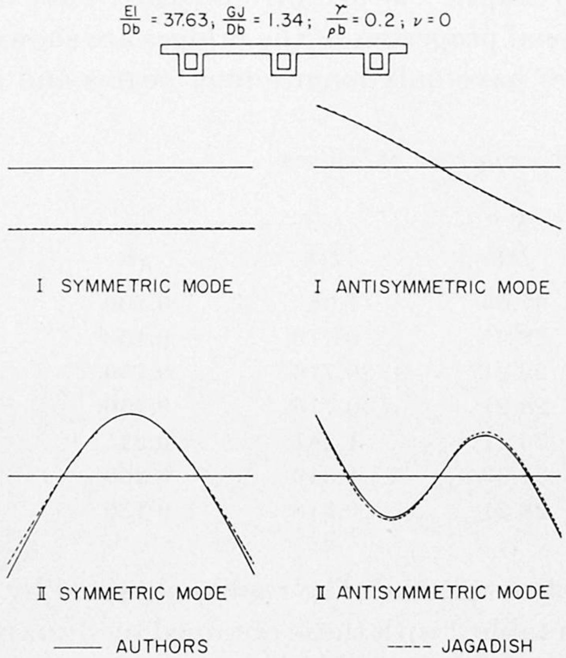


Fig. 7. Modal Shapes of Bridge F. $a/b = 4$, $m = 1$.

Table 2. Value of $\lambda_{mn} = \frac{p_{mn} a^2}{m^2} \sqrt{\frac{\bar{\rho}}{D_x}}$ *with* $m = 1$

Bridges	Authors (Orthotropic Plate Theory)				Jagadish (Rayleigh-Ritz method)			
	I SM	II SM	I ASM	II ASM	I SM	II SM	I ASM	II ASM
A	9.87	11.19	10.15	13.61	9.21	11.22	9.93	13.01
B	9.87	11.01	10.10	13.23	9.68	10.52	9.76	12.78
C	9.87	10.89	10.09	12.84	9.31	11.35	9.96	13.39
D	9.87	15.82	10.80	28.53	9.86	15.73	10.66	29.70
E	9.87	12.61	10.34	18.66	9.42	12.51	10.27	19.03
F	9.87	41.97	14.31	98.35	9.86	40.50	14.02	124.80
G	9.87	39.73	13.52	97.09	9.86	40.60	13.23	103.10

Experimental Work

Experiments were conducted on five aluminium model beam and slab bridges, for checking the use of orthotropic plate theory in the determination of natural frequencies. The models were fabricated by attaching aluminium beams to the plate by means of small brass screws. Complete integral action was assumed and this was taken into account by noting that the beam-plate combination behaves as T-beam and bends about a common neutral axis. Thus, based on a suggestion of TIMOSHENKO [12] the moment of inertia 'I' in equation (25) was taken as

$$I = I_b + \frac{h\,c\,e^2}{(1-\nu^2)}, \tag{28}$$

where I_b = moment of inertia of the beam about its own neutral axis,
h = thickness of deck slab,
c = spacing of beams,
e = depth of the common neutral axis below the middle surface of the deck slab.

The models were driven by an electromagnetic device, continuously changing the excitation frequency. A strain gage fastened to each of the models was used as a vibration pick up to get a display on a cathode ray oscilloscope. The frequencies of excitation at which the display on the oscilloscope shooted up were taken as the resonant frequencies. The different modes were identified by studying the nodal patterns, which were found by strewing fine saw dust on the models. The results of only two models are presented here. The complete details of the experiments and the results are given elsewhere [10]. The sectional details of two models B_1 and B_4 are shown in figures 8 and 9. The experimental and theoretical natural frequencies of these two models are given in tables 3 and 4. Figures 10 to 14 show the actual nodal patterns of B_1 and B_4 in different modes.

Table 3. Natural frequencies of B_1 in c.p.s. $D_x/D_y = 8.396$, $\mu = 0.6735$, $\bar{\rho}/\rho = 1.400$

	$m = 1$		$m = 2$	
n	Experimental	Theoretical	Experimental	Theoretical
1	148	148.8	550	595.8
2	178	179.4	610	627.5
3	286	281.8	660	731.9
4	505	475.6	770	921.0
5	900	809.5	—	—

Table 4. Natural frequencies of B_4 in c.p.s. $D_x/D_y = 36.645$, $\mu = 0.6256$, $\bar{\rho}/\rho = 1.625$

	$m = 1$		$m = 2$	
n	Experimental	Theoretical	Experimental	Theoretical
1	115	112.2	440	448.8
2	160	143.5	488	494.0
3	296	297.1	620	641.3
4	598	578.8	890	913.7
5	1058	1013.0	—	—
6	1820	1599.0	—	—

The theoretical nodal lines have been shown in these figures by broken lines. The comparison instituted in tables 3 and 4 between the theoretical and experimental frequencies of models B_1 and B_4 shows that the orthotropic plate theory predicts the natural frequencies with sufficient accuracy. In fact this good comparison between the theoretical and experimental results was found for all the five bridge models studied [10]. It may also be seen that even the nodal patterns obtained theoretically are not very much different from the experimental nodal patterns.

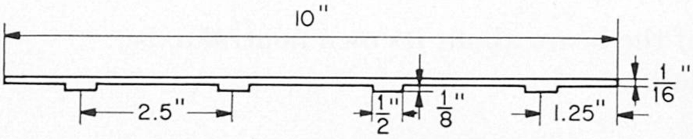


Fig. 8. Model B_1.

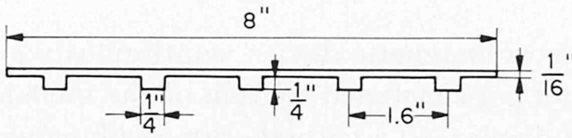


Fig. 9. Model B_4.

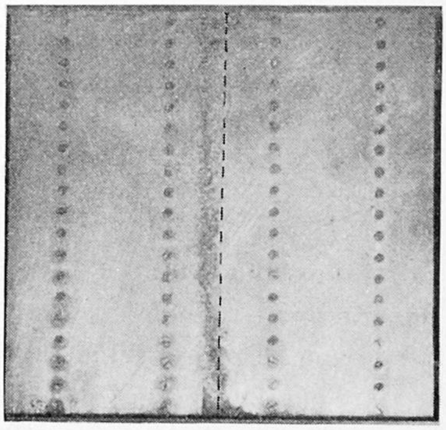


Fig. 10. Model B_1 $m = 1, n = 2$.

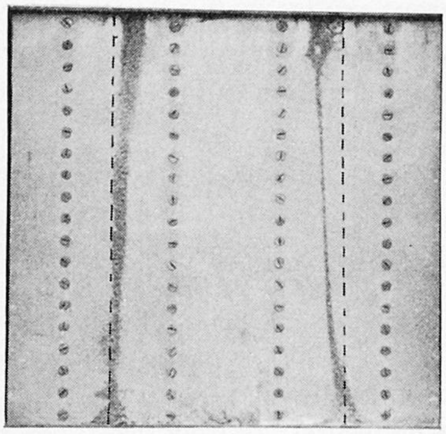


Fig. 11. Model B_1 $m = 1, n = 3$.

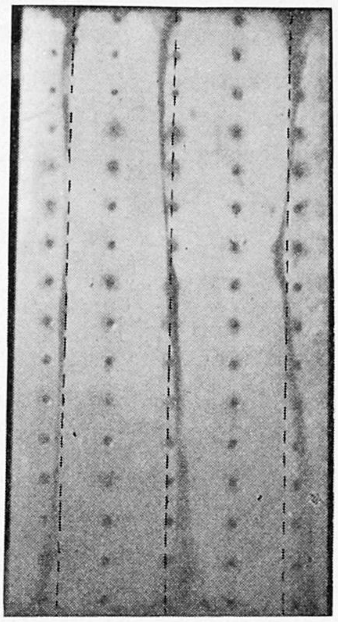


Fig. 13. Model B_4 $m = 1, n = 4$.

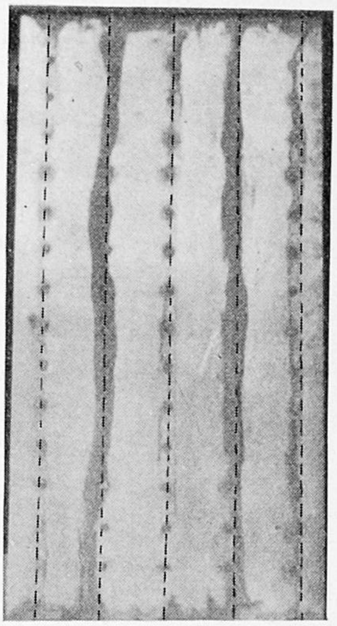


Fig. 14. Model B_4 $m = 1, n = 6$.

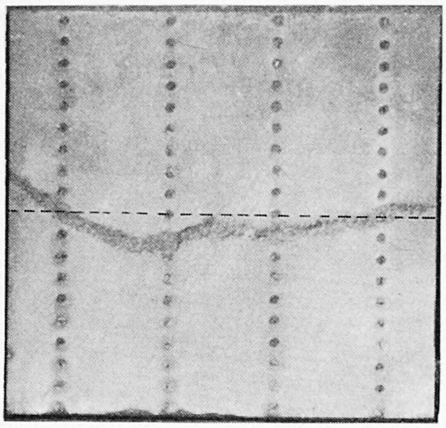


Fig. 12. Model B_1 $m = 2, n = 1$.

Acknowledgements

The authors express their gratitude to Professor N. S. GOVINDA RAO, Professor and Head of the Department of Civil Engineering, Indian Institute of Science for his encouragement during the course of the present work. The authors also thank the authorities of the Department of Mechanical Engineering, Indian Institute of Science, for making available the vibration testing set-up.

References

1. C. E. INGLIS: A mathematical treatise on vibrations in railway bridges, Cambridge University Press, London, 1934.
2. K. MISE and S. KUNII: A theory for the forced vibrations of a railway bridge under the action of moving loads, Quart. Jour. of Mech. and App. Math. Vol. 9, 1956, p. 195.
3. C. MASSONNET: Méthode de calcul des ponts à poutres multiples tenant compte de leur résistance à la torsion, Publications I.A.B.S.E., 1950, p. 147.
4. R. E. ROWE: Concrete Bridge Design, C. R. Books Ltd., London 1962.
5. M. NARUOKA and H. YONEZAWA: A study on the free lateral vibration of the beam bridge by the theory of the orthotropic rectangular plate, Ingr. Archiv. Vol. 26, 1958, p. 20.
6. Y. YAMADA and A. S. VELETSOS: Free vibration of simple span I-beam bridges, Part B, 8th Progress Report, Highway bridge impact investigation, Univ. of Illinois, 1958.
7. C. L. HUANG and W. H. WALKER: Free Vibration of simple span I-beam bridges, Part D, 9th Progress Report, Highway bridge impact investigation, Univ. of Illinois, 1959.
8. K. T. SUNDARA RAJA IYENGAR and K. S. JAGADISH: On the vibrations of beam and slab bridges. Bulletin of the Indian Society of Earthquake technology, Vol. 1, No. 1, 1964, p. 15.
9. K. S. JAGADISH: Free vibrations of beam and slab bridges. Not yet published.
10. R. NARAYANA IYENGAR: On the free vibration of beam and slab and interconnected girder bridges. Thesis submitted for the degree of M. Sc. (Engg.), Indian Institute of Science, Bangalore 1965.
11. T. R. OEHLER: Vibration susceptibilities of various highway bridge types, proc. A.S.C.E., Strl. Division, 1957, p. 1318.
12. S. P. TIMOSHENKO and J. M. GERE: Theory of Elastic Stability, McGraw Hill, 1964.

Summary

The free vibration equation of an orthotropic plate with two opposite edges simply supported, with other two edges free has been completely solved. The results for the natural frequency parameters have been presented in the form of charts. The application of these results to the free vibrations of beam and slab bridges has been explained. The results obtained by this theory for a few bridge examples have been compared with the exact results and good comparison has been found between the two. Experimental work conducted on aluminium model bridges also show that the orthotropic plate theory gives sufficiently accurate results.

Résumé

Après la résolution complète de l'équation des oscillations libres d'une plaque orthotrope à deux bords opposés simplement appuyés et deux bords libres, on présente, sous forme d'abaques, les résultats obtenus pour les paramètres de la fréquence propre. L'application de ces résultats au problème des oscillations libres des ponts à dalle nervurée est expliquée. La comparaison, avec les résultats exacts, des résultats obtenus de cette façon pour différents ponts pris comme exemples rélève une bonne concordance. Des essais exécutés sur des modèles de pont en aluminium, il ressort également que la théorie des plaques orthotropes donne des résultats d'une précision suffisante.

Zusammenfassung

Die Gleichung der freien Schwingung einer orthotropen Platte, bei der zwei gegenüberliegende Ränder einfach unterstützt und die übrigen beiden frei sind, ist vollständig gelöst worden. Die Ergebnisse der Eigenwerte sind in Tabellenform dargestellt. Die Anwendung dieser Ergebnisse auf die freie Schwingung von Balken- und Plattenbrücken wird erläutert. Die durch diese Theorie für einige Brücken erhaltenen Ergebnisse wurden den genauen Werten gegenübergestellt; die dabei erzielte Übereinstimmung ist zufriedenstellend. Versuche an Aluminiummodellbrücken lassen ebenfalls erkennen, daß die Theorie der orthotropen Platte hinreichend genaue Ergebnisse liefert.